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CENG 222 REPORT

In this homework, we looked method of moments (MoM) and Maximum Likehood Estimation.

a) In part a =

our $f(x)$ is =>

$$f(x) = \begin{cases} \frac{2\theta^2}{x^3}, & \theta \leq x < \infty \\ 0, & \text{otherwise} \end{cases}$$

↳ We need to find MoM and MLE est. for θ

Estimating θ using;

MoM

1) Given X (sample set) → $X = \{0.3, 0.6, 0.8, 0.9\}$

2) Finding θ =

$$\mu_1 = E[X] = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{\theta}^{\infty} x \cdot \frac{2\theta^2}{x^3}\,dx = 2\theta^2 \int_{\theta}^{\infty} \frac{1}{x^2}\,dx =$$

$$-2\theta^2 \left(-x^{-1}\Big|_{\theta}^{\infty}\right) = -2\theta^2 \cdot \left(-\frac{1}{\theta}\right) = 2\theta$$

$$\theta = \frac{\bar{X}}{2} = \frac{0.65}{2} = \boxed{0.325} \quad \left(\text{since mean}(\bar{X}) \text{ is } \frac{0.3+0.6+0.8+0.9}{4} = 0.65\right)$$

≈ 0.33 also.

MLE

1) Given X (sample set) → $X = \{0.3, 0.6, 0.8, 0.9\}$

2) Finding θ =

$$f(x_1, x_2, x_3, x_4) = \prod_{i=1}^{4} L(\theta \mid x_i) = \prod_{i=1}^{4} \frac{2\theta^2}{x^3}$$

the log. likehood function

$$\ln f(x) = \sum_{i=1}^{4} \ln\left(\frac{2\theta^2}{x^3}\right) = \sum_{i=1}^{4} \ln 2 + 2\ln\theta - 3\ln x_i$$

$$= 4\ln 2 + 8\ln\theta - 3\sum_{i=1}^{4} \ln x_i$$

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$$\frac{\partial \ln f(x)}{\partial \theta} = \frac{8}{\theta} + \sum_{i=1}^{4} \ldots$$ (Since X doesn't include any critical point.)

(Note: $\ln 2$ and $3\ln x_i$'re gone since it's derivative process with θ.)

With this, for reaching the maximum value, derivative should equal to zero.

- Even $\theta \to \infty$ satisfies the condition, we can not define our θ that way. It's because our θ is not define in f(x) definition like that.

- So what we need to do at that point for max. is choosing minimum value in our sample set (X),

$\min(X) = \min\{x_i\}$ is $\underline{0.3}$. ($X = \{0.3, 0.6, 0.8, 0.9\}$)

$\boxed{\hat{\theta} = 0.3}$ for MLE estimation.

b) In part b, we need to use Inverse Transform Method to generate random samples.

↳ For using this method, we need to find F(x) for =

$$f(x) = \begin{cases} \frac{2\theta^2}{x^3}, & \theta \leq x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

$$F(x) = \int_{\theta}^{x} \frac{2\theta^2}{a^3} \, da$$

$$= 2\theta^2 \left(-\frac{1}{2a^2} \Big|_{\theta}^{x} \right)$$

$$= \cancel{2}\theta^2 \left(-\frac{1}{\cancel{2}x^2} + \frac{1}{\cancel{2}\theta^2} \right)$$

$$= \frac{-\theta^2}{x^2} + \frac{\theta^2}{\theta^2}$$

$$F(x) = 1 - \left(\frac{\theta}{x}\right)^2, \text{ for } x \geq \theta$$

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After finding $F(x)$, we need to find $F^{-1}(x)$:

$$y = 1 - \left(\frac{\theta}{x}\right)^2 \rightarrow y - 1 = -\frac{\theta^2}{x^2} \rightarrow 1 - y = \frac{\theta^2}{x^2}$$

$$x^2 = \frac{\theta^2}{1-y} \rightarrow x = \frac{\theta}{\sqrt{1-y}}, \xrightarrow{\text{replacing } y \text{ and } x} y = \frac{\theta}{\sqrt{1-x}}$$

$$F^{-1}(x) = \frac{\theta}{\sqrt{1-x}}$$

in part b, θ is given as $2.4 \Rightarrow F^{-1}(x) = \frac{2.4}{\sqrt{1-x}}$,

↳ with forming Inverse CDF method shape $F^{-1}(u) = \frac{2.4}{\sqrt{1-u}} = x$,

which is one of our values. (u stands for generated value from standart uniform distribution.)