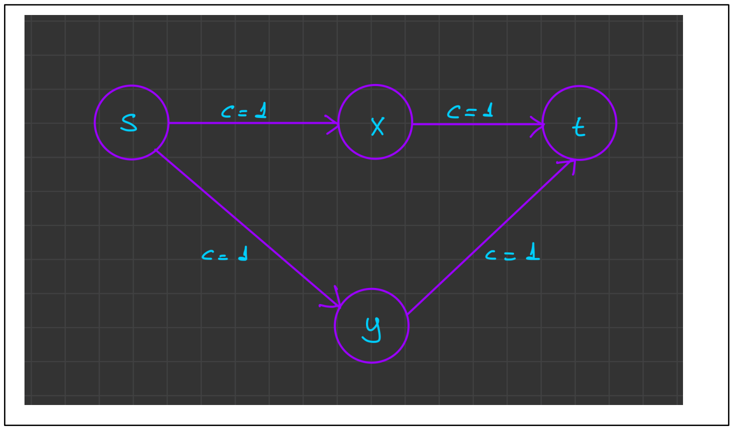
CEREN DİNÇ 28220

CS301 HOMEWORK4

Question1)

G = (V, E, s, t, c)
Source = s
Sink = t
Capacity function = c
Vertices V = {s, t, x, y}
Edges E and capacities c:

- (s, x), c=1
- (x, t), c=1
- (s, y), c=1
- (y, t), c=1



Consider the flow path that gives max flow 1: s → x → t
Flow function f:

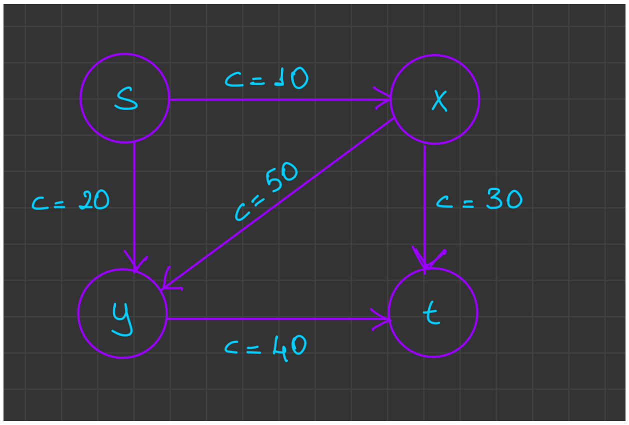
- f(s, x)=1
- f(x, t)=1
- f(s, y)=0
- f(y, t)=0

Consider the flow path that gives max flow 1: s → y → t
Flow function f:

- f(s, x)=0
- f(x, t)=0
- f(s, y)=1
- f(y, t)=1

Question2)

Theorem A is **incorrect**, below is a **counter-example** in which edges have unique capacity :



Flow function f1:

- f1(s,x) = 10
- **f1(x,t) = 0**
- f1(s,y) = 20
- **f1(y,t) = 30**
- f1(x,y) = 10

→ Maximum flow 30

Flow function f2:

- f2(s,x) = 10
- **f2(x,t) = 10**
- f2(s,y) = 20
- **f2(y,t) = 20**
- f2(x,y) = 0

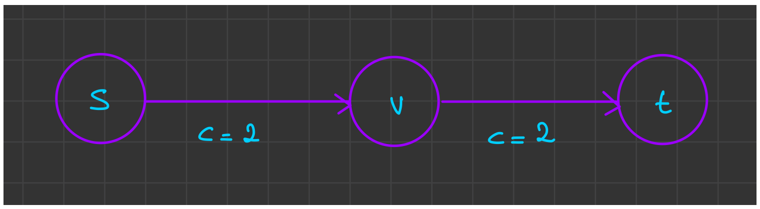
→ Maximum flow 30

Question3)

F is **not guaranteed** to be a flow on G.

The capacity constraint, which is a must for a flow in a network, may not hold for F.
Meaning that the combined flow F may exceed the capacity of an edge in the network.

Counter example:



1st node s → source
2nd node v
3rd node t → sink
capacity c

Flow function 1:
- f1(s,v) = 2
- f1(v,t) = 2

Flow function 2:
- f2(s,v) = 1
- f2(v,t) = 1

Combination of the above flow functions f1 and f2:
- F(s,v) = f1(s,v) + f2(s,v) = 3
- F(v,t) = f1(v,t) + f2(v,t) = 3

F exceeds the capacity of edges as it has flow of 3 on both edges.
The capacity constrained is violated by F, this viaolation shows that F is not a valid flow on G.