

Advanced Systems Lab

Assignment 1

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1.0 Disclaimers and assumptions

This little subsection is here to clarify the assumptions and shortcuts I had to take for this assignment. I figured it would be better to have them all in one place, rather than scattered throughout the document.

Machine used

The machine used to complete this assignment is a 2021 Macbook Pro M1 (10 cores CPU - 16 cores GPU), which is an ARM-based machine.

Microarchitectural parameters

While the handout recommends to use the [🔗 Dougall Johnson's documentation](#) of the Firestorm architecture for gathering information about the microarchitectural parameters, the article does not suffice in painting a complete picture of the machine, as M1 devices have both Performance cores (running Firestorm) and Efficiency cores (running Icestorm).

Frequency and plots

The `c_clock` function is unreliable on my machine. Therefore the cycles for each benchmark have been calculated by using the time passed in seconds and the frequency of the machine. Performance and Efficiency cores run at different clock speeds, so the frequency used in the calculation has been obtained by performing a weighted average of the Performance and Efficiency cores (2.53 GHz). This is obviously not ideal, resulting in a higher margin of error.

Compiler

The compiler used is Apple clang version 15.0.0 (clang-1500.3.9.4), compiling for target arm64-apple-darwin23.2.0.

1.1 (15 pts) Get to know your machine

a) - d)

Below you can see microarchitectural parameters of the laptop used to complete this Homework. **Note:** Apple is notoriously secretive about the detailed specifications of their devices, so the table has been filled on a best-effort basis.

| | |
|--|---|
| Processor manufacturer | TSMC |
| Processor name | Apple M1 Pro (2021) |
| Serial number | KP9H9FY0KF ¹ |
| Microarchitecture | Firestorm/Icestorm ² |
| CPU Base Frequency | 3.28 GHz (performance cores), 2.06 GHz (efficiency cores) |
| CPU Maximum Frequency | Not available |
| Support for Intel Turbo Boost or similar | No |

Table 1: Microarchitectural parameters

e)

In [Figure 1b](#) we can see the assembly code generated by the GCC compiler for the C code shown in [Figure 1a](#).

Using Apple Clang, the resulting compiled code utilizes the FDIV instruction for performing floating point division. While the instruction shares the same name as the one used in the x86 architecture, it is important to note that the underlying microarchitecture of the Apple M1 Pro is ARM-based, and the instruction set is ARMv8.5-A. Furthermore, SSE/SSE2 is not supported.

¹Serial number: The device serial number is indicated, since there is no apparent serial number for the processor itself provided by Apple.

²Firestorm/Icestorm: The microarchitecture of the Apple M1 Pro (2021) uses *Firestorm* for its performance cores and *Icestorm* for its efficiency cores [Source: [🔗 notebookcheck.net](#)]

```
int fp(float a, float b) {
    return a/b;
}
```

(a) C code performing floating point division

```
.section __TEXT,__text,regular,pure_instructions
.build_version macos, 14, 0      sdk_version 14, 2
.globl _fp                      ; -- Begin function fp
.p2align 2
_fp:                             ; @fp
.cfi_startproc
; %bb.0:
fdiv    s0, s0, s1
fcvtzs  w0, s0
ret
.cfi_endproc
; -- End function
.subsections_via_symbols
```

(b) Compiled assembly code, using GCC
(Apple Clang version 15.0.0) and -O3 optimization level

Figure 1: Floating point division compilation

f)

The x86 floating point operations are included in the standard x86 instruction set architecture while SSE/SSE2 is an extension of the x86 instruction set. Both operate on special dedicated registers, with the difference that the x87 FPU is an 80 bit precision floating point and thus has a higher precision, while the SIMD registers are 128 bits, allowing to perform arithmetic operations on multiple floating points in parallel (either 4 single-precisions FP or 2 double-precision FP).

g)

For the following exercises the [Dougall Johnson's documentation](#) about the Firestorm architecture has been used. The article mentions that Firestorm has a pipeline width of 8 instructions per cycle, and 4 SIMD ports each of which can execute both floating-point and SIMD instructions. So the maximum would be 4 flops/cycle.

h)

Latency: 4 cycles. Throughput: 0.25 cycles per instruction. Instruction: FMUL.

i)

j)

Latency: 10. Throughput: 1 cycle per instruction. Instruction: FDIV.

1.2 (20 pts) LU Factorization

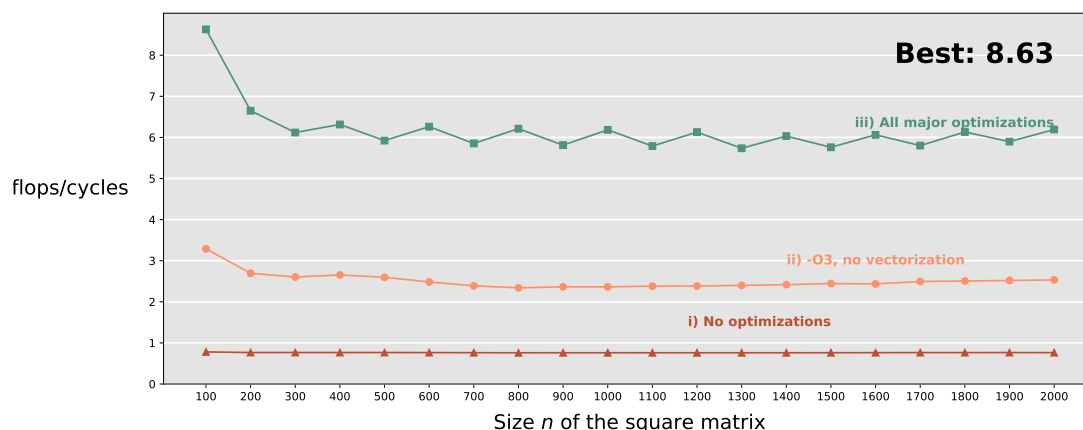
a)

b)

The `compute` function performs $\frac{n^2}{2} - \frac{n}{2}$ FP divisions and $n^3 - n^2$ multiplications and additions, for a total of $n^3 - \frac{n^2}{2} - \frac{n}{2}$ floating point operations. **Note:** Subtractions are reported since they are typically implemented using additions, due to 2s complement arithmetic.

$$W(n) = n^3 - \frac{n^2}{2} - \frac{n}{2} \text{ and } Q(n) \geq 16n^2. \quad I(n) \leq \frac{n}{16}$$

c)

Figure 2: flops/cycle performance of the `compute` function compiled with different optimization flags

d)

1. No optimizations: The code is neither optimized nor vectorized. The performance is the worst and flat across the line.
2. Optimized, but no vectorization: The compiler performs some optimizations resulting in a performance improvement, but no vectorization is performed.
3. All major optimizations: The compiler both performs optimizations and vectorizes the code, resulting in the best performance. We also notice how for the matrix sizes we tested, the performance does not seem to vary greatly. The peak performance recorded is around 8.63 flops/cycles.

1.3 (25 pts) Performance analysis and bounds

a)

$$C(n) = C_{add} * N_{add} + C_{mult} * N_{mult}$$

b)

$$\begin{aligned} N_{add} &= 3n, \\ N_{mult} &= 2n, \\ C(n) &= C_{add} * (3n) + C_{mult} * (2n) \end{aligned}$$

c)

- i) We have a total of $3n$ floating point additions and $2n$ floating point multiplications. All operations can be scheduled either on Port 0 or Port 1. This results in a lower bound of $2.5n$ cycles.
- ii) Now that FMA instructions are enabled, we can fuse the last two operations into one single FMA instruction (i.e. $(u_i * (w_i + x_i)) * (u_i - x_i) + y_i$). We therefore have now a total of n FMA instructions, $2n$ floating point additions and n floating point multiplications, and a lower bound of $2n$ cycles.
- iii) In our computation we have to read at least $5n$ doubles from the arrays x, w, u, z respectively. Therefore, $r_{L3} \geq \frac{5n}{4}$ and $r_{RAM} \geq \frac{5n}{2}$.

d)

The operation intensity is $I(N) \leq \frac{5n \text{ flops}}{8(5n) \text{ bytes}} = \frac{1}{8} \text{ flops/bytes}$.

1.4 (25 pts) Basic optimization

a)

```
void compute(double *x, double *y, double *z, int n) {
    for (int i = 0; i < n; i++) {
        for (int k = 0; k < 2; k++) {
            z[k] += x[i + 1 - k] * y[i + k];
        }
    }
}
```

```
void compute(double *x, double *y, double *z, int n) {
    double z0 = z[0];
    double z1 = z[1];
    int i;
    for (i = 0; i < n; i+=3) {
        int next = i+1;
        int nnext = i+2;
        int nnnext = i+3;
        z0 += (x[next] * y[i]) + (x[nnext] * y[next])
            + (x[nnnext] * y[nnext]);
        z1 += (x[i] * y[next]) + (x[next] * y[nnext])
            + (x[nnext] * y[nnnext]);
    }
    for (; i < n; i++) {
        int next = i+1;
        z0 += x[next] * y[i];
        z1 += x[i] * y[next];
    }
    z[0] = z0;
    z[1] = z1;
}
```

Figure 3: Unoptimized and optimized comp functions

b)

c)

d)

Figure 3 shows a comparison between the unoptimized comp function and its optimized counterpart. In order to improve ILP I perfomed 3x loop unrolling and introduced accumulators for reducing memory accesses to the array z. Initially I implemented 4x loop unrolling, but the performance overhead became noticeable so I resorted to a lower 3x loop unrolling.

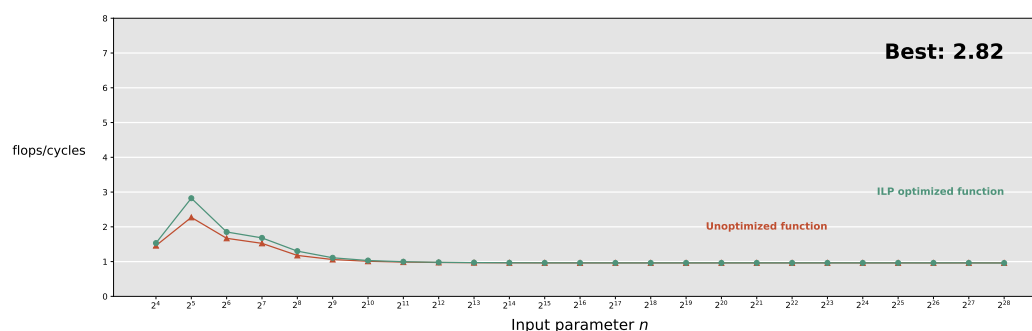


Figure 4: flops/cycle performance of the comp functions

When running the optimized function on Code Expert, we obtain a 2.73 speedup, but as you can see from the plot in Figure 4, when compiling and benchmarking on my Macbook M1 machine, there isn't any performance improvement over the unoptimized version. This is due to the Apple Clang compiler and that M1 is a different architecture to the one used to run the benchmarks on Code Expert. The best performance recorded on my machine is around 2.82 flops/cycles.

1.5 (10 pts) ILP analysis

```
double artcomp(double a, double b, double c, double d) {
    double r;
    r = (a*a*a) / (a*b + (c - d));
    return r;
}
```

a)

In Figure 5a we can see a runtime of at least **23 cycles**. Note that the two mults and 1 subtraction can't start at the same time since they can be only executed on Port 0 and Port 1, introducing additional delay.

b)

In Figure 5b we can see a runtime of at least **22 cycles**. This time around there is no additional delay as we can execute the $a * a$ and $c - d$ operation at the same time.

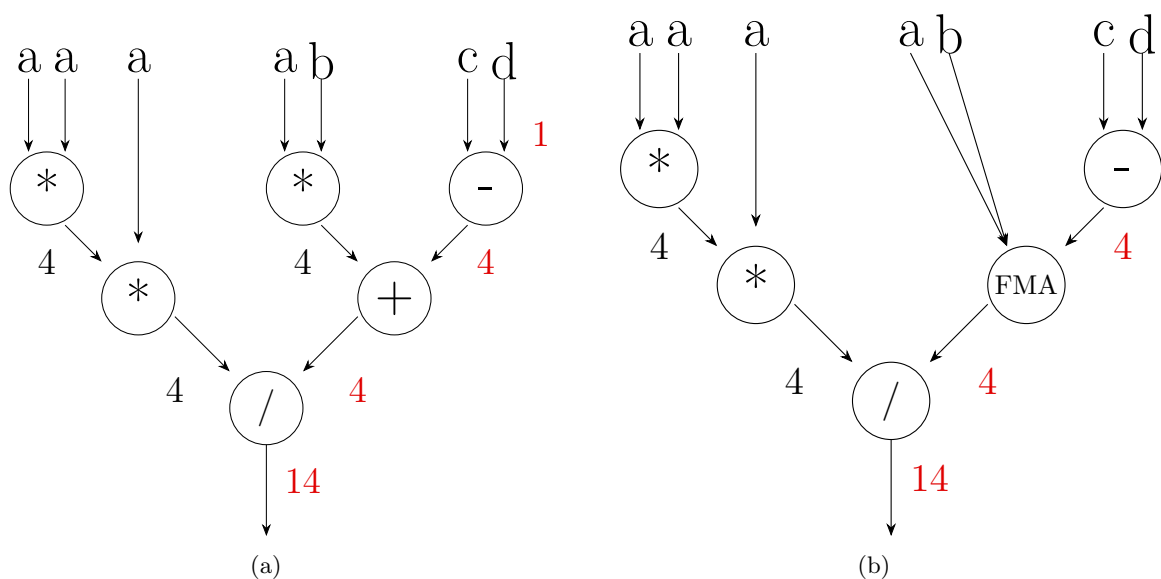


Figure 5: Dependency trees