

## The yam's game

*Yam's* is a kind of dice-poker game. We study here a simplified version of this game.

Two players face up to generate the best **5** dices combinations and to cumulate a maximum of points. In **4** rounds, each player roll **5** standard dice (**6** faces identified with numbers from **1** to **6**). If the player is not pleased with his combination, he can choose, **2** times, to roll again any number of the **5** dice.

Each combination worst a certain number of points depending on the difficulty to generate it.

In this version, we consider those combinations :

Combinations	Description	Value
Full	3+2 identic dice	$30 + \text{sum}(\text{dice})$
Quads	4 identic dice	$40 + \text{sum}(\text{dice})$
Small Flush	Flush with 4 dice	45
Quinte Flush	Flush with 5 dice	$50 + \text{sum}(\text{dice})$
Yam's	4 identic dice	$50 + \text{sum}(\text{dice})$
Nothing	<i>else...</i>	0

Player turn ends after he rolls 2 times the dice or he chose to stop. Then, its combination value is added to his score and the dice-set goes to his opponent.

Example of a first round :

Player-1, the first to play, get **2** dice **one**, **1** die **two**, **1** die **three** and **1** die **four**. He decides to roll again **1** die **one** to reach a *Quinte Flush*. He get a *four*, roll again and get a final *six*. With, **1** die in each value: **one**, **two**, **three**, **four** and **six**, he scores **45** points (*Small Flush*).

At Player-2 turn, dice give: **1** - **one**, **1** - **three** and **3** - **five**. Player-2 keeps the **fives** and roll again the **2** other dice. He gets **2** dice **one** he chooses to keep them for a *Full* worthing **47** points ( $30 + 3 \times 5 + 2$ ).

Then, the game continues for **3** other rounds.

### Question 1

Enumerate all the variables useful for decision-making (roll again or not 1 to 5 dice). Provide the variable domain.

### Question 2

Calculate the number of possible states considering the enumerated variables (provide both the equation and the result).

### Question 3

How many possible actions a player can choose at each time step of its turn ?

## Question 4

The next script provides an autonomous behavior based on function:

- $nb(x)$ : returning the number of dice on the face  $x$ .

The script returns a list of dice face values for which the player aims to roll again the dice. For instances, returning the list **[1, 2]** mean that all the dice on **1** or **2** would be rolling again.

```
if nb(6) > 3 :  
    return [1,2,3,4,5]  
elif nb(5) > 3 :  
    return [1,2,3,4,6]  
elif nb(4) > 3 :  
    return [1,2,3,5,6]  
elif nb(6) > 2 :  
    return [1,2,3,4,5]  
elif nb(5) > 2 :  
    return [1,2,3,4,6]  
elif nb(4) > 2 :  
    return [1,2,3,5,6]  
else :  
    return [1,2,3]
```

- **Question 4-a:** Model this script as a decision tree.
- **Question 4-b:** What is the purpose of this behavior ?

## Question 5

The *Combinations Value* depends on the **5** dice together. I would like to model it on a Bayesian network (the *Combinations Value* and all the variable permitting to reach it).

## Question 6

What is the size of the condition table associated to *Combination Value node* in the proposed Bayesian network. Provide 3 lines of this table as an example.

## Question 7

Let consider only one of the **5** dice (*die-1*). The evolution of the face of *die-1* depends on an action *roll-die-1* that could be *true* or *false*. Finally, rolling the die depends of the last gotten face. In this scenario the player is waiting a 5 or a 6.

- **Question 7.a:** Propose a *dynamic Bayesian network* modeling the evolution of *die-1* with all the condition tables.
- **Question 7.b:** At the begining, the player get a *one*. What is the distributions of probabilities over each of the variables modeled on the *dynamic Bayesian network*, for the time step 0 and the time step 1.

## Question 8

Let consider again the **5** dice. The player get a full (3 - *fives* and 2 - *ones*). The player consider rolling again the 2 one-dot-dice.

- **Question 8.a:** What are the reachable states and their probabilities ?
- **Question 8.b:** What is the value of its current state and the value of "rolling the 2 one-dot-dice once" ? Considering this calculus, is it worthwhile for the player to roll those dice again ?
- **Question 8.c:** What would be the impact on the value of "rolling 2 one-dot-dice" if we consider it would be possible to roll it 2 times if necessary ? (Argue your position without providing the exact calculation).