The Curse of Dimensionality

Factorized Transitions

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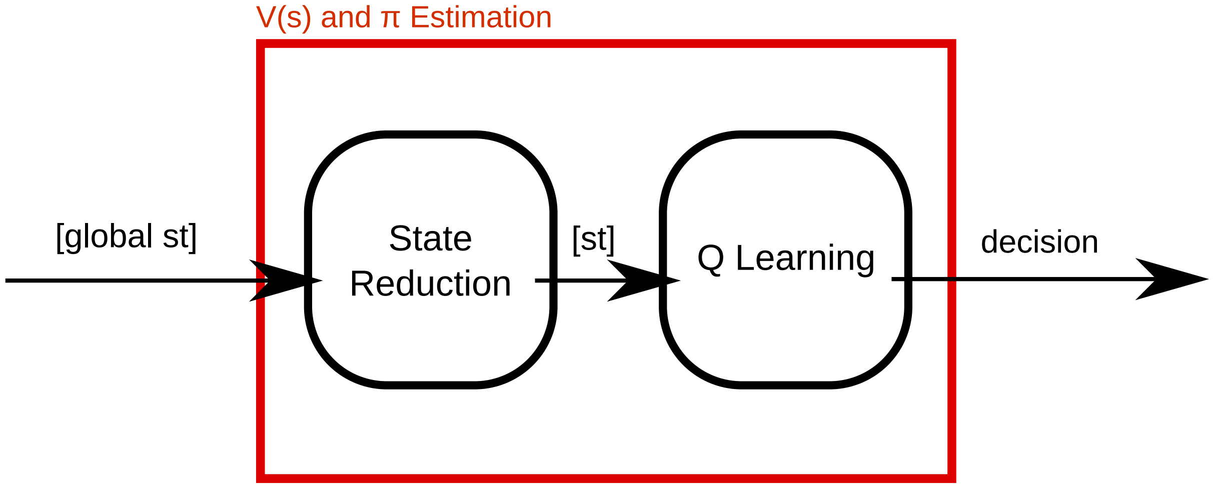
1. **Bayesian Network**
2. **Factorized Transition Function**

Before to go...

Q-Learning over reduced state-space (and actions...) nessearly generate averaged decisions...



Need a model to refine decision making from simulations

1. **Bayesian Network**
2. Factorized Transition Function

Bayesian Network: General idea

Variables are not necessarily correlated over all the others.

Example:

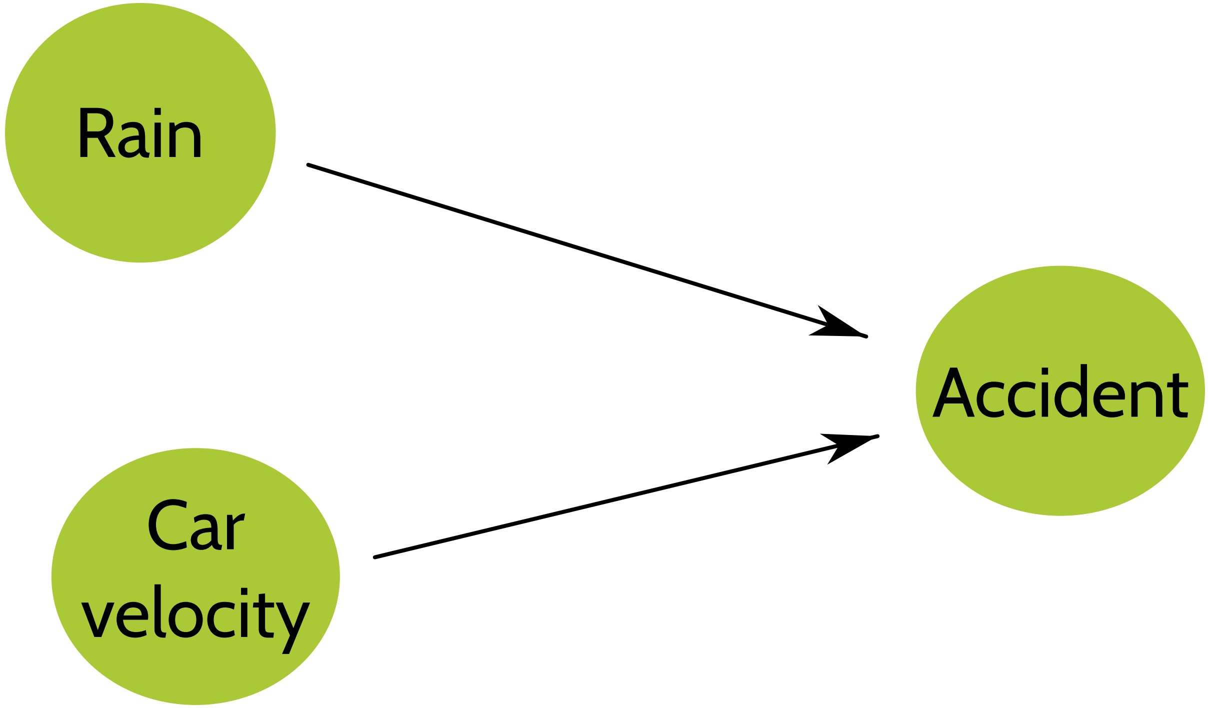
- The probability of *Rain* does not depend from *Car Velocity*.
- The *Car Velocity* does not depend from *Rain*.
- Owever, the probability of *Accident* depends from both *Rain* and *Car Velocity*.

Bayesian Network:

A probabilistic graphical model that represents
a set of variables and their conditional dependencies

Bayesian Network: Car accident Example

Graphical model: *Node:* variable, *Orriented Edge*, dependancy



ATTENTION: *Correlation* is not *Conditional Dependancy*

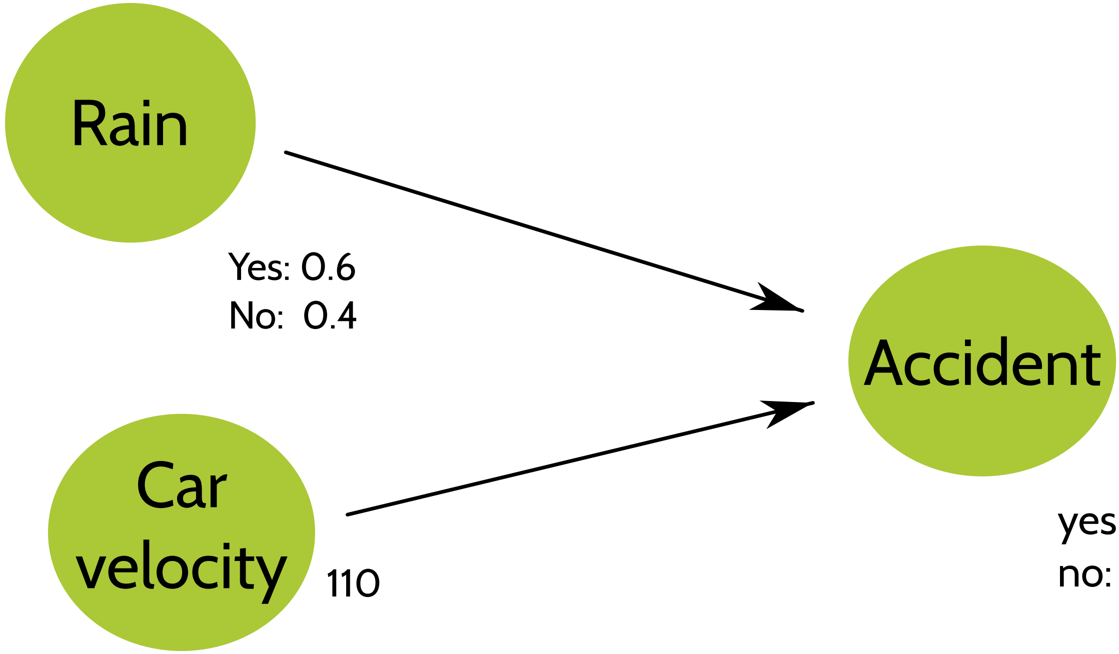
Bayesian Network: Node's Probability Table

Propbality dependencies are defined node by node.

Rain	C.Vel.	**Yes**	**No**
Yes	50	0.1	0.9
	70	0.2	0.8
	90	0.2	0.8
	110	0.3	0.7
	130	0.8	0.2
No	50	0.05	0.95
	70	0.1	0.9
	90	0.1	0.9
	110	0.1	0.9
	130	0.3	0.7

Bayesian Network: Inference

Resulting propbality is computed recursivelly



Yes: 0.6
No: 0.4

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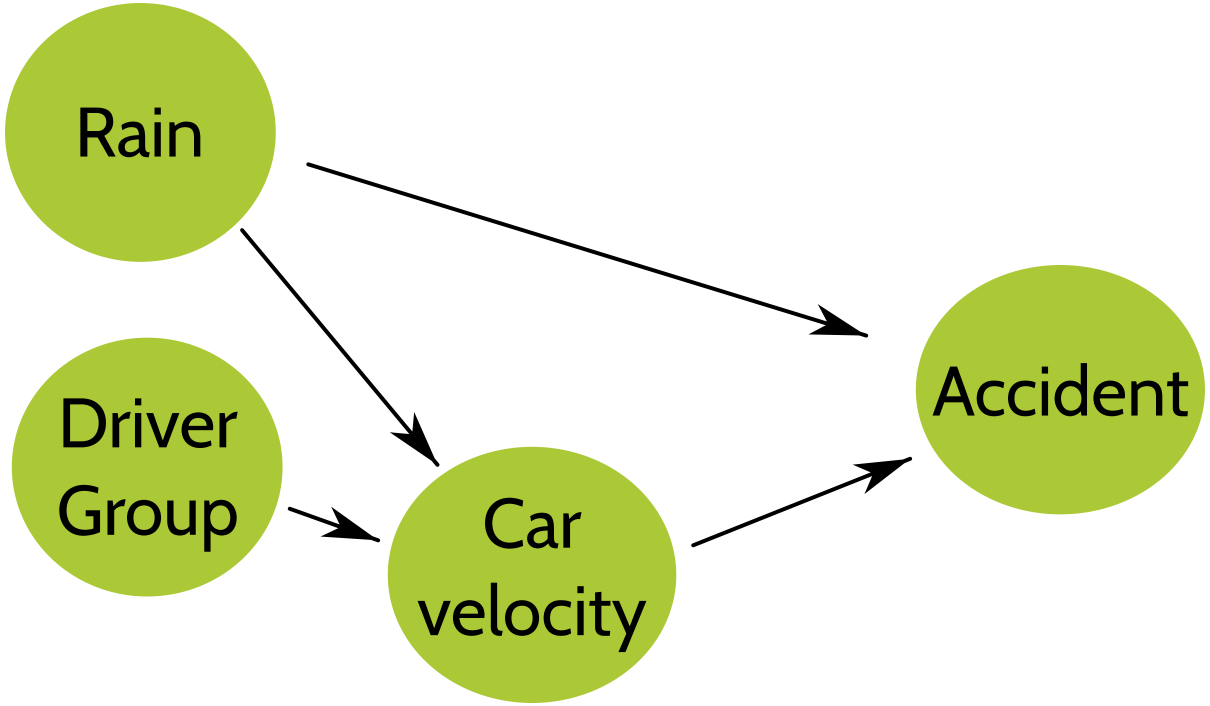
yes: 0.6 x 0.3 + 0.4 x 0.1 = 0.22
no: 0.78

Bayesian Network: Level-up

Growing the graph → no consequencies over existing nodes



ATTENTION: *Correlation* is not *Conditional Dependancy* → **Directed Acyclic Graph**

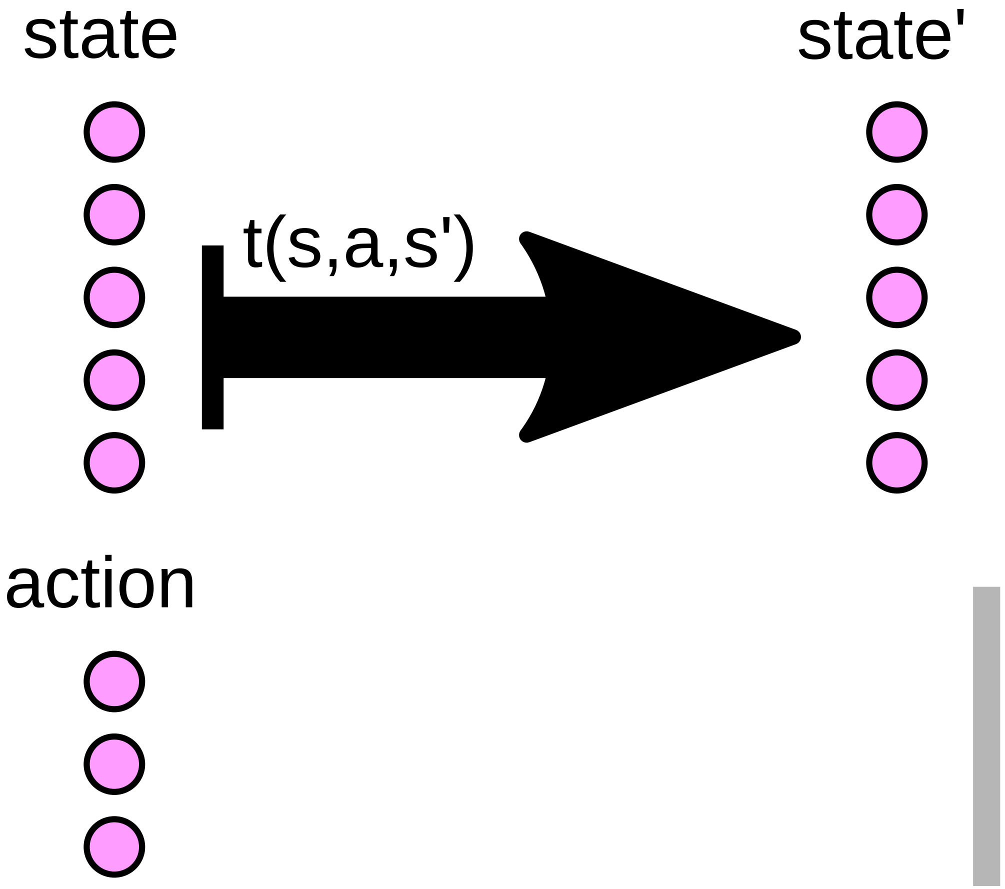
Bayesian Network: Toolbox

- Bayesian Network on Wikipedia
- In python - numerous implementations
  - pomegranate - define compute inferences and more.
  - bnlearn - Learning the bayesian structure (ie. detect the dependencies)

1. Bayesian Network
2. **Factorized Transition Function**

Factorized Transition Function: Problem

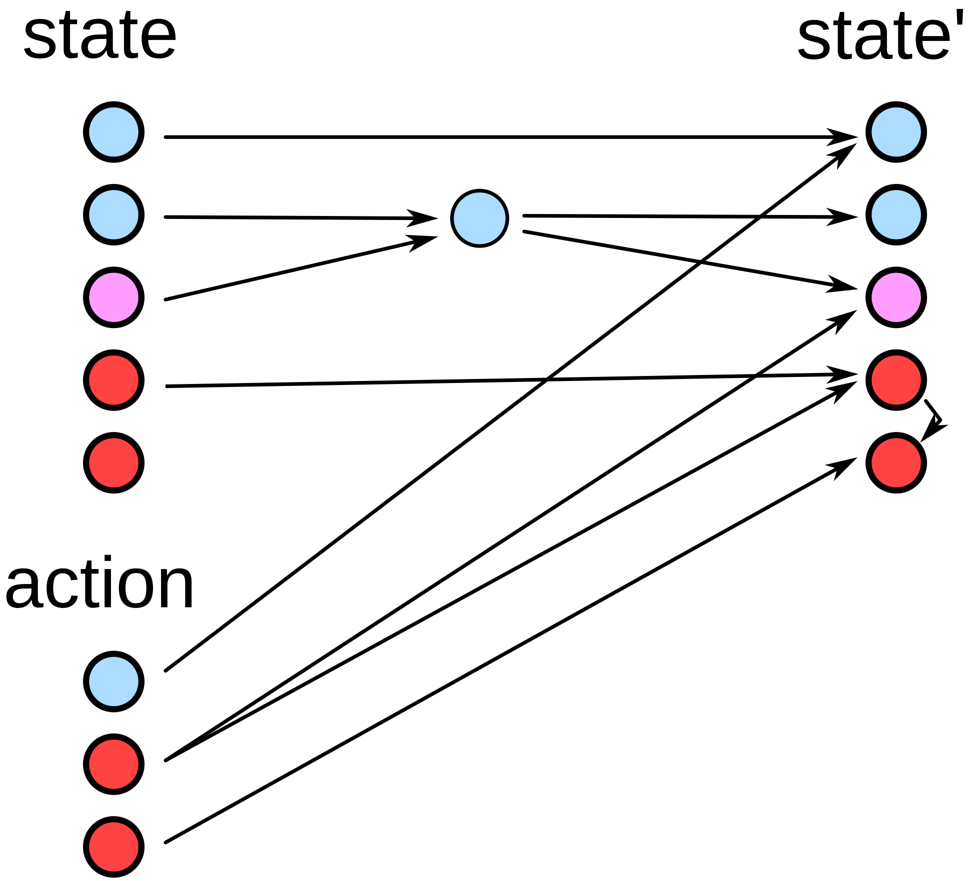
Classically a multi-varibles probabilistic evolution problem



$t\colon |S|\times|A|\times|S| \rightarrow [0, 1]$

$\sim \mathrm{Dom}(\mathrm{Vars})^{|\mathrm{Vars}|\times 2}$

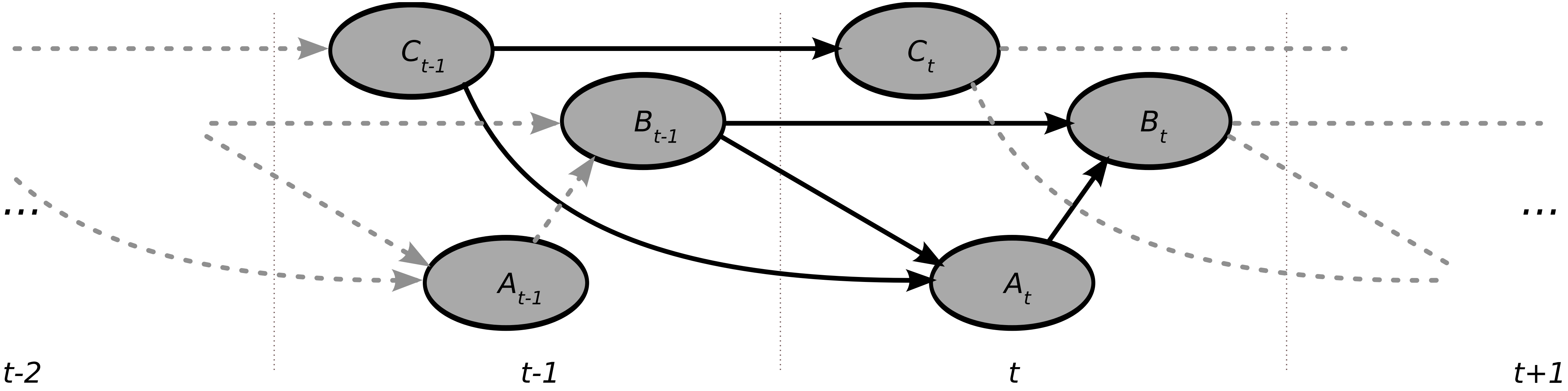
Factorized Transition Function: As Bayesian Network



Each varaible evolution is dependant from few parents
Potentially with intermediate varaibles.

Factorized Transition Function: Dynamic Bayesian Network

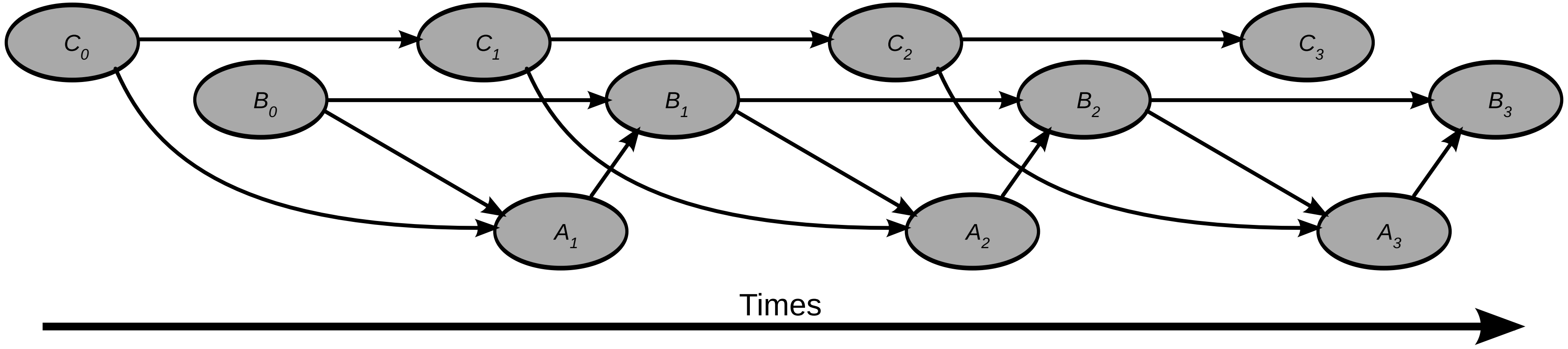
Bayesian Network with duplicated varaibles regarding their evolution in time.



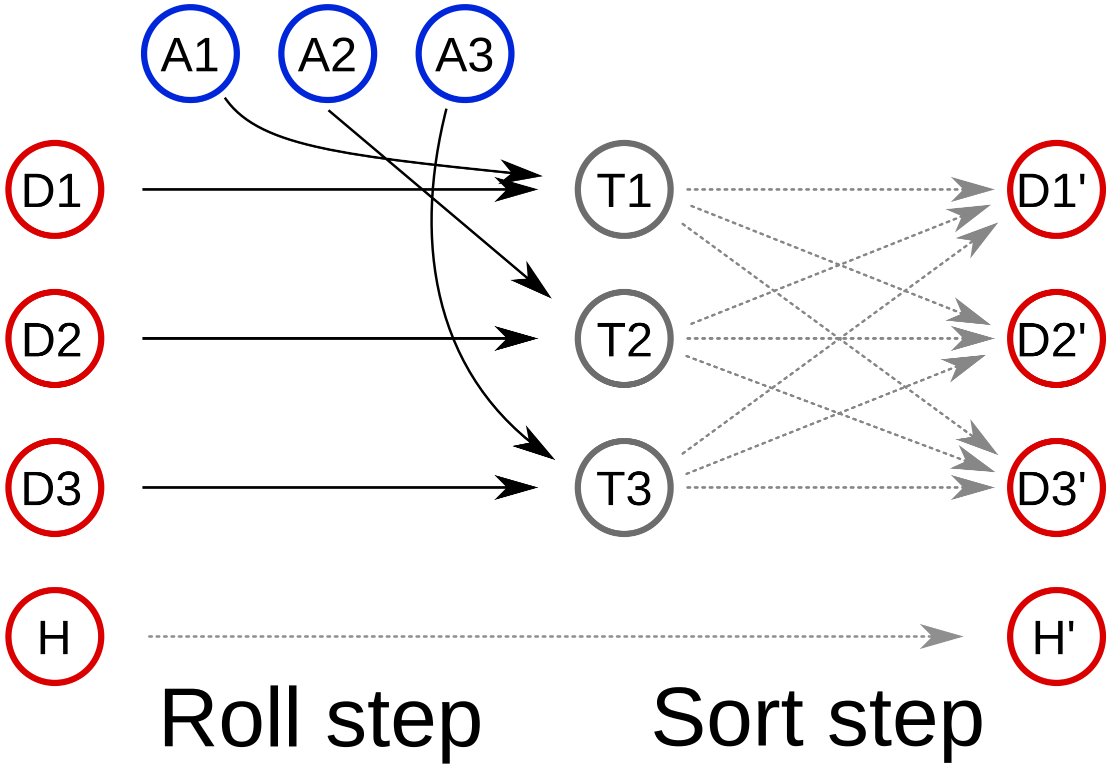
Considering a distribution of probability at time t,
it is possible to infers over the distribution of probabilities at time $t + N$

Factorized Transition Function: Dynamic Bayesian Network

Extended definition over 3 times step.

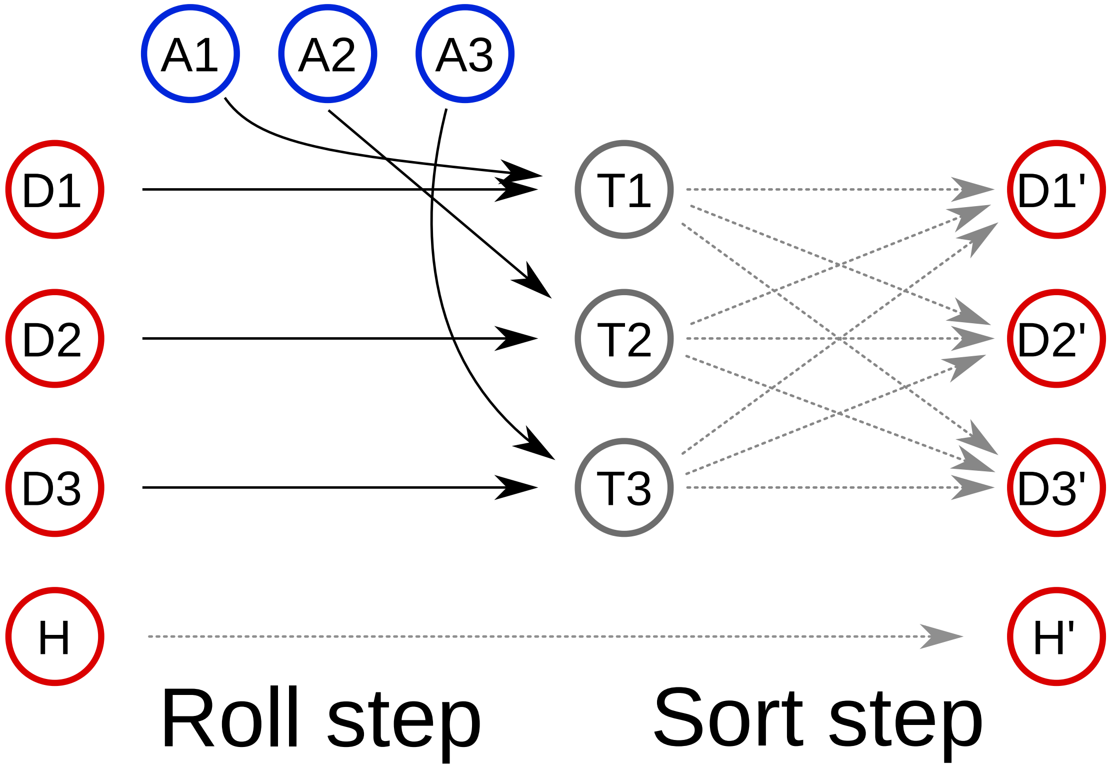


Factorized Transition Function: 421 example



- Roll step: 2 entrances - Small tables
- Sort step: 3 entrances but deterministic

Factorized Transition Function: 421 example



- Let consider cheat dice:
  Learning transitions → computing 3×12 probabilities *vs* $(168^2 \times 8)$

Factorized Transition Function: Zombie Dice Exemple

Naturrally the complexity of the networks grow linearly with the complexity of the game

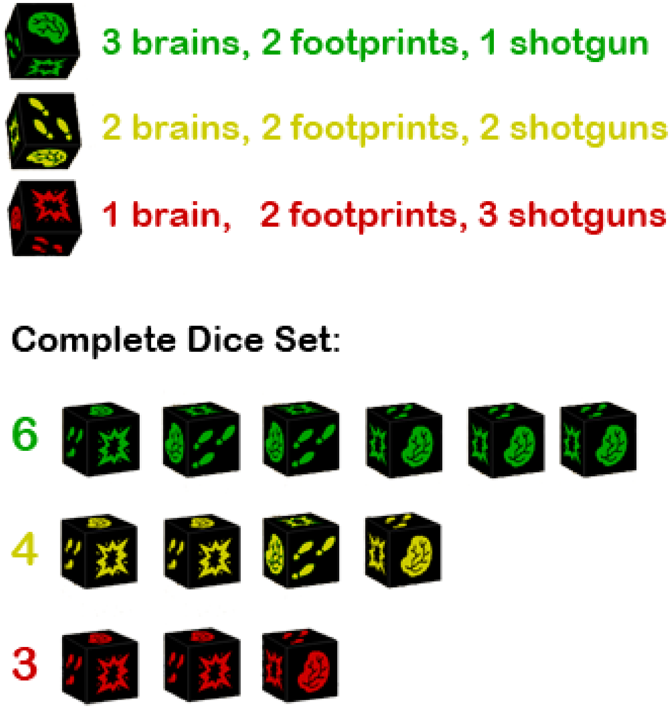
A stop and go game:

2 players or more (2 Zombies)

- The first one to cumulate 13 brains win.
- Player need to pass, before to get 3 damages

Factorized Transition Function: Zombie Dice Exemple

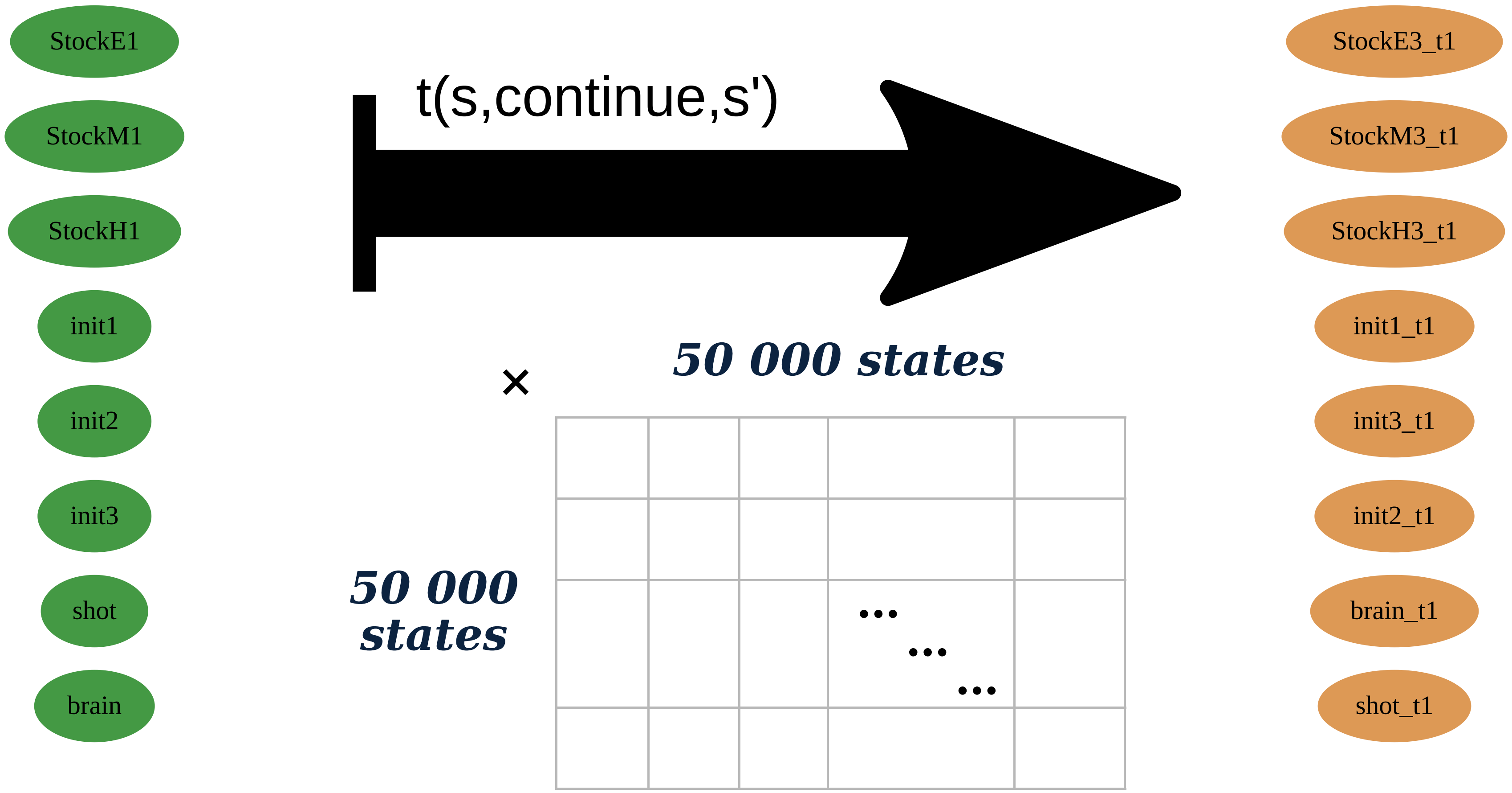
A 4 steps' game engine:


3 brains, 2 footprints, 1 shotgun
2 brains, 2 footprints, 2 shotguns
1 brain, 2 footprints, 3 shotguns
Complete Dice Set:
6
4
3

1. player decides to continue or not
2. new dice are picked randomly (to complete the hand)
3. dice a rolled
4. the player cumulates brains and damages

- **State Space ? Branching ?**

Zombie Dice : Brut Transition Function (action: go)



StockE1

StockM1

StockH1

init1

init2

init3

shot

brain

t(s,continue,s')

50 000 states

×

50 000 states

…

…

…

StockE3_t1

StockM3_t1

StockH3_t1

init1_t1

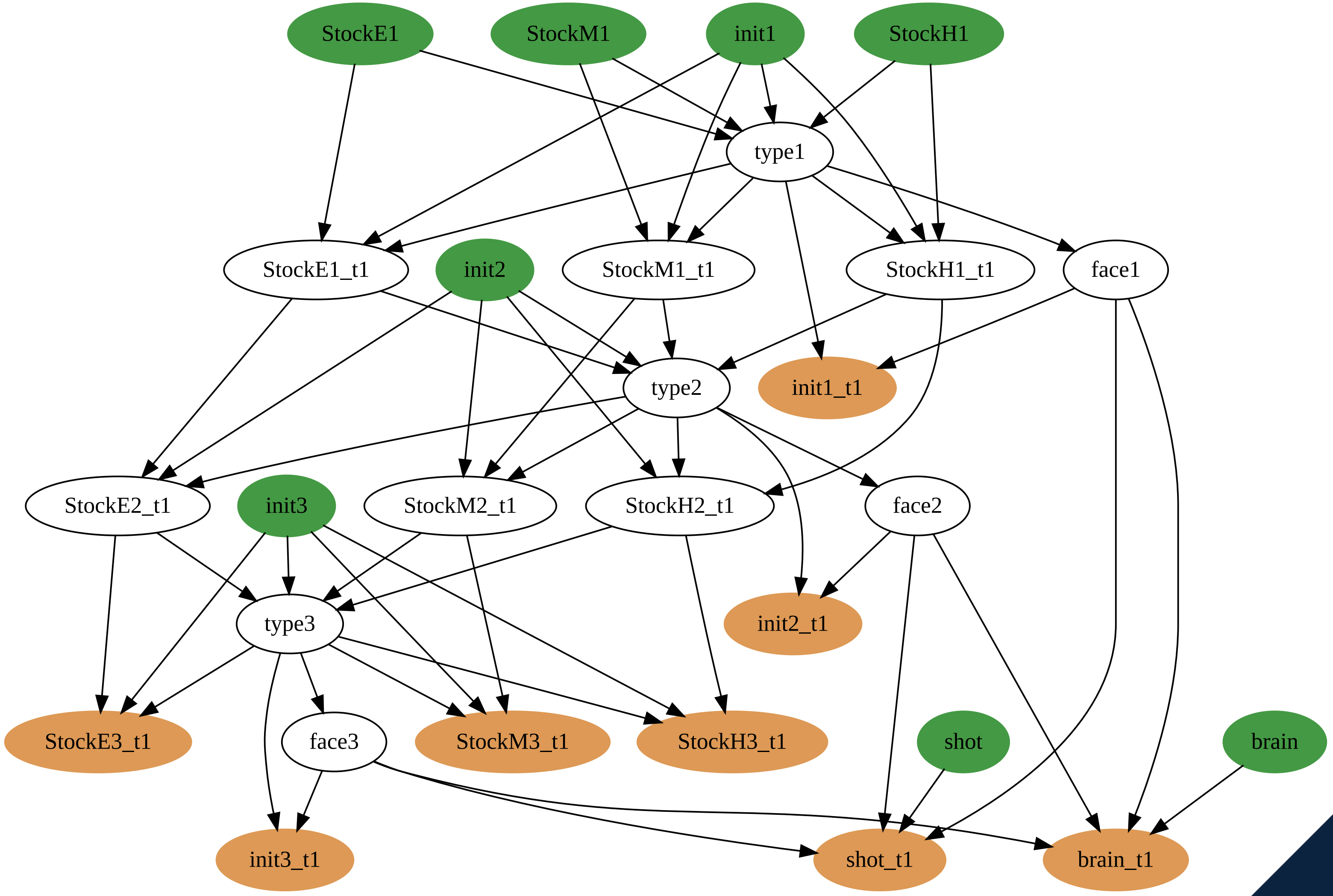
init3_t1

init2_t1

brain_t1

shot_t1

Zombie Dice : Factorized Transition Function (action: go)



1. Bayesian Network
2. Factorized Transition Function