States, Actions, and Policies

Decision Under Uncertainty

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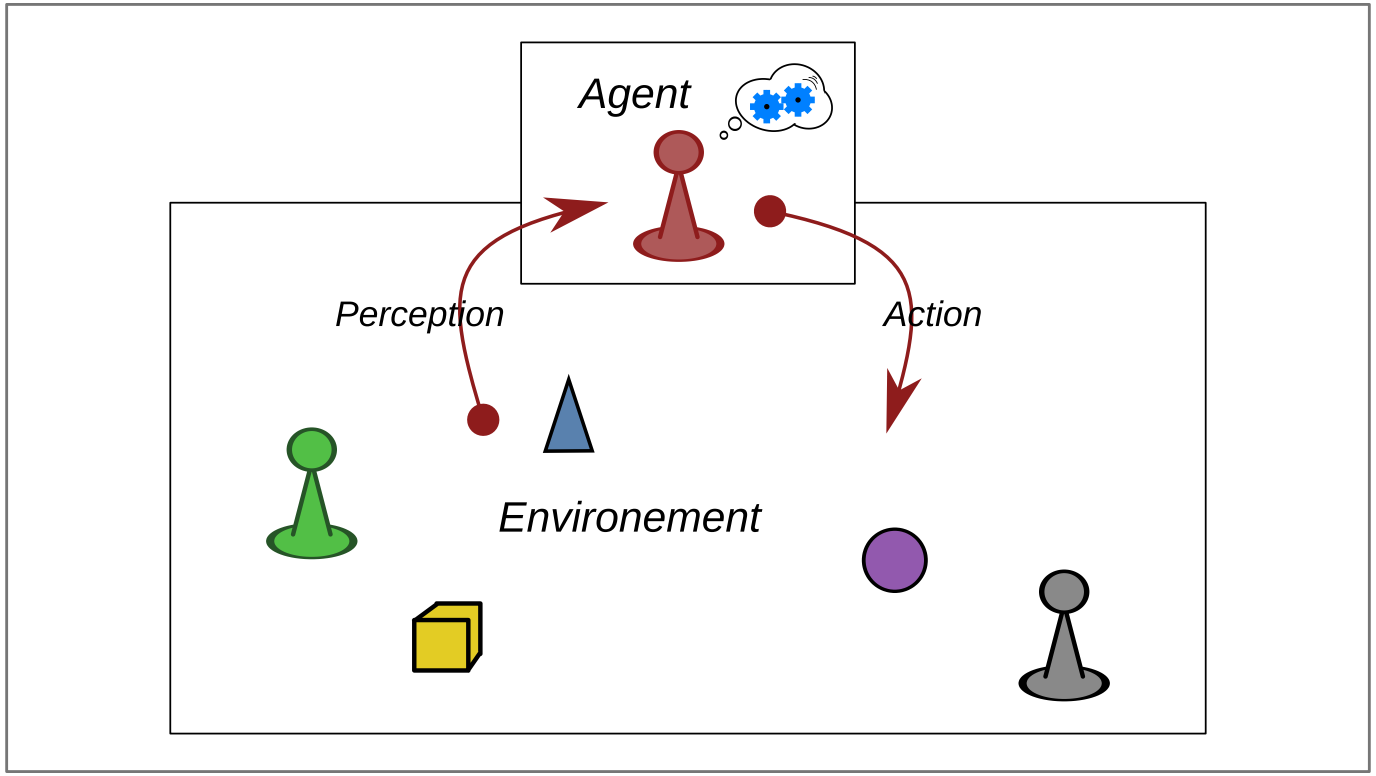
Acting over a dynamic system: the agent



Rarely deterministic, Mostly uncertain

Rational Agent

"I act, therefore I am."

- My actions have an effect over the world **AND** I have the choice to act or not.

cf. "BullShit Jobs" - David Graeber (2019)
(p.132-133 in French version)

Deliberativ Architecture - BDI:

- *Believe*: refers to the knowledge of the agent
- *Desire*: The agent's goals (classically states to reach)
- *Intention*: the succession of actions to perform oriented toward the goals

Acting over a system : formally

Markov Chain (Andreï Markov 1856-1922)

A tuple: $\langle \mathcal{States}\ (S),\ \mathcal{Transitions}\ (T)\rangle$

- **States**: set of configurations defining the studied system
- **Transitions**: Describe the possible evolution of the system state

$$T : S \times S \to [0, 1]$$

$$T(s_t,\ s_{t+1}) = P(s_{t+1}|s_t)$$

Vocabulary Parrenthesis: Hidden Markov Chain

> The system state is not directly observable.

Acting over a system : formally

Impact of the actions

- **Actions**: finite set of possible actions to perform

Updated Transition function:

The probabilistic evolution depends on the performed action.

$$T : S \times A \times S \rightarrow [0, 1]$$

$T(s^t,\ a,\ s^{t+1})$ return the probability to reach s^{t+1} by doing a from s^t:

$$T(s^t,\ a,\ s^{t+1}) = P(s^{t+1} | s^t, a)$$

Multi-variable system

State and Action space:

> Cartesian produc over State and Action variables

Multi-variable Transition function:

The probabilistic evolution depends on the performed action.

$$T : S \times A \times S \to [0, 1] \qquad T\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, \begin{bmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_n \end{bmatrix} \right) \in [0, 1]$$

Model of 421: States and actions

- **States**:
  - The value of each die's face ($d_n \in [1,6]$) and the re-roll number ($h \in [2,0]$)
  - So: **168** states (56 combinations over a horizon of 3).
- **Actions**:
  - The choice of roll again each die: $[roll,\ keep]$
  - so **8** actions (2^3)

Action Example :

By choosing to "roll-*keep*-roll" in state: "6-*4*-3 (2)" to expect a "4-2-1 (1)"

Model of 421: Transition function with 421-game

- **Transitions**:
  - All reachable states by rolling some dice with the probability to reach them.

Model of 421: Transition function with 421-game

Transitions Example :

Choosing to "roll-*keep*-roll" from "6-*4*-3 (2)" implies *21* reachable states:

P(...)	=	[0, 1]	P(...)	=	[0, 1]
4-1-1 (1)	=	$1/36$	…		
4-2-1 (1)	=	$1/18$	6-*4*-4	=	$1/18$
4-2-2 (1)	=	$1/36$	6-5-*4*	=	$1/18$
…			6-6-*4*	=	$1/36$

Choosing : building a policy of actions

- *a policy* (π) : a function returning the action to perform
  Considering the current state of the system:

$$\pi : S \to A$$

$$\pi(s) : \text{ the action to perform in } s$$

Choosing : building a policy of action

Example of policy :

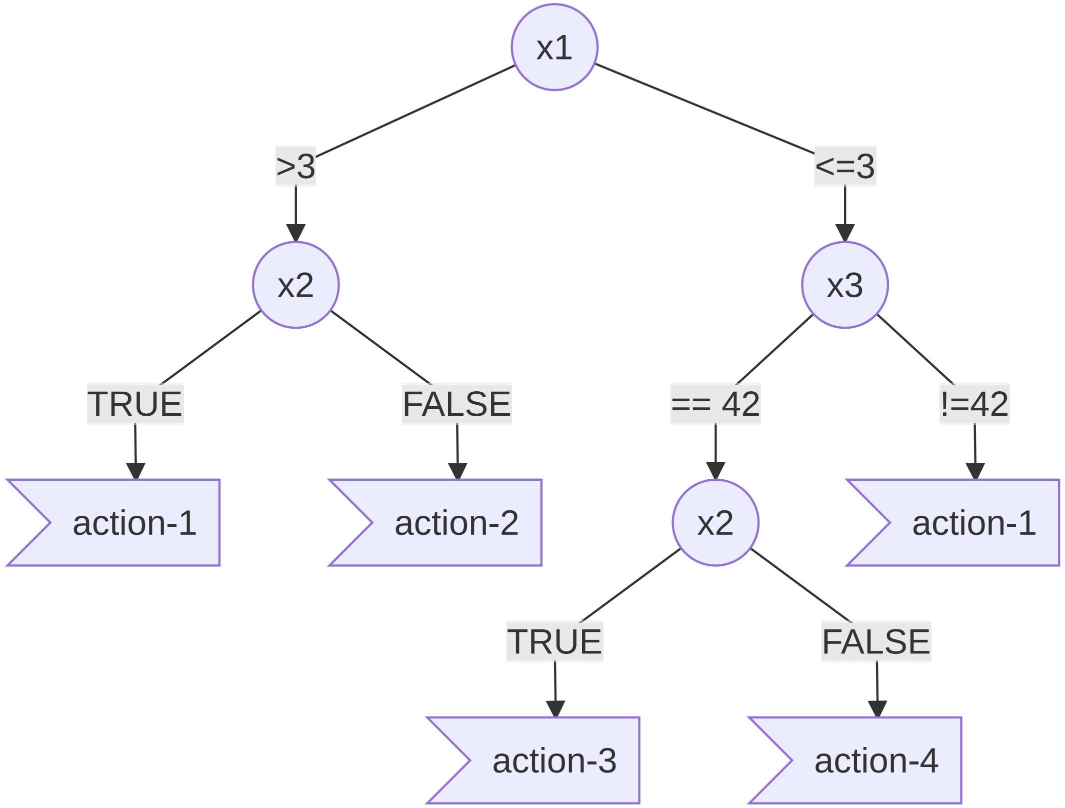
Always target a 4-2-1: keeping only one **4**, one **2** and one **1**

s	$\pi^{421}(s)$	s	$\pi^{421}(s)$
1-1-1	*keep*-roll-roll	…	
2-*1*-1	*keep*-*keep*-roll	*4*-*2*-*1*	*keep*-*keep*-*keep*
3-*1*-1	roll-*keep*-roll	…	
4-*1*-1	*keep*-*keep*-roll	6-6-5	roll-roll-roll
…		6-6-6	roll-roll-roll

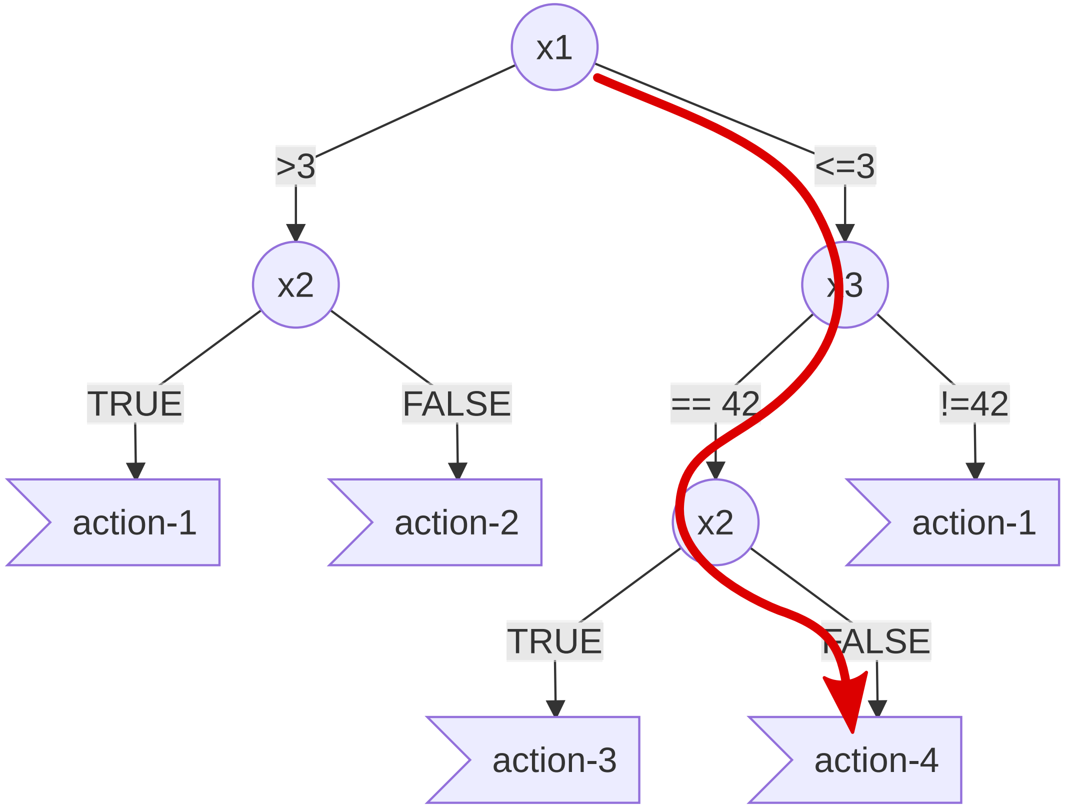
(Invariant over the horizon h)

Policy as decision tree

Nodes: variables ; **Edges:** assignment ; **leaf:** Action to perform



Policy as decision tree



- $\pi(2, False, 42)$ = **Action-4**

Choosing to optimize

Require to evaluate the interest of each action on the system evolution:

- *Reward/Cost function* (R) :

$$R : S \times A \rightarrow \mathbb{R}$$

$R(s_t,\ a)$ is the reward by doing a from s_t.

- *Objective* : Maximazing the gains (sum of percived rewards)

reward in 421-game

Over the final combination only with the action "*keep*-*keep*-*keep*" or when the horizon is 0

$$
\begin{aligned}
score(\text{4-2-1}) &= 800 \\
score(\text{1-1-1}) &= 700 \\
score(\text{x-1-1}) &= 400 + x \\
score(\text{x-x-x}) &= 300 + x \\
score(\text{(x+2)-(x+1)-x}) &= 202 + x \\
score(\text{2-2-1}) &= 0 \\
score(\text{x-x-y}) &= 100 + x \\
score(\text{y-x-x}) &= 100 + y
\end{aligned}
$$

Let's go....