The Curse of Dimensionality

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System Difficulty

Directly correlated to the state space:

The number of states: the Cartesian product of variable domains $|S|$ (minus some unreachable states)

- **421 game:** 3 dice-6 at the horizon 3: $\left(3 \times 6^3 = 648\right)$ but 168 effectives.

Then the branching:

Finnally the number of games:

System Difficulty

Directly correlated to the state space

The number of states: $|S|$

Then the branching:

The number of possible actions and actions' outcomes.

- **421 game:** 2^3 actions, 6^r action outcomes (r, the number of rolled dice).

Finnally the number of games:

The number of all possible succesion of states until reaching an end.
Potentially $|S|^h$ (h the horizon).

Reminder over Combinatorics

With a Classical 32-card game: Possible distribution $32! = \quad 2.6 \times 10^{35}$

Human life: around 5×10^{7} seconds

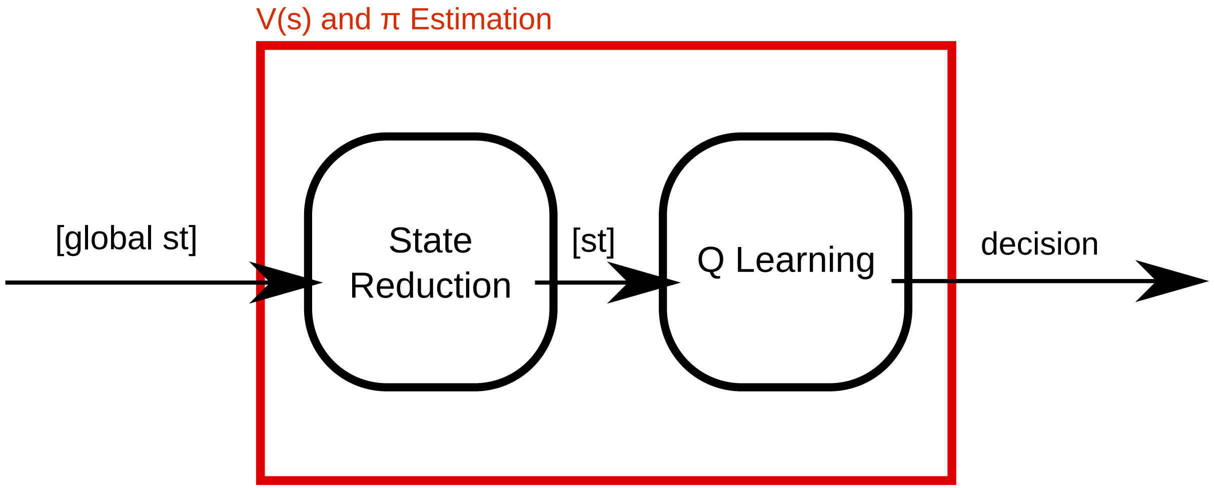
Probability to play 2 times the same distribution in a human life is very close to 0

The root problem: handle large systems

A first basic solution: reduce the state space definition

State reduction in QLearning

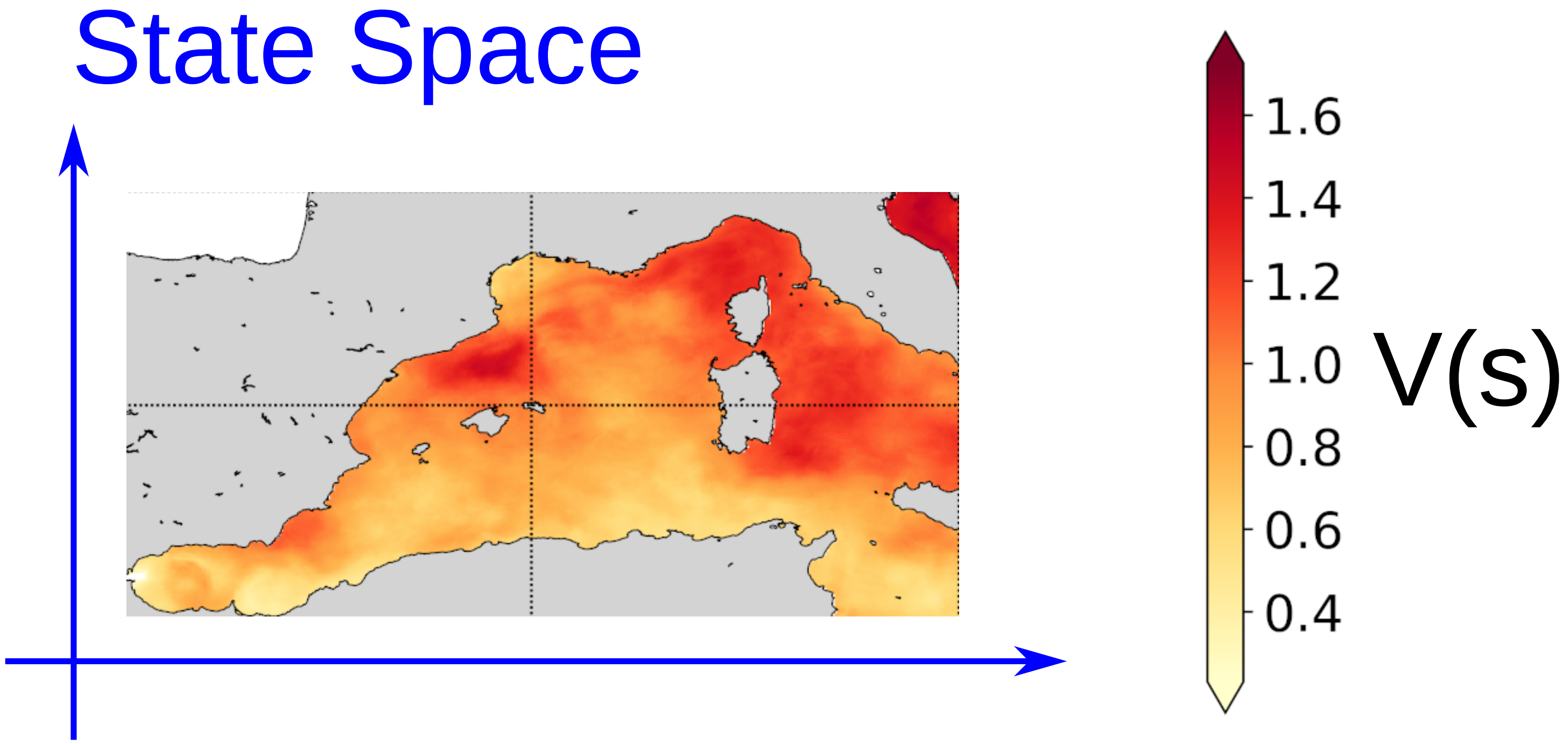
Project the states in a smallest space (dimention and size)



By mitigate the negative impact on the resulting built policy.

State reduction in QLearning

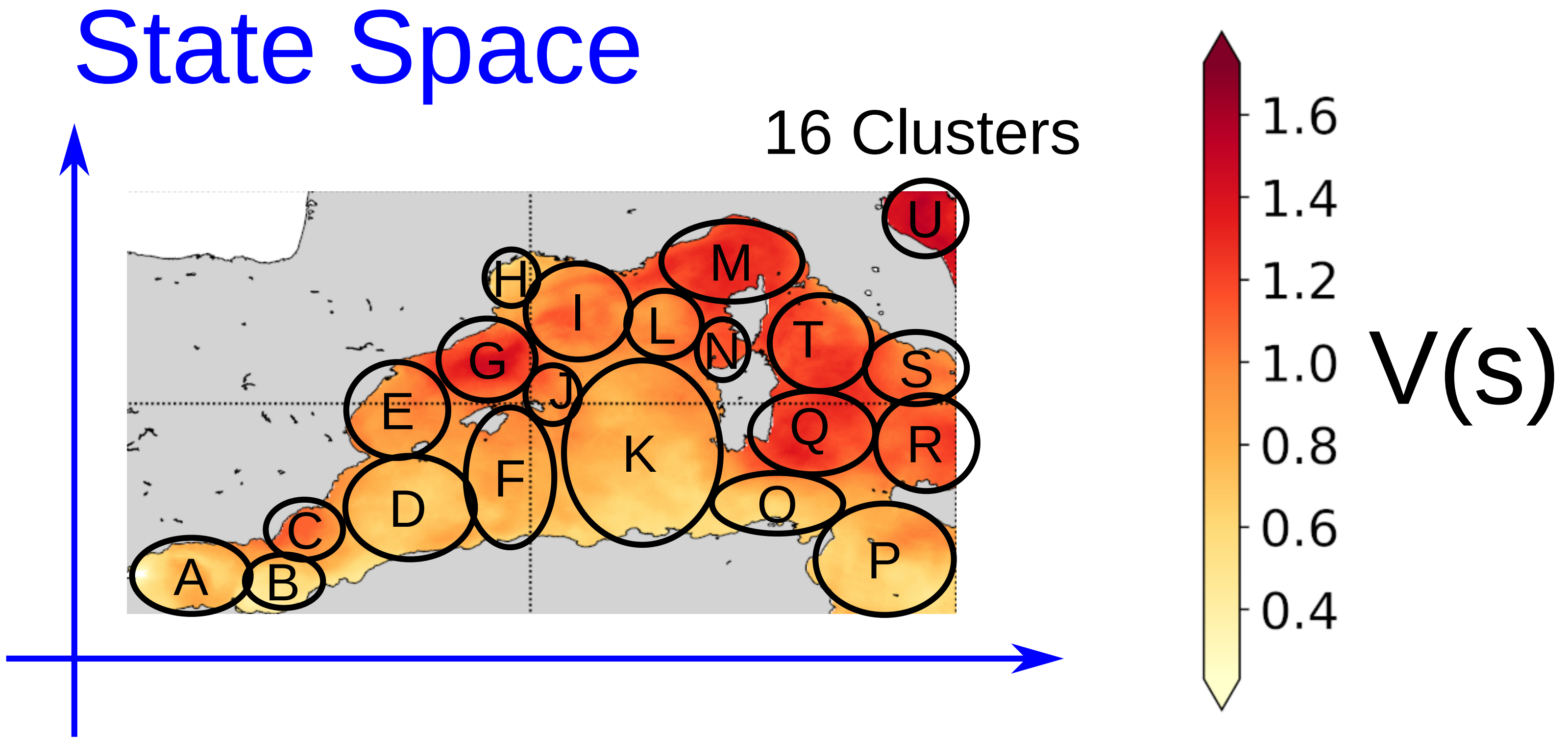
Project the states in a smallest space (dimention and size)



- From evaluated observations.

State reduction in QLearning

Project the states in a smallest space (dimention and size)



- Group together similar states.

State reduction in QLearning

A classical unsupervised learning problem

- Group similar states :
  - close state (in the transition succession)
  - similar reward distributions.

Potentially: a supervised learning problem

- Group similar states :
  - similar Value
  - similar action outcome

(suppose to have some valued states)

With a geometric approach

Principal Component Analysis (PCA)

Searching the hyper-plan that better separate the data, in a given dimension.

K-means

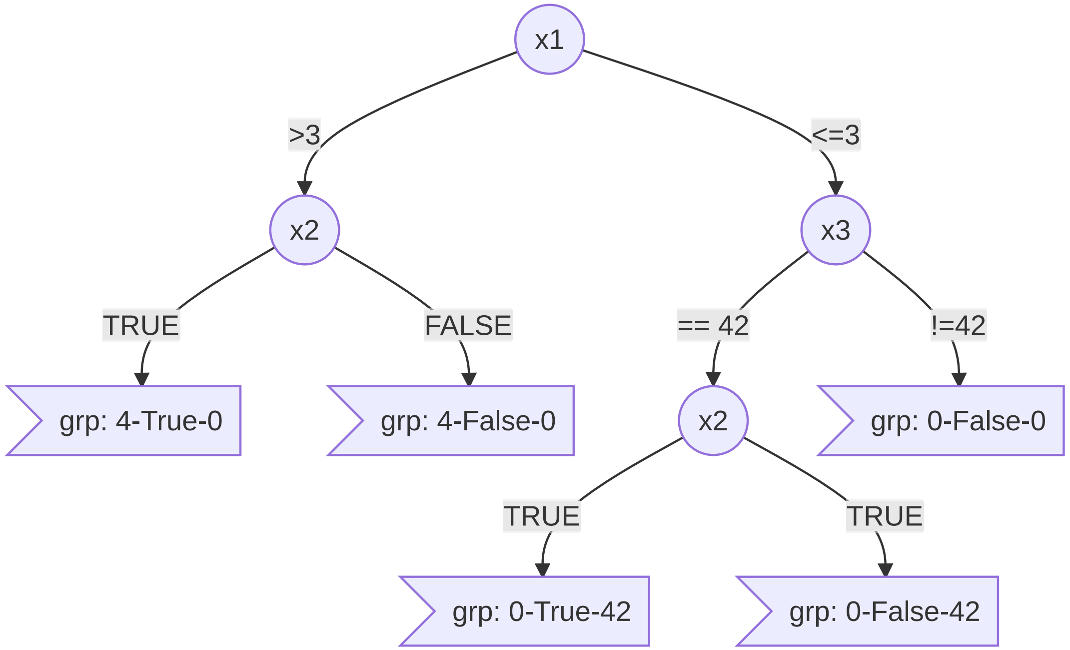
Searching the optimal k center positions that better group the data together.

- Work well with 'linear state transitions' and different states density.
- Suppose a data set (trace)

Based on state variable prevalence

Decision Tree

Nodes: variables ; **Edges:** assignment ; **leaf:** group of states

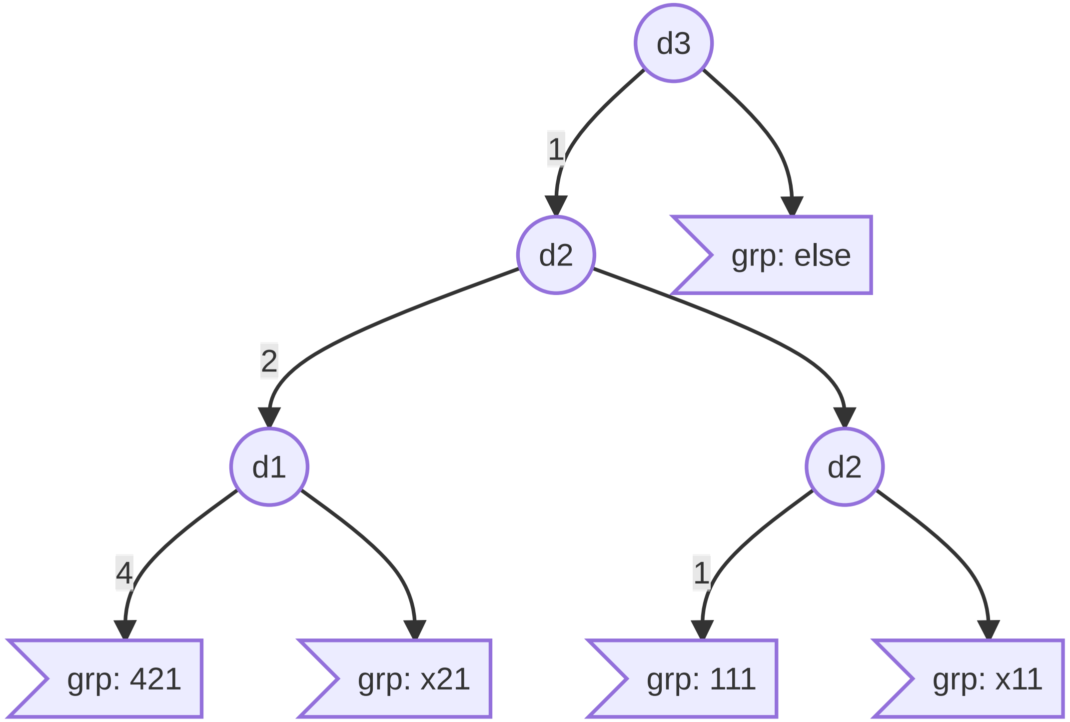


- Expert based Decision tree or learned (ID3 algorithm)

Based on state variable prevalence

Decision Tree

(Example for 421 game)



Learning: an iterative process:

- **1** - Define a first state reduction $red_0(s)$ with a first Q_0 estimation
- **2** - Optimise Q_i and learn a behavior accordingly to $red_i(s)$
- **3** - Generate a new reduction $red_{i+1}(s)$ (more accurate)
- **4** - Propagate value from Q_i to Q_{i+1}
- **5** - goto **2**

Let's go

Trying state reduction in ZombieDice