The Curse of Dimensionality

And how to ward off it

Guillaume Lozenguez

@imt-nord-europe.fr

IMT Nord Europe
École Mines-Télécom
IMT-Université de Lille

1. **The Curse of Dimensionality**
2. **Geometric reduction**
3. **State Decomposition**
4. **Quid of the set of actions**

1. **The Curse of Dimensionality**
    - Example With Risky
2. Geometric reduction
3. State Decomposition
4. Quid of the set of actions

System Difficulty

Directly correlated to the state space:

The number of states: the Cartesian product of variable domains $|S|$ (minus some unreachable states)

- **421 game:** 3 dice-6 at the horizon 3: $\left(3 \times 6^3 = 648\right)$ but 168 effectives.

Then the branching:

Finally, the number of games:

System Difficulty

Directly correlated to the state space

The number of states: $|S|$

Then the branching:

The number of possible actions and actions' outcomes.

- **421 game:** 2^3 actions, 6^r action outcomes (r, the number of rolled dice).

Finally, the number of games:

The number of all possible succession of states until reaching an end.
$|Branching|^h$ (h the horizon) Potentially $|S|^h$ (h the horizon).

Reminder over Combinatorics

With a Classical 32-card game: Possible distribution $32! = \quad 2.6 \times 10^{35}$

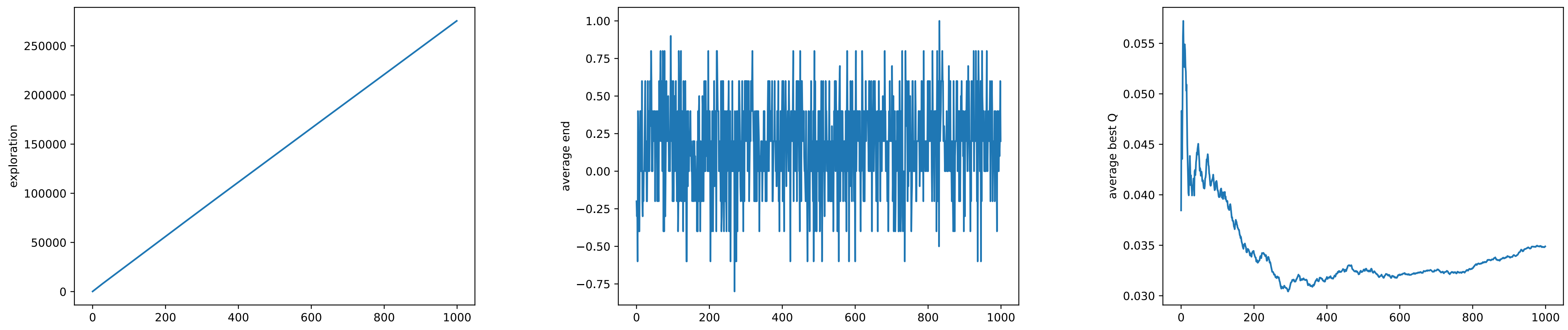
Human life: around 5×10^{7} seconds

Probability to play 2 times the same distribution in a human life is very close to 0

Learning Risky game

A strategic game over 12 cells to conquer by 2 armies: State Space: ~$(4 * 30)^{12}$

Exploration / average End / average Q Values



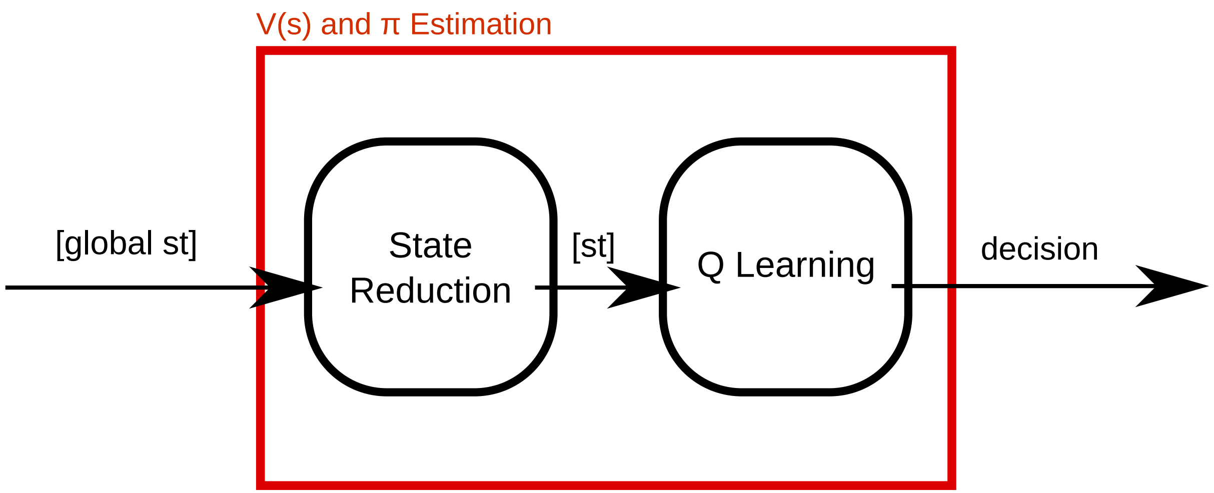
- python code (10000 games, one point each 10 games)

The root problem: handle large systems

A first basic solution: reduce the state space definition

State reduction in QLearning

Project the states in a smallest space (dimension and size)

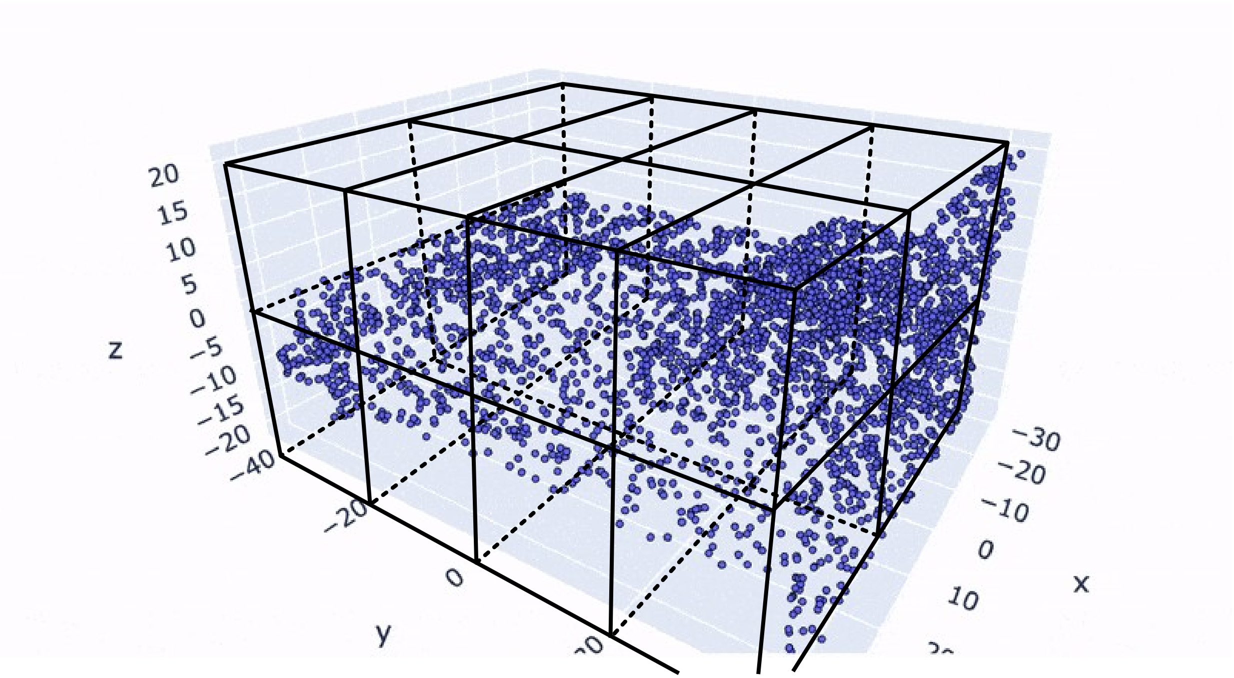


By mitigate the negative impact on the resulting built policy.

1. The Curse of Dimensionality
2. **Geometric reduction**
   - Reduce the dimension (PCA)
   - Clustering (K-means)
3. State Decomposition
4. Quid of the set of actions

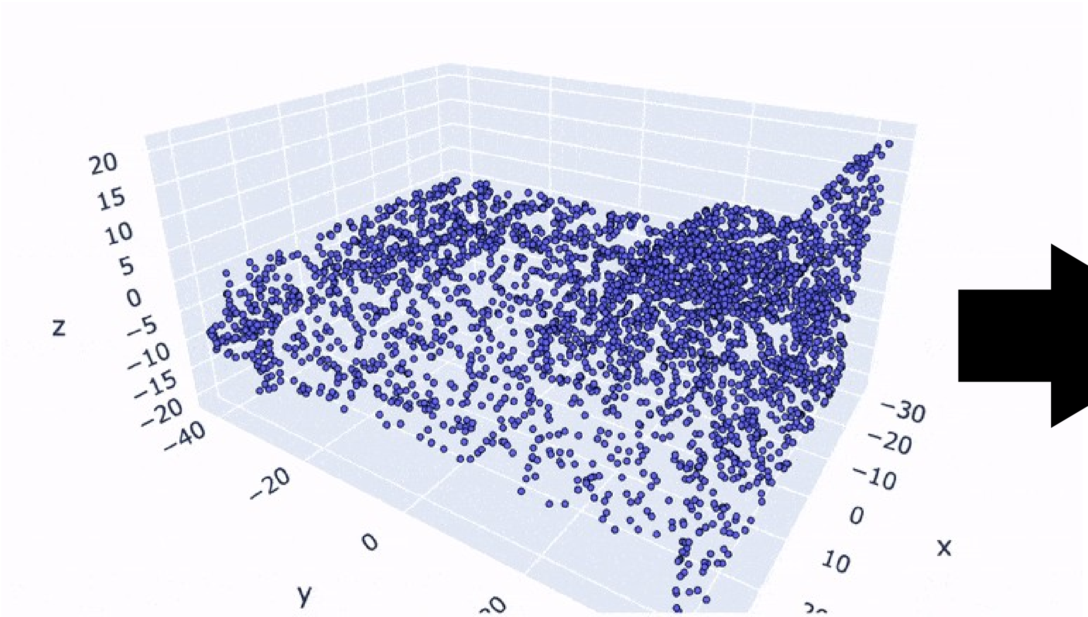
Geometry Reduction

- Consider that **close** states are similar.
- Based on the assumption that: *it is possible to define a distance between States*
- By using regular discretization or adaptative clustering

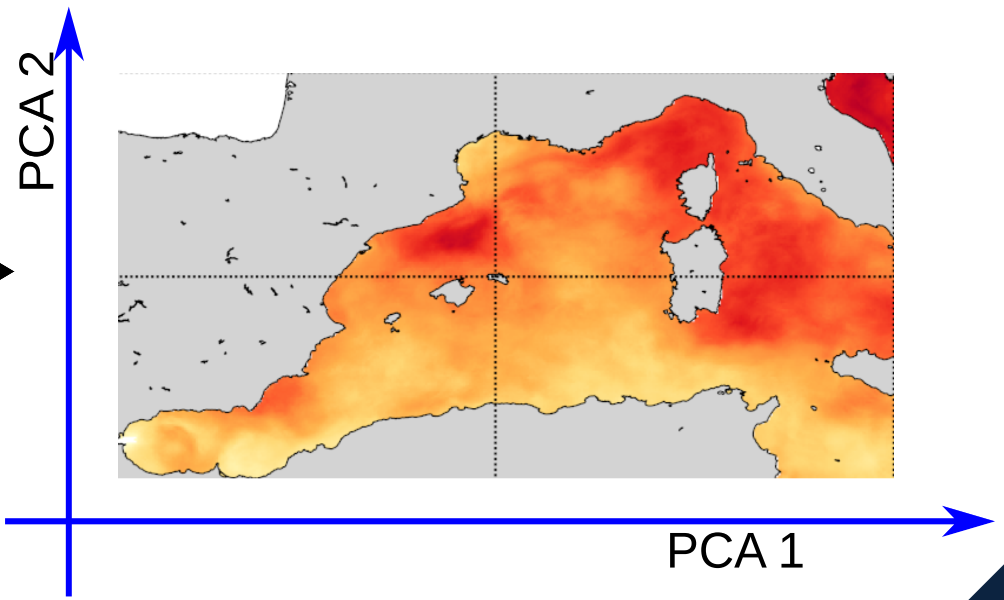


Reduce the dimension - (Principal Component Analysis)

Searching the hyper-plan that better separate the data, in a given dimension.



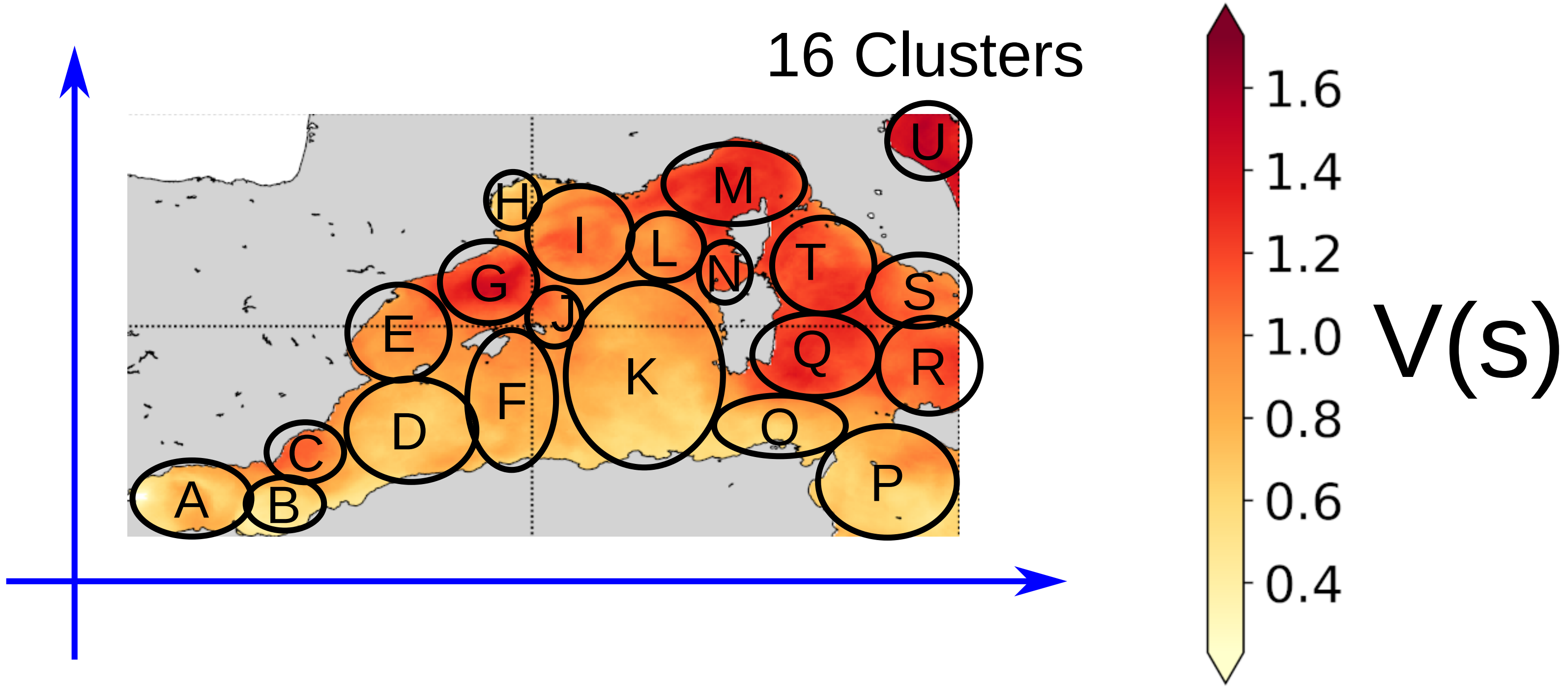
State Space



https://en.wikipedia.org/wiki/Principal_component_analysis

Clustering - (K-means)

regroup the states in coherent sets



K-means:
Searching the optimal *k* center positions that better group/separate the data

Basic 'simple' classification method

Principal Component Analysis (PCA)

Searching the hyper-plan that better separate the data, in a given dimension.

Python scikit-learn module: **sklearn.decomposition.PCA**

K-means

Searching the optimal *k* center positions that better group the data together.

Python scikit-learn module: **sklearn.cluster.KMeans**

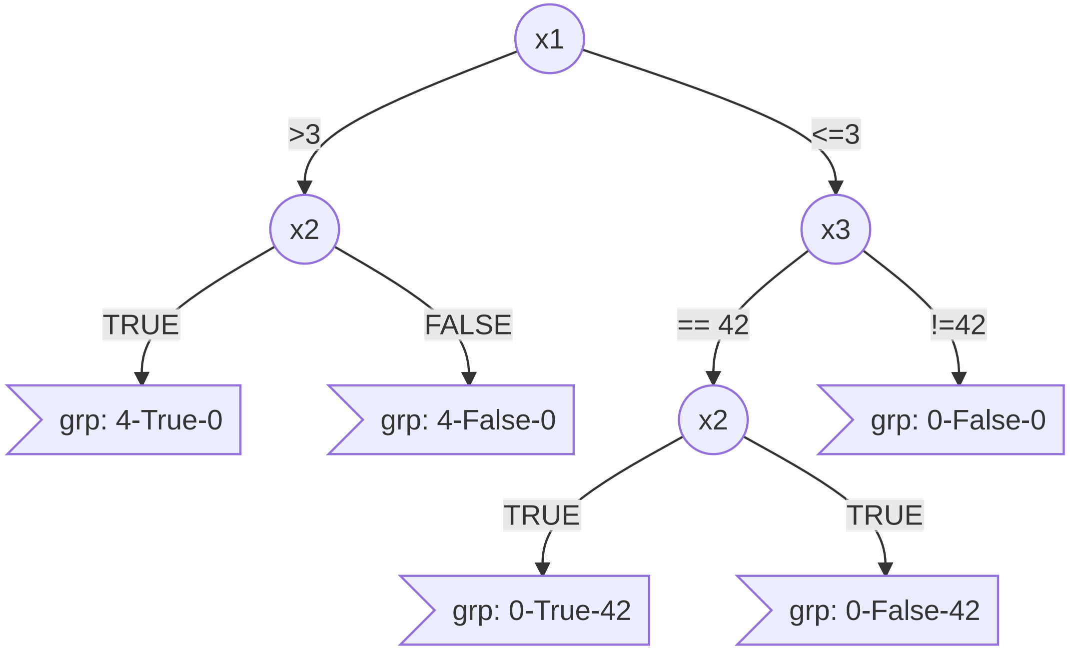
- Work well with 'linear state transitions' and different states density.
- Suppose a data set (trace) ideally with proper values

1. The Curse of Dimensionality
2. Geometric reduction
3. **State-Space Decomposition**
   - Decision Tree (Again)
   - Example With 421
4. Quid of the set of actions

State-Space Decomposition

Factorized method: Based on state variable prevalence

- Decision tree (Again) **Nodes:** variables ; **Edges:** assignment ; **leaf:** group of states



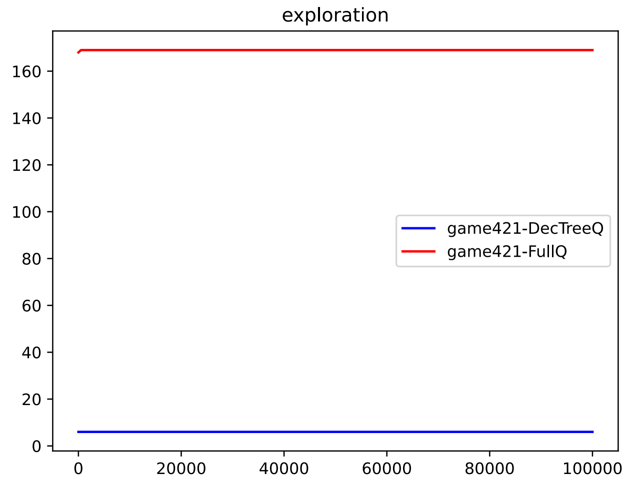
Decision Tree On 421 Q-Learning

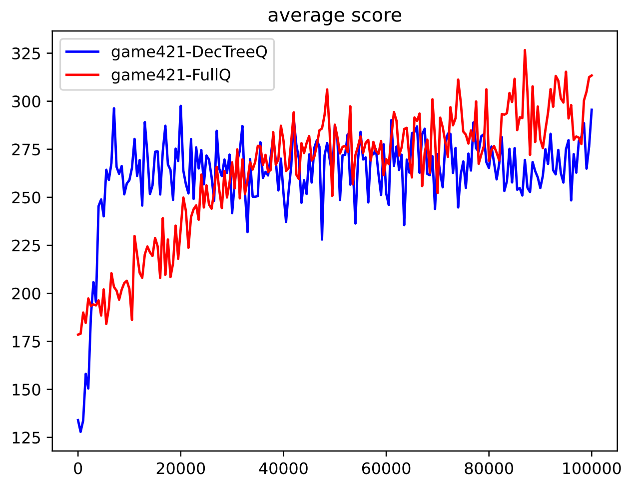
Simply reduce the state definition to 6 states...

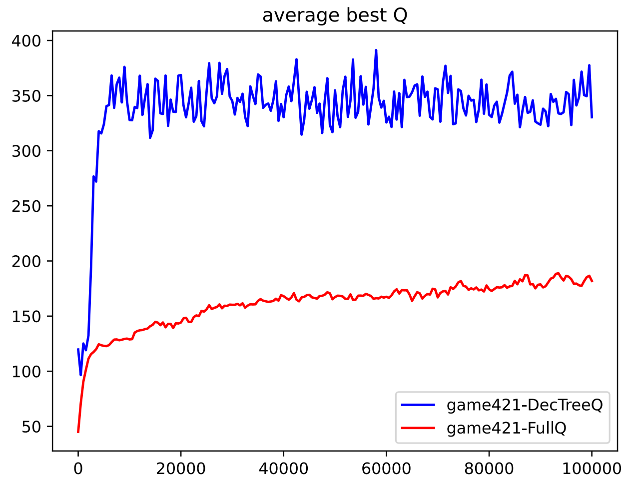
```
def state(self):
   if self.turn == 0 :
      return 'end'
   if self.dices[2] == 1 :
      if self.dices[1] == 2 :
            if self.dices[0] == 4 :
               return "4-2-1"
            return "X-2-1"
      if self.dices[1] == 1 :
            return "X-1-1"
      return "X-X-1"
   return "X-X-X"
```

Decision Tree On 421 Q-Learning

Results:







- python code: [Decision Tree Q-Learning](#) - [plotting](#)

Decision Tree Conclusion...

Conclusion:

It is all about defining the appropriate variable prevalence (Decision Tree Structure)

Learn the structure:

- Expert based Decision Trees or learned (ID3 algorithm)
- Again on python scikit learn: (module tree)

But...

The evaluation of the structure of the tree is performed by deadly execution of Q-Learning !

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3. State Decomposition
4. **Quid of the set of actions**
   - The need of SuperAction

Dealing with combinatorial actions

The same strategy: Decomposition

- Group together 'similar' actions > *SuperAction*
- Geometric or decomposed technic
- Learn Q-Value over *SuperActions*

Dealing with combinatorial actions

At decision steps:

From superaction to local action

Choose one of the actions of the SuperAction:

- randomly
- with the use of an heuristic.
- The 'best' one accordingly to the reached next state...

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Apply Decomposition in Risky game

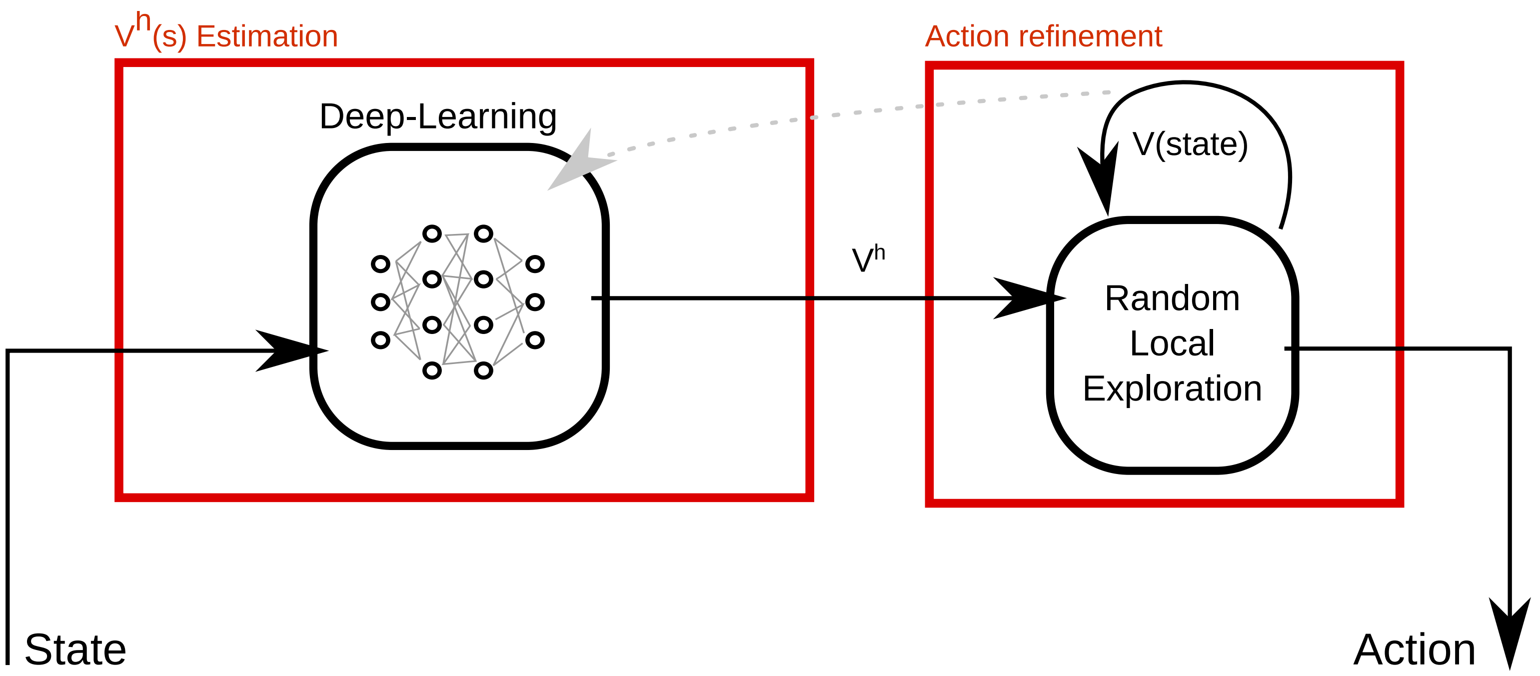
My advice:

- Think iterative: the last increase initializes the next learning phase.
- Start small and grow...

Before to go:

The actual killing strategy: (AlphaGo)

Deep-Learning-based Decision Architecture



DNN (a all-in-one decomposition and evaluation technic) + Monte-Carlo Search