Q-Learning

A classical method of Reinforcement Learning

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Acting over a system evolving under uncertainty

- **States**: set of configurations defining the studied system
- **Action**: finite set of possible actions to perform
- **Transitions**: Describe the possible evolution of the system state

Transition function:

The probabilistic evolution depends on the performed action.

$$T : S \times A \times S \to [0, 1]$$

$T(s_t,\ a,\ s_{t+1})$ return the probability to reach s_{t+1} by doing a from s_t:

$$T(s_t,\ a,\ s_{t+1}) = P(s_{t+1} | s_t, a)$$

Acting to optimize Gain

Require to evaluate the interest of each action on the system evolution:

- *Reward/Cost function* (R) :

$$R : S \times A \times S \to \mathbb{R}$$

$$R(s_t,\ a,\ s_{t+1}) \text{ is the reward by reaching } s_{t+1} \text{ from doing } a \text{ in } s_t$$

OR, in a simplified version:

$$R : S \times A \to \mathbb{R}$$

Acting to optimize gain (accumulated rewards)

- Our objective: *a policy* (π) : a function returning the action to perform considering the current state of the system:

$$\pi : S \to A$$

$$\pi(s) : \text{ the action to perform is } s$$

- *Bellman Equation* :

$$V^\pi(s) = R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') \times V^\pi(s')$$

$$\text{with} : a = \pi(s) \text{ and } \gamma \in [0,1[\text{ the discount factor (typically 0.99)}$$

reward in 421-game

Over the final combination only with the action "*keep*-*keep*-*keep*" or when the horizon is 0

$$\begin{aligned}
score(\text{4-2-1}) &= 800 \\
score(\text{1-1-1}) &= 700 \\
score(\text{x-1-1}) &= 400 + x \\
score(\text{x-x-x}) &= 300 + x \\
score(\text{(x+2)-(x+1)-x}) &= 202 + x \\
score(\text{2-2-1}) &= 0 \\
score(\text{x-x-y}) &= 100 + x \\
score(\text{y-x-x}) &= 100 + y
\end{aligned}$$

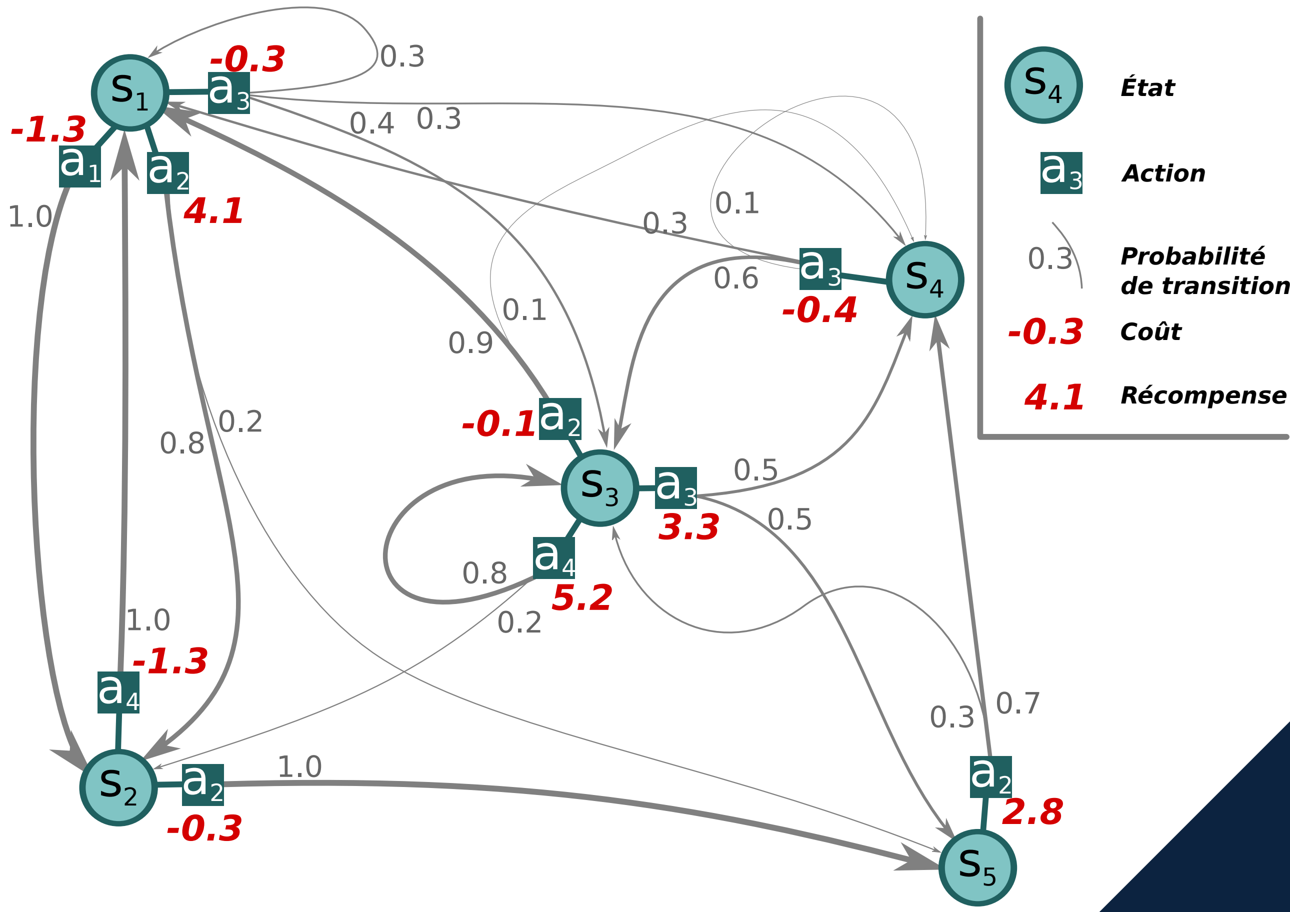
Markov Decision Process

MDP: $\langle S, A, T, R \rangle$:

S : set of system's states
A : set of possible actions
T : S × A × S → [0, 1] : transitions
R : S × A → R : cost/rewards

Optimal policy:

The policy π^* maximizing Bellman



Reinforcement Learning:

Learn the optimal policy

- Without knowledge over the transition probabilities and/or the rewards,
- but, by getting feedback from acting randomly.

2 approaches

- **model-based:** Learn the model, then compute the optimal policy.
- **model-free:** Learn the policy directly.

Model-Free Approaches

Concept

- Learn without generating **transition** and **reward** models.
- Build the **policy** directly from the interactions
- Use only the experience of sequences:

$$state,\ action,\ reward,\ state,\ action,\ \ldots$$

Common approaches:

- **Q-learning**: continuous computing of an expected gain (require rich feedback)
- **Monte-Carlo**: use random explorations until a 'finale' state (slow to converge).

Exploration–Exploitation tradeoff dilemma

The agent build an optimal behavior from trials and errors.

- *Exploration*
  - Try new actions to learn unknown feedback
  - Better understand the dynamics of the system
  - Risky output
- *Exploitation*
  - Use the best-known action
  - Potentially suboptimal

Exploration–Exploitation Tradeoff Dilemma

Examples:

- *Exploitation*: apply a known game strategy **vs** *Exploration* investigate new actions.
- *Exploitation*: go to your favorite restaurant **vs** *Exploration* try a new one.

Classical approach:

- Trigger exploration *when* the old fashion strategy doesn't work anymore
  Problems:
  - Determine that "a strategy doesn't work" ?
  - Determine that "a new policy is well defined" (exploration end) ?
- Continuously Explore and Exploite with a fixed ratio.
  - (take wrong decision periodically)

Continuous Exploration–Exploitation : ϵ-Greedy

A Simple heuristic for the Exploration–Exploitation Tradeoff Dilemma

- Random decision with:
  - a probability ϵ to choose a random action (exploration)
  - a probability $1 - \epsilon$ to choose the best-known action (exploitation)
- Classically ϵ is set to 0.1
- A ϵ-greedy agent behavior punctually takes off-policy action

Then the challenge consists in varying ϵ depending of the knowledge the agent has of the area he is interacting in.

Q-learning

One of the most important discoveries in Reinforcement Learning (simple and efficient)

- At each step, **Q-learning** updates the value attached to a couple (state, action)
- Updates are performed integrate expected future gains
- The update is performed accordingly to a learning rate $\alpha \in]0, 1[$

  $\rightarrow$ α : ratio between new vs old accumulated information.

Q-learning based on a Q function

Considering it is not possible to evaluate state without a policy yet

$$V^{\pi}(s) = R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') \times V^{\pi}(s')$$

the **Q-values** evaluate each action performed from each state:

$$Q : S \times A \to \mathbb{R}, \qquad Q(s,\ a) \text{ is the value of doing } a \text{ from } s$$

and, a **Q-value** is updated iteratively from succession of: $\langle s,\ a,\ s',\ r(s,a,s') \rangle$

$$Q(s,a) = (1-\alpha)Q(s,a) + \alpha \left(r + \gamma \max_{a' \in A} Q(s',a') \right)$$

Q-learning : the algorithm

Input: state and action spaces: *A* ; a step engine *Perform* ;
exploration ratio: ϵ ; learning rate: α ; discount factor γ

```
1. Read the initial state $s$
2. Initialize $Q(s, a)$ to 0 for any action $a$
3. Repeat until convergence
    1. At $\epsilon$ random: get a random $a$ or $a$ maximizing $Q(s, a)$
    2. Perform $a$ and read the reached state $s'$ and the associated reward $r$
    3. If necessary, add $s'$ to $Q$ ( with value $0$ for any action $a$)
    4. Update $Q(s, a)$ accordingly to $\alpha$ and $\gamma$
    5. set $s = s'$
```

Output: the **Q-values**.

Q-learning : the algorithm

In agent-based programming:

- As an initial step :
  1. Initialize Q
- At 'game' start :
  1. Read the initial state s
- At each itereration :
  1. Read the reached state s' and the associated reward r
  2. If necessary, add s' to Q (with value 0 for any action a)
  3. Update $Q(s,a)$ accordingly to α and γ
  4. reccord $s = s'$
  5. At ϵ random: get a random a *or* a maximizing $Q(s,a)$

Q-learning : the main equation

$$Q(s,a) = (1-\alpha)Q(s,a) + \alpha\left(r + \gamma \max_{a' \in A} Q(s',a')\right)$$

- $Q : S \times A \to \mathbb{R}$: the value function we build.
- α : the learning rate
- ϵ : the Exploration-Exploitation ratio
- γ : the discount factor

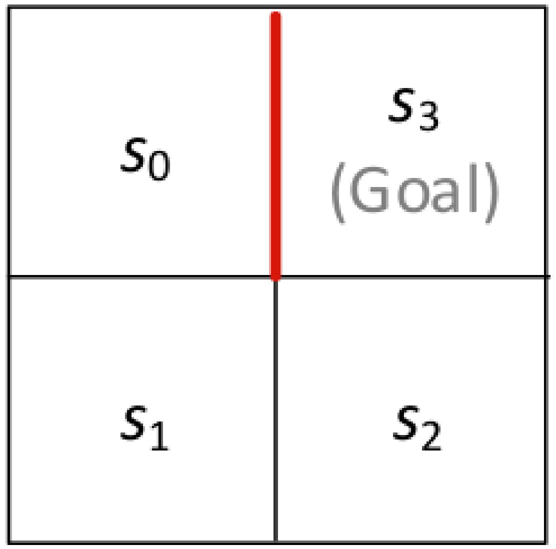
The known optimal policy:

$$\pi^*(s) = \max_{a \in A} Q(s,a)$$

Simple Example

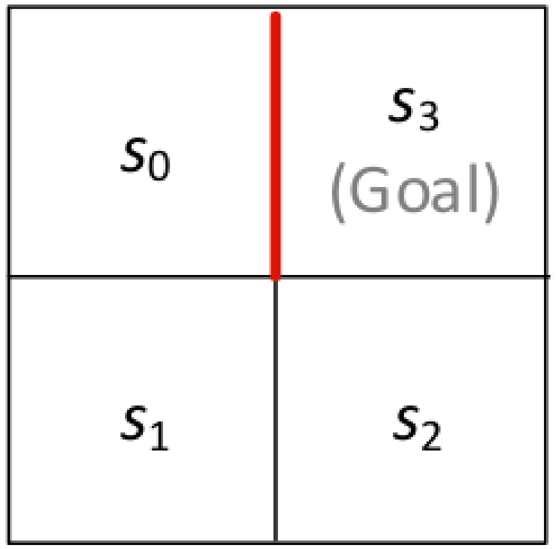
- **States**: 4 positions
  s_0, s_1, s_2 and s_3
- **Actions**: left, right, up, down
- **Transitions**: determinist
- **Rewards**:
  10 for reaching s_3, -1 else

($\epsilon = 0.1, \alpha = 0.1$ and $\gamma = 0.99$)



Simple Example

- From s_0 get action *left* (explore)
  reaches s_0 with -1
  updates $Q(s_0, \mathit{left}) = -0.1$
- s_0 gets *right* (best) $\rightarrow (s_0, -1)$
  updates $Q(s_0, \mathit{right}) = -0.1$
- s_0 gets *down* (exp.) $\rightarrow (s_1, -1)$
  updates $Q(s_0, \mathit{down}) = -0.1$
  ...
- s_2 gets *up* (exp.) $\rightarrow (s_3, 10)$
  updates $Q(s_2, \mathit{up}) = 1$
  End Episode



Simple Example

($\alpha = 0.1, \epsilon = 0.1$ and $\gamma = 0.99$)

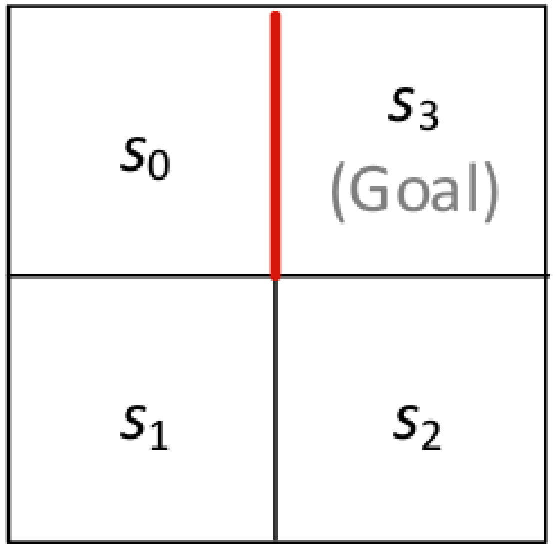
- **Episode 1:** (18 action)

S	s_0	s_1	s_2
$max\,Q$	-0.39	-0.19	1

- **Episode 2:** (15 action)

S	s_0	s_1	s_2
$max\,Q$	-0.43	0.9	1.9

…

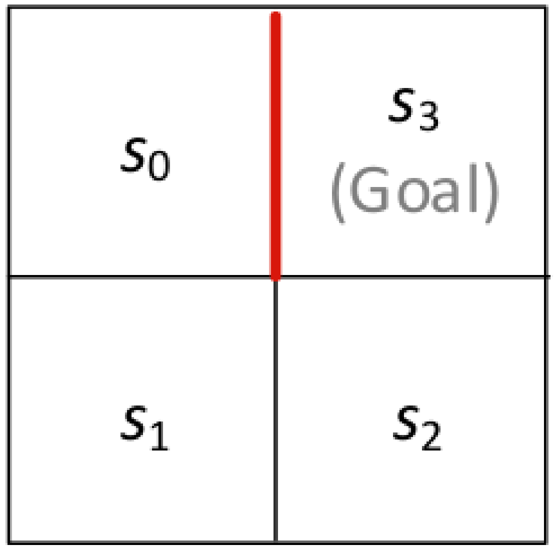


Simple Example

($\alpha = 0.1$, $\epsilon = 0.1$ and $\gamma = 0.99$)

- **Episode N**: (3-4 actions)

S	s_0	s_1	s_2
$max\, Q$	7.8	8.9	10
$argmax\, Q$	$\downarrow$	$\rightarrow$	$\uparrow$



Let's go....