Learning 421 game

Model-Based Learning

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1. **Back to Q-Learning on 421**
2. **Model-Based Learning**
3. **Let's play a more Complicated game**

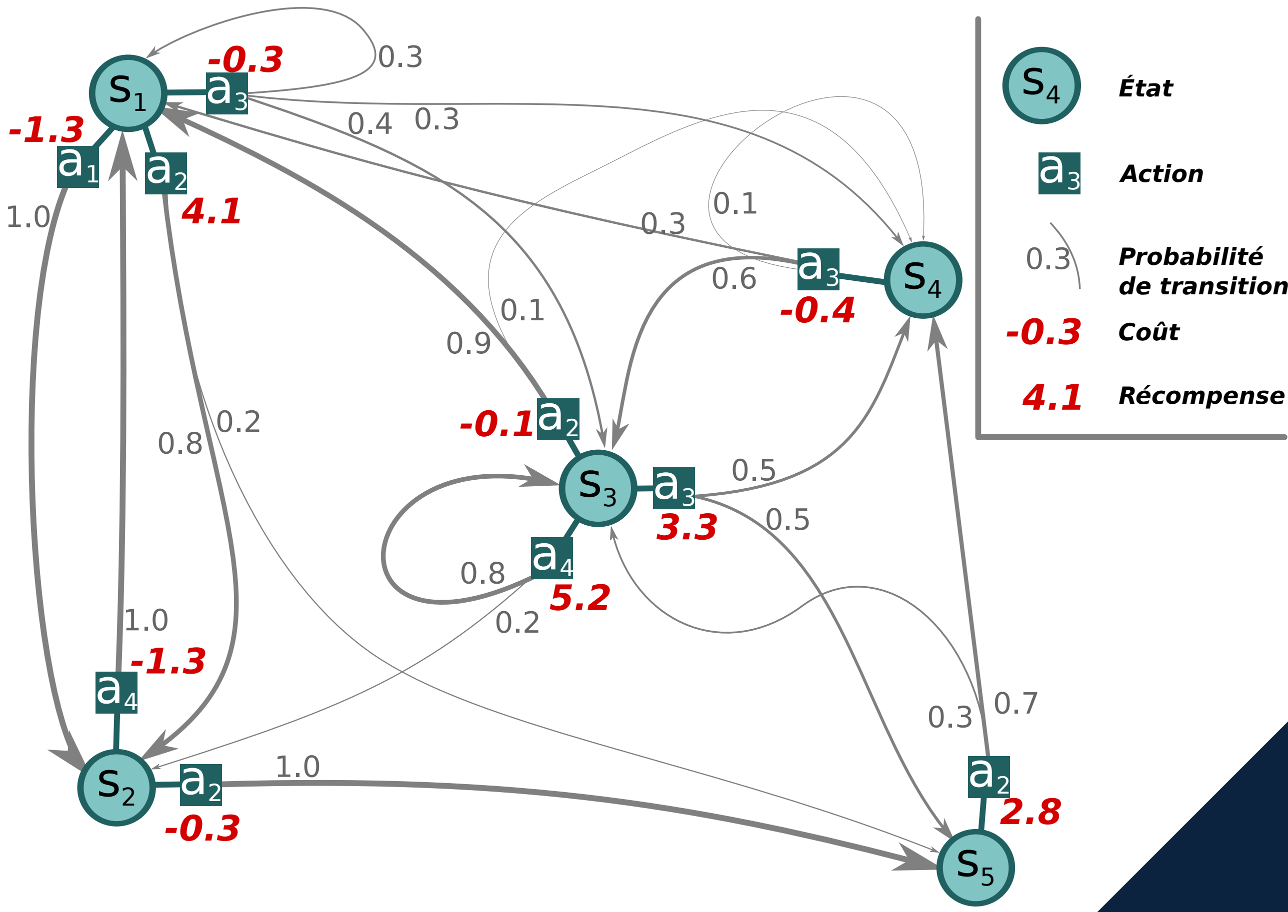
Hypotesis: it's Markovian

The system to control matches a Markov Decision Process

MDP: $\langle S, A, T, R \rangle$:

S : set of system's states
A : set of possible actions
T : S × A × S → [0, 1] : transitions
R : S × A → R : cost/rewards

We do have *S* and *A* but not *t* and *r*



Q-Learning: the basics

- Iterative update on (**state**, **action**) evaluation
- Q-Value equation:

$$Q(s^t, a) = (1-\alpha)Q(s^t, a) + \alpha\left(r + \gamma \max_{a' \in A} Q(s^{t+1}, a')\right)$$

- Few parameters:
  α learning rate ; ϵ Exploration-Exploitation ratio and γ discount factor.

Q-Learning: for instance

- Reaching *4-2-1* at *h-1* from *6-2-1* at *h-2* by doing *roll-keep-keep*.

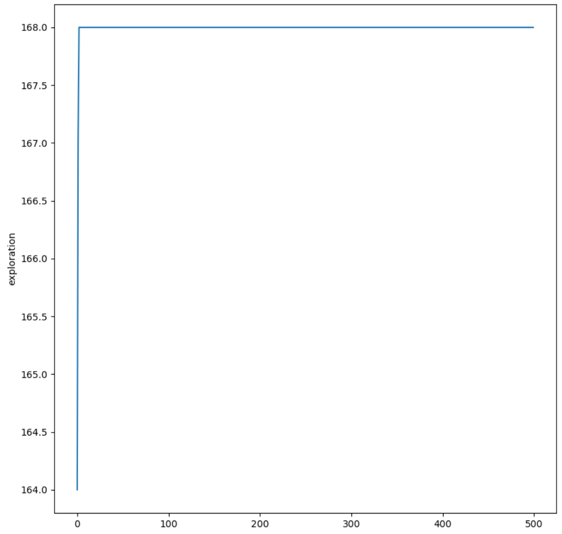
$$Q(\text{2-6-2-1},\ \text{r-k-k}) = (1-\alpha)Q(\text{2-6-2-1},\ \text{r-k-k}) + \alpha\left(r + \gamma \max_{a' \in A} Q(\text{1-4-2-1},\ a')\right)$$

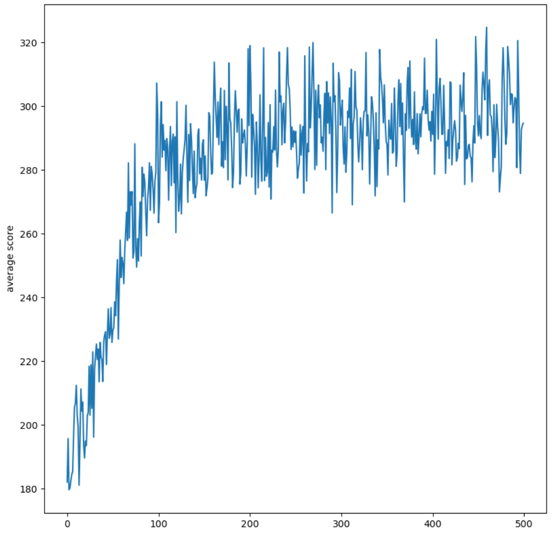
$$Q(\text{2-6-2-1},\ \text{r-k-k}) = (1-\alpha)\,40.0 + \alpha\,(0.0 + 80.0) \qquad (a' = \text{keep}^3)$$

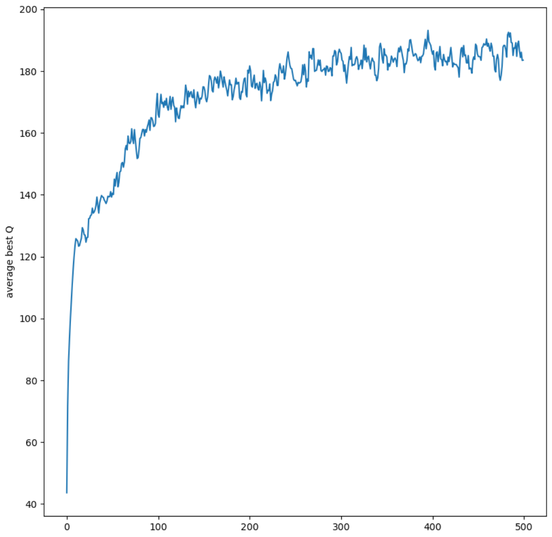
- With α learning rate at *0.1*, $Q(\text{2-6-2-1},\ \text{r-k-k})$ is now equals to *44*

Q-Learning: the basics

- With 500 steps of 500 games:







- α: 0.1 ; ϵ : 0.1 ; γ : 0.99 ;

Drawing plot in Python: pyplot

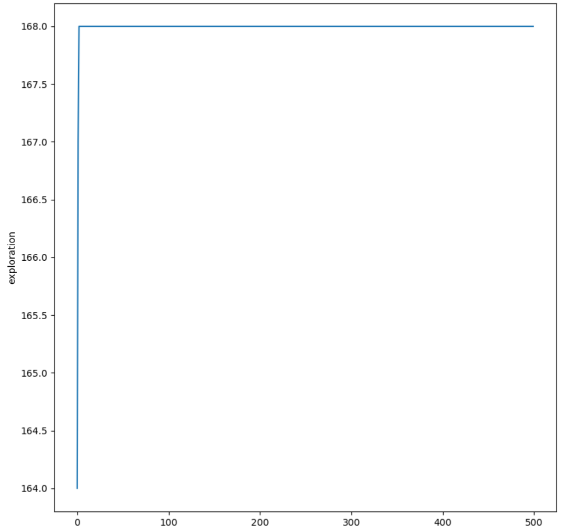
Codes:

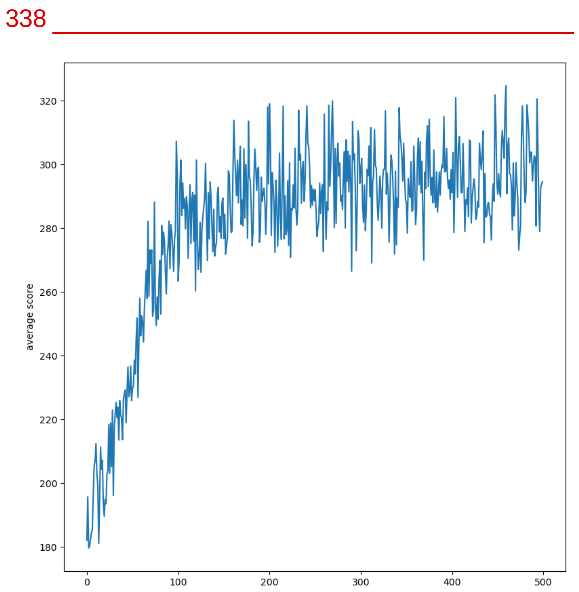
```
import matplotlib.pyplot as plt

...

plt.plot( values )
plt.ylabel( "mean of the y value" )
plt.show()
```

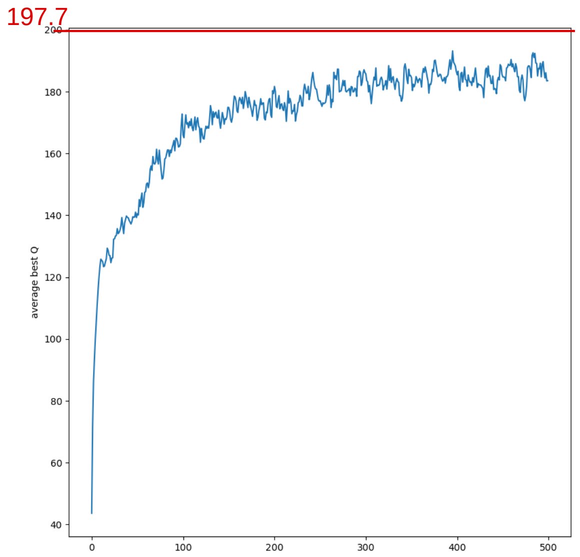
- Where `values` is a list of values in $\mathbb{R}$

Q-Learning: the basics

- With 500 steps of 500 games:



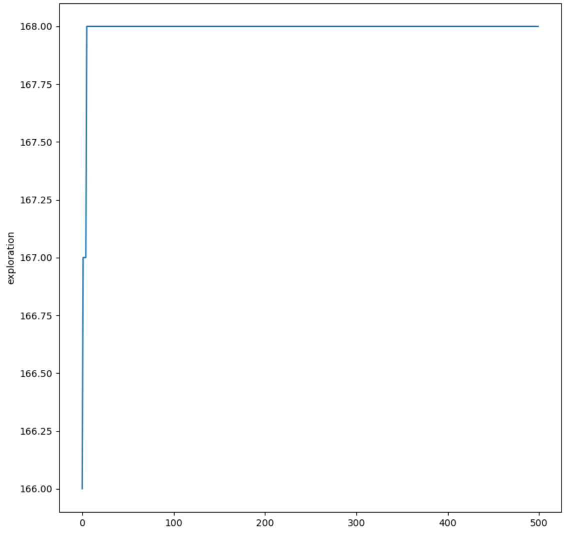


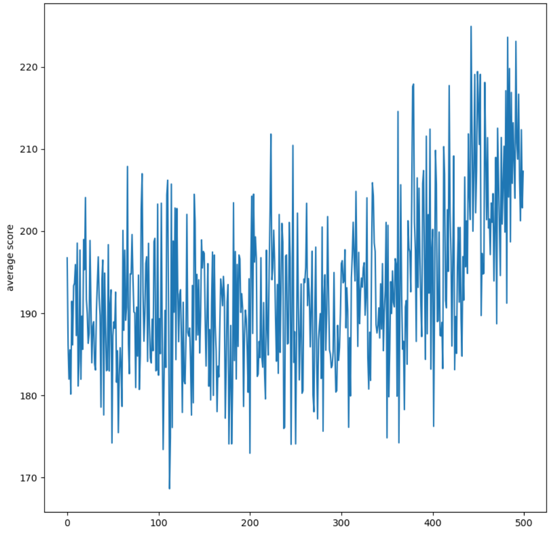


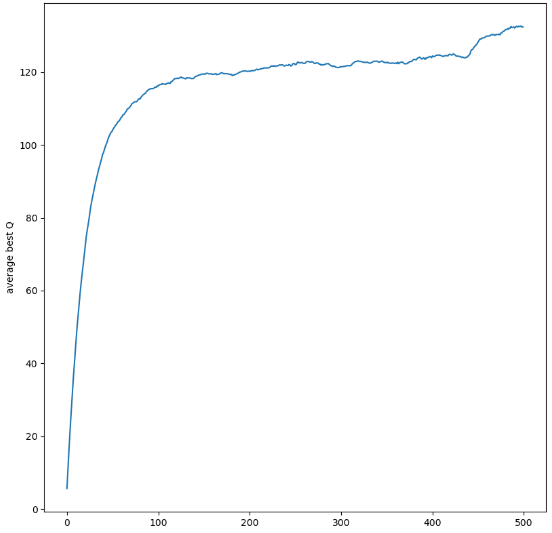
- With optimal threshold

Q-Learning: the basics

- With 500 steps of 500 games:



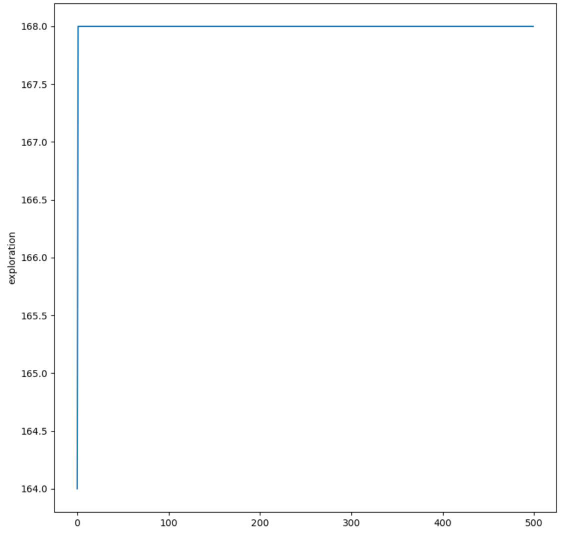


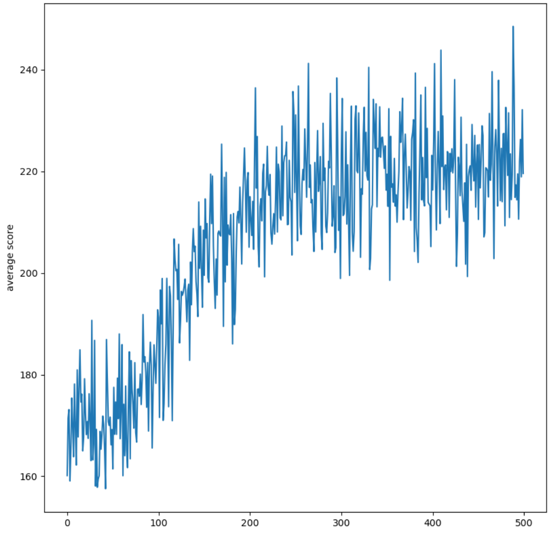


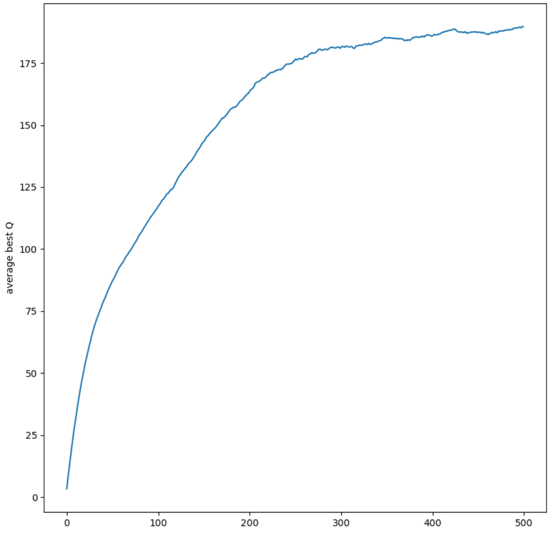
- α: 0.01 ; ϵ : 0.1 ; γ : 0.99 ;

Q-Learning: the basics

- With 500 steps of 500 games:







- α: 0.01 ; ϵ : 0.6 ; γ : 0.99 ;

Playing with the parameters:

- Generate rapidly "good" policies
- Converge on a maximal and stable Q-Values
  (an indicator for optimal policy)
- Potentially: be reactive to system modification (recovery)

Ideally: implement dynamic parameters

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Model-based learning

Main Idea:

- Random trajectories (a lot)
- Until each transition is visited several times.
- Compute an optimal policy.

Potentially:

- Require drived exploration to go in every 'niche'
- But generally: only incomplete exploration can be performed

But first the Model

Markov Decision Process

A framework for modeling stochastic evolution of the system to control.

Bellman equation

Recursive evaluation of states to compute expected gains.

Solving algorithms

- Value iteration
- Policy iteration

Markov Decision Process

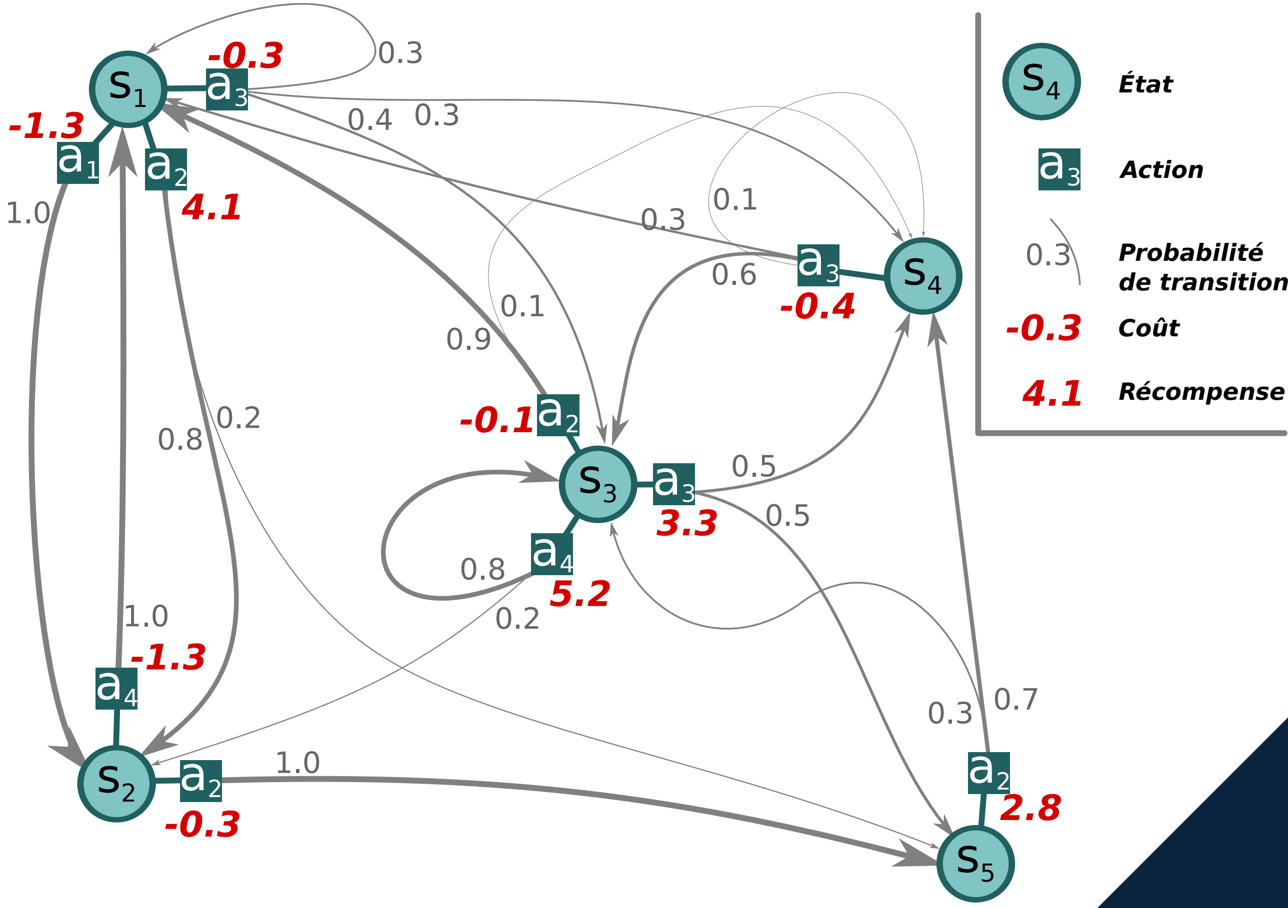
MDP: $\langle S, A, T, R \rangle$:

S : set of system's states
A : set of possible actions
T : S × A × S → [0, 1] : transitions
R : S × A → R : cost/rewards

Optimal policy:

π: a function returning the action to perform in each crossed states.

π^*: the optimal policy maximizing the gains (expected cumulated rewards).



Choosing : building a policy of action

Example of policy in 421:

π^{421}: Always target a 4-2-1 (keep only one **4**, one **2** and one **1**).

s	$\pi^{421}(s)$	s	$\pi^{421}(s)$
h-*1*-1-1	*keep*-roll-roll	…	
h-*2*-*1*-1	*keep*-*keep*-roll	h-*4*-*2*-*1*	*keep*-*keep*-*keep*
h-3-*1*-1	roll-*keep*-roll	…	
h-*4*-*1*-1	*keep*-*keep*-roll	h-6-6-5	roll-roll-roll
…		h-6-6-6	roll-roll-roll

(Invariant over the horizon h)

Bellman Equation

State evaluation for a given policy π :

$$V^{\pi}(s) = R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') \times V^{\pi}(s')$$

$$\text{with} : a = \pi(s) \text{ and } \gamma \in [0,1] \text{ the discount factor (typically } 0.99)$$

As a sum of gains:

- The immediate reward: $R(s,a)$.
- The future gains $V^{\pi}(s')$, for all possible next states $s' \in S$,
- proportionally to the probability to reach them $T(s,a,s')$

Solving MDP: Value Iteration

Input: an **MDP:** $\langle S, A, T, R \rangle$; precision error: ϵ ; discount factor: γ ; initial **V(s)**

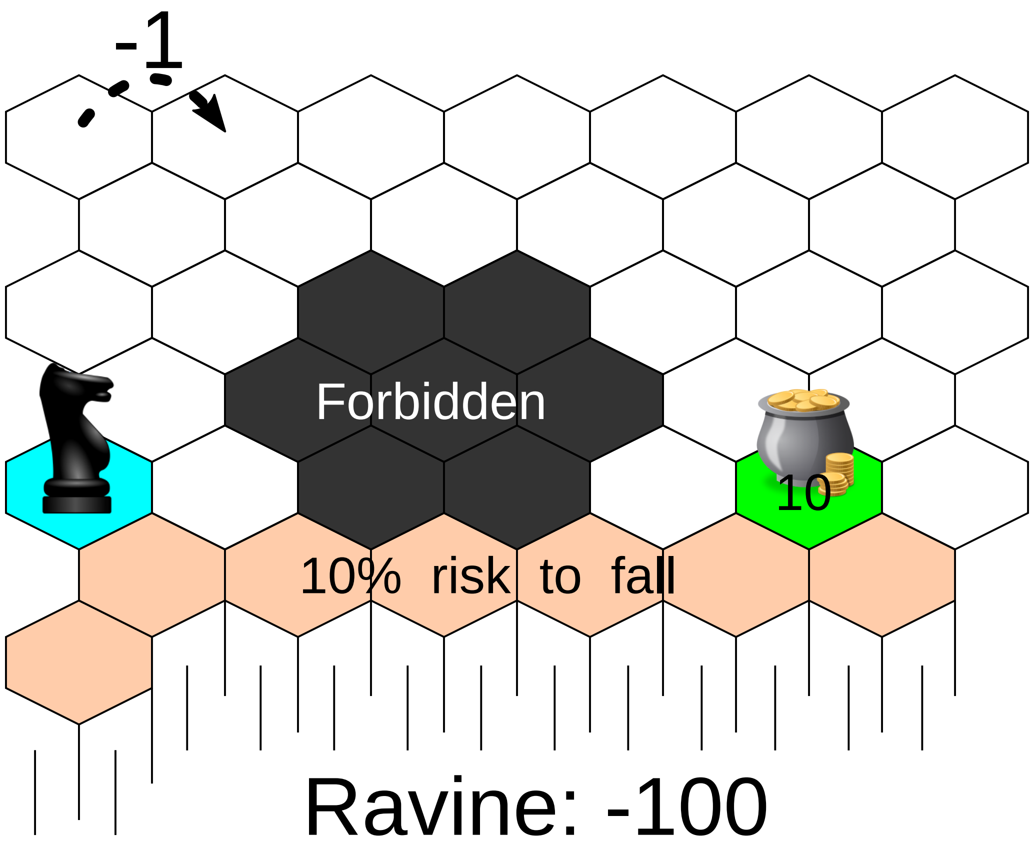
1. Repeat until the **maximal delta* $< \epsilon$

   For each state $s \in S$

   - Search the action a^* maximizing the Bellman Equation on s
   - Update $\pi(s)$ and **V()** by considering action a^*
   - Compute the delta value between the previous and the new **V(S)**

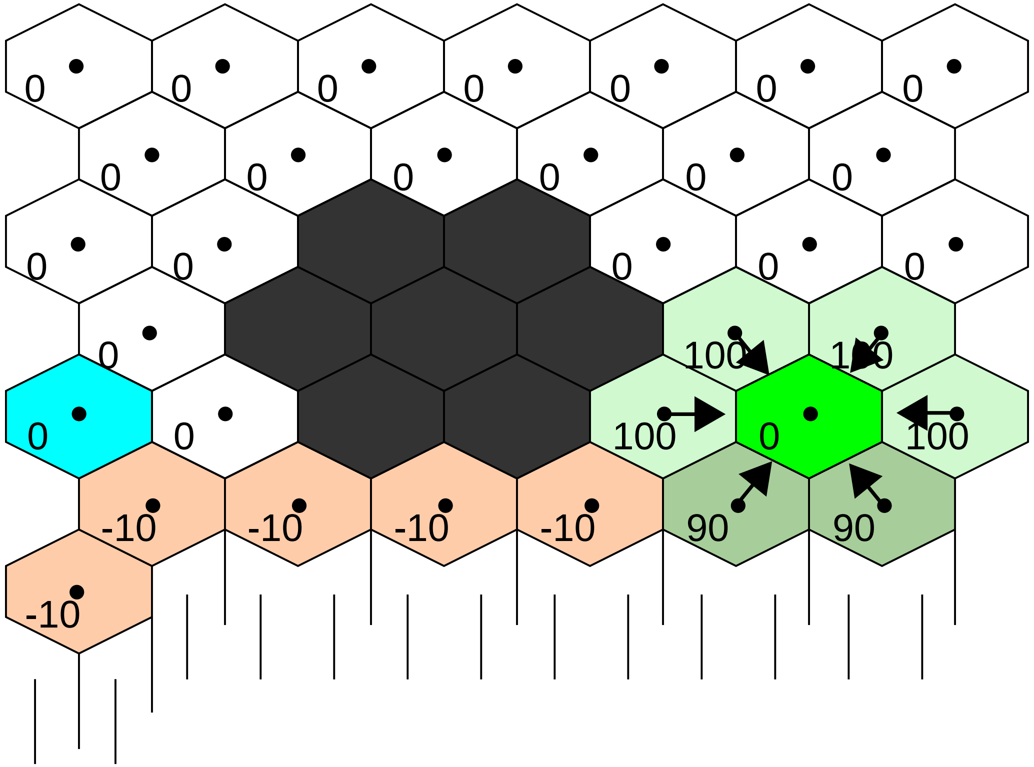
Output: an optimal π^* and associated V-values

Example: Dangerous move.



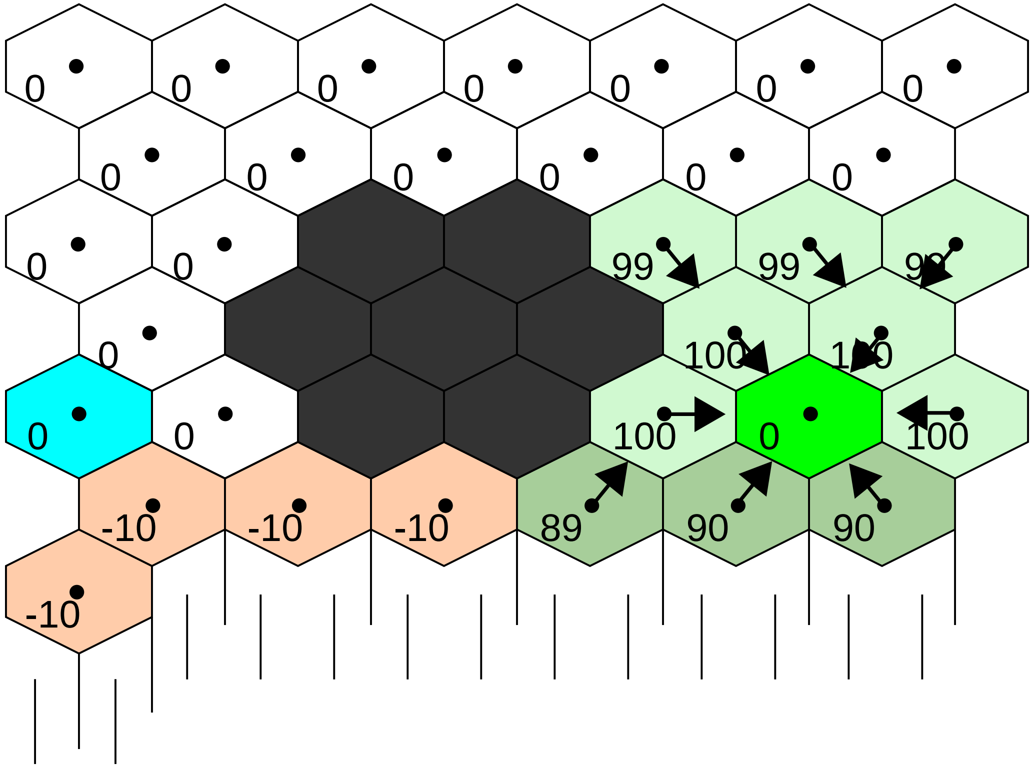
Probleme definition

Example: Dangerous move.



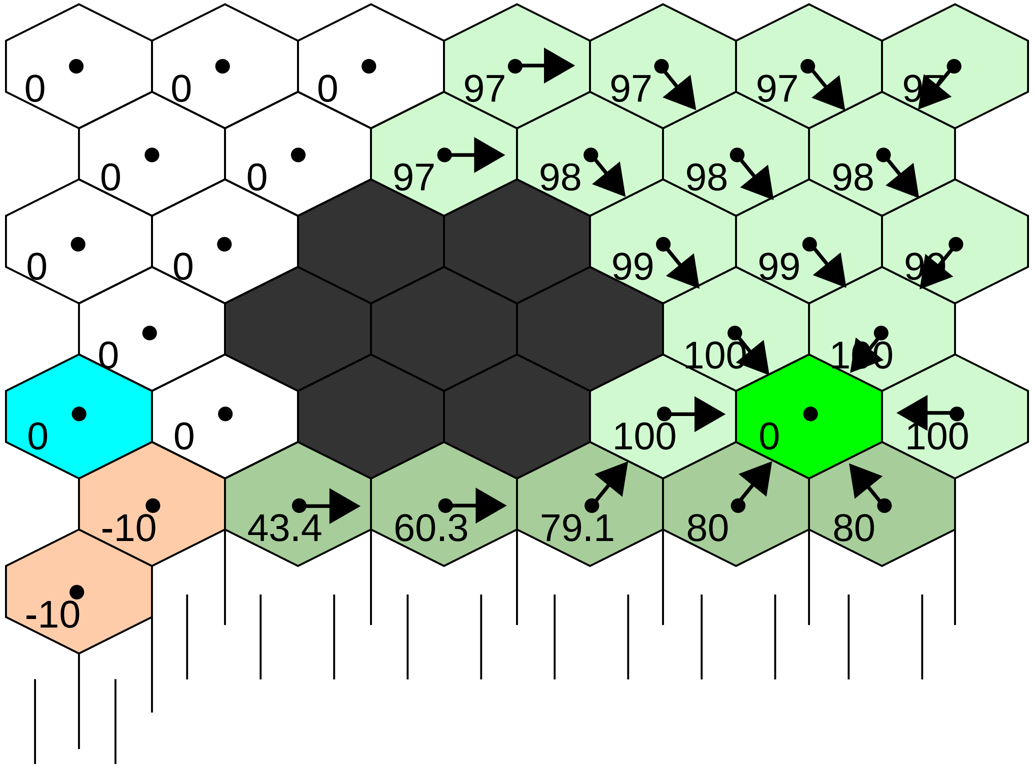
Value-Iteration: first iteration

Example: Dangerous move.



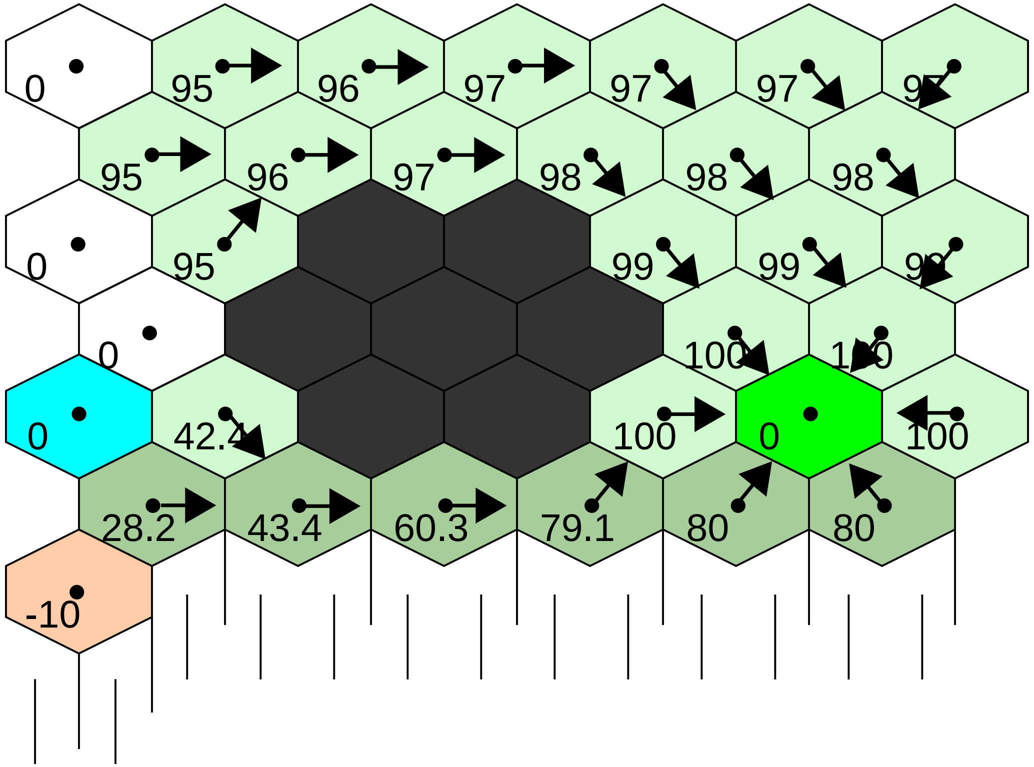
Value-Iteration: second iteration

Example: Dangerous move.



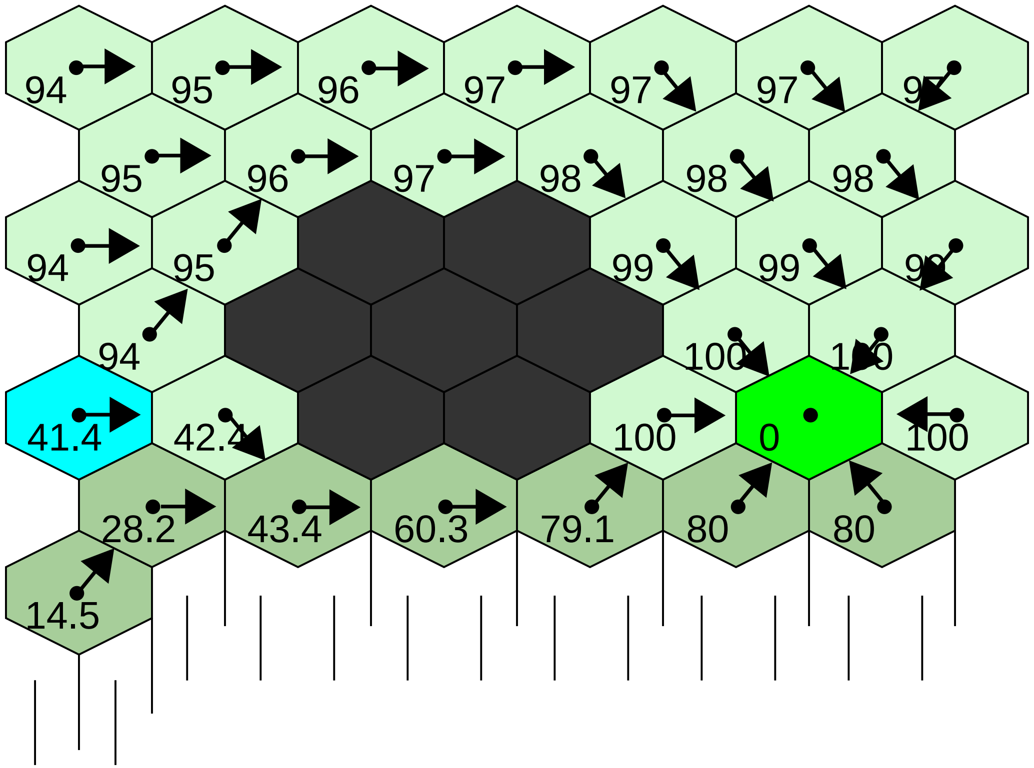
Value-Iteration: 4^{th} iteration

Example: Dangerous move.



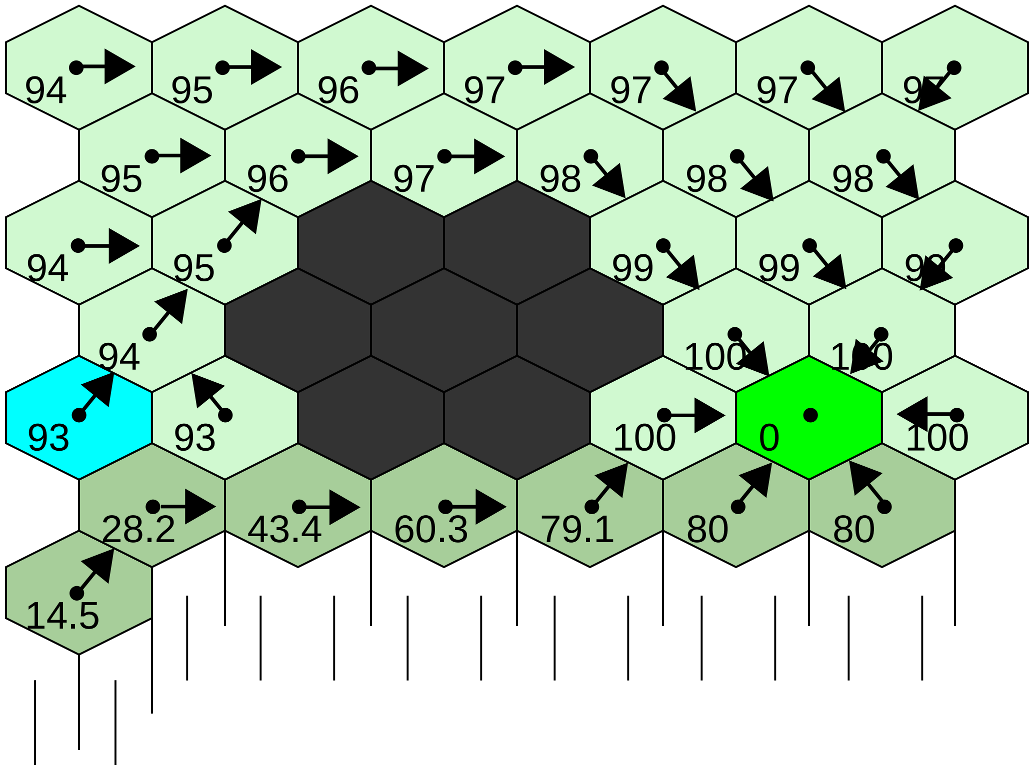
Value-Iteration: 6^{th} iteration

Example: Dangerous move.



Value-Iteration: 7^{th} iteration

Example: Dangerous move.



Value-Iteration: 8^{th} iteration

Solving MDP: Policy Iteration

Input: an **MDP:** $\langle S, A, T, R \rangle$; precision error: ϵ ; discount factor: γ ; initial **V(s)**

1. Compute $\pi(s)$ according to $V(s)$, for each state $s \in S$
2. Repeat until $\pi(s)$ is stable:
   - Update $V(s)$ with $\pi(s)$ at ϵ error, for each state $s \in S$
   - Update $\pi(s)$ according to $V(s)$, for each state $s \in S$

Output: an optimal π^* and associated V-values

Ok now learn the model...

- Define the state-space (small but covering).
- Define the action-space.
- Explore the system:
  - Compute the average rewards $R(s, a)$.
  - Compute all transition probability $T(s, a, s')$

Learn the transition

The transition function is the core object to learn.

It is a 3-dimension structure of floating point values (probabilities).

$$|S|^2 \times |A| \text{ values.}$$

A simple game as **421** with **168** states and **8** actions
would requires **225 792** values.

Luky for us, in application, most of the transitions are null (ie. imposible),
and it is possible to take advandages from structures in the systems mechanism.

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