Q-Learning

A classical method of Reinforcement Learning

Guillaume.Lozenguez

@imt-nord-europe.fr

IMT Nord Europe
École Mines-Télécom
IMT-Université de Lille

1. **A teoritical framework: Markov Decision Process**
2. **On the go, model free learning**
    - **Compute QValues**
    - **Choose an Action**
3. **Exercice**

Acting over a system evolving under uncertainty

- **States**: set of configurations defining the studied system
- **Action**: finite set of possible actions to perform
- **Transitions**: Describe the possible evolution of the system state

Transition function:

The probabilistic evolution depends on the performed action.

$$T : S \times A \times S \rightarrow [0, 1]$$

$T(s^t,\ a,\ s^{t+1})$ return the probability to reach s^{t+1} by doing a from s^t:

$$T(s^t,\ a,\ s^{t+1}) = P(s^{t+1} | s^t, a)$$

Transition in 421-game

- For instance, doing ***Keep-Kepp-Roll*** in ***4-2-2 (2)*** :
  - *6-4-2 (1)* = 1/6
  - *5-4-2 (1)* = 1/6
  - *4-4-2 (1)* = 1/6 Or $T\left(422(2),\ \text{k-k-r},\ 442(1)\right) = 1/6$
  - *4-3-2 (1)* = 1/6
  - *4-2-2 (1)* = 1/6
  - *4-2-1 (1)* = 1/6

- For instance, doing ***Keep-Kepp-Kepp*** in ***1-1-1 (2)*** : *1-1-1 (0)* = 1

Acting to optimize Gain

Require to evaluate the interest of each action on the system evolution:

- *Reward/Cost function* (R) :

$$R : S \times A \times S \to \mathbb{R}$$

$$R(s^t,\ a,\ s^{t+1}) \text{ is the reward by reaching } s^{t+1} \text{ from doing } a \text{ in } s^t$$

OR, in a simplified version:

$$R : S \times A \to \mathbb{R}$$

reward in 421-game

Over the final combination when the horizon reaches 0

$$
\begin{aligned}
score(\text{4-2-1}) &= 800\\
score(\text{1-1-1}) &= 700\\
score(\text{x-1-1}) &= 400 + x\\
score(\text{x-x-x}) &= 300 + x\\
score(\text{(x+2)-(x+1)-x}) &= 202 + x\\
score(\text{2-2-1}) &= 0\\
score(\text{x-x-y}) &= 100 + x\\
score(\text{y-x-x}) &= 100 + y
\end{aligned}
$$

Reward function: : $r(s, a, s') = score(s') \quad \text{if} \quad h = 0\ ; \quad 0 \quad \text{else}$

Acting to optimize gain (accumulated rewards)

- Our objective: *a policy* (π) : a function returning the action to perform considering the current state of the system:

$$\pi : S \to A$$
$$\pi(s) : \text{ the action to perform is } s$$

- *Bellman Equation* :

$$V^{\pi}(s) = R(s^t, a) + \gamma \sum_{s^{t+1} \in S} T(s^t,\ a,\ s^{t+1}) \times V^{\pi}(s^{t+1})$$

$$\text{with} : a = \pi(s) \text{ and } \gamma \in [0, 1[\text{ the discount factor (typically } 0.99)$$

Markov Decision Process

MDP: $\langle S, A, T, R \rangle$:

S : set of system's states

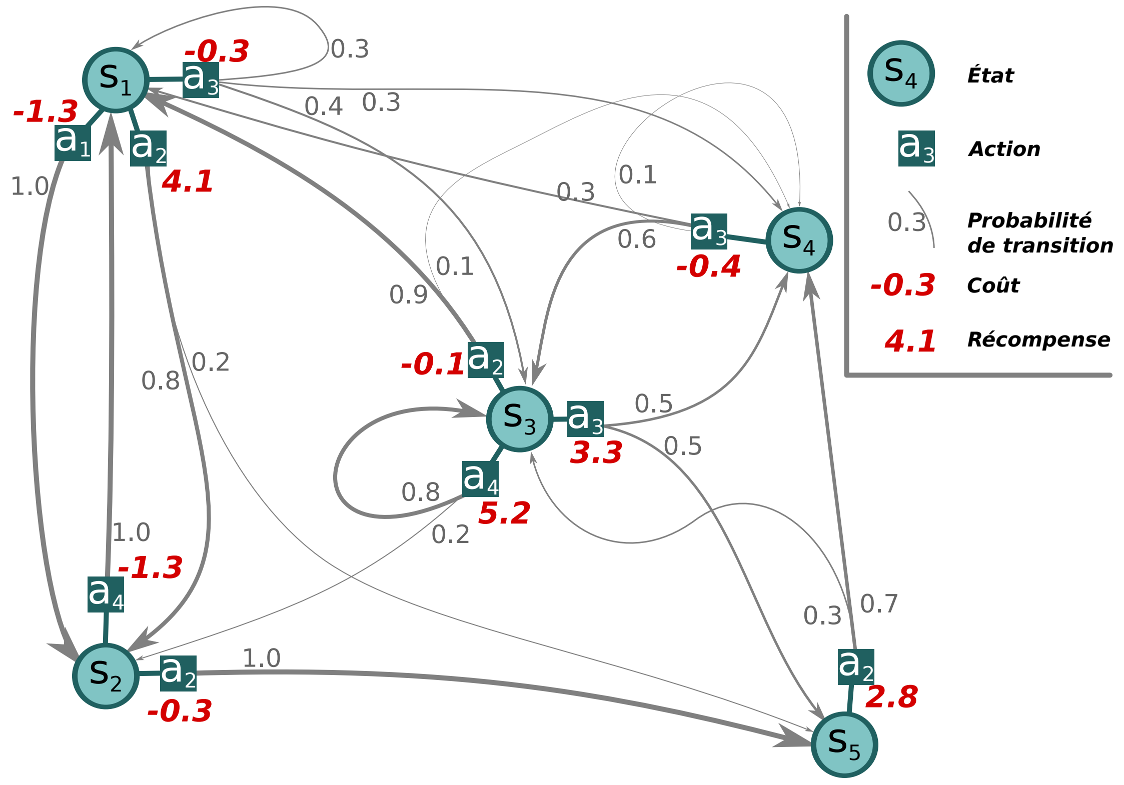
A : set of possible actions

T : S × A × S → [0, 1] : transitions

R : S × A → R : cost/rewards

Optimal policy:

The policy π^* maximizing Bellman



Reinforcement Learning:

Learn the optimal policy

- Without knowledge over the transition probabilities and/or the rewards,
- but, by getting feedback from acting randomly.

2 approaches

- **model-based:** Learn the model (T, R), and compute a policy.
- **model-free:** Learn the policy directly.

1. A teoritical framework: Markov Decision Process
2. **On the go, model free learning**
    - **Compute QValues**
    - Choose an Action
3. Exercice

Model-Free Approaches

Concept

- Learn without generating **transition** and **reward** models.
- Build the **policy** directly from the interactions
- Use only the experience of sequences:

$$state,\ action,\ reward,\ state,\ action,\ \ldots$$

Common approaches:

- **Q-learning**:
  continuous computing of an expected gain (require rich feedback)
- **Monte-Carlo**: use random explorations until a 'finale' state (slow to converge).

Q-learning

One of the core discoveries in Reinforcement Learning (simple and efficient)

- At each step, **Q-learning** updates the value attached to a couple (state, action)
- Updates are performed integrate expected future gains
- The update is performed accordingly to a learning rate $\alpha \in]0, 1[$
  $\rightarrow \alpha$: ratio between new vs old accumulated information.

Q-learning based on a Q function

Considering it is not possible to evaluate state without a policy yet

$$V^{\pi}(s) = R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') \times V^{\pi}(s')$$

the **Q-values** evaluate each action performed from each state:

$$Q : S \times A \to \mathbb{R}, \qquad Q(s,\ a) \text{ is the value of doing } a \text{ from } s$$

and, a **Q-value** is updated iteratively from succession of: $\langle s,\ a,\ s',\ r \rangle$

$$Q(s,a) = (1-\alpha)Q(s,a) + \alpha\left(r + \gamma Q(s',a')\right)$$

Q-learning : the algorithm

Input: state and action spaces: *A* ; a step engine *Perform* ;
exploration ratio: ϵ ; learning rate: α ; discount factor γ

1. Read the initial state s
2. Initialize $Q(s, a)$ to 0 for any action a
3. Repeat until convergence
   i. Select an action a with random
   ii. *Perform* a and read the reached state s' and the associated reward r
   iii. If necessary, add s' to Q (with value 0 for any action a)
   iv. Update $Q(s, a)$ accordingly to α and γ
   v. Set $s = s'$

Output: the **Q-values**.

Q-learning : the main equation

Update Q each time a tuple $\langle s^t, a, s^{t+1}, r \rangle$ is read

$$newQ(s, a) = (1 - \alpha)\, Q(s, a) + \alpha\, (\text{incomming-feedback})$$

$$\text{incomming-feedback} = r(s, a, s') + \gamma Q(s', a')$$

- α : the learning rate ($= 0.1$)
- γ : the discount factor ($= 0.999$)

The known optimal policy:

$$\pi^*(s) = \max_{a \in A} Q(s, a)$$

1. A teoritical framework: Markov Decision Process
2. **On the go, model free learning**
    - Compute QValues
    - **Choose an Action**
3. Exercice

Exploration–Exploitation tradeoff dilemma

The agent build an optimal behavior from trials and errors.

- *Exploration*
  - Try new actions to learn unknown feedback
  - Better understand the dynamics of the system
  - Risky output
- *Exploitation*
  - Use the best-known action
  - Potentially suboptimal

Exploration–Exploitation Tradeoff Dilemma

Examples:

- *Exploitation*: apply a known game strategy **vs** *Exploration* investigate new actions.
- *Exploitation*: go to your favorite restaurant **vs** *Exploration* try a new one.

Classical approach:

- Trigger exploration *when* the old fashion strategy doesn't work anymore
  Problems:
  - Determine that "a strategy doesn't work" ?
  - Determine that "a new policy is well defined" (exploration end) ?
- Continuously Explore and Exploite with a fixed ratio.
  - (take wrong decision periodically)

Continuous Exploration–Exploitation : ϵ-Greedy

A Simple heuristic for the Exploration–Exploitation Tradeoff Dilemma

- Random decision with:
  - a probability ϵ to choose a random action (exploration)
  - a probability $1 - \epsilon$ to choose the best-known action (exploitation)
- Classically ϵ is set to 0.1
- A ϵ-greedy agent behavior punctually takes off-policy action

Then the challenge consists in varying ϵ depending of the knowledge the agent has of the area he is interacting in.

Q-learning : the algorithm

Input: state and action spaces: *A* ; a step engine *Perform* ;
exploration ratio: ϵ ; learning rate: α ; discount factor γ

1. Read the initial state s
2. Initialize $Q(s, a)$ to 0 for any action a
3. Repeat until convergence
    i. **At ϵ random: get a random a *or* a maximizing $Q(s, a)$**
    ii. *Perform* a and read the reached state s' and the associated reward r
    iii. If necessary, add s' to Q (with value 0 for any action a)
    iv. Update $Q(s, a)$ accordingly to α and γ
    v. set $s = s'$

Output: the **Q-values**.

Q-learning : In Agent-Based Architecture

- As an initial step (**wakeUp**) :
  - Initialize Q
  - Initialize state and action variables ($s,\ a$).
- At each itereration (**perceive**):
  - Read the reached state s' and the associated reward r
  - If necessary, add s' to Q (with value 0 for any action a)
  - Update $Q(s, a)$ accordingly to α and γ
  - reccord $s =\ s'$
- Taking decisions (**decide**):
  - At ϵ random: get a random a *or* a maximizing $Q(s, a)$

1. A teoritical framework: Markov Decision Process
2. **On the go, model free learning**
    - Compute QValues
    - **Choose an Action**
3. Exercice

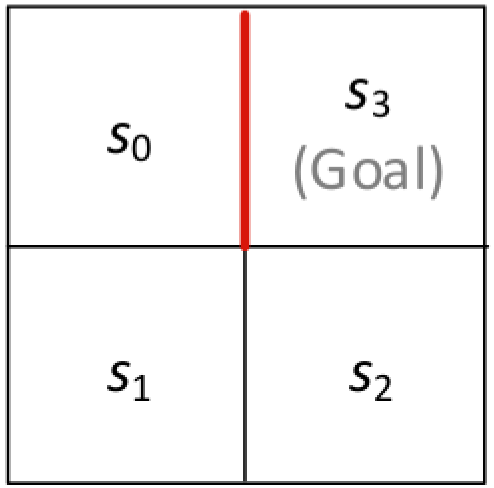
Exercice

Applying Q-Learning…

Simple Example

- **States**: 4 positions
  s_0, s_1, s_2 and s_3
- **Actions**: left, right, up, down
- **Transitions**: determinist
- **Rewards**:
  10 for reaching s_3, -1 else

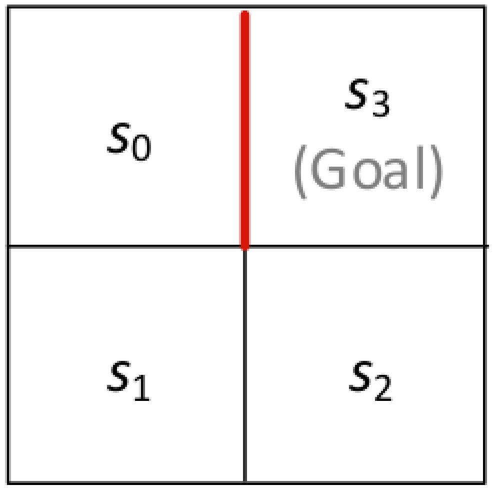
($\epsilon = 0.1$, $\alpha = 0.1$ and $\gamma = 0.99$)



($\alpha = 0.1$, $\epsilon = 0.1$ and $\gamma = 0.99$)

Simple Example

- From s_0 get action *left* (explore)
  reaches s_0 with -1
  updates $Q(s_0, \mathit{left}) = -0.1$
- s_0 gets *right* (best) $\rightarrow (s_0, -1)$
  updates $Q(s_0, \mathit{right}) = -0.1$
- s_0 gets *down* (exp.) $\rightarrow (s_1, -1)$
  updates $Q(s_0, \mathit{down}) = -0.1$
  ...
- s_2 gets *up* (exp.) $\rightarrow (s_3, 10)$
  updates $Q(s_2, \mathit{up}) = 1$
  End Episode



($\alpha = 0.1$, $\epsilon = 0.1$ and $\gamma = 0.99$)

Simple Example

($\alpha = 0.1$, $\epsilon = 0.1$ and $\gamma = 0.99$)

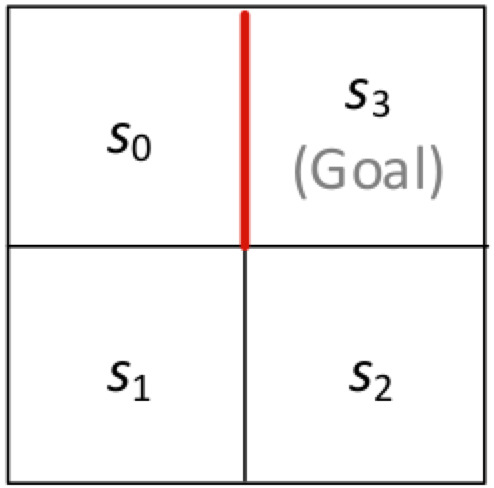
- **Episode 1:** (18 action)

S	s_0	s_1	s_2
$max\, Q$	-0.39	-0.19	1

- **Episode 2:** (15 action)

S	s_0	s_1	s_2
$max\, Q$	-0.43	0.9	1.9

...

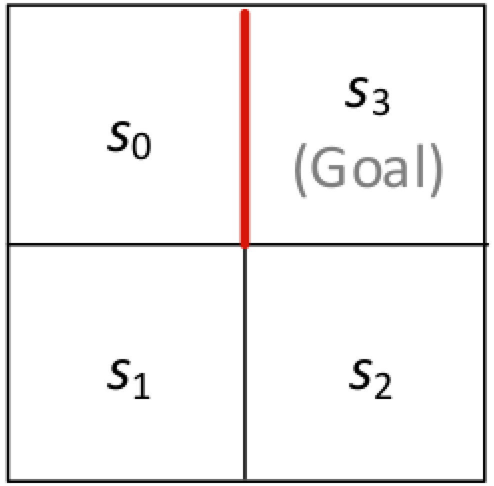


($\alpha = 0.1$, $\epsilon = 0.1$ and $\gamma = 0.99$)

Simple Example

- **Episode N**: (3-4 actions)

S	s_0	s_1	s_2
$max\, Q$	7.8	8.9	10
$argmax\, Q$	$\downarrow$	$\rightarrow$	$\uparrow$



($\alpha = 0.1$, $\epsilon = 0.1$ and $\gamma = 0.99$)

Exercice: Apply Q-Learning

Agent Version:

- On 421 game of hackagame

Classical version:

- On Lunar-Lander game of farama::gymnasium