1.3: Separable Equations

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0.1 Method

Def: An equation in the form $\frac{dy}{dx} = f(x)g(y)$ is called a **separable equation**.

We can put this equation in the form $\frac{1}{g(y)}\frac{dy}{dx} = f(x)$, then multiply over the dx to get $\frac{1}{g(y)}dy = f(x)dx$. Then, we can integrate on both sides to get a solution.

0.2 Proof

Generally, you can not "multiply" over dx because you can't treat a derivative in Leibniz notation as a fraction. However, what we did is a shortcut, and below is an equivalent, more well defined solution.

Proof: Suppose we have a separable equation in the form $g(y)\frac{dy}{dx} = f(x)$. Then, let $u = y(x)$, so $du = fracdydxdx$. If we perform integration on both sides of the separable equation, we get $\int g(y)\frac{dy}{dx}dx = \int f(x)dx$, and we can substitute u for y to get $\int g(u)du = \int f(x)dx$. This takes the same form as our shortcut above, so if we just rename the variables, so the shortcut and this well defined method are equivalent.

0.3 Implicit Solutions

Sometimes it is hard to separate out y from a given equation, so we just leave it as-is and call it an **implicit solution**.

0.4 Exercises

1.3.1 $\frac{dy}{dx} = \frac{x}{y}$
Solution: $\frac{dy}{dx} = x\frac{1}{y} \rightarrow \int ydy = \int xdx \rightarrow \frac{1}{2}y^2 = \frac{1}{2}x^2 \rightarrow y^2 = x^2$. This is the best we can do as $y^2 = x^2$ is not a function.