# 8.3: Matrices

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A linear operator which transforms an $N$ dimensional vector space with basis $\mathbf{e}_j$ into an $M$ dimensional vector space with basis $\mathbf{f}_i$ can be represented by an $M$ by $N$ matrix ($M$ rows, $N$ columns).

$$\mathcal{A}(\vec{r}) = \begin{bmatrix} A_{11} & A_{12} & A_{13} & ... & A_{1N} \\ A_{21} & A_{22} & A_{23} & ... & A_{2N} \\ A_{31} & A_{32} & A_{33} & ... & A_{3N} \\ ... & ... & ... & ... & ... \\ A_{M1} & A_{M2} & A_{M3} & ... & A_{MN} \end{bmatrix}$$

where $A_{ij}$ is the coefficient that transforms the $j$th basis vector in $\mathbf{e}_j$ into the $i$th basis vector component in $\mathbf{f}_i$.

If the dimensions the linear operator is transforming are the same, then the matrix is a square matrix.

> **Definition:** (Vectors as Matrices) We can write $N$ dimension vectors as $N$ by 1 matrices, in terms of their components $x_i$ with respect to a basis $\mathbf{e}_i$.
>
> $$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ ... \\ x_N \end{bmatrix}$$
>
> This type of matrix is called a **column matrix** or a **vector matrix**.
>
> Alternatively, the vector can be written as a 1 by $N$ transposed matrix:
>
> $$\vec{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & ... & x_N \end{bmatrix}^T$$