8.4: Elastic Collisions

Alex L.

October 22, 2024

Elastic collisions conserve both momentum and kinetic energy.

Proposition: There are two equations governing elastic collisions: The conservation of kinetic energy:

$$\frac{1}{2}m_A v_{A0}^2 + \frac{1}{2}m_B v_{B0}^2 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2$$

And the conservation of momentum:

$$m_A v_{A0} + m_B v_{B0} = m_A v_{A1} + m_B v_{B1}$$

Example: A neutron with mass 1.0 atomic mass units and velocity 2.7×10^7 collides with a carbon nucleus with mass 12.0 units at rest. Find the velocities after the collision. (1 amu $= 1.66 \times 10^{-27}$kg).

Solution: Let m_N be the mass of the neutron and m_C be the mass of the carbon nucleus. Then,

$$m_N v_{N0} + m_C v_{C0} = m_N v_{N1} + m_C v_{C1}$$

and substituting for the inital velocities, we get that

$$m_N v_{N0} = m_N v_{N1} + m_C v_{C1}$$

Doing the same for kinetic energy, we get that

$$m_N v_{N0}^2 = (m_N v_{N1}^2 + m_C v_{C1}^2)$$

Lets now put all the like terms for our energy equation on one side, getting

$$m_N(v_{N0}^2 - v_{N1}^2) = m_C v_{C1}^2$$

And applying difference of two squares, we get

$$m_N(v_{N0} - v_{N1})(v_{N0} + v_{N1}) = m_C v_{C1}^2$$

Lets rearrange the momentum equation in the same way:

$$m_N(v_{N0} - vN1) = m_C v_{C1}$$

Dividing the two prior equations, we get

$$v_{N0} + v_{N1} = v_{C1}$$

Now we substitute this into our momentum equation to get

$$m_N(v_{N0} - v_{N1}) = m_C(v_{N0} + v_{N1})$$

And solving for v_{N1}, we get

$$v_{N1} = \frac{v_{N0}(m_N - m_C)}{m_N + m_C}$$