Rudin Chapter 1: The Extended Real Number System

Alex L.

May 25, 2025

Definition: (The Extended Real Number System)

The **extended reals** is the reals with two extra elements, $-\infty$ and ∞, with the property that

$$-\infty < x < \infty$$

for any $x \in \mathbb{R}$

Definition: (Customary Definitions of the Reals)

1. If x is real then $x + \infty = \infty$ and $x - \infty = -\infty$ and $\frac{x}{\infty} = \frac{x}{-\infty} = 0$
2. If $x > 0$ then $x * \infty = \infty$ and $x * -\infty = -\infty$
3. If $x < 0$ then $x * \infty = -\infty$ and $x * -\infty = \infty$