3.4: De Moivre's Theorem

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October 4, 2024

Theorem: De Moivre's Theorem: $(\cos\theta + i\sin\theta)^n = (e^{i\theta})^n = e^{in\theta} = \cos n\theta + i\sin n\theta$

0.1 Roots of Unity

Def: The nth **roots of unity** are complex numbers z that fulfill the equation $z^n = 1$

To solve, rewrite the equation as $z^n = e^{ik2\pi}$ with $k \in \mathbb{Z}$ (any number with modulus 1 and argument as a multiple of 2π is equal to 1). Now, we take the nth root of each side to get $z = e^{\frac{ik2\pi}{n}}$, and you can find solutions by plugging in n and iterating over values for k.

0.2 Solving Polynomial Equations

Ex: Solve the equation $z^6 - z^5 + 4z^4 - 6z^3 + 2z^2 - 8z + 8 = 0$
Solution: We factorize to get $(z^3 - 2)(z^2 + 4)(z - 1) = 0$. This means that $z^3 = 2 = 2e^{ik2\pi}$. By taking the cube root of both sides, we get $z = 2^{\frac{1}{3}} e^{\frac{ik2\pi}{3}}$, and we find that we get three solutions, $z_1 = 2^{\frac{1}{3}}$ $z_2 = 2^{\frac{1}{3}}(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)$, and $z_3 = 2^{\frac{1}{3}}(-\frac{1}{2} - \frac{\sqrt{3}}{2}i)$. Paired with the other three solutions from our other terms, $z_4 = 2i$, $z_5 = -2i$, $z_6 = 1$, we have six solutions for a sixth order equation.