Classification of Differential Equations

Alex L.

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Def: An **ordinary differential equation** is an equation with only one independent variable.

Ex: $\frac{dy}{dx} = x^2$

Def: A **partial differential equation** is an equation with several independent variables

Ex: $\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = 0$

Def: If there are several simultaneous true differential equations, they are called a **system of equations**.

Ex: $y' = x \quad x' = y$

Ex: $\nabla \cdot \vec{D} = \rho, \nabla \cdot \vec{B} = 0, \nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}, \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Def: The **order** of an equation or a system is the number of the highest appearing derivative in that equation or system.

Ex: $a^4 \frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = 0$ is a fourth order PDE.

Def: An equation is **linear** when the dependent variable (or variables) and their derivatives are not multiplied together, and no other functions of them appear. They will appear in the form

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + ... + a_1(x)\frac{dy}{dx} + a_0(x)y = b(x)$$

The functions $a_0, a_1, ..., a_n$ are called the **coefficients**.

Ex: $\frac{dx}{dt} = x^2$ is nonlinear because the dependent variable, x, is multiplied with itself.

Ex: $\frac{dx}{dt} = \sin x$ is nonlinear because a function of x, $\sin x$, appears.

Def: A *homogeneous* differential equation is an equation where all terms depend on the dependent variable. There are no terms with only the independent variable or a function of it.

Ex: $\frac{dx}{dt} + x\sin t = 0$ is a homogeneous equation.

Ex: $\frac{dx}{dt} + x\sin t = 1$ is not a homogeneous equation. The 1 term does not depend on the dependent variable.

Def: A linear equation has **constant coefficients** if every coefficient is a constant.

Ex: $3\frac{dy}{dx} = 12x$

Def: An **autonomous** equation is a differential equation that depends only on the dependent variable. The independent variable never appears in the equation.

Ex: $\frac{dy}{dx} = f(y)$ is a general first order autonomous equation.