6.3: Work and Energy with Varying Forces

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Work is the integral of the x component of force with respect to x:

$$W = \int F_x dx$$

Def: The force required to stretch a string past its resting position is given by $F = kx$, where k is the **spring constant**. The force increases linearly with stretch distance. The work required to stretch a string to a given position is given by $W = \frac{1}{2}kx^2$.

Ex: Work Done by a Spring Scale:

A 600 N woman steps on a spring scale, and the spring is compressed by 1 cm. Find the spring constant and the total work done during compression.

Solution: The spring and the woman are at equilibrium after the 1 cm compression, so the spring constant is 600 N/cm. The total work done is $\frac{1}{2} * 600 * 1^2 = 300$ centijoules, or 3 joules of work.

Work along a curve is given by

$$W = \int \vec{F} \cdot \vec{dl} = \int F \cos\theta dl = \int F_{||} dl$$

where $\vec{dl}$ is the vector tangent to the curve and $F_{||}$ is the component of $\vec{F}$ parallel to the curve.