3.2: Manipulation of Complex Numbers

Alex L.

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0.1 Addition and Subtraction

To add or subtract complex numbers, just add or subtract like terms.

$z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$

The addition of complex numbers is commutative and associative

0.2 Modulus and Argument

Def: The **modulus** of a complex number is the distance from the origin in the Argand diagram and is denoted

$$|z| = \sqrt{a^2 + b^2}$$

Def: The **argument** of a complex number is the angle the complex number makes from the positive x-axis on an Argand diagram, and is denoted

$$\arg z = \arctan(\frac{b}{a})$$

Remember, if the complex number is in quadrant II or III, the angle the arctan function gives is in relation to the negative x-axis, and you will have to add π radians to correct for it.

0.3 Multiplication

The product of two complex numbers is the same as multiplying two binomials, remember, $i \cdot i = -1$.

$$z_1 \cdot z_2 = (a_1 + ib_1)(a_2 + ib_2) = a_1a_2 + ia_1b_2 + ia_2b_1 - b_1b_2$$

Multiplying a complex number by i rotates it around the origin by $\frac{\pi}{2}$ radians anticlockwise.

0.4 Complex Conjugate

Def: If $z = a + bi$, then the **comlex conjugate** of z is denoted

$$z^* = a - bi$$

A complex conjugate is a reflection of a complex number about the real axis in an Argand diagram, and multiplying a complex number with its complex conjugate leaves a real number with no imaginary component.

0.5 Division

The division of two complex numbers is

$$\frac{z_1}{z_2} = \frac{a_1 + ib_1}{a_2 + ib_2}$$

and we multiply both the top and the bottom by the complex conjugate of the denominator to get

$$\frac{z_1}{z_2} = \frac{(a_1a_2 + b_1b_2) + i(a_2b_1 - a_1b_2)}{a_2^2 + b_2^2}$$