4.5: Power Series

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Def: Power series are series in the form

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$$

The ratio test shows us that if

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}x| < 1 \to |x| \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| < 1$$

the series converges.

As you can see, the convergence depends on x, so we will often get an **interval of convergence** where $P(x)$ converges.

0.1 Operations

If $P(x)$ and $Q(x)$ are power series, then the interval of convergence of $P(x) \pm Q(x)$ is the overlap of the two intervals of convergence.

If two power series converge for all values x, you may substitute one into the other to get another series that converges for all values x. For example, e^x and $\sin x$ both converge for all x, and so does $e^{\sin x}$.

The derivative or integral of a power series also converges in the same range.