4.4: Expected Values of Functions of Random Variables

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1 Motivation & Goals:

Suppose I have a discrete random variable X, and a function $g(X) = Y$ transforming that random variable into another random variable Y? What is the expected value of Y?

2 Content

Proposition: If X is a discrete random variable with values x_i, and with respective probabilities $p(x_i)$, then $E(g(X))$, the expected value of the transformed variable, can be calculated by $\sum_i g(x_i)p(x_i)$.

Proof: The events in the original variable still occur in the same proportions, so $g(x_i)$ occurs just as frequently as x_i, as such, to get the expected value for g_X, we just take the expected value of $X = \sum_i x_i p(x_i)$ and swap x_i for $g(x_i)$ to get $E(g(X)) = \sum_i g(x_i)p(x_i)$

Example: (4a) Let X denote a random variable with $P(\{X = -1\}) = .2$ $P(\{X = 0\}) = .5$ $P(\{X = 1\}) = 1$. What is $E(X^2)$?

Solution:

$$E(X^2) = -1^2 P(\{X = -1\}) + 0^2 P(\{X = 0\}) + 1^2 P(\{X = 1\}) = 1 * .2 + .5 + 0 + 1 * .3 = .5$$