2.3: Higher Order Linear ODEs

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Theorem: Superposition: If $y_1, y_2, y_3, ..., y_n$ are linearly independent solutions of an equation, then $y = C_1y_1 + C_2y_2 + C_3y_3 + ... + C_ny_n$.

Def: Linear independence of multiple functions $y_1, y_2, ..., y_n$, is when there is only one soluttion to the equation $c_1y_1 + c_2y_2 + c_3y_3 + ... + c_ny_n = 0$, the trivial solution, where $[c_1, c_2, c_3, ..., c_n] = 0$

0.1 Constant Coefficient Higher ODEs

The process for solving constant coefficient higher odes is the same. Just let $y = e^{rx}$, and substitute all of the derivatvies. An nth degree ODE will give you an nth degree characteristic equation. If you have repeated roots, the first root will be e^{rx}, the second root will be xe^{rx}, the third root will be x^2e^{rx}, and so on.

0.2 The Wronskian

Def: The **Wronskian** is a method of determining linear independence of a bunch of equations.

Given equations $x_1, x_2, x_3, x_4, ..., x_n$, the Wronskian is equal to

$$det \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & ... & x_n \\ x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & x_4^{(1)} & ... & x_n^{(1)} \\ ... & ... & ... & & & \\ x_1^{(n-1)} & x_2^{(n-1)} & x_3^{(n-1)} & x_4^{(n-1)} & ... & x_n^{(n-1)} \end{bmatrix}$$

where each column is a function and $n-1$ of its derivatives. Since derivation is a linear function, if the Wronskian is nonzero, then the entire system is linearly independent.