1.1: Integrals as Solutions

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Def: The general form of a first order ordinary differential equation is $\frac{dy}{dx} = f(x, y)$

Technique: When a first order ODE takes the form $\frac{dy}{dx} = f(x)$, we can integrate both sides with respect to x to get $y(x) = \int f(x)dx + C$. $y(x)$ is the general solution for the ODE.

Technique: If we are given an **initial value problem** (IVP) with starting values $y(x_0) = y_0$, we can directly solve for the particular solution by adding a lower bound of integration x_0 and an upper bound of integration x, and adding y_0 outside the integral. The particular solution ca be found by the formula $y(x) = \int_{x_0}^{x} f(x)dx + y_0 = (f(x) - f(x_0)) + y_0$

Technique: If we are given a first order ODE in the fomr $\frac{dy}{dx} = f(y)$, we can swap the roles of the dependent and independent variable by taking the reciprocal of both sides. We get $\frac{dx}{dy} = \frac{1}{f(y)}$. By integrating, we get $x(y) = \int \frac{1}{f(y)}dy + C$. Then, simply rewrite the resulting equation in terms of x. Keep in mind that this change of variables only works if the function $f(y)$ is invertible, meaning there exists a well defined inverse $f^{-1}(y)$.

Exercises:

1.1.2. Solve $\frac{dy}{dx} = x^2 + x$ for $y(1) = 3$
Solution: $y(x) = \int_1^x x^2 + x \; dx + 3 = (\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{6}) + 3 = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{13}{6}$