8.7: Complex and Hermitian Conjugate

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Definition: (Complex Conjugate of a Matrix) The **complex conjugate** of a matrix is found by taking the complex conjugate of each component. It is denoted A^*.

If A is real, then $A^* = A$

Definition: (Hermitian Conjugate of a Matrix) The **hermitian conjugate** of a matrix is found by taking the complex conjugate of the transpose of a matrix, and is denoted $A^\dagger$

$A^\dagger = (A^*)^T = (A^T)^*$

If A is real, then $A^\dagger = A^T$

The inner product of two vectors in an orthonormal basis, $\langle a|b\rangle = a^\dagger b$, or the matrix product of the hermitian conjugate of a times b.

If the basis is not orthonormal, then $\langle a|b\rangle = a^\dagger G b$, where G is the matrix with components $G_{ij} = \langle \mathbf{e}_i|\mathbf{e}_j\rangle$