# 3.6: Applications to Differentiation and Integration

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## 0.1 Differentiation

Suppose we want to find $\frac{d}{dx}(e^{3x}\cos 4x)$. This is hard to do by itself, but we can bake this into the real component of a complex number to make it easier. Take the complex number $z = e^{3x}(\cos 4x + i\sin 4x)$. This number is equal to $e^{3x}e^{4ix}$ by De Moivre's Theorem, and by rearranging we get $e^{(3+4i)x}$. Differentiating, we get $(3+4i)e^{(3+4i)x} = (3+4i)e^{3x}(\cos 4x+i\sin 4x)$ We multiply through, then keep only the real component, and get $e^{3x}(3\cos 4x - 4\sin 4x)$. By keeping the imaginary parts, we get the derivative for $e^{3x}\sin 4x$ as well!

## 0.2 Integration

Suppose we want to find $\int e^{ax}\cos bxdx$. Lets put this as a real component of a complex number $e^{ax}(\cos bx + i\sin bx) = e^{ax}e^{ibx} = e^{a+ib}x$. Integrating, we get $\int e^{(a+ib)x} = \frac{e^{(a+ib)x}}{a+ib} + c = \frac{(a-ib)e^{(a+ib)x}}{(a+ib)(a-ib)} + c = \frac{e^{ax}}{a^2+b^2}(ae^{ibx} - ibe^{ibx}) + c$. Taking the real component, we get $\frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c_1$