Simple Functions and Equations

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Def: A **polynomial equation** is an expression in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0 = 0$$

and the values of x that satisfy this equation are called the **roots** of $f(x)$. n is an integer that is greater than zero and is called the **degree** of the polynomial, and $a_0, a_1 ... a_n$ are called **coefficients**, and are real numbers with $a_n \neq 0$.

For polynomials with degree > 4, there is not a general method for finding roots of polynomials, and the methods for polynomials of degree 3 and 4 are so complicated that its better to use an approximation most of the time.

Def: The general formula for a polynomial of degree one is:

$$a_1 x + a_0 = 0$$

and the formula for finding a root α_1 is $\alpha_1 = -a_0/a_1$.

Def: The general formula for a polynomial of degree two is:

$$a_2 x^2 + a_1 x + a_0 = 0$$

and the formula for finding roots α_1, α_2 is

$$\alpha_{1,2} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2 a_0}}{2a_2}$$

and is called the **quadratic formula**.

Def: The **discriminant** $b^2 - 4ac$ determines what form the roots will take. If the discriminant is positive, both roots are real and the quadratic will cross the x-axis in two points. If both are negative, both roots are complex and the curve never crosses the x-axis, and if the value is zero, both roots are equal and the curve touches the x-axis in exactly one location.

Theorem: The fundamental theorem of algebra: a nth degree polynomial has n roots, though they may be real or complex.