8.9: Determinant of a Matrix

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Definition: (Minors) The **minor** of an element A_{ij} in a matrix is the determinant of the matrix if you removed the ith row and the jth column from the matrix. It is denoted M_{ij}

Definition: (Cofactor) The **cofactor** of an element A_{ij} in a matrix is found by multiplying the minor of A_{ij} by $(-1)^{i+j}$. It is denoted C_{ij}

Definition: (Determinant) The **determinant** of a matrix is found by multiplying all cofactors of a single row or column in a matrix by their corresponding elements, then adding them all up. The determinant of a matrix A is denoted $\det A = |A| = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}$ but this can be extended to any n by n square matrix.

For example, for an m by m matrix, the determinant could be found by taking row 1, and doing the operation above to get

$$A_{11}C_{11} + A_{21}C_{21} + A_{31}C_{31} + ... + A_{m1}C_{m1}$$

Proposition: If the rows of a 3 by 3 matrix are the vectors a, b, and c, then the determinant of the matrix is $a \cdot (b \times c)$.

0.1 Properties of Determinants

Proposition:

1. $|A^T| = |A|$
2. $|A^\dagger| = |(A^*)^T| = |A^*| = |A|^*$
3. If we swap two rows or columns in a matrix, the determinant doesn't change in magnitude, but changes in sign.
4. If all elemenst in a row or column in matrix A have a common factor λ, and that factor is removed, then the corresponding determinant is $\frac{|A|}{\lambda}$
5. If two rows or columns are identical, then $|A| = 0$
6. The determinant of a matrix is unchanged if we add a multiple of one row or column to another row or column.
7. If A and B are square matrices, then $|AB| = |A||B| = |BA|$