3.3: The Isomorphism Theorems

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Theorem: The First Isomorphism Theorem: If $\varphi : G \to H$ is a homomrophism of groups, then $\ker\varphi \trianglelefteq G$ and $G/\ker\varphi \simeq \varphi G$

Corrolary 17: Let $\varphi : G \to H$ be a homomorphism of groups.

1. φ is injective iff $\ker\varphi = 1$
2. $|G : \ker\varphi| = |\varphi(G)|$

Theorem: The Second Isomorphism Theorem Let G be a group, let A and B be subgroups of G and assume $A \leq N_G(B)$. Then, AB is a subgroup of G, $B \trianglelefteq AB$, $A \cap B \trianglelefteq A$, and $AB/B \simeq A/(A \cap B)$. (Remember that $N_G(A)$ is the set of elements in G that commute with all elements in A)

Proof: Note: all elements of A do normalize B. Then, by definition, aba^{-1} is in B. Therefore, $a(aba^{-1})$ is in AB.