3.7: Hyerbolic Functions

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0.1 Definitions

Def: $\cosh x = \frac{1}{2}(e^x + e^{-x})$

Def: $\sinh x = \frac{1}{2}(e^x - e^{-x})$

Def: $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Def: $\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$

Def: $\operatorname{csch} x = \frac{2}{e^x - e^{-x}}$

Def: $\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

0.2 Identities

Imaginary Identities:

$\cosh x = \cos ix$

$i \sinh x = \sin ix$

$\cos x = \cosh ix$

$i \sin x = \sinh ix$

Pythagorean Identities:

$\operatorname{sech}^2 x = 1 - \tanh^2 x$

$\operatorname{csch}^2 x = \coth^2 x - 1$

$\sinh 2x = 2 \sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

0.3 Calculus

$\frac{d}{dx} \sinh x = \cosh x$

$\frac{d}{dx} \cosh x = \sinh x$

$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$

$\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$

$\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

$\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$

$\frac{d}{dx} \cosh^{-1} \frac{x}{a} = \frac{1}{\sqrt{x^2 - a^2}}$

$\frac{d}{dx} \sinh^{-1} \frac{x}{a} = \frac{1}{\sqrt{x^2 + a^2}}$

$\frac{d}{dx} \tanh^{-1} \frac{x}{a} = \frac{a}{a^2 - x^2}$ for $x^2 < a^2$

$\frac{d}{dx} \coth^{-1} \frac{x}{a} = \frac{-a}{x^2 - a^2}$ for $x^2 > a^2$