1.2: Slope Fields

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0.1 Slope Fields

A general first order ODE looks like $\frac{dy}{dx} = f(x, y)$. By plugging in values for x, y, $f(x, y)$ tells you the slope of $y(x)$ at every point in the plane. If we draw a line with the slope that $f(x, y)$ gives us at every point in the plane, we have made a slope field.

To find a particular solution to a slope field, just pick a starting point, and draw a curve that is tangent to every line you encounter on the slope field.

0.2 Existence and Uniqueness

Def: A solution **exists** if there is a valid solution at a given point.

Def: A solution is **unique** if there is only one solution at that point.

Theorem: Picard's Theorem on Existence and Uniqueness: If $\frac{dy}{dx} = f(x, y)$ is defined at (x_0, y_0) and $\frac{\partial f}{\partial y}$ is also defined at (x_0, y_0), then there exists a unique solution at and around (x_0, y_0).