0.2: Introduction to Differential Equations

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Def: A **differential equation** is an equation with one or more derivatives in it.

Ex: $\frac{dx}{dt} + x = 2\cos t$

In the above scenario, t is the independent variable and x is the dependent variable.

Def: A **solution** to a differential equation is a function that satisfies the equation.

Ex: We claim that $x = x(t) = \cos t + \sin t$ is a solution for $\frac{dx}{dt} + x = 2\cos t$.

To verify, we plug x into the equation. Then, $\frac{dx}{dt} = \cos t - \sin t$, and the entire equation evaluates to $\cos t - \sin t + \cos t + \sin t = 2\cos t$ which is true.

Ex: We can also claim that $x(t) = \cos t + \sin t + e^{-t}$ is a solution. Then, $\frac{dx}{dt} = -\sin t + \cos t - e^{-t}$ and the entire equation evaluates to $\cos t - \sin t - e^{-t} + \cos t + \sin t + e^{-t} = 2\cos t$ which is true.

As it turns out, solutions to this equation take the form $x(t) = \cos t + \sin t + Ce^{-t}$ for a constant C.

There can be multiple solutions to a differential equation.

Def: A **general solution** to a differential equation is a set of equations which includes all possible solutions, and often has variables that can be changed. A **particular solution** is a solution to a differential equation that often satisfies a set of starting conditions, like $x(0) = 0$.