1.4: Partial Fractions

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0.1 Method

Def: Partial fractions is a way to break apart a complex fraction into its constitutent parts, which is useful for integration.

The behavior of a polynomial fraction is determined by the location of the zeroes in its denominator, so we will try to emulate the location of these zeroes.

We can write a polynomial fraction in the form

$$f(x) = \frac{A_1}{(x-\alpha_1)_1^n} + \frac{A_2}{(x-\alpha_2)_2^n} + ...$$

We can then set $\alpha_1, \alpha_2...$ to be the zeroes of the fraction.

Ex: Write $\frac{4x+2}{x^2+3x+2}$ as a partial fraction.
Solution: This fraction has two zeroes so we use two terms. We factor the bottom to find that our zeroes are $x = -1, -2$, so our fraction becomes $\frac{A_1}{(x+1)} + \frac{A_2}{(x+2)}$. We then put the partial fraction over a common denominator $(x+1)(x+2)$ multiply everything by $(x+1)(x+2)$ to get $4x+2 = A_1(x+2) + A_2(x+1)$. Then, by setting $x = -1$, we can solve for $A_1 = -2$ and by setting $x = -2$, we can solve for $A_2 = 6$. We get our final fraction as $\frac{6}{(x+2)} - \frac{2}{(x+1)}$.

0.2 Complications:

The degree of the numerator is greater than the degree of the denominator:

Divide the numerator into the denominator to get a polynomial and a remainder, then perform the partial fraction on the remainder.

A repeated zero in the denominator:

If you have a quadratic such as $\frac{x-4}{(x+1)(x-2)^2}$, with a repeated zero, you have to put your partial fraction in the form $\frac{A}{(x-4)} + \frac{Bx+C}{(x-2)^2}$.