22.3: Gauss's Law

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Definition: (Gauss's Law) **Gauss's Law** states that the total electric flux through any closed surface is equal to the sum of the charges inside.

Example: Suppose we have a single point charge q inside the center of a sphere of radius R. The field strength at any point of the sphere is given by

$$E = \frac{q}{4\pi\epsilon_0 R^2}$$

Since the field is always normal to the sphere surface, the flux is given by

$$\Phi_E = EA = \frac{q}{4\pi\epsilon_0 R^2} \cdot 4\pi R^2 = \frac{q}{\epsilon_0}$$

We see that the flux is not dependent on the sphere's radius, only on the charge enclosed.

Example: Now, let's apply this reasoning to irregular surfaces. Given an irregular closed surface around a point charge, we want to transform it into a sphere. Suppose now that we take an area dA of the irregular surface. This area will have a normal vector that is offset from the radial electric field vector by an angle we call ϕ. If we bend it into a sphere, it turns out that two sides of the area are shortened by a factor of $\cos(\phi)$. As such, the projected area becomes $dA\cos(\phi)$. This is because as $dA \to 0$, the sphere becomes locally flat, so modifying dA is equivalent to projecting onto a flat surface. Now, lets integrate all the dAs using the equation

$$\Phi_E = \oint E\cos(\phi)dA$$

However, now notice that this integral is the exact same as the total flux of an uneven electric field through a sphere! Therefore,

$$\Phi_E = \oint E\cos(\phi)dA = \frac{q}{\epsilon_0}$$

The general form of Gauss's Law is given by

$$\Phi_E = \oint E\cos(\phi)dA = \oint E_\perp dA = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$