12.2: Pressure in a Fluid

Alex L.

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Definition: (Pressure) **Pressure** within a fluid at a small area is the normal force of the fluid exerted at that area over the area of the area.

$$p = \frac{dF_\perp}{dA}$$

we can imagine a thin sheet and calculate the normal force on one side.

If a fluid isn't flowing, the pressure within the fluid must be equal.

Theorem: The deeper you go within a still fluid, the more pressure there is.

Proof: Imagine we analyze a small rectangular prism of fluid. It has top and bottom faces with area A and a height dy, where we take the positive vertical direction to be upward. Since the fluid is still, the forces must be balanced. We will only look at the forces in the y axis.

The force on the bottom of the prism of fluid acting upwards is given by pA, the pressure of the fluid times the area of the bottom surface of the prism.

The force on the prism acting downwards is given by $(p + dp)A + dw$, where we are assuming dp is a small change in pressure and dw is the weight of our prism of fluid.

In all,

$$pA - (p + dp)A - dw = 0$$

However, notice that weight is just density times volume times g, and volume is height times base area, so

$$pA - (p + dp)A - Adyg\rho = 0$$

Cancelling the A, we get

$$p - p - dp - dyg\rho = 0$$

We can cancel the p and $-p$ to get

$$-dp - dyg\rho = 0$$

Adding $dyg\rho$ to both sides gets us $-dp = dyg\rho$, and finally, dividing by $-dy$ gives

$$\frac{dp}{dy} = g\rho$$

Lemma: If we integrate both sides with respect to dy, we get

$$\int_{y_1}^{y_2} \frac{dp}{dy} dy = \int_{y_1}^{y_2} g\rho dy$$

we get

$$p_1 - p_2 = g\rho(y_2 - y_1)$$

(remember that we took upwards to be positive, and upwards means less pressure). From this we get

$$p_1 = p_2 + g\rho h$$

Definition: (Gauge Pressure) Most ways of measuring pressure involve comparing two different pressures. As such, when we use a pressure gauge, we are actually comparing the pressure inside to the atmospheric pressure. This is called **gauge pressure**.