7.4: Basis Vectors and Components

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In 3D space, given three non-coplanar vectors e_1, e_2, e_3, it is possible to describe any vector in 3D space in the form $a = a_1e_1 + a_2e_2 + a_3e_3$.

Def: The **basis** of an n-dimensional space is a set of n linearly independent vectors such that every vector in the space may be described as a linear combination of the vectors. The coefficients of the basis vectors are called **components**.

Def: A **linear combination** of quantities $x_1, x_2, x_3, ..., x_n$ is a quantity $c = a_1x_1 + a_2x_2 + a_3x_3 + ... + a_nx_n$, where $a_1, a_2, a_3, ..., a_n$ are scalar quantities.

Theorem: Any set of n linearly independent vectors forms a basis for an n-dimensional space.

By convention, in 3D space, we use the vectors, $\vec{i}, \vec{j}, \vec{k}$, which align with the x, y, and z axes respectively. However, for brevity, the coefficients of the linear combination are usually expressed in terms of its components only: (a_i, a_j, a_k).

The sum of two vectors is the sum of its components.