3.8: Matrix Exponentials

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November 23, 2024

Definition: (Matrix Exponential) We define a matrix exponential using a Taylor series:

$$e^A = 1 + A + \frac{1}{2}A^2 + \frac{1}{6}A^3 + ... = \sum_{n=0}^{\infty} \frac{1}{n!}A^n$$

Proposition: The solution to a differential equation of the form

$$\vec{x}' = P\vec{x}$$

where P is a constant coefficient square matrix is

$$\vec{x} = e^{Pt}$$

However, calculating the matrix exponential can be very hard, because we have to take infinite powers of matrices. There is an easier way. If we have a matrix A, we can put it in the form $A = EDE^{-1}$, where D is a diagonal matrix, then

$$A^n = ED^nE^{-1}$$

Since D is a diagonal matrix, raising D to a power is equivalent to raising each element on the diagonal to that power.