Physics Letters B ••• (••••) ••••••

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

Cosmological stability in $f(\phi, \mathcal{G})$ gravity

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ARTICLE INFO

Article history:
Received 20 December 2022
Received in revised form 2 February 2023
Accepted 2 February 2023
Available online xxxx
Editor: M. Trodden

ABSTRACT

In gravitational theories where a canonical scalar field ϕ with a potential $V(\phi)$ is coupled to a Gauss-Bonnet (GB) term $\mathcal{G}$ with the Lagrangian $f(\phi, \mathcal{G})$, we study the cosmological stability of tensor and scalar perturbations in the presence of a perfect fluid. We show that, in decelerating cosmological epochs with a positive tensor propagation speed squared, the existence of nonlinear functions of $\mathcal{G}$ in f always induces Laplacian instability of a dynamical scalar perturbation associated with the GB term. This is also the case for $f(\mathcal{G})$ gravity, where the presence of nonlinear GB functions $f(\mathcal{G})$ is not allowed during the radiation- and matter-dominated epochs. A linearly coupled GB term with ϕ of the form $\xi(\phi)\mathcal{G}$ can be consistent with all the stability conditions, provided that the scalar-GB coupling is subdominant to the background cosmological dynamics.

1. Introduction

General Relativity (GR) is a fundamental theory of gravity whose validity has been probed in Solar System experiments [1] and sub-millimeter laboratory tests [2,3]. Despite the success of GR describing gravitational interactions in the Solar System, there have been long-standing cosmological problems such as the origins of inflation, dark energy, and dark matter. To address these problems, one typically introduces additional degrees of freedom (DOFs) beyond those appearing in GR [4–10]. One of such new DOFs is a canonical scalar field ϕ with a potential $V(\phi)$ [11–22]. If the scalar field evolves slowly along the potential, it is possible to realize cosmic acceleration responsible for inflation or dark energy. An oscillating scalar field around the potential minimum can be also the source for dark matter.

The other way of introducing a new dynamical DOF is to modify the gravitational sector from GR. The Lagrangian in GR is given by an Einstein-Hilbert term $M_{\rm Pl}^2 R/2$, where $M_{\rm Pl}$ is the reduced Planck mass and R is the Ricci scalar. If we consider theories containing nonlinear functions of R of the form $f(R)$, there is one scalar DOF arising from the modification of gravity [23,24]. One well known example is the Starobinsky's model, in which the presence of a quadratic curvature term R^2 drives cosmic acceleration [25]. It is also possible to construct $f(R)$ models of late-time cosmic acceleration [26–32], while being consistent with local gravity constraints.

The Einstein tensor $G_{\mu\nu}$ obtained by varying the Einstein-Hilbert action satisfies the conserved relation $\nabla^\mu G_{\mu\nu} = 0$ (∇^μ is a covariant derivative operator), with the property of second-order field equations of motion in metrics. If we demand such conserved and second-order properties for 2-rank symmetric tensors, GR is the unique theory of gravity in 4 dimensions [33]. In spacetime dimensions higher than 4, there is a particular combination known as a Gauss-Bonnet (GB) term $\mathcal{G}$ consistent with those demands [34]. In 4 dimensions, the GB term is a topological surface term and hence it does not contribute to the field equations of motion. In the presence of a coupling between a scalar field ϕ and $\mathcal{G}$ of the form $\xi(\phi)\mathcal{G}$, the spacetime dynamics is modified by the time or spatial variation of ϕ. Indeed, this type of scalar-GB coupling appears in the context of low energy effective string theory [35–37]. The cosmological application of the coupling $\xi(\phi)\mathcal{G}$ has been extensively performed in the literature [38–63]. Moreover, it is known that the same coupling gives rise to spherically symmetric solutions of hairy black holes and neutron stars [64–80]. The Lagrangian $f(\mathcal{G})$ containing nonlinear functions of $\mathcal{G}$ also generates nontrivial contributions to the spacetime dynamics [81–90].

In Ref. [91], De Felice and Suyama studied the stability of scalar perturbations in $f(R, \mathcal{G})$ gravity on a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) background. In theories with $f_{,R\mathcal{G}}^2 - f_{,RR} f_{,\mathcal{G}\mathcal{G}} \neq 0$, where $f_{,R\mathcal{G}} = \partial^2 f/\partial\mathcal{G}\partial R$, $f_{,RR} = \partial^2 f/\partial R^2$, and $f_{,\mathcal{G}\mathcal{G}} = \partial^2 f/\partial\mathcal{G}^2$, there is an unusual scale-dependent sound speed which propagates superluminally in the short-wavelength limit, unless the vacuum is in a de Sitter state (see also Ref. [92] for the analysis in an anisotropic cosmological background). We note that this problem

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https://doi.org/10.1016/j.physletb.2023.137751

does not arise for $f(R)$ gravity or $M_{\rm Pl}^2 R/2 + f(\mathcal{G})$ gravity. In Ref. [93], the same authors extended the analysis to a more general Lagrangian $f(\phi, R, \mathcal{G})$ with a canonical scalar field ϕ and showed that the property of the scale-dependent sound speed is not modified by the presence of ϕ. Taking a perfect fluid (radiation or nonrelativistic matter) into account in $f(R, \mathcal{G})$ gravity, the cosmological stability and evolution of matter perturbations were studied in Refs. [94–96].

In Einstein-scalar-GB gravity given by the Lagrangian $M_{\rm Pl}^2 R/2 + f(\phi, \mathcal{G})$, where ϕ is a canonical scalar field, the problem of scale-dependent sound speeds mentioned above is not present. In this theory, the propagation of scalar perturbations on the flat FLRW background was studied in Ref. [93] without taking into account matter. While the sound speed associated with the field ϕ is luminal for theories with $f_{,\mathcal{G}\mathcal{G}} \neq 0$, the propagation speed squared c_s^2 arising from a nonlinear GB term deviates from that of light and it can be even negative. In Ref. [93], the authors discussed the possibility for satisfying the Laplacian stability condition $c_s^2 > 0$. In the presence of matter, however, the stability conditions are subject to modifications from those in the vacuum. To understand what happens for the dynamics of cosmological perturbations during radiation- and matter-dominated epochs, we need to study their stabilities by incorporating radiation or nonrelativistic matter.

In this letter, we will derive general conditions for the absence of ghosts and Laplacian instabilities in $M_{\rm Pl}^2 R/2 + f(\phi, \mathcal{G})$ gravity, where ϕ is a canonical scalar field with a potential $V(\phi)$. In theories where the scalar field ϕ is coupled to the linear GB term, i.e., $f(\phi, \mathcal{G}) = \xi(\phi)\mathcal{G}$, there is only one dynamical scalar DOF ϕ. In theories with $f_{,\mathcal{G}\mathcal{G}} \neq 0$, the Lagrangian $f(\phi, \mathcal{G})$ can be expressed in terms of two scalar fields ϕ and χ coupled to the linear GB term, where χ arises from the nonlinearity in $\mathcal{G}$. Hence the latter theory has two dynamical scalar DOFs. To study the cosmological stability of $f(\phi, \mathcal{G})$ theories with $f_{,\mathcal{G}\mathcal{G}} \neq 0$, we take a perfect fluid into account as a form of the Schutz-Sorkin action [97–99]. We will show that the squared sound speed arising from nonlinear functions of $\mathcal{G}$ is negative during decelerating cosmological epochs including radiation and matter eras. To reach this conclusion, we exploit the fact that the propagation speed squared c_t^2 of tensor perturbations must be positive to avoid Laplacian instability of gravitational waves.

The same Laplacian instability of scalar perturbations is also present in $M_{\rm Pl}^2 R/2 + f(\mathcal{G})$ gravity with any nonlinear function of $\mathcal{G}$ in f. We note that, in $f(\mathcal{G})$ models of late-time cosmic acceleration, violent instabilities of matter density perturbations during the radiation and matter eras were reported in Ref. [100]. This can be regarded as the consequence of a negative sound speed squared of the scalar perturbation $\delta\mathcal{G}$ arising from the nonlinearity of $\mathcal{G}$ in f. Since $\delta\mathcal{G}$ is coupled to the matter perturbation $\delta\rho$, the background cosmological evolution during the radiation and matter eras is spoiled by the rapid growth of $\delta\rho$. Our analysis in this letter shows that similar catastrophic instabilities persist for more general scalar-GB couplings $f(\phi, \mathcal{G})$ with $f_{,\mathcal{G}\mathcal{G}} \neq 0$.

This letter is organized as follows. In Sec. 2, we revisit cosmological stability conditions in $M_{\rm Pl}^2 R/2 + \xi(\phi)\mathcal{G}$ gravity with a canonical scalar field ϕ, which can be accommodated in a subclass of Horndeski theories with a single scalar DOF [101–104]. This is an exceptional case satisfying the condition $f_{,\mathcal{G}\mathcal{G}} = 0$, under which the Laplacian instability of scalar perturbations can be avoided. In Sec. 3, we derive the background equations and stability conditions of tensor perturbations in $M_{\rm Pl}^2 R/2 + f(\phi, \mathcal{G})$ gravity with $f_{,\mathcal{G}\mathcal{G}} \neq 0$ by incorporating a perfect fluid. In Sec. 4, we proceed to the derivation of a second-order action of scalar perturbations and obtain conditions for the absence of ghosts and Laplacian instabilities in the scalar sector. In particular, we show that an effective cosmological equation of state $w_{\rm eff}$ needs to be in the range $w_{\rm eff} < -(2 + c_t^2)/6$ to ensure Laplacian stabilities of the perturbation $\delta\mathcal{G}$. Sec. 5 is devoted to conclusions.

2. $\xi(\phi)\mathcal{G}$ gravity

We first briefly revisit the cosmological stability in $\xi(\phi)\mathcal{G}$ gravity given by the action

$$\mathcal{S} = \int {\rm d}^4 x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R + \eta X - V(\phi) + \xi(\phi)\mathcal{G} \right] + \mathcal{S}_m(g_{\mu\nu}, \Psi_m)\,, \tag{2.1}$$

where g is a determinant of the metric tensor $g_{\mu\nu}$, η is a constant, $X = -(1/2)g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ is a kinetic term of the scalar field ϕ, $V(\phi)$ and $\xi(\phi)$ are functions of ϕ, and $\mathcal{G}$ is a GB term defined by

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\,, \tag{2.2}$$

with $R_{\mu\nu}$ and $R_{\mu\nu\rho\sigma}$ being the Ricci and Riemann tensors, respectively. For the matter action $\mathcal{S}_m$, we consider a perfect fluid minimally coupled to gravity.

The action (2.1) contains one scalar DOF ϕ besides the matter field Ψ_m. If we consider Horndeski theories [101] given by the action

$$\begin{aligned}\mathcal{S} = \int {\rm d}^4 x \sqrt{-g} \Big[& G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4,X}(\phi, X)\left\{(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right\} \\ & + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}(\phi, X)\left\{(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right\}\Big] \\ & + \mathcal{S}_m(g_{\mu\nu}, \Psi_m)\,, \end{aligned} \tag{2.3}$$

then the theory (2.1) can be accommodated by choosing the coupling functions [103]

$$\begin{aligned} & G_2(\phi, X) = \eta X - V(\phi) + 8\xi_{,\phi\phi\phi\phi}(\phi)X^2(3 - \ln|X|)\,, \qquad G_3(\phi, X) = 4\xi_{,\phi\phi\phi}(\phi)X(7 - 3\ln|X|)\,, \\ & G_4(\phi, X) = \frac{M_{\rm Pl}^2}{2} + 4\xi_{,\phi\phi}(\phi)X(2 - \ln|X|)\,, \qquad G_5(\phi, X) = -4\xi_{,\phi}(\phi)\ln|X|\,, \end{aligned} \tag{2.4}$$

where we use the notations $F_{,X} = \partial F/\partial X$ and $F_{,\phi} = \partial F/\partial\phi$ for any arbitrary function F.

Let us consider a spatially flat FLRW background given by the line element ${\rm d}s^2 = -{\rm d}t^2 + a^2(t)\delta_{ij}{\rm d}x^i{\rm d}x^j$, where $a(t)$ is a time-dependent scale factor. The perfect fluid has a density ρ and pressure P. The background equations as well as the perturbation equations in full Horndeski theories were derived in Refs. [103,105–107]. On using the correspondence (2.4), the background equations of motion in theories given by the action (2.1) are

$$3\tilde{q}_t H^2 = \frac{1}{2}\eta\dot{\phi}^2 + V(\phi) + \rho\,, \tag{2.5}$$

$$2\tilde{q}_t \dot{H} = -\eta\dot{\phi}^2 - H^2\tilde{q}_t\left(\tilde{c}_t^2 - 1\right) - \rho - P\,, \tag{2.6}$$

$$\eta\left(\ddot{\phi} + 3H\dot{\phi}\right) + V_{,\phi} - \xi_{,\phi}\mathcal{G} = 0\,, \tag{2.7}$$

$$\dot{\rho} + 3H(\rho + P) = 0\,, \tag{2.8}$$

where $H = \dot{a}/a$ is the Hubble expansion rate, a dot represents the derivative with respect to t, and

$$\tilde{q}_t = M_{\rm Pl}^2 + 8\xi_{,\phi}H\dot{\phi}\,, \tag{2.9}$$

$$\tilde{c}_t^2 = \frac{M_{\rm Pl}^2 + 8(\xi_{,\phi}\ddot{\phi} + \xi_{,\phi\phi}\dot{\phi}^2)}{M_{\rm Pl}^2 + 8\xi_{,\phi}H\dot{\phi}}\,, \tag{2.10}$$

$$\mathcal{G} = 24H^2\left(H^2 + \dot{H}\right)\,. \tag{2.11}$$

In the presence of tensor perturbations h_{ij} with the perturbed line element ${\rm d}s^2 = -{\rm d}t^2 + a^2(t)(\delta_{ij} + h_{ij}){\rm d}x^i{\rm d}x^j$, the second-order action of traceless and divergence-free modes of h_{ij} was already derived in full Horndeski theories [103,106,107]. In the current theory, the conditions for the absence of ghosts and Laplacian instabilities are

$$\tilde{q}_t > 0\,, \tag{2.12}$$

$$\tilde{c}_t^2 > 0\,, \tag{2.13}$$

where $\tilde{q}_t$ and $\tilde{c}_t^2$ are defined by Eqs. (2.9) and (2.10), respectively. Note that $\tilde{q}_t$ determines the sign of a kinetic term of h_{ij}, while $\tilde{c}_t^2$ corresponds to the propagation speed squared of tensor perturbations.

For the scalar sector, we choose the perturbed line element ${\rm d}s^2 = -(1+2\alpha){\rm d}t^2 + 2\partial_i B{\rm d}t{\rm d}x^i + a^2(t)\delta_{ij}{\rm d}x^i{\rm d}x^j$ in the flat gauge, where α and B are scalar metric perturbations. There is also a scalar-field perturbation $\delta\phi$ besides the matter perturbation $\delta\rho$ and the fluid velocity potential v. After deriving the quadratic-order action of scalar perturbations, we can eliminate nondynamical variables α, B, and v from the action. Then, we are left with the two dynamical perturbations $\delta\phi$ and $\delta\rho$ in the second-order action. In the short-wavelength limit, there is neither ghost nor Laplacian instability for $\delta\phi$ under the conditions [103,106,107]

$$\tilde{q}_s = 2\left(\eta\tilde{q}_t + 96H^4\xi_{,\phi}^2\right) > 0\,, \tag{2.14}$$

$$\tilde{c}_s^2 = \frac{\eta\tilde{q}_t - 32(2 + \tilde{c}_t^2 + 6w_{\rm eff})H^4\xi_{,\phi}^2}{\eta\tilde{q}_t + 96H^4\xi_{,\phi}^2} > 0\,, \tag{2.15}$$

where $\tilde{c}_s$ corresponds to the propagation speed of $\delta\phi$, and $w_{\rm eff}$ is the cosmological effective equation of state defined by

$$w_{\rm eff} \equiv -1 - \frac{2\dot{H}}{3H^2}\,. \tag{2.16}$$

The stability conditions for $\delta\rho$ are given by $\rho + P > 0$ and $c_m^2 > 0$, where c_m^2 is the matter sound speed squared.

Under the stability condition (2.12) with $\eta > 0$, the scalar no-ghost condition (2.14) is satisfied. Let us consider the case in which contributions of the scalar-GB coupling are suppressed, such that

$$\{|\xi_{,\phi}H\dot{\phi}|,\, |\xi_{,\phi}\ddot{\phi}|,\, |\xi_{,\phi\phi}\dot{\phi}^2|\} \ll M_{\rm Pl}^2\,, \qquad H^4\xi_{,\phi}^2 \ll \eta\tilde{q}_t\,. \tag{2.17}$$

Then, it follows that $\tilde{q}_t \simeq M_{\rm Pl}^2$, $\tilde{c}_t^2 \simeq 1$, $\tilde{q}_s \simeq 2\eta M_{\rm Pl}^2$, and $\tilde{c}_s^2 \simeq 1$. In such cases, provided that $\eta > 0$, all the stability conditions are consistently satisfied. If the scalar-GB coupling contributes to the late-time cosmological dynamics, there is an observational bound on $\tilde{c}_t$ constrained from the GW170817 event together with the electromagnetic counterpart, i.e., $-3 \times 10^{-15} \le \tilde{c}_t - 1 \le 7 \times 10^{-16}$ [108] for the redshift $z \le 0.009$. This translates to the limit

$$\left|\xi_{,\phi}\ddot{\phi} + \xi_{,\phi\phi}\dot{\phi}^2 - \xi_{,\phi}H\dot{\phi}\right| \lesssim 10^{-15}M_{\rm Pl}^2\,, \tag{2.18}$$

which gives a tight constraint on the amplitude of $\xi(\phi)$. In this case, contributions of the scalar-GB coupling to the background Eqs. (2.5) and (2.6) are highly suppressed relative to the field density $\rho_\phi = \eta\dot{\phi}^2/2 + V(\phi)$ and the matter density.

The bound (2.18) is not applied to early cosmological epochs including inflation, radiation, and matter eras. We note, however, that the dominance of the scalar-GB coupling to the background equations prevents the successful cosmic expansion history. This can also give rise to the violation of either of the stability conditions (2.12)-(2.15). Provided the scalar-GB coupling is suppressed in such a way that inequalities (2.17) hold, the linear stabilities are ensured for both tensor and scalar perturbations.

3. $f(\phi, \mathcal{G})$ gravity

We extend $\xi(\phi)\mathcal{G}$ gravity to more general theories in which a canonical scalar field ϕ with a potential $V(\phi)$ is coupled to the GB term of the form $f(\phi, \mathcal{G})$. The action in such theories is given by

$$\mathcal{S}=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R+\eta X-V(\phi)+f(\phi,\mathcal{G})\right]+\mathcal{S}_m(g_{\mu\nu},\Psi_m)\,, \tag{3.1}$$

where a matter field Ψ_m is minimally coupled to gravity. It is more practical to introduce a scalar field χ and resort to the following action

$$\mathcal{S}=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R+\eta X-U(\phi,\chi)+\xi(\phi,\chi)\mathcal{G}\right]+\mathcal{S}_m(g_{\mu\nu},\Psi_m)\,, \tag{3.2}$$

where

$$U(\phi,\chi)\equiv V(\phi)-f(\phi,\chi)+\chi\xi(\phi,\chi)\,,\qquad \xi(\phi,\chi)\equiv f_{,\chi}(\phi,\chi)\,, \tag{3.3}$$

with the notation $f_{,\chi}=\partial f/\partial\chi$. Varying the action (3.2) with respect to χ, it follows that

$$(\chi-\mathcal{G})\,\xi_{,\chi}=0\,. \tag{3.4}$$

So long as $\xi_{,\chi}\neq 0$, we obtain $\chi=\mathcal{G}$. In this case, the action (3.2) reduces to (3.1). Thus, the equivalence of (3.2) with (3.1) holds for

$$f_{,\mathcal{G}\mathcal{G}}\neq 0\,, \tag{3.5}$$

under which there is a new scalar DOF χ arising from the gravitational sector.

Theories with $f_{,\mathcal{G}\mathcal{G}}=0$ correspond to the coupling $f=\xi(\phi)\mathcal{G}$, in which case the cosmological stability conditions were already discussed in Sec. 2. In $\xi(\phi)\mathcal{G}$ gravity, we do not have the additional scalar DOF χ arising from $\mathcal{G}$, so the term $\xi_{,\chi}$ in Eq. (3.4) does not have the meaning of $f_{,\mathcal{G}\mathcal{G}}$. Thus, the action (3.2) with the new dynamical DOF χ does not reproduce the action (2.1) in $\xi(\phi)\mathcal{G}$ gravity.

In the following, we will focus on theories with $f_{,\mathcal{G}\mathcal{G}}\neq 0$, i.e., those containing the nonlinear dependence of $\mathcal{G}$ in f. For the matter field Ψ_m, we incorporate a perfect fluid without a dynamical vector DOF. This matter sector is described by the Schutz-Sorkin action [97–99]

$$\mathcal{S}_m=-\int \mathrm{d}^4x\left[\sqrt{-g}\,\rho(n)+J^\mu\partial_\mu\ell\right]\,, \tag{3.6}$$

where the fluid density ρ is a function of its number density n. The vector field J^μ is related to n according to the relation $n=\sqrt{J^\mu J^\nu g_{\mu\nu}/g}$, where $u^\mu=J^\mu/(n\sqrt{-g})$ is the fluid four velocity. A scalar quantity ℓ in $\mathcal{S}_m$ is a Lagrange multiplier, with the notation of a partial derivative $\partial_\mu\ell=\partial\ell/\partial x^\mu$. Varying the matter action (3.6) with respect to ℓ and J^μ, respectively, we obtain

$$\partial_\mu J^\mu=0\,, \tag{3.7}$$

$$\partial_\mu\ell=u_\mu\rho_{,n}\,, \tag{3.8}$$

where $\rho_{,n}=\mathrm{d}\rho/\mathrm{d}n$.

3.1. Background equations

We derive the background equations of motion on the spatially flat FLRW background given by the line element

$$\mathrm{d}s^2=-N^2(t)\,\mathrm{d}t^2+a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\,, \tag{3.9}$$

where $N(t)$ is a lapse function. Since the fluid four velocity in its rest frame is given by $u^\mu=(N^{-1},0,0,0)$, the vector field J^μ has components $J^\mu=(na^3,0,0,0)$. From Eq. (3.7), we obtain

$$\mathcal{N}_0\equiv na^3=\text{constant}\,, \tag{3.10}$$

which means that the total fluid number $\mathcal{N}_0$ is conserved. This translates to the differential equation $\dot{n}+3Hn=0$, which can be expressed as a form of the continuity equation

$$\dot{\rho}+3H\,(\rho+P)=0\,, \tag{3.11}$$

where P is a fluid pressure defined by $P=n\rho_{,n}-\rho$.

On the background (3.9), the total action (3.2) is expressed in the form

$$\mathcal{S}=\int \mathrm{d}t\,\mathrm{d}^3x\left[\frac{\eta a^3\dot{\phi}^2}{2N}-\frac{3M_{\rm Pl}^2a\dot{a}^2}{N}-Na^3U(\phi,\chi)-\frac{8\dot{a}^3\dot{\xi}(\phi,\chi)}{N^3}-Na^3\rho-na^3\dot{\ell}\right]. \tag{3.12}$$

From Eq. (3.8), we obtain the following relation

$$\dot{\ell}=-N\rho_{,n}\,. \tag{3.13}$$

Varying the action (3.12) with respect to N, a, ϕ, χ respectively and setting $N=1$ at the end, we obtain the background equations of motion

$$3q_t H^2 = \frac{1}{2}\eta\dot{\phi}^2 + U(\phi,\chi) + \rho\,, \tag{3.14}$$

$$2q_t \dot{H} = -\eta\dot{\phi}^2 - H^2 q_t \left(c_t^2 - 1\right) - \rho - P\,, \tag{3.15}$$

$$\eta\left(\ddot{\phi} + 3H\dot{\phi}\right) + V_{,\phi} - f_{,\phi} = 0\,, \tag{3.16}$$

$$\chi = \mathcal{G} = 24H^2\left(H^2 + \dot{H}\right)\,, \tag{3.17}$$

where

$$q_t = M_{\rm Pl}^2 + 8H(\xi_{,\phi}\dot{\phi} + \xi_{,\chi}\dot{\chi})\,, \tag{3.18}$$

$$c_t^2 = \frac{M_{\rm Pl}^2 + 8(\xi_{,\phi}\ddot{\phi} + \xi_{,\phi\phi}\dot{\phi}^2 + \xi_{,\chi}\ddot{\chi} + \xi_{,\chi\chi}\dot{\chi}^2 + 2\xi_{,\phi\chi}\dot{\phi}\dot{\chi})}{M_{\rm Pl}^2 + 8H(\xi_{,\phi}\dot{\phi} + \xi_{,\chi}\dot{\chi})}\,. \tag{3.19}$$

We recall that the perfect fluid obeys the continuity Eq. (3.11). We notice that Eqs. (3.14)-(3.16) are of similar forms to Eqs. (2.5)-(2.7) in $\xi(\phi)\mathcal{G}$ gravity, but the expressions of q_t and c_t^2 are different from $\tilde{q}_t$ and $\tilde{c}_t^2$, respectively, because of the appearance of time derivatives of χ. These χ derivatives do not vanish for $\xi_{,\chi} \neq 0$, i.e., for $f_{,\mathcal{G}\mathcal{G}} \neq 0$. As we will show in Sec. 4, nonlinearities of $\mathcal{G}$ in f are responsible for the appearance of a new scalar propagating DOF $\delta\chi$.

3.2. Stabilities in the tensor sector

We proceed to the derivation of stability conditions for tensor perturbations in theories given by the action (3.2). The perturbed line element including the tensor perturbation h_{ij} is

$$ds^2 = -dt^2 + a^2(t)\left(\delta_{ij} + h_{ij}\right)dx^i dx^j\,, \tag{3.20}$$

where we impose the traceless and divergence-free gauge conditions ${h^i}_i = 0$ and $\partial^i h_{ij} = 0$. For the gravitational wave propagating along the z direction, nonvanishing components of h_{ij} are expressed in the form

$$h_{11} = h_1(t,z)\,, \qquad h_{22} = -h_1(t,z)\,, \qquad h_{12} = h_{21} = h_2(t,z)\,, \tag{3.21}$$

where the two polarized modes h_1 and h_2 are functions of t and z.

The second-order action arising from the matter action (3.6) can be expressed as

$$(\mathcal{S}_m^{(2)})_t = -\int dt\, d^3x \sum_{i=1}^{2} \frac{1}{2}a^3 P h_i^2\,, \tag{3.22}$$

where P can be eliminated by using the background Eq. (3.15). Expanding the total action (3.2) up to quadratic order in tensor perturbations and integrating it by parts, the resulting second-order action reduces to

$$\mathcal{S}_t^{(2)} = \int dt\, d^3x \sum_{i=1}^{2} \frac{a^3}{4} q_t \left[\dot{h}_i^2 - \frac{c_t^2}{a^2}(\partial h_i)^2\right]\,, \tag{3.23}$$

where $(\partial h_i)^2 = (\partial h_i/\partial z)^2$. We recall that q_t and c_t^2 are given by Eqs. (3.18) and (3.19), respectively.

To avoid the ghost and Laplacian instabilities in the tensor sector, we require the two conditions $q_t > 0$ and $c_t^2 > 0$, which translate to

$$M_{\rm Pl}^2 + 8H(\xi_{,\phi}\dot{\phi} + \xi_{,\chi}\dot{\chi}) > 0\,, \tag{3.24}$$

$$M_{\rm Pl}^2 + 8(\xi_{,\phi}\ddot{\phi} + \xi_{,\phi\phi}\dot{\phi}^2 + \xi_{,\chi}\ddot{\chi} + \xi_{,\chi\chi}\dot{\chi}^2 + 2\xi_{,\phi\chi}\dot{\phi}\dot{\chi}) > 0\,. \tag{3.25}$$

In $f(\mathcal{G})$ gravity without the scalar field ϕ, tensor stability conditions can be obtained by setting $\dot{\phi} = 0$ and $\ddot{\phi} = 0$ in Eqs. (3.24) and (3.25).

We vary the action (3.23) with respect to h_i (with $i = 1, 2$) in Fourier space with a comoving wavenumber $\boldsymbol{k}$. Then, we obtain the tensor perturbation equation of motion

$$\ddot{h}_i + \left(3H + \frac{\dot{q}_t}{q_t}\right)\dot{h}_i + c_t^2\frac{k^2}{a^2}h_i = 0\,, \tag{3.26}$$

where $k = |\boldsymbol{k}|$. Since $\xi = f_{,\chi} = f_{,\mathcal{G}}$, the $\mathcal{G}$ dependence in f leads to the modified evolution of gravitational waves in comparison to GR. If the energy densities of ϕ and χ are relevant to the late-time cosmological dynamics after the matter dominance, the observational constraint on the tensor propagation speed c_t arising from the GW170817 event [108] ($|c_t - 1| \lesssim 10^{-15}$) gives a tight bound on the scalar-GB coupling $f(\phi,\mathcal{G})$. Such a stringent limit is not applied to the cosmological dynamics in the early Universe, but the conditions (3.24) and (3.25) need to be still satisfied.

4. Stabilities of $f(\phi, \mathcal{G})$ gravity in the scalar sector

In this section, we will derive conditions for the absence of scalar ghosts and Laplacian instabilities in theories given by the action (3.2). A perturbed line element containing scalar perturbations α, B, ζ, and E is of the form

$$ds^2 = -(1+2\alpha)dt^2 + 2\partial_i B dt dx^i + a^2(t)\left[(1+2\zeta)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i dx^j \,. \tag{4.1}$$

For the scalar fields ϕ and χ, we consider perturbations $\delta\phi$ and $\delta\chi$ on the background values $\bar{\phi}(t)$ and $\bar{\chi}(t)$, respectively, such that

$$\phi = \bar{\phi}(t) + \delta\phi(t, \boldsymbol{x}) \,, \qquad \chi = \bar{\chi}(t) + \delta\chi(t, \boldsymbol{x}) \,, \tag{4.2}$$

where we will omit a bar from the background quantities in the following.

In the matter sector, the temporal and spatial components of J^μ are decomposed into the background and perturbed parts as

$$J^0 = \mathcal{N}_0 + \delta J \,, \qquad J^i = \frac{1}{a^2(t)}\delta^{ik}\partial_k \delta j \,, \tag{4.3}$$

where δJ and δj are scalar perturbations. In terms of the velocity potential v, the spatial component of fluid four velocity is expressed as $u_i = -\partial_i v$. From Eq. (3.8), the scalar quantity ℓ has a background part obeying the relation $\dot{\ell} = -\rho_{,n}$ besides a perturbation $-\rho_{,n} v$. Then, we have

$$\ell = -\int^t \rho_{,n}(\tilde{t})\, d\tilde{t} - \rho_{,n} v \,. \tag{4.4}$$

Defining the matter density perturbation

$$\delta\rho \equiv \frac{\rho_{,n}}{a^3}\left[\delta J - \mathcal{N}_0(3\zeta + \partial^2 E)\right] , \tag{4.5}$$

the fluid number density n has a perturbation [107,109]

$$\delta n = \frac{\delta\rho}{\rho_{,n}} - \frac{(\mathcal{N}_0 \partial\chi + \partial\delta j)^2}{2\mathcal{N}_0 a^5} - \frac{(3\zeta + \partial^2 E)\delta\rho}{\rho_{,n}} - \frac{\mathcal{N}_0(\zeta + \partial^2 E)(3\zeta - \partial^2 E)}{2a^3} \,, \tag{4.6}$$

up to second order. The matter sound speed squared is given by

$$c_m^2 = \frac{P_{,n}}{\rho_{,n}} = \frac{n\rho_{,nn}}{\rho_{,n}} \,. \tag{4.7}$$

Expanding (3.6) up to quadratic order in perturbations, we obtain the second-order matter action same as that derived in Refs. [107,109]. Varying this matter action with respect to δj leads to

$$\partial\delta j = -a^3 n (\partial v + \partial B) \,, \tag{4.8}$$

whose relation will be used to eliminate δj.

In the following, we choose the gauge

$$E = 0 \,, \tag{4.9}$$

under which a scalar quantity ξ associated with the spatial gauge transformation $x^i \to x^i + \delta^{ij}\partial_j \xi$ is fixed. A scalar quantity ξ^0 associated with the temporal part of the gauge transformation $t \to t + \xi^0$ can be fixed by choosing a flat gauge ($\zeta = 0$) or a unitary gauge ($\delta\phi = 0$). We do not specify the temporal gauge condition in deriving the second-order action, but we will do so at the end.

Expanding the total action (3.2) up to quadratic order in scalar perturbations and integrating it by parts, the resulting second-order action is given by

$$\mathcal{S}_s^{(2)} = \int dt\, d^3x \left(L_{\rm flat} + L_\zeta\right) , \tag{4.10}$$

where

$$\begin{aligned} L_{\rm flat} = a^3 \Bigg[& \frac{\eta}{2}\dot{\delta\phi}^2 - \frac{\eta}{2}\frac{(\partial\delta\phi)^2}{a^2} + \frac{1}{2}\left(f_{,\phi\phi} - V_{,\phi\phi}\right)\delta\phi^2 + \left\{\frac{1}{2}\eta\dot{\phi}^2 - 3H^2(2q_t - M_{\rm Pl}^2)\right\}\alpha^2 - H(3q_t - M_{\rm Pl}^2)\frac{\partial^2 B}{a^2}\alpha \\ & - \frac{C_4}{16H^2}\delta\chi^2 + \left\{C_1\dot{\delta\phi} + C_2\delta\phi - C_3\frac{\partial^2\delta\phi}{a^2} + 3HC_4\dot{\delta\chi} - C_4\frac{\partial^2\delta\chi}{a^2} + 3(HC_5 - \dot{H}C_4)\delta\chi\right\}\alpha \\ & + \left(C_3\dot{\delta\phi} + C_6\delta\phi + C_4\dot{\delta\chi} + C_5\delta\chi\right)\frac{\partial^2 B}{a^2} + (\rho+P)v\frac{\partial^2 B}{a^2} - v\dot{\delta\rho} - 3H(1+c_m^2)v\delta\rho \\ & - \frac{1}{2}(\rho+P)\frac{(\partial v)^2}{a^2} - \frac{c_m^2}{2(\rho+P)}\delta\rho^2 - \alpha\delta\rho \Bigg] , \end{aligned} \tag{4.11}$$

$$\begin{aligned} L_\zeta = a^3 \Bigg[& \left\{3H(3q_t - M_{\rm Pl}^2)\alpha - 3\left(C_3\dot{\delta\phi} + C_6\delta\phi + C_4\dot{\delta\chi} + C_5\delta\chi\right) - 3(\rho+P)v + 2q_t\frac{\partial^2 B}{a^2}\right\}\dot{\zeta} - 3q_t\dot{\zeta}^2 \\ & + q_t c_t^2\frac{(\partial\zeta)^2}{a^2} - 2\left\{q_t\alpha + \left(1 + \frac{\dot{H}}{H^2}\right)(C_3\delta\phi + C_4\delta\chi)\right\}\frac{\partial^2\zeta}{a^2} \Bigg] , \end{aligned} \tag{4.12}$$

where q_t and c_t^2 are given by Eqs. (3.18) and (3.19), respectively, and

$$C_1 = 24H^3\xi_{,\phi} - \eta\dot{\phi}\,, \qquad C_2 = -24H^2\left[(H^2+\dot{H})\xi_{,\phi} - H(\xi_{,\phi\phi}\dot{\phi} + \xi_{,\phi\chi}\dot{\chi})\right] - V_{,\phi} + f_{,\phi}\,,$$
$$C_3 = 8H^2\xi_{,\phi}\,, \qquad C_4 = 8H^2\xi_{,\chi}\,,$$
$$C_5 = -8H^2\left(H\xi_{,\chi} - \xi_{,\phi\chi}\dot{\phi} - \xi_{,\chi\chi}\dot{\chi}\right), \qquad C_6 = \eta\dot{\phi} - 8H^2\left(H\xi_{,\phi} - \xi_{,\phi\phi}\dot{\phi} - \xi_{,\phi\chi}\dot{\chi}\right). \tag{4.13}$$

Now, we switch to the Fourier space with a comoving wavenumber $\boldsymbol{k}$. Varying the total action (4.10) with respect to α, B, and v, respectively, we obtain

$$C_1\dot{\delta\phi} + C_2\delta\phi + 3HC_4\dot{\delta\chi} - 3(\dot{H}C_4 - HC_5)\delta\chi + 3\left(3q_t - M_{\rm Pl}^2\right)H\dot{\zeta} + \left[\eta\dot{\phi}^2 - 6H^2(2q_t - M_{\rm Pl}^2)\right]\alpha$$
$$+\frac{k^2}{a^2}\left[2q_t\zeta + H\left(3q_t - M_{\rm Pl}^2\right)B + C_3\delta\phi + C_4\delta\chi\right] - \delta\rho = 0\,, \tag{4.14}$$
$$C_3\dot{\delta\phi} + C_6\delta\phi + 2q_t\dot{\zeta} + C_4\dot{\delta\chi} + C_5\delta\chi - H\left(3q_t - M_{\rm Pl}^2\right)\alpha + (\rho+P)v = 0\,, \tag{4.15}$$
$$\dot{\delta\rho} + 3H(1+c_m^2)\delta\rho + 3(\rho+P)\dot{\zeta} + \frac{k^2}{a^2}(\rho+P)(v+B) = 0\,. \tag{4.16}$$

In the following, we choose the flat gauge given by

$$\zeta = 0\,, \tag{4.17}$$

to obtain stability conditions for scalar perturbations. We will discuss the two cases: (A) $f(\phi,\mathcal{G})$ gravity and (B) $f(\mathcal{G})$ gravity in turn.

4.1. $f(\phi,\mathcal{G})$ gravity

In $f(\phi,\mathcal{G})$ gravity with $f_{,\mathcal{G}\mathcal{G}} \neq 0$, we can construct gauge-invariant scalar perturbations $\delta\phi_{\rm f} = \delta\phi - \dot{\phi}\,\zeta/H$, $\delta\chi_{\rm f} = \delta\chi - \dot{\chi}\,\zeta/H$, and $\delta\rho_{\rm f} = \delta\rho - \dot{\rho}\,\zeta/H$. For the gauge choice (4.17), they reduce, respectively, to $\delta\phi$, $\delta\chi$, and $\delta\rho$, which correspond to the dynamical scalar DOFs. Note that the perturbation $\delta\chi = \delta\mathcal{G}$ arises from nonlinearities in the GB term. We solve Eqs. (4.14)-(4.16) for α, B, v and substitute them into Eq. (4.10). Then, the resulting quadratic-order action in Fourier space is expressed in the form

$$\mathcal{S}_s^{(2)} = \int {\rm d}t\,{\rm d}^3x\,a^3\left(\dot{\vec{\mathcal{X}}}^t\boldsymbol{K}\dot{\vec{\mathcal{X}}} - \frac{k^2}{a^2}\vec{\mathcal{X}}^t\boldsymbol{G}\vec{\mathcal{X}} - \vec{\mathcal{X}}^t\boldsymbol{M}\vec{\mathcal{X}} - \vec{\mathcal{X}}^t\boldsymbol{B}\dot{\vec{\mathcal{X}}}\right), \tag{4.18}$$

where $\boldsymbol{K}$, $\boldsymbol{G}$, $\boldsymbol{M}$, $\boldsymbol{B}$ are 3×3 matrices, and

$$\vec{\mathcal{X}}^t = (\delta\phi, \delta\chi, \delta\rho/k)\,. \tag{4.19}$$

The leading-order contributions to $\boldsymbol{M}$ and $\boldsymbol{B}$ are of order k^0. Taking the small-scale limit $k\to\infty$, nonvanishing components of the symmetric matrices $\boldsymbol{K}$ and $\boldsymbol{G}$ are

$$K_{11} = \frac{\eta[C_3\dot{\phi} - H(3q_t - M_{\rm Pl}^2)]^2 + 6C_3^2H^2q_t}{2H^2(3q_t - M_{\rm Pl}^2)^2}\,, \qquad K_{22} = \frac{C_4^2(\eta\dot{\phi}^2 + 6H^2q_t)}{2H^2(3q_t - M_{\rm Pl}^2)^2}\,,$$
$$K_{12} = K_{21} = \frac{C_4[C_3(\eta\dot{\phi}^2 + 6H^2q_t) - \eta H\dot{\phi}(3q_t - M_{\rm Pl}^2)]}{2H^2(3q_t - M_{\rm Pl}^2)^2}\,, \qquad K_{33} = \frac{a^2}{2(\rho+P)}\,, \tag{4.20}$$

and

$$G_{11} = -\frac{\eta H(3q_t - M_{\rm Pl}^2)[2C_3\dot{\phi} - H(3q_t - M_{\rm Pl}^2)] + C_3^2[\rho + P - 6q_t\dot{H} + 3H^2q_t(c_t^2 - 3)]}{2H^2(3q_t - M_{\rm Pl}^2)^2}\,,$$
$$G_{22} = \frac{C_4^2[3H^2q_t(3 - c_t^2) + 6q_t\dot{H} - \rho - P]}{2H^2(3q_t - M_{\rm Pl}^2)^2}\,,$$
$$G_{12} = G_{21} = -\frac{C_4[\eta H\dot{\phi}(3q_t - M_{\rm Pl}^2) + C_3\{\rho + P - 6q_t\dot{H} + 3H^2q_t(c_t^2 - 3)\}]}{2H^2(3q_t - M_{\rm Pl}^2)^2}\,, \qquad G_{33} = \frac{a^2c_m^2}{2(\rho+P)}\,. \tag{4.21}$$

To derive these coefficients, we have absorbed k^2-dependent terms present in $\boldsymbol{B}$ into the components of $\boldsymbol{G}$ and used the relation $C_1 = 3HC_3 - \eta\dot{\phi}$, and

$$\dot{C}_3 = C_6 + C_3\left(H + \frac{2\dot{H}}{H}\right) - \eta\dot{\phi}\,, \qquad \dot{C}_4 = C_4\left(H + \frac{2\dot{H}}{H}\right) + C_5\,, \qquad \dot{q}_t = Hq_t(c_t^2 - 1) + \left(H + \frac{\dot{H}}{H}\right)(q_t - M_{\rm Pl}^2)\,. \tag{4.22}$$

The scalar ghosts are absent under the following three conditions

$$K_{33}=\frac{a^2}{2(\rho+P)}>0\,, \tag{4.23}$$

$$K_{11}K_{22}-K_{12}^2=\frac{3C_4^2\,\eta q_t}{2(3q_t-M_{\rm Pl}^2)^2}>0\,, \tag{4.24}$$

$$\det \boldsymbol{K}=\frac{3C_4^2\,\eta q_t a^2}{4(\rho+P)(3q_t-M_{\rm Pl}^2)^2}>0\,. \tag{4.25}$$

Under the no-ghost condition $q_t>0$ of tensor perturbations, inequalities (4.23)-(4.25) are satisfied for

$$\rho+P>0\,, \tag{4.26}$$

$$\eta>0\,. \tag{4.27}$$

In the limit of large k, dominant contributions to the second-order action (4.18) arise from $\boldsymbol{K}$ and $\boldsymbol{G}$. Then, the dispersion relation can be expressed in the form

$$\det\left(c_s^2\boldsymbol{K}-\boldsymbol{G}\right)=0\,, \tag{4.28}$$

where c_s is the scalar propagation speed. Solving Eq. (4.28) for c_s^2, we obtain the following three solutions

$$c_{s1}^2=1\,, \tag{4.29}$$

$$c_{s2}^2=-\frac{\eta\dot{\phi}^2+\rho+P+3q_t[(c_t^2-3)H^2-2\dot{H}]}{6H^2q_t}\,, \tag{4.30}$$

$$c_{s3}^2=c_m^2\,, \tag{4.31}$$

which correspond to the squared propagation speeds of $\delta\phi$, $\delta\chi$, and $\delta\rho$, respectively. The scalar perturbation $\delta\phi$ has a luminal propagation speed, so it satisfies the Laplacian stability condition. For $c_m^2>0$, the matter perturbation $\delta\rho$ is free from Laplacian instability. On using the background Eq. (3.15), the sound speed squared (4.30) can be expressed as[1]

$$c_{s2}^2=\frac{1}{3}\left(4-c_t^2+\frac{4\dot{H}}{H^2}\right)=-\frac{1}{3}\left(2+c_t^2+6w_{\rm eff}\right)\,, \tag{4.33}$$

where $w_{\rm eff}$ is the effective equation of state defined by Eq. (2.16). The Laplacian stability of $\delta\chi$ is ensured for $c_{s2}^2>0$, i.e.,

$$w_{\rm eff}<-\frac{1}{6}\left(2+c_t^2\right)\,. \tag{4.34}$$

Since we need the condition $c_t^2>0$ for the absence of Laplacian instability in the tensor sector, $w_{\rm eff}$ must be in the range $w_{\rm eff}<-1/3$. This translates to the condition $\dot{H}+H^2=\ddot{a}/a>0$, so the Laplacian stability of $\delta\chi$ requires that the Universe is accelerating. In decelerating cosmological epochs, the condition (4.34) is always violated for $c_t^2>0$. During the radiation-dominated ($w_{\rm eff}=1/3$) and matter-dominated ($w_{\rm eff}=0$) eras, we have $c_{s2}^2=-(4+c_t^2)/3$ and $c_{s2}^2=-(2+c_t^2)/3$, respectively, which are both negative for $c_t^2>0$.

We thus showed that, for scalar-GB couplings $f(\phi,\mathcal{G})$ containing nonlinear functions of $\mathcal{G}$, $\delta\chi$ is prone to the Laplacian instability during the radiation and matter eras. Hence nonlinear functions of $\mathcal{G}$ should not be present in decelerating cosmological epochs. Even if c_{s2}^2 is positive in the inflationary epoch, c_{s2}^2 changes its sign during the transition to a reheating epoch (in which $w_{\rm eff}\simeq 0$ for a standard reheating scenario). During the epoch of late-time cosmic acceleration, c_{s2}^2 can be positive, but it changes the sign as we go back to the matter era. Since $\delta\chi$ is coupled to $\delta\phi$ and $\delta\rho$, the instability of $\delta\chi$ leads to the growth of $\delta\phi$ and $\delta\rho$ for perturbations deep inside the Hubble radius. This violates the successful background evolution during the decelerating cosmological epochs.

The squared propagation speeds (4.29)-(4.31) have been derived by choosing the flat gauge (4.17), but they are independent of the gauge choices. Indeed, we will show in Appendix A that the same values of c_{s1}^2, c_{s2}^2, and c_{s3}^2 can be obtained by choosing the unitary gauge. We also note that the scalar propagation speed squared (2.15) in $\xi(\phi)\mathcal{G}$ gravity is not equivalent to the value (4.29). As we observe in Eq. (2.15), the propagation of ϕ is affected by the coupling $\xi(\phi)$ with the linear GB term $\mathcal{G}$. In $f(\phi,\mathcal{G})$ theory with $f_{,\mathcal{G}\mathcal{G}}\neq 0$, the new scalar field χ plays a role of the dynamical DOF arising from the nonlinear GB term. In this latter case, the propagation of the other field ϕ does not practically acquire the effect of a coupling with the GB term and hence c_{s1} reduces to the luminal value.

[1] If we eliminate c_t^2 by using Eq. (3.15), we can express Eq. (4.30) in the form

$$c_{s2}^2=1+\frac{2\dot{H}}{H^2}+\frac{\eta\dot{\phi}^2+\rho+P}{3q_tH^2}\,. \tag{4.32}$$

From this expression, it seems that the existence of the last term can lead to $c_{s2}^2>0$ even in the decelerating Universe. In the absence of matter ($\rho=0=P$), this possibility was suggested in Ref. [93]. Eliminating q_t instead of c_t^2 from Eq. (4.30), it is clear that this possibility is forbidden even in the presence of matter.

4.2. $f(\mathcal{G})$ gravity

Finally, we also study the stability of scalar perturbations in $f(\mathcal{G})$ gravity given by the action

$$\mathcal{S}=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R+f(\mathcal{G})\right]+\mathcal{S}_m(g_{\mu\nu},\Psi_m)\,. \tag{4.35}$$

In this case, there is no scalar field ϕ coupled to the GB term. The action (4.35) is equivalent to Eq. (3.2) with $\phi=0$, $X=0$, $V(\phi)=0$, $U=-f(\chi)+\chi\xi(\chi)$, and $\xi=f_{,\chi}(\chi)$. As shown in Ref. [103], this theory belongs to a subclass of Horndeski theories with one scalar DOF χ besides a matter fluid.

In $f(\mathcal{G})$ gravity, the second-order action of scalar perturbations is obtained by setting ϕ, $\delta\phi$, and their derivatives 0 in Eqs. (4.11) and (4.12). We choose the flat gauge (4.17) and eliminate α, B, v from the action by using Eqs. (4.14)-(4.16). Then, the second-order scalar action reduces to the form (4.18) with 2×2 matrices $\boldsymbol{K}$, $\boldsymbol{G}$, $\boldsymbol{M}$, $\boldsymbol{B}$ and two dynamical perturbations

$$\vec{\mathcal{X}}^t=(\delta\chi,\delta\rho/k)\,. \tag{4.36}$$

In the small-scale limit, nonvanishing components of $\boldsymbol{K}$ and $\boldsymbol{G}$ are given by

$$K_{11}=\frac{3C_4^2q_t}{(3q_t-M_{\rm Pl}^2)^2}\,,\qquad K_{22}=\frac{a^2}{2(\rho+P)}\,, \tag{4.37}$$

$$G_{11}=-\frac{C_4^2[\rho+P+3q_t[(c_t^2-3)H^2-2\dot{H}]}{2H^2(3q_t-M_{\rm Pl}^2)^2}\,,\qquad G_{22}=\frac{a^2c_m^2}{2(\rho+P)}\,. \tag{4.38}$$

The no-ghost conditions correspond to $K_{11}>0$ and $K_{22}>0$, which are satisfied for $q_t>0$ and $\rho+P>0$. The propagation speed squared for $\delta\chi$ is

$$c_{s1}^2=\frac{G_{11}}{K_{11}}=-\frac{\rho+P+3q_t[(c_t^2-3)H^2-2\dot{H}]}{6H^2q_t}=-\frac{1}{3}\left(2+c_t^2+6w_{\rm eff}\right)\,, \tag{4.39}$$

where, in the last equality, we used the background Eq. (3.15) with $\dot{\phi}=0$. The other matter propagation speed squared is given by $c_{s2}^2=G_{22}/K_{22}=c_m^2$. Since the last expression of Eq. (4.39) is of the same form as Eq. (4.33), the Laplacian instability of $\delta\chi$ is present in decelerating cosmological epochs. In Ref. [100], violent growth of matter perturbations was found during the radiation and matter eras for $f(\mathcal{G})$ models of late-time cosmic acceleration. This is attributed to the Laplacian instability of $\delta\chi$ coupled to $\delta\rho$, which inevitably occurs for nonlinear functions of $f(\mathcal{G})$.

5. Conclusions

In this letter, we studied the stability of cosmological perturbations on the spatially flat FLRW background in scalar-GB theories given by the action (3.1). Provided that $f_{,\mathcal{G}\mathcal{G}}\neq 0$, the action (3.1) is equivalent to (3.2) with a new scalar DOF χ arising from nonlinear GB terms. Theories with $f_{,\mathcal{G}\mathcal{G}}=0$ correspond to a linear GB term coupled to a scalar field ϕ of the form $\xi(\phi)\mathcal{G}$, which belongs to a subclass of Horndeski theories. To make a comparison with the scalar-GB coupling $f(\phi,\mathcal{G})$ containing nonlinear functions of $\mathcal{G}$, we first revisited stabilities of cosmological perturbations in $\xi(\phi)\mathcal{G}$ gravity in Sec. 2. In this latter theory, provided that the scalar-GB coupling is subdominant to the background equations of motion, the stability conditions of tensor and scalar perturbations can be consistently satisfied.

In Sec. 3, we derived the background equations and stability conditions of tensor perturbations for the scalar-GB coupling $f(\phi,\mathcal{G})$ with $f_{,\mathcal{G}\mathcal{G}}\neq 0$. Besides a canonical scalar field ϕ with the kinetic term ηX and the potential $V(\phi)$, we incorporate a perfect fluid given by the Schutz-Sorkin action (3.6). The absence of ghosts and Laplacian instabilities requires that the quantities q_t and c_t^2 defined by Eqs. (3.18) and (3.19) are both positive. In terms of q_t and c_t^2, the background equations of motion in the gravitational sector can be expressed in a simple manner as Eqs. (3.14) and (3.15), where the latter is used to simplify a scalar sound speed later.

In Sec. 4, we expanded the action in $f(\phi,\mathcal{G})$ gravity with $f_{,\mathcal{G}\mathcal{G}}\neq 0$ up to quadratic order in scalar perturbations. After eliminating nondynamical variables α, B, and v, the second-order action is of the form (4.18) with three dynamical perturbations (4.19). With the no-ghost condition $q_t>0$ of tensor perturbations, the scalar ghosts are absent for $\eta>0$ and $\rho+P>0$. The sound speeds of perturbations $\delta\phi$ and $\delta\rho$ have the standard values 1 and c_m, respectively. However, the squared propagation speed of $\delta\chi$, which arises from nonlinear GB functions in f, has a nontrivial value $c_{s2}^2=-(2+c_t^2+6w_{\rm eff})/3$. Since the positivity of c_{s2}^2 requires that $w_{\rm eff}<-(2+c_t^2)/6$, we have $w_{\rm eff}<-1/3$ under the absence of Laplacian instability in the tensor sector ($c_t^2>0$). This means that the scalar perturbation associated with nonlinearities of the GB term is subject to Laplacian instability during decelerating cosmological epochs including radiation and matter eras. The same property also holds for $f(\mathcal{G})$ gravity with $f_{,\mathcal{G}\mathcal{G}}\neq 0$.

We thus showed that a canonical scalar field ϕ coupled to a nonlinear GB term does not modify the property of negative values of c_{s2}^2 in the decelerating Universe. During inflation or the epoch of late-time cosmic acceleration, it is possible to avoid Laplacian instability of the perturbation $\delta\chi$ in $f(\phi,\mathcal{G})$ gravity with $f_{,\mathcal{G}\mathcal{G}}\neq 0$. However, in the subsequent reheating period after inflation or in the preceding matter era before dark energy dominance, the Laplacian instability inevitably emerges to violate the successful background cosmological evolution. We have shown this for a canonical scalar field ϕ, but it may be interesting to see whether the same property persists for the scalar field ϕ arising in Horndeski theories and its extensions like DHOST theories [110,111]. While we focused on the analysis on the FLRW background, it will be also of interest to study whether some instabilities are present for perturbations on a static and spherically symmetric background in $f(\phi,\mathcal{G})$ gravity with $f_{,\mathcal{G}\mathcal{G}}\neq 0$. The latter is important for the construction of stable hairy black hole or neutron star solutions in theories beyond the scalar-GB coupling $\xi(\phi)\mathcal{G}$. These issues are left for future works.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgements

ST is supported by the Grant-in-Aid for Scientific Research Fund of the JSPS Nos. 19K03854 and 22K03642.

Appendix A. Stability conditions in $f(\phi, \mathcal{G})$ gravity in unitary gauge

In this Appendix, we derive stability conditions of scalar perturbations in $f(\phi, \mathcal{G})$ gravity by choosing the unitary gauge

$$\delta\phi = 0\,. \tag{A.1}$$

Then, the gauge-invariant perturbations $\mathcal{R} = \zeta - H\delta\phi/\dot{\phi}$, $\delta\chi_{\rm u} = \delta\chi - \dot{\chi}\,\delta\phi/\dot{\phi}$, and $\delta\rho_{\rm u} = \delta\rho - \dot{\rho}\,\delta\phi/\dot{\phi}$ reduce, respectively, to ζ, $\delta\chi$, and $\delta\rho$. After the elimination of nondynamical variables α, B, v from Eq. (4.10), the second-order action reduces to the form (4.18) with the dynamical perturbations

$$\vec{\mathcal{X}}^t = (\zeta, \delta\chi, \delta\rho/k)\,, \tag{A.2}$$

where nonvanishing matrix components of $\boldsymbol{K}$ and $\boldsymbol{G}$ are $K_{11}, K_{22}, K_{12} = K_{21}, K_{33}$ and $G_{11}, G_{22}, G_{12} = G_{21}, G_{33}$. In the short-wavelength limit, the ghosts are absent for

$$K_{33} = \frac{a^2}{2(\rho + P)} > 0\,, \tag{A.3}$$

$$K_{11}K_{22} - K_{12}^2 = \frac{3C_4^2\,\eta q_t \dot{\phi}^2}{2H^2(3q_t - M_{\rm Pl}^2)^2} > 0\,, \tag{A.4}$$

$$\det \boldsymbol{K} = \frac{3C_4^2\,\eta q_t a^2 \dot{\phi}^2}{4(\rho + P)H^2(3q_t - M_{\rm Pl}^2)^2} > 0\,. \tag{A.5}$$

Under the tensor no-ghost condition $q_t > 0$, inequalities (A.3)-(A.5) are satisfied for $\rho + P > 0$ and $\eta > 0$. These conditions are the same as those derived by choosing the flat gauge.

The scalar propagation speed squared c_s^2 can be derived by solving the dispersion relation (4.28). On using the background Eq. (3.15), we obtain the three values of c_s^2 exactly the same as Eqs. (4.29)-(4.31). Thus, the propagation speeds in the small-scale limit are independent of the gauge choices. In $f(\mathcal{G})$ gravity, we also obtain the same scalar propagation speeds as those derived in the flat gauge.

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