Table of Common Distributions

Bernoulli (p)

$P(X=x \mid p)=p^x(1-p)^{1-x}$; $x=0,1$; $0\le p\le 1$

$EX=p$, $Var\,X=p(1-p)$ $M_X(t)=(1-p)+pe^t$

Binomial (n, p)

$P(X=x \mid n,p)=\binom{n}{x}p^x(1-p)^{n-x}$

$x=0,1,2,\dots,n$; $0\le p\le 1$

$EX=np$, $Var\,X=np(1-p)$ $M_X(t)=[pe^t+(1-p)]^n$

Multinomial (m, p_1, ..., p_n)

$f(x_1,\dots,x_n)=\frac{m!}{x_1!\dots x_n!}p_1^{x_1}\cdots p_n^{x_n}=m!\prod_{i=1}^{n}\frac{p_i^{x_i}}{x_i!}$

Geometric (p)

$P(X=x \mid p)=p(1-p)^{x-1}$; $x=1,2,\dots$; $0\le p\le 1$

$EX=\frac{1}{p}$, $Var\,X=\frac{1-p}{p^2}$

$M_X(t)=\frac{pe^t}{1-(1-p)e^t}$, $t<-log(1-p)$

Hypergeometric

$P(X=x \mid N,M,K)=\frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}$; $x=0,1,\dots,K$

$M-(N-K)\le x\le M$; $N,M,K\ge 0$

$EX=\frac{KM}{N}$, $Var\,X=\frac{KM}{N}\frac{(N-M)(N-K)}{N(N-1)}$

Negative binomial (r, p)

$P(X=x \mid r,p)=\binom{r+x-1}{x}p^r(1-p)^x$;$x=0,1,\dots$;$0\le p\le 1$

$EX=\frac{r(1-p)}{p}$, $Var\,X=\frac{r(1-p)}{p^2}$

$M_X(t)=\left(\frac{p}{1-(1-p)e^t}\right)^r$, $t<-log(1-p)$

Poisson (λ)

$P(X=x \mid \lambda)=\frac{e^{-\lambda}\lambda^x}{x!}$; $x=0,1,\dots$; $0\le\lambda<\infty$

$EX=\lambda$, $Var\,X=\lambda$ $M_X(t)=e^{\lambda(e^t-1)}$

Beta (α, β)

$f(x \mid \alpha,\beta)=\frac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}$, $0\le x\le 1$, $\alpha>0$, $\beta>0$

$EX=\frac{\alpha}{\alpha+\beta}$, $Var\,X=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

$M_X(t)=1+\sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1}\frac{\alpha+r}{\alpha+\beta+r}\right)\frac{t^k}{k!}$

Cauchy (θ, σ)

$f(x \mid \theta,\sigma)=\frac{1}{\pi\sigma}\frac{1}{1+\left(\frac{x-\theta}{\sigma}\right)^2}$, $\sigma>0$

Mean and variance **Do not exist**

If X and Y are independent N(0,1), X/Y is Cauchy

Chi squared (p)

$f(x \mid p)=\frac{x^{p/2-1}e^{-x/2}}{\Gamma(p/2)2^{p/2}}$; $0\le x<\infty$; $p=1,2,\dots$

$EX=p$, $Var\,X=2p$ $M_X(t)=\left(\frac{1}{1-2t}\right)^{p/2}$, $t<\frac{1}{2}$

$\chi^2_{(m)}\sim Gamma\left(\frac{m}{2},2\right)$

Double exponential (μ, σ)

$f(x \mid \mu,\sigma)=\frac{1}{2\sigma}e^{-|x-\mu|/\sigma}$, $\sigma>0$

$EX=\mu$, $Var\,X=2\sigma^2$ $M_X(t)=\frac{e^{\mu t}}{1-(\sigma t)^2}$, $|t|<\frac{1}{\sigma}$

Exponential (*β*)

$f(x \mid \beta)=\frac{1}{\beta}e^{-x/\beta}$, $0\le x<\infty$, $\beta>0$

$EX=\beta$, $Var\,X=\beta^2$ $M_X(t)=\frac{1}{1-\beta t}$, $t<\frac{1}{\beta}$

F

$f(x \mid \nu_1,\nu_2)=\frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}}\cdot\frac{x^{(\nu_1-2)/2}}{\left(1+\left(\frac{\nu_1}{\nu_2}\right)x\right)^{(\nu_1+\nu_2)/2}}$

$EX=\frac{\nu_2}{\nu_2-2}$, $\nu_2>2$

$Var\,X=2\left(\frac{\nu_2}{\nu_2-2}\right)^2\cdot\frac{\nu_1+\nu_2-2}{\nu_1(\nu_2-4)}$, $\nu_2>4$

$EX^n=\frac{\Gamma\left(\frac{\nu_1+2n}{2}\right)\Gamma\left(\frac{\nu_2-2n}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\left(\frac{\nu_2}{\nu_1}\right)^n$, $n<\frac{\nu_2}{2}$

$F_{\nu_1,\nu_2}=\left(\frac{\chi^2_{\nu_1}}{\nu_1}\right)/\left(\frac{\chi^2_{\nu_2}}{\nu_2}\right)$ $F_{1,\nu}=t_\nu^2$

Gamma (α, β)

$f(x \mid \alpha,\beta)=\frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$, $0\le x<\infty$, $\alpha,\beta>0$

$EX=\alpha\beta$, $Var\,X=\alpha\beta^2$ $M_X(t)=\left(\frac{1}{1-\beta t}\right)^\alpha$, $t<\frac{1}{\beta}$

Logistic (μ, β)

$f(x \mid \mu,\beta)=\frac{1}{\beta}\frac{e^{-(x-\mu)/\beta}}{[1+e^{-(x-\mu)/\beta}]^2}$, $\beta>0$

$EX=\mu$, $Var\,X=(\pi^2\beta^2)/3$

$M_X(t)=e^{\mu t}\Gamma(1-\beta t)\Gamma(1+\beta t)$, $|t|<\frac{1}{\beta}$

Lognormal (μ, α)

$f(x \mid \mu,\sigma^2)=\frac{1}{\sqrt{2\pi}\,\sigma}\frac{e^{-(log(x)-\mu)^2/(2\sigma^2)}}{x}$, $0\le x<\infty$

$EX=e^{\mu+\sigma^2/2}$, $Var\,X=e^{2(\mu+\sigma^2)}-e^{2\mu+\sigma^2}$

$EX^n=e^{n\mu+n^2\sigma^2/2}$

Normal (μ, σ²)

$f(x \mid \mu,\sigma^2)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/(2\sigma^2)}$

$EX=\mu$, $Var\,X=\sigma^2$ $M_X(t)=e^{\mu t+\sigma^2t^2/2}$

Pareto (α, β)

$f(x \mid \alpha,\beta)=\frac{\beta\alpha^\beta}{x^{\beta+1}}$, $0<\alpha x<\infty$, $\alpha>0$, $\beta>0$

$EX=\frac{\beta\alpha}{\beta-1}$, $\beta>1$, $Var\,X=\frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}$, $\beta>2$

t

$f(x \mid \nu)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\left(\frac{x^2}{\nu}\right)\right)^{(\nu+1)/2}}$, $\nu=1,\dots$

$EX=0$, $\nu>1$, $Var\,X=\frac{\nu}{\nu-2}$

$EX^n=\frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{\nu-n}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\nu^{n/2}$ if n<v and even

$EX^n=0$ if n<v and odd $F_{1,\nu}=t_\nu^2$

Uniform (a, b)

$f(x \mid a,b)=\frac{1}{b-a}$, $a\le x\le b$

$EX=\frac{b+a}{2}$, $Var\,X=\frac{(b-a)^2}{12}$ $M_X(t)=\frac{e^{bt}-e^{at}}{(b-a)^t}$

Weibull (γ, β)

$f(x \mid \gamma,\beta)=\frac{\gamma}{\beta}x^{\gamma-1}e^{-x^\gamma/\beta}$, $0\le x<\infty$, $\gamma>0$, $\beta>0$

$EX=\beta^{1/\gamma}\Gamma\left(1+\frac{1}{\gamma}\right)$, $Var\,X=\beta^{2/\gamma}\left[\Gamma\left(1+\frac{2}{\gamma}\right)-\Gamma^2\left(1+\frac{1}{\gamma}\right)\right]$

$EX^n=\beta^{n/\gamma}\Gamma\left(1+\frac{n}{\gamma}\right)$