6.4. Figure 6.12 shows some data on the number of coal-mining disasters per year between 1851 and 1962, available from the website for this book. These data originally appeared in [434] and were corrected in [349]. The form of the data we consider is given in [91]. Other analyses of these data include [445, 525].

The rate of accidents per year appears to decrease around 1900, so we consider a change-point model for these data. Let $j = 1$ in 1851, and index each year thereafter, so $j = 112$ in 1962. Let X_j be the number of accidents in year j, with $X_1, \ldots, X_\theta \sim$ i.i.d. Poisson(λ_1) and $X_{\theta+1}, \ldots, X_{112} \sim$ i.i.d. Poisson(λ_2). Thus the change-point occurs after the θth year in the series, where $\theta \in \{1, \ldots, 111\}$. This model has parameters θ, λ_1, and λ_2. Below are three sets of priors for a Bayesian analysis of this model. In each case, consider sampling from the priors as the first step of applying the SIR algorithm for simulating from the posterior for the model parameters. Of primary interest is inference about θ.

a. Assume a discrete uniform prior for θ on $\{1, 2, \ldots, 111\}$, and priors $\lambda_i | a_i \sim$ Gamma($3, a_i$) and $a_i \sim$ Gamma(10, 10) independently for $i = 1, 2$. Using the SIR approach, estimate the posterior mean for θ, and provide a histogram and a credible interval for θ. Provide similar information for estimating λ_1 and λ_2. Make a scatterplot of λ_1 against λ_2 for the initial SIR sample, highlighting the points resampled at the second stage of SIR. Also report your initial and resampling sample sizes, the number of unique points and highest observed frequency in your resample, and a measure of the effective sample size for importance sampling in this case. Discuss your results.

Gamma	$X \sim$Gamma(r, λ) $\lambda > 0$ and $r > 0$	$f(x) = \dfrac{\lambda^r x^{r-1}}{\Gamma(r)} \exp\{-\lambda x\}$ $x > 0$	$E\{X\} = r/\lambda$ $\text{var}\{X\} = r/\lambda^2$
Poisson	$X \sim$Poisson(λ) $\lambda > 0$	$f(x) = \dfrac{\lambda^x}{x!} \exp\{-\lambda\}$ $x \in \{0, 1, \ldots\}$	$E\{X\} = \lambda$ $\text{var}\{X\} = \lambda$