Math 537 HW 1

July 9, 2024

Problem 1

Let X have covariance matrix:

$$\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

a.) Find $v^{\frac{1}{2}}$ then find ρ

b.) Compute the correlation matrix between x_1 and $(\frac{1}{2}x_2 + \frac{1}{2}x_3$

Problem 2

If $\vec{x}$ is multivariate normally distributed with $\vec{\mu} = (-1, 1)$ and $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Find $F_{\vec{x}}(0, 0)$

Problem 3

If $f(\vec{x}) = (x_1 + x_2^2 + x_3)^2$ for $\vec{x} = (x_1, x_2, x_3)$, find the gradient and Hessian of f() with respect to $\vec{x}$.

Problem 4

If

$$A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 3 & -1 \\ 4 & -1 & 1 \end{bmatrix}$$

diagonalize A into $\Gamma\Lambda\Gamma^t$. Simply report Γ and Λ.

Problem 5

Download the HW1.csv file. In this file you will find three variables: y, x_1 and x_2.

a.) Produce r^2 for the following three linear regression models:

i. y $\sim x_1 + x_2$
ii. y $\sim x_1$
iii. y $\sim x_2$

b.) Compute the standardized eigen vectors for the covariance matrix of x_1 and x_2. Store eigen vectors in a matrix Γ.

c.) Create a scatter plot for x1,x2. Add the vectors $\lambda_1 e_1$ and $\lambda_2 e_2$ to your plot. (You might want to stretch them a bit, maybe multiply them by their eigen values or some constant times their eigen values so you can see them more clearly).

d.) Let c_1 and c_2 be new eigen transformed predictors. That is to say, c_1 and c_2 are the columns of (x1,x2)Γ.

Produce r^2 for the following models

iv. y $\sim c_1 + c_2$
v. y $\sim c_1$
vi. y $\sim c_2$

e.) Which model using only 1 variable ii, iii, v, vi was best? Which model using two variables i, iv was best?

Side note, you've just performed principle component analysis...more on this later!