

# Homework 5.1

Michael Pena

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```
# make EM function
EM <- function(th0,maxit,tolerr){
  # print header
  header = paste0("Iteration", "          Theta", "          Modified Relative Err")
  print(header)
  # set initials
  it = 1
  mre = 1

  #run loop
  while(it <= maxit && mre >= tolerr){
    # E part
    EX2 = 1997*th0/(th0 + 2)
    # M part
    th1 = (EX2 + 32)/(EX2 + 1843)
    # reset mre
    mre = abs(th1 - th0)/pmax(1,abs(th1))
    # print
    print(sprintf('%2.0f          %12.12f          %.1e',it, th0, mre))
    it = 1 + it
    th0 <- th1
  }
  th1
}
```

part (a).

```
EM(0.02,200,1e-06)
```

## [1] "Iteration	Theta	Modified Relative Err"
## [1] " 1	0.020000000000	7.8e-03"
## [1] " 2	0.027793132773	3.9e-03"
## [1] " 3	0.031742941132	2.0e-03"
## [1] " 4	0.033721123764	9.8e-04"
## [1] " 5	0.034705946241	4.9e-04"
## [1] " 6	0.035194771667	2.4e-04"
## [1] " 7	0.035437045211	1.2e-04"
## [1] " 8	0.035557033560	5.9e-05"
## [1] " 9	0.035616437341	2.9e-05"
## [1] "10	0.035645841640	1.5e-05"
## [1] "11	0.035660395186	7.2e-06"
## [1] "12	0.035667598091	3.6e-06"

```
## [1] "13"          0.035671162906      1.8e-06"
## [1] "14"          0.035672927162      8.7e-07"
## [1] 0.0356738
```

part (b).

```
# make EM function
EM_conv <- function(th0,maxit,tolerr){
  # print header
  header = paste0("Iteration", "      Linear Convergence", "      Quad. Convergence")
  print(header)
  # set initials
  it = 1
  mre = 1

  #run loop
  while(it <= maxit && mre >= tolerr){
    # E part
    EX2 = 1997*th0/(th0 + 2)
    # M part
    th1 = (EX2 + 32)/(EX2 + 1843)
    # reset mre
    mre = abs(th1 - th0)/pmax(1,abs(th1))
    #convergence formulaes
    th_star = (sqrt(3728689) - 1657)/7680
    # check linear convergence
    l_conv = abs(th1 - th_star)/abs(th0 - th_star)
    # check quadratic convergence
    q_conv = abs(th1 - th_star)/(abs(th0 - th_star)^2)
    # print
    print(sprintf('%2.0f      %12.12f      %12.12f',it, l_conv, q_conv))
    it = 1 + it
    th0 <- th1
  }
  th1
}

EM_conv(0.02,200,1e-06)
```

```
## [1] "Iteration      Linear Convergence      Quad. Convergence"
## [1] " 1"          0.502819528447      32.078505207879"
## [1] " 2"          0.498852146527      63.293876500508"
## [1] " 3"          0.496865162528      126.373656677162"
## [1] " 4"          0.495875958360      253.835587735340"
## [1] " 5"          0.495384958043      511.386453971229"
## [1] " 6"          0.495141606571      1031.794029415581"
## [1] " 7"          0.495021084368      2083.329025856852"
## [1] " 8"          0.494961416286      4208.058958630665"
## [1] " 9"          0.494931881161      8501.284582371960"
## [1] "10"         0.494917262862      17176.168705584194"
## [1] "11"         0.494910027911      34704.624195777113"
## [1] "12"         0.494906447236      70122.590270009168"
## [1] "13"         0.494904675128      141688.069683164387"
```

```
## [1] "14                0.494903798105          286293.150414111209"
## [1] 0.0356738
```

Algorithm seems to be linearly convergent at given the conditions where  $\text{tolerr} = 10^{-6}$ ,  $\theta^{(0)} = 0.02$ , and maximum iterations is 200.