Name (please print)____

Note: Show your work on all problems. Each part of each problem is worth 5 points. A total of 25 points is possible.

1. Let $X \sim \text{gamma}(\alpha, \beta)$. Show that $EX^n = \beta^n \Gamma(n+\alpha)/\Gamma(\alpha)$, where n is a positive integer.

2. Let X be a continuous random variable with pdf f_X and cdf F_X. Moreover, assume that f_X is symmetric about a point a.

   (a) Show that the random variables $U = X - a$ and $W = a - X$ have the same distribution.

   (b) Assuming that the k-th central moment of X exists, show that for an odd positive integer $E[X-a]^k = 0$.

3. Let X be a random variable with pmf $f_X(x) > 0$ for $x = 1, 2, 3, \cdots$ (positive integers), and $f_X(x) = 0$ for all other values of x. Then, the pmf of X_T, the random variable X truncated at $X = 1$, is given by

$$f_{X_T}(x) = \frac{f_X(x)}{P(X > 1)}, \quad \text{for} \quad x = 2, 3, \cdots .$$

(a) Verify that $f_{X_T}(x)$ is a pmf.

(b) Assume that $f_X(1) = 1/4$, $E(X) = \mu$. Obtain $E(X_T)$ as a function of μ.