homework 3 (part 2)

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part (a).

```
#function to square root a matrix "A"
sqrtm <- function(A){
  a <- eigen(A)
  sqm <- a$vectors %*% diag(sqrt(a$values)) %*% t(a$vectors)
  sqm <- (sqm+t(sqm))/2
}

#function for generating data
gen <- function(n,p,mu,sigma,seed){
  #generate data from a p-variate normal with mean mu and covaraince sigma
  #set seed to 2024
  set.seed(seed)
  #generate data from normal
  z <- matrix(rnorm(n*p),n,p)
  datan <- z %*% sqrtm(sigma) + matrix(mu,n,p,byrow = TRUE)
  return(datan)
}
```

```
# putting in the data
sig <- matrix(c(1,0.7,0.7,0.7,1,0.7,0.7,0.7,1), nrow = 3, ncol = 3)
mu <- matrix(c(-1,1,2), nrow =3)

gen(200,3,mu,sig,2024)
```

```
##               [,1]        [,2]        [,3]
##   [1,]  0.53417448  1.99752690  4.09201078
##   [2,] -0.16493028  1.83871169  3.01017072
##   [3,] -1.29141617  0.34173506  1.87173671
##   [4,] -1.39598410  0.83310406  1.26631856
##   [5,]  0.68205483  2.60780901  3.57243184
##   [6,] -0.88565598 -0.14674974  0.27808135
##   [7,] -0.69750624  0.59780452  2.14451035
##   [8,] -1.09882754  1.04972969  1.91336619
##   [9,] -1.67822970  1.47496380  2.16921295
##  [10,] -1.60920380  1.24125190  2.39108885
##  [11,] -2.23788061  0.29942350  2.44353291
##  [12,] -0.37432156  1.67520507  2.41081941
##  [13,] -0.18077742  1.05461261  2.81267717
##  [14,] -1.41258992 -1.07511362  1.82815926
##  [15,] -3.42502001  0.72039198  1.77343558
##  [16,] -0.97174318  1.32804593  2.15130492
##  [17,] -1.94712110  0.94492819  0.97059740
##  [18,] -0.69371906  1.34365902  1.61689178
##  [19,] -3.13562528 -0.58265318 -0.11015700
##  [20,] -2.20899656  1.48406340  2.22712714
##  [21,] -0.78603333  0.23242330  0.64893186
##  [22,]  0.30336506  2.94955445  3.99320947
##  [23,] -0.06996822  1.43254200  2.30312797
##  [24,] -1.35670019  0.71968853  1.52302234
##  [25,] -0.35647072  1.44041960  2.89314453
##  [26,] -2.37864381  0.59793097  1.30649220
##  [27,] -0.64519983  1.37318653  3.21899921
##  [28,] -0.70533563  1.17105610  1.16134800
##  [29,] -2.62876466  0.35704355  1.93835358
##  [30,] -0.41827341  1.97903141  3.41703125
##  [31,] -1.11895437  2.40750024  3.81914969
##  [32,] -0.57885694  1.96950298  3.39072687
##  [33,] -1.19136124  0.96504973  2.62259713
##  [34,] -0.36872400  0.76746205  1.46984338
##  [35,] -1.11950500  1.13627539  2.69696372
##  [36,] -0.29754482  1.53687037  3.03030480
##  [37,] -1.26784451  0.51258051  0.65481773
##  [38,] -1.90176634  0.08377974  0.87673787
##  [39,] -1.67447138  0.68091749  1.31430989
##  [40,] -0.93277894  0.33050505  1.82539488
##  [41,] -1.90206267  0.69491143  1.23856342
##  [42,] -0.11823969  2.48227265  3.73992398
##  [43,] -0.17940723  2.29689770  2.43151181
##  [44,]  0.96918773  2.50984871  2.64487925
##  [45,] -2.17204100 -0.01504248  1.79016806
##  [46,] -0.38716798  2.57749334  2.32212305
##  [47,] -1.30735554  0.20312069  0.84145435
##  [48,] -0.49953301  1.27572342  1.84799859
##  [49,] -0.74827188  0.28721706  2.00504157
##  [50,]  0.62503731  1.64607417  2.33795950
##  [51,] -1.28795576  1.96286552  1.39831112
##  [52,] -1.31952940  1.19428985  2.58870160
##  [53,] -1.51353522  1.40261706  1.37152472
##  [54,] -2.75477702 -0.34408773 -0.43279946
##  [55,] -1.25358968  1.26836740  2.53911282
##  [56,] -2.46785102 -0.54482729  1.84475950
##  [57,] -0.00454116  1.58599299  2.76202171
##  [58,] -1.01839948  1.02430144  2.51636749
##  [59,] -0.33745920  1.41716382  1.45211222
##  [60,] -1.00195535  0.38553567  2.69018408
##  [61,]  0.54277890  1.53952955  3.02832361
##  [62,] -0.14426192  1.19642577  2.73261717
##  [63,] -0.16915165  1.82597089  2.83049098
##  [64,] -0.70396617  0.78587364  2.84646519
##  [65,] -0.92176518 -1.21489644  0.31611075
##  [66,] -0.92493855  1.47889525  2.12199835
##  [67,] -2.86520514 -0.02129291  1.30280256
##  [68,]  0.30634462  2.31408224  3.62549458
##  [69,]  0.23150997  2.25789949  3.97590141
##  [70,] -2.75473081  0.91243825  1.08471282
##  [71,]  0.55429315  2.42477433  3.35838825
##  [72,] -2.19462304  0.36875699  1.35301088
##  [73,] -1.29245031 -0.18959328  1.37280793
##  [74,] -1.21935581  0.29308283  1.71697512
##  [75,] -0.70821705  1.49738904  2.06631190
##  [76,] -1.27793761  0.91003926  1.87304307
##  [77,]  0.02554407  0.90729673  2.21375689
##  [78,] -1.17342877 -0.41768952  1.04900111
##  [79,] -2.40898382 -0.35234000  0.86318846
##  [80,] -0.80592107  1.12046806  2.54626271
##  [81,] -1.77181141  1.27334672  2.07672487
##  [82,] -0.24609954  1.34196371  1.88165006
##  [83,] -0.01494726  2.25882683  2.13144207
##  [84,] -2.27705195  0.80665837  2.00494659
##  [85,] -0.96250949  1.29031754  1.46792863
##  [86,] -0.03605578  1.14133061  2.54651516
##  [87,] -1.49875520  0.17466638  1.47401813
##  [88,] -1.12516454  1.04962788  2.14332675
##  [89,] -1.01899354  1.71938585  1.84930571
##  [90,]  0.89882697  3.83903234  3.99840582
##  [91,] -2.31906640 -0.02464187  1.34214562
##  [92,] -1.23583906  1.48984347  1.97641106
##  [93,] -0.93887078  2.35512386  2.65870308
##  [94,] -0.60162704  0.79342395  1.90959407
##  [95,] -0.68866068  1.19656646  1.54504681
##  [96,] -2.53746831  0.98025972  1.67327514
##  [97,] -1.78867663  1.56876443  1.52318923
##  [98,] -2.13379593 -0.16106652  2.03066704
##  [99,] -1.95924821 -0.64085443  0.22411093
## [100,]  0.24587843  1.68994571  2.91900082
## [101,]  1.44325119  2.62305919  3.65814080
## [102,] -1.06719251  0.34537004  1.29124880
## [103,]  0.08021062  1.33471846  2.94649601
## [104,] -0.52755558  1.98830317  2.41230004
## [105,] -0.83824562  2.72687412  4.07810303
## [106,] -0.79639447  1.24468421  1.54502801
## [107,] -1.90977471  0.14031724  0.05479277
## [108,]  0.84637816  2.60575890  4.18117044
## [109,] -1.46230681  0.94385158  1.80077219
## [110,] -1.92180166  1.42359133  1.11235713
## [111,] -1.98595370  0.93587148  1.76206064
## [112,] -0.30549654  0.34711752  2.32230042
## [113,] -2.46752046 -0.96517178  0.40412594
## [114,] -1.01905617  0.52034662  2.48367242
## [115,] -0.76109632  3.18633644  2.27204306
## [116,] -3.03629510 -0.26467024  0.50353320
## [117,]  0.44092260  1.37580877  2.02854590
## [118,] -0.64823310  1.08240109  1.85132308
## [119,] -0.96226481  1.75380108  2.05664128
## [120,] -1.12267467  0.02919072  0.21835240
## [121,] -1.85977296 -0.29970211  0.84375170
## [122,] -1.37411824  0.91530867  1.55778655
## [123,] -2.27525074  0.65140734  1.01946224
## [124,] -2.05577727 -0.31986784 -0.02247862
## [125,] -0.86302961  2.98203453  2.32501203
## [126,] -0.46657921  1.42820772  3.44053608
## [127,] -2.66802362 -0.64111353  0.29690758
## [128,] -1.47067765  1.50301766  2.55852777
## [129,] -1.90947032 -0.66741146  1.80258812
## [130,] -0.55270429  0.96300811  2.34484906
## [131,] -2.11264892  0.46458306  0.95057062
## [132,] -1.47826638  0.10629154  0.81307434
## [133,] -1.20862965  1.88032002  2.01158980
## [134,] -0.91695524  0.80098500  2.40799693
## [135,]  0.94462080  2.63813270  3.14398318
## [136,] -1.57195398  0.05996463  3.00047059
## [137,] -1.65030854  1.75792664  1.92663735
## [138,] -1.50778643 -0.93363550  0.74261737
## [139,] -1.32148822  0.50461575  0.49591686
## [140,] -1.06183519  1.34504391  2.19215202
## [141,] -1.10631082  0.23356737  0.97694808
## [142,] -2.79228027 -0.98089048  0.62406285
## [143,] -1.48268144  1.13289953  2.13423664
## [144,]  0.18896500  1.25338282  1.64402065
## [145,] -0.12792780  1.79566703  2.18304842
## [146,]  0.67564123  3.10388967  4.13810103
## [147,]  0.37223558  0.79163942  2.79380725
## [148,] -0.94581973  1.24068626  1.60483732
## [149,] -0.74005245  0.15488652  1.57488264
## [150,]  0.36992049  2.50484117  2.81313251
## [151,] -1.89869252  0.50032172  2.11895347
## [152,] -0.63726520  1.82290418  2.60869386
## [153,]  1.49690638  2.58187727  3.42476094
## [154,] -0.22510949  0.67812471  1.97988632
## [155,] -2.42856763 -1.12565227  0.87472564
## [156,] -3.69132707 -0.35344441  1.05727902
## [157,] -1.03356977  1.18546348  0.85544194
## [158,] -0.48729595  2.12671614  3.51711233
## [159,] -1.33544477  1.69822263  1.89165774
## [160,] -0.93774094  0.29828194  2.18553914
## [161,]  0.89919911  1.81235840  3.68203641
## [162,] -0.16307013  2.13957414  3.91401822
## [163,] -1.46538926  0.12662730  1.19035226
## [164,]  0.12502128  0.91230487  2.21327551
## [165,] -1.74530493  0.70509998  0.54861528
## [166,] -1.64846160  0.40837197  1.83943668
## [167,] -0.84973955  1.16905754  2.02972232
## [168,] -2.30720389  0.70160692  1.92466461
## [169,]  0.33273950  1.98637674  2.75736244
## [170,]  0.07972521  0.48755435  0.86146463
## [171,] -1.07601757  2.08111516  1.57885303
## [172,] -0.64389483  0.61633125  1.32663795
## [173,] -1.29852450  0.74364606  2.41367662
## [174,] -1.35865261  0.44592140  0.72074640
## [175,] -0.35749859  1.45130272  2.87590767
## [176,]  0.48754501  1.78319492  3.69412204
## [177,] -0.16427264  1.04098563  1.84438641
## [178,] -1.06522932  0.59035915  1.75043930
## [179,]  0.78795268  3.66990263  4.02162093
## [180,] -2.09526681 -0.34942338  0.32367128
## [181,] -0.73661441  0.97771660  1.21339749
## [182,] -0.57946606  1.14706266  2.73683512
## [183,] -0.94094261  0.83690957  0.67496214
## [184,] -1.87480930  0.50630338  0.99868941
## [185,] -0.55684300  2.48546276  3.08179078
## [186,] -0.14855433  1.65327463  2.53453956
## [187,] -0.61464148 -0.04501877  1.91492489
## [188,] -1.36795765  1.27910592  1.29708701
## [189,] -1.03034282  1.26416214  3.32855640
## [190,]  0.32123301  2.71977765  3.87425246
## [191,] -1.51021602  0.82007101  1.81869464
## [192,] -2.08503457  0.89468386  0.81182893
## [193,] -0.66631943  1.51901460  1.28424215
## [194,] -0.28126730  0.50860880  2.39972814
## [195,]  0.51629449  2.58789458  2.65955464
## [196,] -0.40656349  1.57616165  2.10129119
## [197,] -0.13614163  0.94454070  1.98732848
## [198,] -0.12621821  1.51875127  3.42099776
## [199,]  0.74474338  0.74682402  2.76788231
## [200,] -2.76214849  0.13068300  1.06253603
```

part (b).

```
# compile Sigma and Mu into a single theta vector
to.theta <- function(mu,sig){
  p <- length(mu)
  theta <- matrix(0,nrow = (p + p*(1+p)/2),ncol = 1)
  theta[1:p] <- mu

  k = p + 1
  for(i in 1:p){
    for(j in 1:i){
      theta[k] <- sig[i,j]
      k = k + 1
    }
  }
  return(theta)
}
```

```
# turning theta back into mu and Sigma
from.theta <- function(p,theta){
  mu <- theta[1:p]
  sig <- matrix(0, nrow = p, ncol = p)

  k = p + 1
  for (i in 1:p){
    for (j in 1:i){
      sig[i,j] <- theta[k]
      sig[j,i] <- sig[i,j]
      k = k + 1
    }
  }
 list(mu = mu, sig = sig)
}
```

```
# make gradient
gradient <- function(x,mu,sig){
  n <- nrow(x)
  p <- ncol(x)

  inv.sig <- solve(sig) # inverse sigma

  # make initials
  xi.sum <- matrix(0, nrow = p, ncol = 1)
  grad.mu <- xi.sum
  C.mu <- matrix(0, nrow = p, ncol = p)

  # take care of C.mu
  for (i in 1:n){
    xi <- as.numeric(x[i,] - mu)
    xi.sum <- xi.sum + xi
    C.mu <- C.mu + (xi %*% t(xi))
  }
  grad.mu <- inv.sig %*% xi.sum
  A <- (n* inv.sig) - inv.sig %*% C.mu  %*% inv.sig
  grad.sig <- matrix(0, nrow = nrow(A), ncol = ncol(A))

  for (i in 1:nrow(sig) - 1){
    grad.sig[i,i] <- -0.5*A[i,i]
  }
  for (i in 1:nrow(sig) - 1){
    for (j in (i+1):ncol(sig)){
      grad.sig[i,j] <- -1*A[i,j]
      grad.sig[j,i] <- -1*A[i,j]
    }
  }
  grad.norm <- norm(to.theta(grad.mu,grad.sig), type = '2')
  list(grad.mu = grad.mu, grad.sig=grad.sig, grad.norm = grad.norm)
}
```

```
likemvn <- function (x,mu,sig) {
  # computes the likelihood and the gradient for multivariate normal
  # if gcomp=FALSE, then the gradient is not computed
  # x is the n by p data matrix
  # mu is the mean
  # sig is the covariance
  # gcomp if TRUE, the gradient with respect to mu will be output
  n = nrow(x)
```

```r
  p = ncol(x)

  C.mu = matrix(0,p,p) # initializing sum of (xi-mu)(xi-mu)^T
  xi.sum = matrix(0,p,1) # initializing sum of xi-mu
  grad.mu = xi.sum; # initializing this sum is used for the gradient w.r.t. mu
  for (i in 1:n){
    xi = as.numeric(x[i,] - mu)
    xi = xi + 1
    C.mu = C.mu + xi %*% t(xi)
  }

  ell = -(n*p*log(2*pi)+n*log(det(sig)) + sum(solve(sig) %*% C.mu ))/2
  return(ell)
}
```

```r
# Steepest ascent

optmvn <- function (x,mu,sig,maxit,tolerr,tolgrad) {
  header = paste0("Iteration",
                  "      Halving",
                  "       log-likelihood",
                  "       ||gradient||")
  print(header)

  for(it in 1:maxit){
    theta0 <- to.theta(mu,sig)
    L <- likemvn(x,mu,sig)
    grad.mu0 <- gradient(x,mu,sig)$grad.mu
    grad.sig0 <- gradient(x,mu,sig)$grad.sig
    grad.norm0 <- gradient(x,mu,sig)$grad.norm

    if (it == 1 | it ==2 | it == 477 | it == 478){
      print(sprintf('%2.0f                 --          %3.4f               %.1e',it,L,grad.norm0))
    }

    direc <- to.theta(grad.mu0,grad.sig0) # get direction
    # get new components
    theta1 <-  theta0 + direc
    mu1 <- from.theta(length(mu), theta1)$mu
    sig1 <- from.theta(length(mu), theta1)$sig
    grad.norm1 <- gradient(x,mu1,sig1)$grad.norm

    if(all(eigen(sig1)$values > 0)){atmp = likemvn(x,mu1,sig1)}
    else{atmp = -Inf}

    halve = 0

    if(it == 1 | it ==2 | it == 477 | it == 478){
    print(sprintf('%2.0f                 %2.0f          %3.4f               %.1e',
                  it,  halve, atmp, grad.norm1))}

    while((all(eigen(sig1)$values <= 0) && halve < 20) || atmp < L){
      halve = halve + 1
      # mathematics
      theta1 <- theta0 + direc/(2^halve)
      mu1 = from.theta(length(mu), theta1)$mu
      sig1 = from.theta(length(mu), theta1)$sig

      if(all(eigen(sig1)$values > 0)){atmp = likemvn(x,mu1,sig1)}
      else{atmp = -Inf}

      grad.norm1 <-  gradient(x,mu1,sig1)$grad.norm

      if(it == 1 | it ==2 | it == 477 | it == 478){
          print(sprintf('%2.0f                %2.0f          %3.4f               %.1e',it,  halve,atmp
      }
    }
    if(it == 1 | it == 2 | it == 477){
      print("-----------------------------------------------------------------")
      print(header)
    }
    r.e <- max(abs(theta0 - theta1)/abs(pmax(1,abs(theta0))))
    theta0 <- theta1

    if (r.e < tolerr & grad.norm1 < tolgrad){break}

    mu <- mu1
    sig <- sig1
  }
  return(list("mu.estimator" = mu,
              "sigma.estimator" = sig,
              "iteration" = it))
}
```

```r
# putting in parameters
x <- gen(200,3,mu,sig,2024)
m <- c(0,0,0)
s <- diag(3)

optmvn(x,m,s,500,1e-6,1e-5)
```

```
## [1] "Iteration      Halving       log-likelihood       ||gradient||"
## [1] " 1                 --          -3846.7751               8.1e+02"
## [1] " 1                  0          -Inf               5.2e+02"
## [1] " 1                  1          -Inf               5.0e+02"
## [1] " 1                  2          -Inf               4.5e+02"
## [1] " 1                  3          -Inf               3.8e+02"
## [1] " 1                  4          -Inf               3.0e+02"
## [1] " 1                  5          -Inf               2.1e+02"
## [1] " 1                  6          -Inf               1.3e+02"
## [1] " 1                  7          -Inf               7.6e+01"
## [1] " 1                  8          -Inf               1.6e+02"
## [1] " 1                  9          -Inf               1.2e+04"
## [1] " 1                 10          -2965.7040               3.5e+02"
## [1] "-----------------------------------------------------------------"
## [1] "Iteration      Halving       log-likelihood       ||gradient||"
## [1] " 2                 --          -2965.7040               3.5e+02"
## [1] " 2                  0          -Inf               4.7e+04"
## [1] " 2                  1          -Inf               6.0e+04"
## [1] " 2                  2          -Inf               1.1e+05"
## [1] " 2                  3          -Inf               2.5e+06"
## [1] " 2                  4          -Inf               4.7e+04"
## [1] " 2                  5          -Inf               3.5e+03"
## [1] " 2                  6          -Inf               6.9e+02"
## [1] " 2                  7          -Inf               9.1e+02"
## [1] " 2                  8          -Inf               2.5e+04"
## [1] " 2                  9          -2156.4052               2.3e+02"
## [1] "------------------------------------------------------------------"
## [1] "Iteration      Halving       log-likelihood       ||gradient||"
## [1] "477                 --          -501.0274               8.9e+05"
## [1] "477                  0          -Inf               2.9e+00"
## [1] "477                  1          -Inf               2.9e+00"
## [1] "477                  2          -Inf               2.9e+00"
## [1] "477                  3          -Inf               2.9e+00"
## [1] "477                  4          -Inf               2.9e+00"
## [1] "477                  5          -Inf               2.9e+00"
## [1] "477                  6          -Inf               2.9e+00"
## [1] "477                  7          -Inf               2.8e+00"
## [1] "477                  8          -Inf               2.8e+00"
## [1] "477                  9          -Inf               2.7e+00"
## [1] "477                 10          -Inf               2.6e+00"
## [1] "477                 11          -Inf               3.2e+00"
## [1] "477                 12          -Inf               5.8e+00"
## [1] "477                 13          -Inf               1.3e+01"
## [1] "477                 14          -Inf               3.7e+01"
## [1] "477                 15          -Inf               1.7e+02"
## [1] "477                 16          -Inf               5.0e+03"
## [1] "477                 17          -Inf               1.4e+02"
## [1] "477                 18          -Inf               9.4e+02"
## [1] "477                 19          -1707.3445               1.9e+02"
## [1] "477                 20          -1446.5475               1.6e+02"
## [1] "477                 21          -1333.9782               2.1e+02"
## [1] "477                 22          -1244.8847               5.4e+02"
## [1] "477                 23          -1160.4694               2.4e+03"
## [1] "477                 24          -1072.9957               9.7e+03"
## [1] "477                 25          -977.5635               3.4e+04"
## [1] "477                 26          -873.1483               9.6e+04"
## [1] "477                 27          -766.6403               2.2e+05"
## [1] "477                 28          -672.2395               3.9e+05"
## [1] "477                 29          -601.6567               5.7e+05"
## [1] "477                 30          -556.2846               7.0e+05"
## [1] "477                 31          -530.0992               7.9e+05"
## [1] "477                 32          -515.9554               8.4e+05"
## [1] "477                 33          -508.5937               8.6e+05"
## [1] "477                 34          -504.8367               8.8e+05"
## [1] "477                 35          -502.9386               8.8e+05"
## [1] "477                 36          -501.9847               8.9e+05"
## [1] "477                 37          -501.5064               8.9e+05"
## [1] "477                 38          -501.2670               8.9e+05"
## [1] "477                 39          -501.1472               8.9e+05"
## [1] "477                 40          -501.0873                8.9e+05"
## [1] "477                 41          -501.0573                8.9e+05"
## [1] "477                 42          -501.0423                8.9e+05"
## [1] "477                 43          -501.0348                8.9e+05"
## [1] "477                 44          -501.0311                8.9e+05"
## [1] "477                 45          -501.0292                8.9e+05"
## [1] "477                 46          -501.0283                8.9e+05"
## [1] "477                 47          -501.0278                8.9e+05"
## [1] "477                 48          -501.0276                8.9e+05"
## [1] "477                 49          -501.0275                8.9e+05"
## [1] "477                 50          -501.0274                8.9e+05"
## [1] "477                 51          -501.0274                8.9e+05"
## [1] "477                 52          -501.0274                8.9e+05"
## [1] "477                 53          -501.0274                8.9e+05"
## [1] "477                 54          -501.0274                8.9e+05"
## [1] "477                 55          -501.0274                8.9e+05"
## [1] "477                 56          -501.0274                8.9e+05"
## [1] "477                 57          -501.0274                8.9e+05"
## [1] "477                 58          -501.0274                8.9e+05"
## [1] "477                 59          -501.0274                8.9e+05"
## [1] "477                 60          -501.0274                8.9e+05"
## [1] "477                 61          -501.0274                8.9e+05"
## [1] "477                 62          -501.0274                8.9e+05"
## [1] "477                 63          -501.0274                8.9e+05"
## [1] "477                 64          -501.0274                8.9e+05"
## [1] "477                 65          -501.0274                8.9e+05"
## [1] "477                 66          -501.0274                8.9e+05"
## [1] "477                 67          -501.0274                8.9e+05"
## [1] "477                 68          -501.0274                8.9e+05"
## [1] "477                 69          -501.0274                8.9e+05"
## [1] "477                 70          -501.0274                8.9e+05"
## [1] "477                 71          -501.0274                8.9e+05"
## [1] "477                 72          -501.0274                8.9e+05"
## [1] "477                 73          -501.0274                8.9e+05"
## [1] "477                 74          -501.0274                8.9e+05"
## [1] "477                 75          -501.0274                8.9e+05"
## [1] "------------------------------------------------------------------"
## [1] "Iteration      Halving       log-likelihood        ||gradient||"
## [1] "478                 --          -501.0274                8.9e+05"
## [1] "478                  0          -Inf                2.9e+00"
## [1] "478                  1          -Inf                2.9e+00"
## [1] "478                  2          -Inf                2.9e+00"
## [1] "478                  3          -Inf                2.9e+00"
## [1] "478                  4          -Inf                2.9e+00"
## [1] "478                  5          -Inf                2.9e+00"
## [1] "478                  6          -Inf                2.9e+00"
## [1] "478                  7          -Inf                2.8e+00"
## [1] "478                  8          -Inf                2.8e+00"
## [1] "478                  9          -Inf                2.7e+00"
## [1] "478                 10          -Inf                2.6e+00"
## [1] "478                 11          -Inf                3.2e+00"
## [1] "478                 12          -Inf                5.8e+00"
## [1] "478                 13          -Inf                1.3e+01"
## [1] "478                 14          -Inf                3.7e+01"
## [1] "478                 15          -Inf               1.7e+02"
## [1] "478                 16          -Inf               5.0e+03"
## [1] "478                 17          -Inf               1.4e+02"
## [1] "478                 18          -Inf               9.4e+02"
## [1] "478                 19          -1707.3445               1.9e+02"
## [1] "478                 20          -1446.5475               1.6e+02"
## [1] "478                 21          -1333.9782               2.1e+02"
## [1] "478                 22          -1244.8847               5.4e+02"
## [1] "478                 23          -1160.4694               2.4e+03"
## [1] "478                 24          -1072.9957               9.7e+03"
## [1] "478                 25          -977.5635               3.4e+04"
## [1] "478                 26          -873.1483               9.6e+04"
## [1] "478                 27          -766.6403               2.2e+05"
## [1] "478                 28          -672.2395               3.9e+05"
## [1] "478                 29          -601.6567               5.7e+05"
## [1] "478                 30          -556.2846               7.0e+05"
## [1] "478                 31          -530.0992               7.9e+05"
## [1] "478                 32          -515.9554               8.4e+05"
## [1] "478                 33          -508.5937               8.6e+05"
## [1] "478                 34          -504.8367               8.8e+05"
## [1] "478                 35          -502.9386               8.8e+05"
## [1] "478                 36          -501.9847               8.9e+05"
## [1] "478                 37          -501.5064               8.9e+05"
## [1] "478                 38          -501.2670               8.9e+05"
## [1] "478                 39          -501.1472               8.9e+05"
## [1] "478                 40          -501.0873               8.9e+05"
## [1] "478                 41          -501.0573               8.9e+05"
## [1] "478                 42          -501.0423               8.9e+05"
## [1] "478                 43          -501.0348               8.9e+05"
## [1] "478                 44          -501.0311               8.9e+05"
## [1] "478                 45          -501.0292               8.9e+05"
## [1] "478                 46          -501.0283               8.9e+05"
## [1] "478                 47          -501.0278               8.9e+05"
## [1] "478                 48          -501.0276               8.9e+05"
## [1] "478                 49          -501.0275               8.9e+05"
## [1] "478                 50          -501.0274               8.9e+05"
## [1] "478                 51          -501.0274               8.9e+05"
## [1] "478                 52          -501.0274               8.9e+05"
## [1] "478                 53          -501.0274               8.9e+05"
## [1] "478                 54          -501.0274               8.9e+05"
## [1] "478                 55          -501.0274               8.9e+05"
## [1] "478                 56          -501.0274               8.9e+05"
## [1] "478                 57          -501.0274               8.9e+05"
## [1] "478                 58          -501.0274               8.9e+05"
## [1] "478                 59          -501.0274               8.9e+05"
## [1] "478                 60          -501.0274               8.9e+05"
## [1] "478                 61          -501.0274               8.9e+05"
## [1] "478                 62          -501.0274               8.9e+05"
## [1] "478                 63          -501.0274               8.9e+05"
## [1] "478                 64          -501.0274               8.9e+05"
## [1] "478                 65          -501.0274               8.9e+05"
## [1] "478                 66          -501.0274               8.9e+05"
## [1] "478                 67          -501.0274               8.9e+05"
## [1] "478                 68          -501.0274               8.9e+05"
## [1] "478                 69          -501.0274               8.9e+05"
## [1] "478                 70          -501.0274               8.9e+05"
## [1] "478                 71          -501.0274               8.9e+05"
## [1] "478                 72          -501.0274               8.9e+05"
## [1] "478                 73          -501.0274               8.9e+05"
## [1] "478                 74          -501.0274               8.9e+05"
## [1] "478                 75          -501.0274               8.9e+05"
```

```
## $mu.estimator
## [1] -1.2045485  0.6976222  2.3888200
##
## $sigma.estimator
##           [,1]      [,2]      [,3]
## [1,] 1.3859638 0.6606713 0.1281701
## [2,] 0.6606713 1.1462171 0.9588739
## [3,] 0.1281701 0.9588739 1.0000000
##
## $iteration
## [1] 500
```