Homework 2 (Part 2)

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(a).

General Secant Method

$$x_{n+2} = x_{n+1} - f(x_{n+1})\frac{x_{n+1} - x_n}{f(x_{n+1}) - f(x_n)}$$

$$\ell'(\theta) = \frac{1997}{2+\theta} - \frac{1811}{1-\theta} + \frac{32}{\theta}$$

```
# render derivative
f <- function(theta){
  1997/(2 + theta) - 1811/(1-theta) + 32/theta
}

# build secant method


secM <- function(theta0,theta1,maxit,tolerr,tolgrad){
# input initials
  it = 0
  absg <- abs(f(theta1))
  mre <- abs(theta1 - theta0)/max(1,abs(theta1))
  store <- data.frame()
  while(it < maxit && (absg > tolgrad || mre > tolerr)){
    # secant method equation
    theta2 <- theta1 - f(theta1)*(theta1 - theta0)/(f(theta1) - f(theta0))
    # storing into dataframe
    absg <- abs(f(theta2))
    mre <- abs(theta1 - theta0)/max(1,abs(theta1))

    # end of the loop
     theta0 <- theta1
     theta1 <- theta2
     it = it + 1
     row <- c(it,theta2,mre,absg)
     store <- rbind(store,row)
  }
# formatting dataframe
  store <- data.frame(store) %>% set_names("Iteration","Theta","Relative Error","Gradient at Theta") %>%
  Theta = sprintf("%12.12f",Theta),
  `Relative Error` = sprintf("%.1e", `Relative Error`),
```

```
  `Gradient at Theta` = sprintf("%.1e", `Gradient at Theta`)
  )
 return(store)
}
```

```
# start process
secM(theta0 = .02,.01,20,1e-6,1e-9)
```

```
##   Iteration          Theta Relative Error Gradient at Theta
## 1         1 0.024561848011        1.0e-02           4.3e+02
## 2         2 0.027823212918        1.5e-02           2.7e+02
## 3         3 0.033351047002        3.3e-03           6.8e+01
## 4         4 0.035197558267        5.5e-03           1.3e+01
## 5         5 0.035646185180        1.8e-03           7.9e-01
## 6         6 0.035674309037        4.5e-04           9.6e-03
## 7         7 0.035674655571        2.8e-05           6.9e-06
## 8         8 0.035674655823        3.5e-07           6.1e-11
```

(b).

```
secM2 <- function(theta0,theta1,maxit,tolerr,tolgrad,star){
# input initials
  it = 0
  gold = (1 + sqrt(5))/2
  absg <- abs(f(theta1))
  mre <- abs(theta1 - theta0)/max(1,abs(theta1))
  store <- data.frame()
  while(it < maxit && (absg > tolgrad || mre > tolerr)){
    # secant method equation
    theta2 <- theta1 - f(theta1)*(theta1 - theta0)/(f(theta1) - f(theta0))
    # storing into dataframe
    absg <- abs(f(theta2))
    mre <- abs(theta1 - theta0)/max(1,abs(theta1))
    rat <- abs(theta1-star)/(abs(theta1 - theta0))^(gold)
    sigdig <- -log10(abs(theta1 - star)/abs(star))

    # end of the loop
     theta0 <- theta1
     theta1 <- theta2
     it = it + 1
     row <- c(it,theta2,rat,sigdig)
     store <- rbind(store,row)
  }
# formatting dataframe
store <- data.frame(store) %>% set_names("Iteration","Theta","Convergence Ratio","Sig. Digits") %>% muta
  Iteration = sprintf("%02.f",Iteration),
  Theta = sprintf("%12.12f",Theta),
  `Convergence Ratio` = sprintf("%.3e",`Convergence Ratio`)
  )
 return(store)
}
```

```
thetstar <- (-1657 + sqrt(3728689))/7680
secM2(.02,.01,20,1e-6,1e-9,thetstar)
```

```
##   Iteration          Theta Convergence Ratio Sig. Digits
## 1        01 0.024561848011         4.422e+01   0.1428552
## 2        02 0.027823212918         1.042e+01   0.5065360
## 3        03 0.033351047002         8.286e+01   0.6574103
## 4        04 0.035197558267         1.044e+01   1.1861968
## 5        05 0.035646185180         1.264e+01   1.8737526
## 6        06 0.035674309037         7.443e+00   3.0979625
## 7        07 0.035674655571         8.010e+00   5.0122985
## 8        08 0.035674655823         7.143e+00   8.1513843
```

(c).

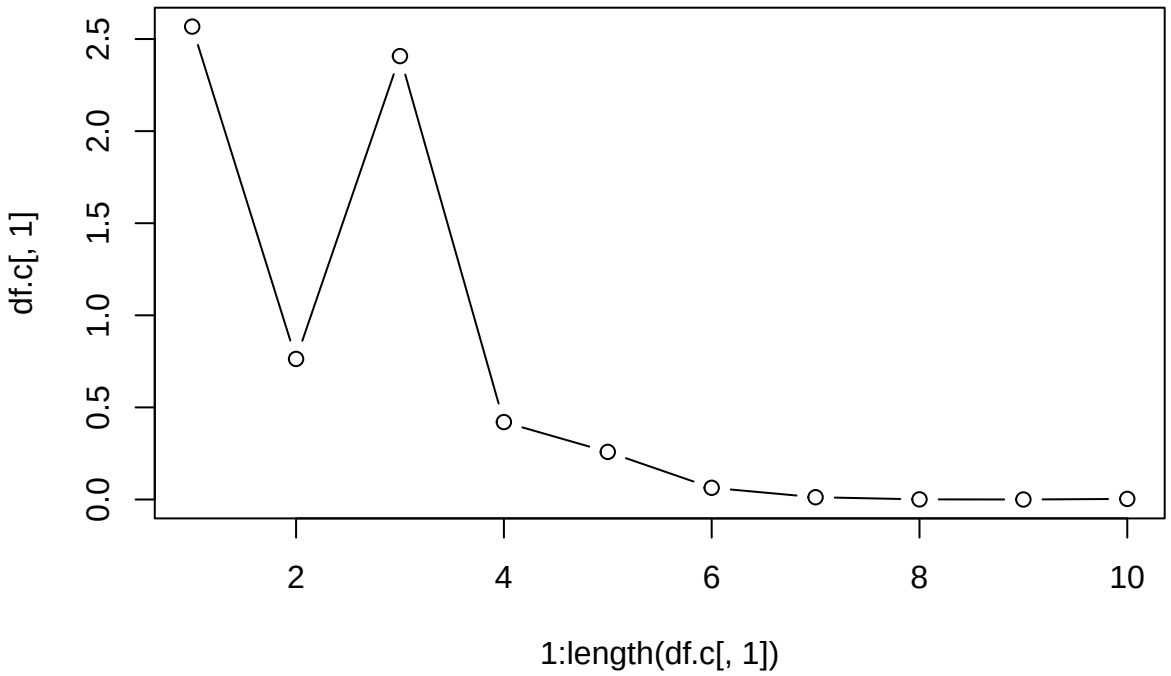
```
secM2.c <- function(theta0,theta1,maxit,tolerr,tolgrad,star){
# input initials
  it = 0
  gold = (1 + sqrt(5))/2
  absg <- abs(f(theta1))
  mre <- abs(theta1 - theta0)/max(1,abs(theta1))
  store <- data.frame()
  while(it < maxit && (absg > tolgrad || mre > tolerr)){
    # secant method equation
    theta2 <- theta1 - f(theta1)*(theta1 - theta0)/(f(theta1) - f(theta0))
    # storing into dataframe
    absg <- abs(f(theta2))
    mre <- abs(theta1 - theta0)/max(1,abs(theta1))
    ratsl <- abs(theta1-star)/(abs(theta1 - theta0))
    ratquad <- abs(theta1-star)/(abs(theta1 - theta0))^2
    sigdig <- -log10(abs(theta1 - star)/abs(star))

    # end of the loop
     theta0 <- theta1
     theta1 <- theta2
     it = it + 1
     row <- c(it,theta2,ratsl,ratquad,sigdig)
     store <- rbind(store,row)
  }
# formatting dataframe
store <- data.frame(store) %>% set_names("Iteration","Theta","Convergence Ratio Lin","Convergence Ratio
  Iteration = sprintf("%02.f",Iteration),
  Theta = sprintf("%12.12f",Theta)
  )
 return(store)
}
```

```
thetstar <- (-1657 + sqrt(3728689))/7680
df.c <- secM2.c(.02,.01,20,1e-10,1e-12,thetstar)[,3:4]
df.c
```

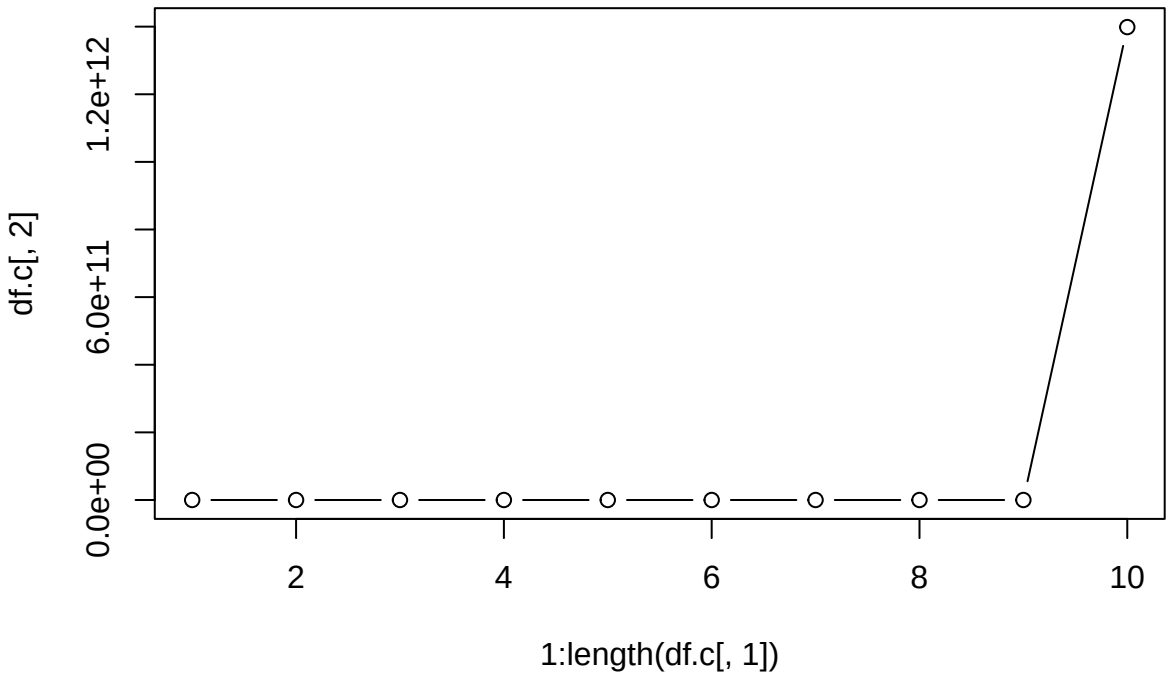
```
##    Convergence Ratio Lin Convergence Ratio Quad
## 1           2.567466e+00           2.567466e+02
## 2           7.631454e-01           5.240718e+01
## 3           2.407410e+00           7.381603e+02
## 4           4.203471e-01           7.604191e+01
## 5           2.583778e-01           1.399276e+02
```

```
## 6           6.346174e-02           1.414577e+02
## 7           1.233066e-02           4.384414e+02
## 8           7.264900e-04           2.096446e+03
## 9           8.819999e-06           3.503457e+04
## 10          3.115265e-03           1.398620e+12
```

```
plot(1:length(df.c[,1]),df.c[,1],type = 'b')
```




```
plot(1:length(df.c[,1]),df.c[,2],type = 'b')
```



this seems to converge linearly as

ratsl <- abs(theta1-star)/(abs(theta1 - theta0))

approaches zero while

ratquad <- abs(theta1-star)/(abs(theta1 - theta0))^2

does diverges. This implies our algorithm is super-linearly convergent.

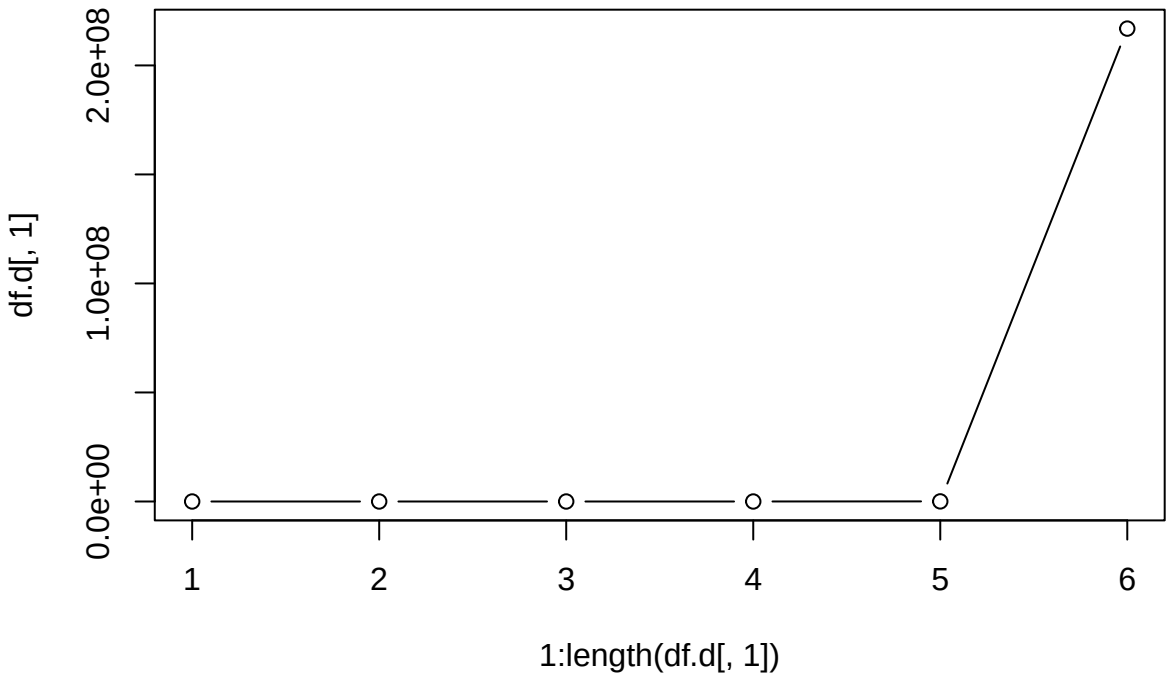
e.g. the plots show that $\lim_{n\to\infty} \frac{|\theta^{n+1}-\theta^*|}{|\theta^n-\theta^*|} = 0$ and $\lim_{n\to\infty} \frac{|\theta^{n+1}-\theta^*|}{|\theta^n-\theta^*|^2} = \infty$

(d).

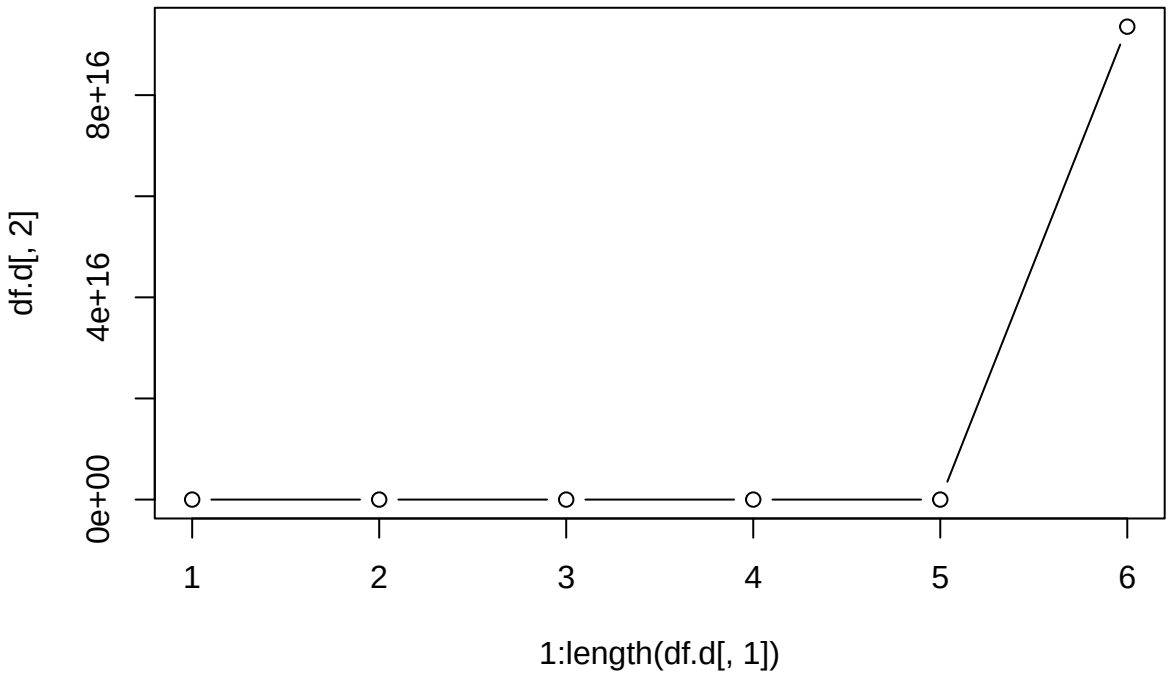
```
df.d <- secM2.c(-.2,.2,10,1e-6,1e-9,thetstar)[,3:4]
df.d
```

```
##   Convergence Ratio Lin Convergence Ratio Quad
## 1          4.108134e-01           1.027033e+00
## 2          7.814417e-01           1.039344e+00
## 3          5.848901e+00           6.818071e+01
## 4          4.483754e+02           3.997853e+05
## 5          4.378993e+04           3.813306e+09
## 6          2.168818e+08           9.354047e+16
```

```
plot(1:length(df.d[,1]),df.d[,1],type = 'b')
```




```
plot(1:length(df.d[,1]),df.d[,2],type = 'b')
```



When we choose two initial points -0.2 and 0.2 then both of our ratio.

The plots show that both $\lim_{n\to\infty} \frac{|\theta^{n+1}-\theta^*|}{|\theta^n-\theta^*|} = \infty$

and $\lim_{n\to\infty} \frac{|\theta^{n+1}-\theta^*|}{|\theta^n-\theta^*|^2} = \infty$ implying a divergent secant algorithm.