MATH 531T: Time Series Analysis and Forecasting (Summer 2024)
Exam #1

Due date: Tuesday, June 4th, by 5:30pm

Please upload your answers in the form of **one pdf file** on to the dropbox folder you have shared with me. In all questions which involve R, make sure relevant R code, output, and graphs are included in the answers to each individual part of the questions. Do not put your R code in the appendix or at the end of the file.

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1 Concepts and Theoretical Questions

1. Based on the i.i.d. noise sequence $X_1, X_2, \cdots$ where $P(X_i = 1) = P(X_i = -1) = 0.5$, define the random walk S_t with the initial point $S_0 = 0$. Write an R code to simulate 500 steps of S_t, i.e. $S_0, \cdots, S_{500}$ and plot S_t vs. t. Generate the sample acf plot of S_t and comment on its stationarity.

2. Consider the stationary model $X_t = 0.2X_{t-1} + 0.4X_{t-2} + Z_t$ in which X_i and Z_j are uncorrelated for all $i < j$ and $\{Z_t\} \sim WN(0, \sigma^2)$. Calculate the ACF of $\{X_t\}$.

3. Consider the MA(2) process defined by $X_t = Z_t + \theta Z_{t-1} + \theta^2 Z_{t-2}$, where $\{Z_t\} \sim WN(0, 9)$. Assuming that θ is a finite number,

   (a) Show that this process is stationary.

   (b) What are the maximum and minimum possible values of the correlation between X_t and X_{t-2}?

2 Data Analysis

4. The dataset *TotalSales.txt* includes the total quarterly sales in thousands of dollars (Winter, Spring, Summer, Fall) for a small business in Orange County during Winter 1995 - Winter 2015. The company would like to fit a model to the data and use it for prediction. For full marks, you should provide the R code and output for all parts of this question.

(a) Perform and exploratory analysis of the data: provide discussion on general/seasonal trends and plots.

(b) Fit two appropriate polynomial regression models to the data (including trend and seasonal components if necessary) with time being the explanatory variable(s). Provide detailed residual diagnostics of your model along with discussion for assessing model assumptions. Also provide and discuss which model fitis better according to standard fit summaries, e.g., AIC, BIC, MSE, etc.