HW6.2

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problem 1

part (a)

```
# general importance sampling
ISE <- function(f,g,h,n){
 return(list(Theta = mean(h*f/g), StandErr = sqrt(var(h*f/g)/n)))
}

# generate values from cauchy
N <-20000
y <- rcauchy(N)
# render function vectors
fy <- rep(0,N)
# making sure f(y) only passes values between 0 and 1
for(i in 1:N){
  if(y[i] >= 0 & y[i] <= 1){fy[i] =  exp(-y[i])/(1-exp(-1))}
}
gy <- 1/(pi*(1+y^2))
hy <- 1/(1 + y^2)

#get estimations
ISE(fy,gy,hy,N)
```

```
## $Theta
## [1] 0.8366677
##
## $StandErr
## [1] 0.01068073
```

part (b).

```
# generate trucated cauchy samples
y <- rep(0,N)
for(i in 1:N){
  repeat{
    Z <- rcauchy(1)
    if(Z <= 1 & Z >= 0){
      Z -> y[i]
      break
    }
```

```
  }
}
# render function vectors
fy <- rep(0,N)
# making sure f(y) only passes values between 0 and 1
for(i in 1:N){
  if(y[i] <= 1){fy[i] =  exp(-y[i])/(1-exp(-1))}
}
hy <- 1/(1 + y^2)
gy <- 4/(pi*(1+y^2))
# get estimations
ISE(fy,gy,hy,N)
```

```
## $Theta
## [1] 0.8273135
##
## $StandErr
## [1] 0.001568982
```

part (c).

```
# generate exp samples
rexp(N) -> y
# render function vectors
fy <- rep(0,N)
# making sure f(y) only passes values below 1 (rexp doesn't gen negatives)
for(i in 1:N){
  if(y[i] <= 1){fy[i] =  exp(-y[i])/(1-exp(-1))}
}
hy <- 1/(1 + y^2)
gy <- exp(-y)
# get estimations
ISE(fy,gy,hy,N)
```

```
## $Theta
## [1] 0.8276334
##
## $StandErr
## [1] 0.00468778
```

problem 2

```
# render X_i from the trivariate - t
cov <-  matrix(c(1,3/5,1/3,3/5,1,11/15,1/3,11/15,1),3,3,byrow = T)
muv <- c(0,0,0)
X <- as.matrix(rmvnorm(N,mean = muv, sigma = cov))

# render pmfs
gX <- dmvnorm(X,mean = muv,sigma = cov)
fX = rep(0,N)
  for (i in 1:N) {
    fX[i] <- (5 + (t(X[i,])) %*% solve(cov)%*%(X[i,]))^-4
```

```
  }
wX <- fX/gX
T0 = sum(wX)

#make indicator
I = as.numeric(X[,1] <= 1 & X[,2] <= 4 & X[,3] <= 2 )

# render
theta = sum(I*wX)/T0
theta
```

```
## [1] 0.7985146
```

problem 3

part (a)

set pdfs equal to eachother case I: $y < 0$

$$\frac{e^y}{2} = \frac{e^{-y^2/2}}{\sqrt{2\pi}}e^y = \frac{2e^{-y^2/2}}{\sqrt{2\pi}}e^y \cdot e^{y^2/2} = \frac{2}{\sqrt{2\pi}}e^{y+\frac{y^2}{2}} = \sqrt{\frac{2}{\pi}}y + \frac{y^2}{2} = ln[\sqrt{\frac{2}{\pi}}]\frac{1}{2}y^2 + y - \frac{1}{2}ln[2/\pi] = 0$$

```
a = .5
b = 1
c = -0.5*log(2/pi)
y1 = (-b-sqrt(b^2 - 4*a*c))/2*a
y2 = (-b+sqrt(b^2 - 4*a*c))/2*a
y1;y2
```

```
## [1] -0.435138

## [1] -0.06486199
```

likewise case II: $y >0 $

$$\frac{1}{2}y^2 - y - \frac{1}{2}ln[2/\pi] = 0$$

```
a = .5
b = -1
c = -0.5*log(2/pi)
y1 = (-b-sqrt(b^2 - 4*a*c))/2*a
y2 = (-b+sqrt(b^2 - 4*a*c))/2*a
y1;y2
```

```
## [1] 0.06486199

## [1] 0.435138
```

part (b).

α will be the maximum of $g(y)/f_Y(y)$, thus

$$0 = \frac{d}{dy}\left(\frac{1}{2}e^{-|y|} \cdot \sqrt{2\pi} \cdot e^{y^2/2}\right)0 = \sqrt{\frac{\pi}{2}}\frac{d}{dy}\left(e^{-|y|+y^2/2}\right)$$

when $y > 0 $

$$0 = \sqrt{\frac{\pi}{2}}e^{-y+y^2/2}(-y+1)y^* = 1$$

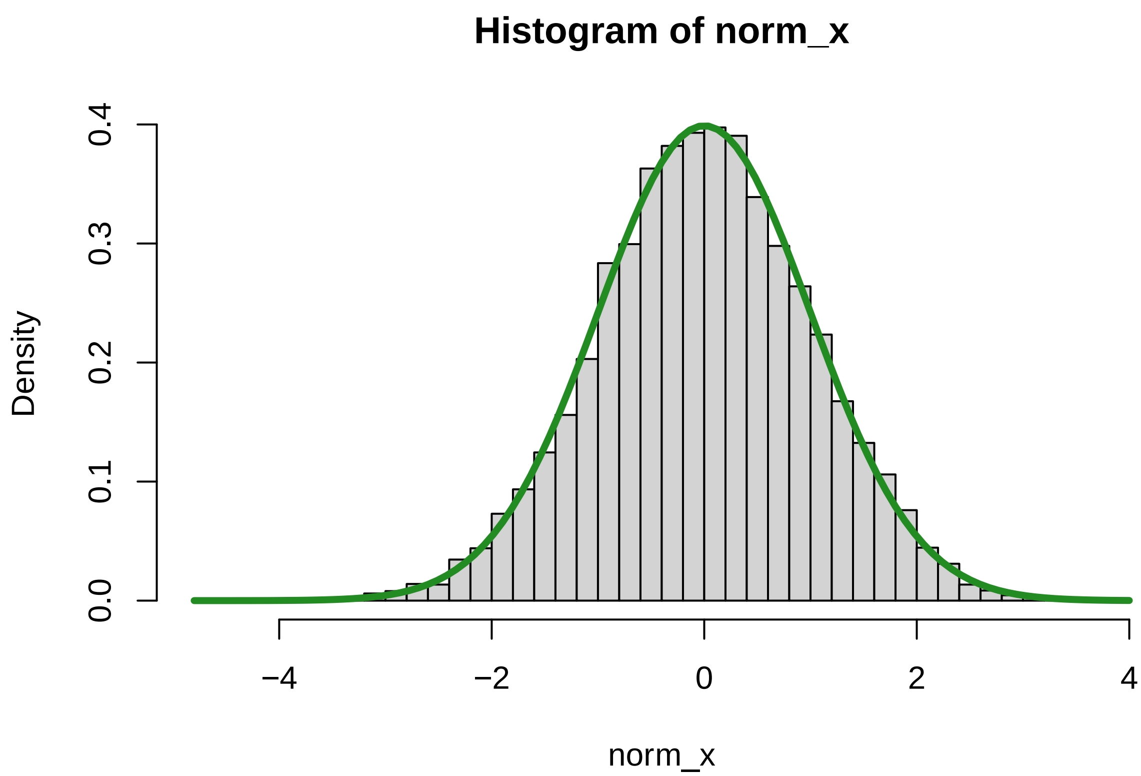
likewise when $y < 0$

$$0 = \sqrt{\frac{\pi}{2}}e^{y+y^2/2}(y+1)y^* = -1$$

notice $g(1)/f_Y(1) = g(-1)/f_Y(-1) = 0.7601735$

part (c).

```r
# render math functions
f <- function(x){
 exp(-(x^2)/2)/sqrt(2*pi)
}

g <- function(x){
 exp(-1*abs(x))/2
}

N = 10000
alpha = g(1)/f(1)
# open norm vector
norm_x <- rep(0,N)

for(i in 1:N){
 repeat{
  y <- -log(runif(1)) # inverse of double exponential
  u <- runif(1)
  if(u < f(y)*alpha/g(y)){break}
 }

 if(runif(1) < .5){
  Z = y
 }else{Z = -y}

  Z -> norm_x[i]
}

hist(norm_x, prob = T, breaks = 50)
curve(dnorm(x),add=T,lwd = 3.5, col = "forestgreen")
```


Histogram of norm_x
Density
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
4
norm_x