Quiz 1 | Math 530 | Fall 2023

8/29 – 30

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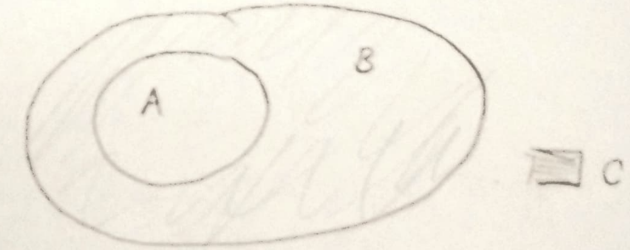
Note: Show your work on all problems. Please do not include any obvious algebraic details. A total of 20 points is possible.

1. **[4 Points]** Using **only** the Axioms of probability and the finite additivity property show that for any two events A and B if $A \subset B$, then $P(A) \leq P(B)$. State the properties that you use at each step. You don't need to prove set-theoretic properties. Note: You will not get any credit if you use other theorems on properties of $P(\cdot)$.

need to show if $A \subset B$
then $P(A) \leq P(B)$



Kolmogorov

(i) $P(A) \geq 0 \quad \forall A \in \mathcal{B}$

(ii) $P(S) = 1$

(iii) if $A_1, A_2, \dots$ are disjoint,
then $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$.

[proof]:

Let A & B be two events such that $A \subset B$
assume $C = B \cap A^c$, thus $B = A \cap C$
we know A and C are disjoint, thus by (Kolmogorov iii)
$P(A \cup C) = P(A) + P(C)$
also by the additivity property we know because $B = A \cap C$
then $P(B) = P(A) + P(C)$
by (kolmogorov i) $P(C) \geq 0$
thus becuz $P(B) = P(A) + P(C)$
then $P(B) \geq P(A)$

$\square$

2. **[5 Points]** Suppose that $\mathcal{B}$ is a Borel field. Let $A_1 \in \mathcal{B}$ and $A_2 \in \mathcal{B}$. Use only the three properties of a Borel field, given in Definition 1.2.1 of your text (page 6), to show that $A_1 \cap A_2 \in \mathcal{B}$. Note you are only allowed to use set theory and the three properties in your proof. Cite the properties that you use at each step of your proof.

[Proof]:

Suppose that $\mathcal{B}$ is a Borel field. Let $A_1 \in \mathcal{B}$ and $A_2 \in \mathcal{B}$.

thus we know by [Def. 1.2.1c]. if $A_1, A_2 \in \mathcal{B}$ then $A_1 \cup A_2 \in \mathcal{B}$.

But notice that the union of A_1 and A_2 also includes the intersection of A_1 and A_2.

thus, we can conclude

$A_1 \cup A_2 \in \mathcal{B}$

$\square$

3. **[5 Points]** There are four children in a family. Their mom purchases six different gifts and decides to divide the gifts randomly between the children. Assuming that a child can receive no gift, or multiple gifts (up to all six gifts), What is the probability that each child receives at least one gift?

$|S| = 4^6 = 4096$ → children and gifts are distinct

$|E| = 4!\,S(6,4)$ ←

$= 4! \cdot \frac{1}{4!} \sum_{i=0}^{4} (-1)^i \binom{4}{i} (4-i)^6$ ← stirling numbers of the 2nd kind {from MATH 471 or MAT 3740 CPP}

$= \binom{4}{0}4^6 - \binom{4}{1}3^6 + \binom{4}{2}2^6 - \binom{4}{3}1^6 + \binom{4}{4}(0)^6$

$S(n,k)$ $n=6$ $k=4$

$= 4096 - 2916 + 384 - 4 + 0$

$= 1560$

$|E|/|S| = \boxed{195/512}$

4. **[3 Points]** In the previous problem, thinking only about the number of gifts received by each child, how many different possibilities are there? [for example, (3, 1, 1, 1) is one possibility where child 1 receives 3 gifts and each of the other children receive one; or (1, 3, 1, 1) is another possibility where the second child receives three gifts and the remaining children receive one; yet (0, 0, 6, 0) is another choice where the third child receives all the gifts. I have given examples of three possibilities. The problem is asking for the total number of such possibilities.]

~~let $n=4$ $k=2$~~

~~1 way~~

~~2 ways~~

~~2 ways~~

~~5 ways~~

$\cancel{\binom{4+2-1}{2-1} = \binom{5}{1} = 5}$

$\cancel{|S| = \binom{n+r-1}{r} = 6+4-1}$

$|S| = 4^6 = 4096$

/ 4096 possible ways. /

Combinatorics

5. [3 Points] Suppose that we have collection of six numbers., $\{1, 2, 7, 8, 14, 20\}$. If we draw six numbers with replacement from this set, what is the probability that the mean of the six numbers drawn is 11?

$\longrightarrow$ Sum will have to be 66

$$\overset{14}{x_1} + \overset{8}{x_2} + \overset{8}{x_3} + \overset{2}{x_4} + \overset{20}{x_5} + \overset{14}{x_6} = 66$$

~~method 1~~

~~(14, 14, 8, 8, 2, 20) is one solution~~

$$\cancel{\frac{6!}{2!\,2!\,1!\,1!} = 180}$$

there is more possibilities

method 2

find coef $[x^{66}]$

of $(x + x^2 + x^7 + x^8 + x^{14} + x^{20})^6$

needs software!

Python ↓

```
>> from sympy import expand, collect
>> from sympy.abc import x
>> polynom = (x + x**2 + x**7 + x**8 + x**14
        + x**20)
>> print(expand(polynom).coeff(x**66))
[0]  580  = |E|
```

$|S| = 6^6 = 46656$

thus $|E| / |S| = 580/46656 \approx \boxed{0.0124}$