MATH 531T: Time Series Analysis and Forecasting (Summer 2024)
Exam #2

Due date: Wednesday, June 12 by 5:30pm

Please upload your answers in the form of **one pdf file** on Canvas. In all questions which involve R, make sure relevant R code, output, and graphs are included in the answers to each individual part of the questions. Do not put your R code in the appendix or at the end of the file. Please include the code source file, e.g., .R or .RMD as well.

1 Concepts and Theoretical Questions

1. Which of the following sample ACF plots represent non-stationary data?
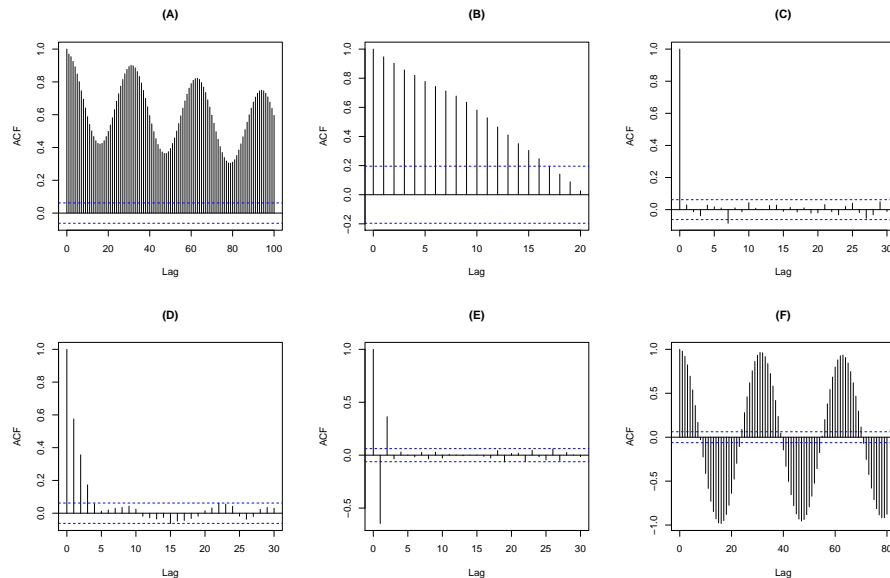
2. Consider the model $X_t = \beta_0 + \beta_1 t + \beta_2 \sin(\frac{\pi}{2}t) + Z_t$ where $\{Z_t\} \sim WN(0, \sigma^2)$. Define the differencing $\nabla_d X_t = X_t - X_{t-d}$.

   (a) Investigate the stationarity of X_t.

   (b) Show that $\nabla_4 X_t$ does not have a time-dependent mean.

3. Consider the AR(1) model $X_t = \phi X_{t-1} + W_t$ with $W_t \sim N(0, \sigma^2)$ for $t = 1, ..., n$. Derive the log likelihood of the model (you may use matrix notation if you prefer). Find the MLE of ϕ. (Note x_1 should serve as your initial value).

2 Simulation/Data Analysis

4. Consider the *Population.csv* data that describes a monthly population count per milliliters of a population of rotifers (yes you may use the internet to look up "rotifers") from January 2010 to December 2015.

   (a) Plot the time series and comment on what you see (e.g. general/seasonal trends/anomalies).

   (b) Use the *decompose*() function to extract potential general and seasonal trends. Provide plots of the general trend, seasonal trend, and the residuals. Comment on the trends.

   (c) Provide an autocorrelation plot (ACF) of the residuals and comment on whether the residuals look stationary.

   (d) Fit the following ARMA models to the residuals: ARMA(1, 0, 1), ARMA(2,0,0), ARMA(0,0,2), ARMA(2,0,1), and ARMA(2,0,2). Provide the AIC and parameter estimates for each model in an organized table. Discuss which model fits the data the best.