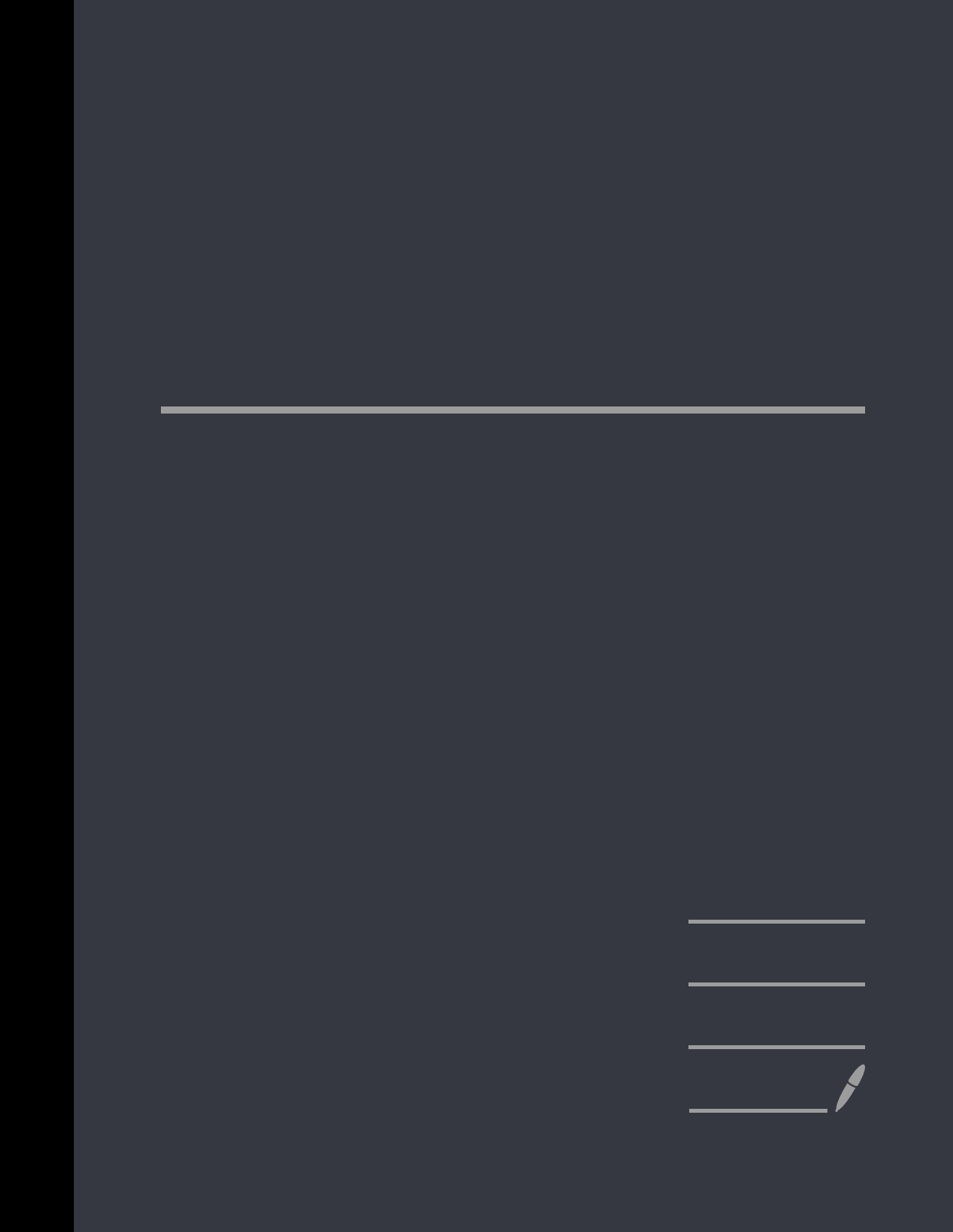

16) $y_{ij} | \theta_j, \sigma^2 \sim N(\theta_j, \sigma^2)$

$\theta_j | \mu, \sigma^2 \sim N(\mu, \sigma^2)$

$\mu \sim N(30, 1)$

$\sigma^2 \sim \Gamma^{-1}(2, 10)$

Write out the joint posterior $f(\theta, \mu, \sigma^2 | y)$

$$f(\theta, \mu, \sigma^2 | y) \propto p(\mu) p(\sigma^2) \prod_{j=1}^{J} p(\theta_j | \mu, \sigma^2) \prod_{j=1}^{J} \prod_{i=1}^{n_j} p(y_{ij} | \theta_j, \sigma^2)$$

$$\propto N(30, 1) \Gamma^{-1}(2, 10) \prod_{j=1}^{J} [N(\mu, \sigma^2)] \prod_{j=1}^{J} \prod_{i=1}^{n_j} N(\theta_j, \sigma^2)$$

1c) $f(\theta_j \mid \theta_{-j}, \mu, \sigma^2, y) \propto p(\theta_j \mid \mu, \sigma^2) \prod_{i=1}^{n_j} p(y_{ij} \mid \theta_j, \sigma^2)$

$\propto N(\mu, \sigma^2) \prod_{i=1}^{n_j} \left[N(\theta_j, \sigma^2) \right]$

$\Rightarrow \theta_j \mid \mu, \sigma^2, y \sim N\left(\frac{n_j \bar{y}_{\cdot j} / \sigma^2 + \mu/\sigma^2}{n_j/\sigma^2 + 1/\sigma^2}, \left[\frac{n_j}{\sigma^2} + \frac{1}{\sigma^2} \right]^{-1} \right)$ derived from class

1d) $f(\mu \mid \theta, \sigma^2, y)$ and $f(\sigma^2 \mid \theta, \mu, y)$

$$p(\mu \mid \theta, \sigma^2, y) \propto p(\mu) \prod_{j=1}^{J} p(\theta_j \mid \mu, \sigma^2)$$

$$\propto N(30, 1) \prod_{j=1}^{J} \left[N(\mu, \sigma^2) \right]$$

$$\mu \mid \theta, \sigma^2, y \sim N\left(\frac{J\bar{\theta}/\sigma^2 + 30}{J/\sigma^2 + 1}, \left[J/\sigma^2 + 1 \right]^{-1} \right)$$

$$f(\sigma^2 \mid \theta, \mu, y) \propto p(\sigma^2) \prod_{j=1}^{J} \prod_{i=1}^{n_j} p(y_{ij} \mid \theta_j, \sigma^2)$$

$$\propto \Gamma^{-1}(2, 10) \prod_{j=1}^{J} \prod_{i=1}^{n_j} \left[N(\theta_j, \sigma^2) \right]$$

$$\sim \Gamma^{-1}\left(1 + \frac{\sum_{j=1}^{J} n_j}{2}, 10 + \frac{\sum_{j=1}^{J} \sum_{i=1}^{n_j} (y_{ij} - \theta_j)^2}{2} \right)$$