HW6.2

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problem 3

part (a)

set pdfs equal to eachother case I: $y < 0$

$$\frac{e^y}{2} = \frac{e^{-y^2/2}}{\sqrt{2\pi}}e^y = \frac{2e^{-y^2/2}}{\sqrt{2\pi}}e^y \cdot e^{y^2/2} = \frac{2}{\sqrt{2\pi}}e^{y+\frac{y^2}{2}} = \sqrt{\frac{2}{\pi}}y + \frac{y^2}{2} = ln[\sqrt{\frac{2}{\pi}}]\frac{1}{2}y^2 + y - \frac{1}{2}ln[2/\pi] = 0$$

```
a = .5
b = 1
c = -0.5*log(2/pi)
y1 = (-b-sqrt(b^2 - 4*a*c))/2*a
y2 = (-b+sqrt(b^2 - 4*a*c))/2*a
y1;y2
```

```
## [1] -0.435138
```

```
## [1] -0.06486199
```

likewise case II: $y >0 $

$$\frac{1}{2}y^2 - y - \frac{1}{2}ln[2/\pi] = 0$$

```
a = .5
b = -1
c = -0.5*log(2/pi)
y1 = (-b-sqrt(b^2 - 4*a*c))/2*a
y2 = (-b+sqrt(b^2 - 4*a*c))/2*a
y1;y2
```

```
## [1] 0.06486199
```

```
## [1] 0.435138
```

part (b).

```
# function to find optimal alphas
opt_alpha <- function(N,Gu,Fx,U){
 # initialize
 incr = 1/N
 alpha = 0
```

```
 STO <- data.frame()


 #begin loop
 for(i in 1:N){
   alpha = alpha + incr
   e_x = Gu/alpha
   R = sum(as.numeric(U <= Fx/e_x))
   row <- c(R,alpha)
   STO <- rbind(STO,row)
 }
Rmin = min(STO[,1])
result = STO[STO[,1] == Rmin,]
return(result[1,])
}

# render discrete functions
a=0.06486199
b=0.435138
N = 20000
U <- runif(N,a,b)
Gu <- exp(-U)/2
Fx <- dnorm(U)
opt_alpha(N,Gu,Fx,U)
```

```
##   X0 X5e.05
## 1  0  5e-05
```

```
U <- runif(N,-b,-a)
Gu <- exp(-U)/2
Fx <- dnorm(U)
opt_alpha(N,Gu,Fx,U)
```

```
##   X20000 X5e.05
## 1  20000  5e-05
```