Name (please print)______________________________

Note: Show your work on all problems. Each part of each problem is worth 5 points. A total of 25 points is possible.

1. Let X be a random variable with pdf

$$f(x) = \frac{1}{2\beta} e^{-|x-\alpha|/\beta}, \quad -\infty < x < \infty, \quad -\infty < \alpha < \infty, \quad \beta > 0.$$

   Derive the mgf of the random X. State the domain where the mgf is defined.

2. Let Y be a geometric random variable with parameter p, where p is the success probability. Show that as p approaches zero, the random variable $W = pY$ converges to the exponential distribution with parameter $\beta = 1$.

3. Theaters A and B compete for the business of 1000 customers. Assume that Theater A shows a more popular movie, and thus the probability that a randomly selected customer chooses Theater A is $3/4$. Let n be the number of seats in Theater A. Write an equation that you would solve for n such that the probability of turning away a customer by Theater A, because of a full house, is less than 5%. Do not solve for n.

4. Let X have the standard normal distribution (i.e. $X \sim N(0,1)$). Use the moment generating function of X to obtain $E(X^4)$.

5. Let Y be a random variable with pmf

$$P(Y = \sqrt{3}) = P(Y = -\sqrt{3}) = 1/6, \quad P(Y = 0) = 2/3.$$

Obtain $E(Y^4)$.