Name (please print)____

Note: Show your work on all problems. Each problem is worth 5 points. A total of 25 points is possible.

1. Prove or give a counter example: If A and B are two events, then $P(A|B) + P(A|B^c) = 1$.

2. We have two coins, each having $P(heads) = \alpha$, where $0 \leq \alpha \leq 1$. We flip these two coins continually and simultaneously until either two heads appear or two tails appear. What is the probability that two heads appear first; that is two heads appear before two tails appear.Compute the probability in terms of α.

3. Use mathematical induction to show that

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \cdots P(A_n|A_1 \cap A_2 \cap \cdots \cap A_{n-1}).$$

You can assume that you know $P(A \cap B) = P(A|B)P(B)$.

4. Let X be a continuous random variable with pdf $f(x)$ and cdf $F(x)$ both of which have support in $(-\infty, \infty)$. Consider the fixed values a and b with $a < b$. Show that the following function is a pdf with support $[a, b]$:

$$g(x) = f(x)/[F(b) - F(a))]$$

5. Suppose that X is a continuous random variable with cumulative distribution function (cdf)

$$F(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{x+1}{5} & \text{if } -1 \leq x \leq 4 \\ 1 & \text{if } x > 4. \end{cases}$$

Obtain the cumulative distribution function of $Y = 4 - (x - 1)^2$.