Homework 11 Verification

ATSC 507

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A19. Given the following fields of 50-kPa height (km). Find the:

Each field (i.e., each weather map) below covers an area from North to South and West to East.

a. mean forecast error

$$ME = \overline{(F - V)} = \bar{F} - \bar{V}$$

```
In [3]: ME = np.mean(d['F']- d['V'])
print(f"Mean Forecast Error: {round(ME,4)} km")
```

Mean Forecast Error: 0.105 km

b. mean persistence error

$$\overline{(A-V)} = \overline{A} - \overline{V} = \text{mean persistence error}$$

```
In [4]: MPE = np.mean(d['A']- d['V'])
    print(f"Mean Persistence Error: {round(MPE,4)} km")
```

Mean Persistence Error: 0.005 km

c. mean absolute forecast error

$$MAE = \overline{|F - V|}$$

```
In [5]: MAE = np.mean(abs(d['F']- d['V']))
    print(f"Mean Absolute Forecast Error: {round(MAE,4)} km")
```

Mean Absolute Forecast Error: 0.105 km

d. mean squared forecast error

$$MSE = \overline{(F - V)^2}$$

```
In [6]: MSE = np.mean((d['F']- d['V'])**2)
    print(f"Mean Squared Forecast Error:{round(MSE,4)} km^2")
```

Mean Squared Forecast Error: 0.0145 km^2

e. mean squared climatology error

$$MSEC = \overline{(C - V)^2}$$

```
In [7]: MSEC = np.mean((d['C']- d['V'])**2)
print(f"Mean Squared Climatology Error: {round(MSEC,4)} km^2")
```

Mean Squared Climatology Error: 0.0055 km^2

f. mean squared forecast error skill score

$$MSESS = 1 - \frac{MSE}{MSEC}$$

```
In [8]: MSESS = 1 - (MSE/ MSEC)
    print(f"Mean Squared Forecast Error Skill Score: {round(MSESS,4)} ")
```

Mean Squared Forecast Error Skill Score: -1.6364

g. RMS forecast error

$$RMSE = \sqrt{\overline{(F - V)^2}}$$

```
In [9]: RMS = np.mean((d['F']- d['V'])**2)**(0.5)
print(f"RMS Forecast Error: {round(RMS,4)} km")
```

RMS Forecast Error: 0.1204 km

h. correlation coefficient between forecast and verification

$$r = \frac{F'V'}{\sqrt{\overline{(F')^2}} \cdot \sqrt{\overline{(V')^2}}}$$

$$F' = F - \overline{F} \quad \text{and} \quad V' = V - \overline{V}$$

Correlation Coefficient between Forecast and Verification: 0.9248

i. forecast anomaly correlation

forecast anomaly correlation =
$$\frac{\overline{[(F-C)-\overline{(F-C)}]\cdot[(V-C)-\overline{(V-C)}]}}{\sqrt{\overline{[(F-C)-\overline{(F-C)}]^2}\cdot\overline{[(V-C)-\overline{(V-C)}]^2}}}$$

Forecast Anomaly Correlation: 0.6699

j. persistence anomaly correlation

```
persistence anomaly correlation = \frac{[(A-C) - \overline{(A-C)}] \cdot [(V-C) - \overline{(V-C)}]}{\sqrt{[(A-C) - \overline{(A-C)}]^2 \cdot [(V-C) - \overline{(V-C)}]^2}}
```

Persistence Anomaly Correlation: -0.0881

k. Draw height contours by hand for each field, to show locations of ridges and troughs.

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5.2	5.3	5.4	5.3
5.3	5.4	5.5	5.4
5.4	5.5	5.6	5.5
5.5	5.6	5.7	5.6
5.6	5.7	5.8	5.7

Forecast:

5.3	5.4	5.5	5.4
5.5	5.4	5.5	5.6
5.6	5.6	5.6	5.6
5.8	5.7	5.6	5.7
5.9	5.8	5.7	5.8

Verification:

5.3	5.3	5.3	5.4
54	5.3	54	5.5
5.5	5.4	5.5	5.5
5.7	5.5	5.6	5.6
5.8	5.7	5.6	5.6

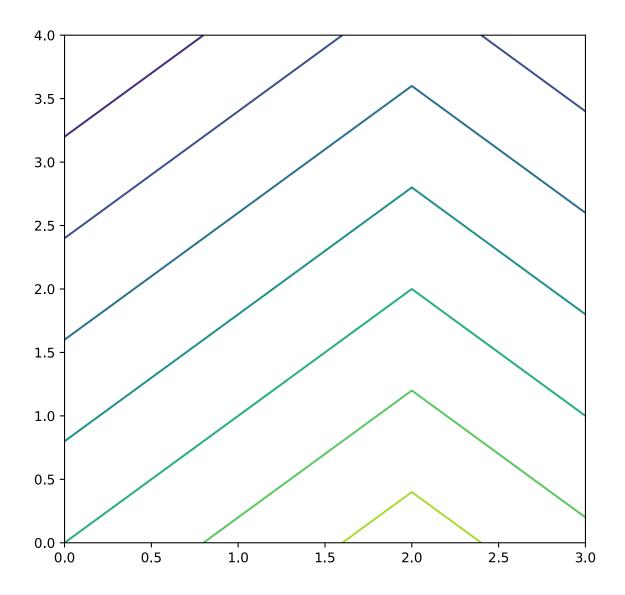
Climate:

5.4	5.4	5.4	5.4
5.4	5.4	5.4	5.4
5.5	5.5	5.5	5.5
5.6	5.6	5.6	5.6
5.7	5.7	5.7	5.7

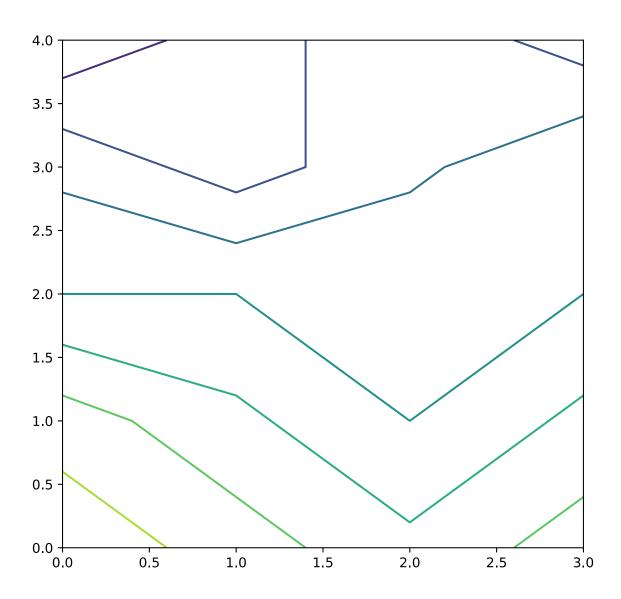
```
In [13]: fig, ax = plt.subplots(figsize=(7,7))
         fig.suptitle('Analysis', fontsize= plt_set.title_size, fontweight="bold"
         level = np.mean(d['A'])
         ax.contour(np.flip(d['A'],0))
         fig, ax = plt.subplots(figsize=(7,7))
         fig.suptitle('Forecast', fontsize= plt_set.title_size, fontweight="bold"
         level = np.mean(d['F'])
         ax.contour(np.flip(d['F'],0))
         fig, ax = plt.subplots(figsize=(7,7))
         fig.suptitle('Verification', fontsize= plt_set.title_size, fontweight="b
         old")
         level = np.mean(d['V'])
         ax.contour(np.flip(d['V'],0))
         fig, ax = plt.subplots(figsize=(7,7))
         fig.suptitle('Climate', fontsize= plt set.title size, fontweight="bold")
         level = np.mean(d['C'])
         ax.contour(np.flip(d['C'],0))
         # plt.show()
```

Out[13]: <matplotlib.contour.QuadContourSet at 0x7fb1083ebfd0>

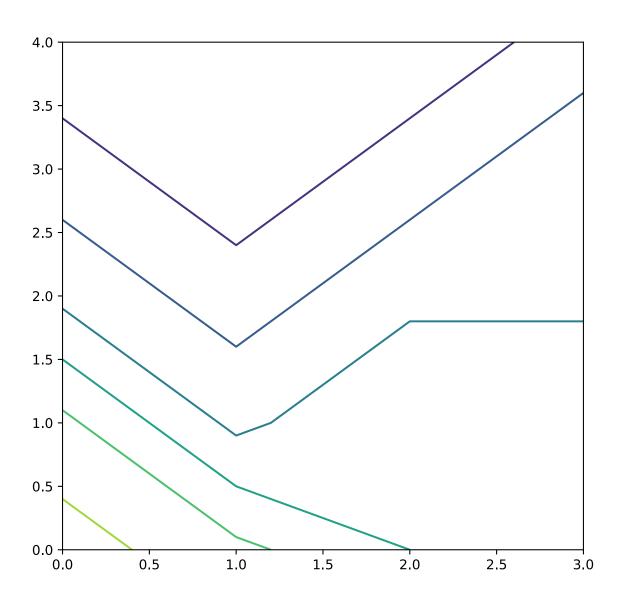
Analysis



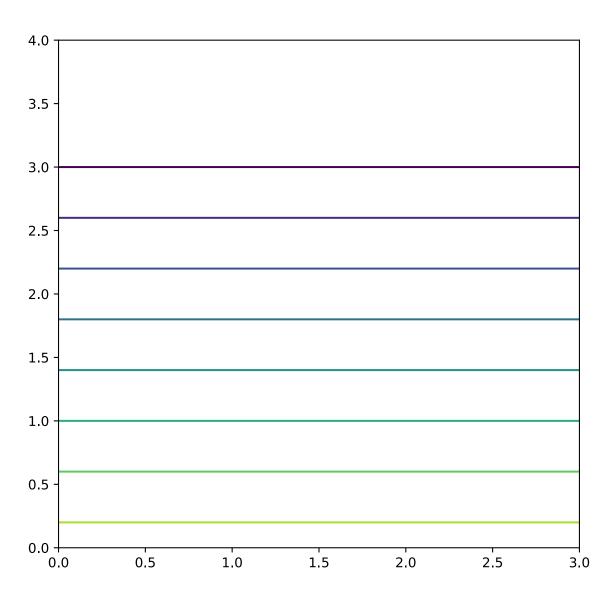
Forecast



Verification



Climate



A20. Given the following contingency table, calculate all the binary verification statistics.

Observation

Yes No

Forecast Yes: 150 65

No: 50 100

In [14]: a, b, c ,d = 150, 65, 50, 100 # Find: B, PC, HSS, H, F, FAR, TSS, CSI, GSS

n = a + b + c + d

365

$$B = \frac{a+b}{a+c}$$

In [16]: B = (a + b) / (a + c)
print("bias score", B)

bias score 1.075

$$PC = \frac{a+d}{n}$$

In [17]: PC = (a + d) / n
print ("portion correct", PC)

portion correct 0.684931506849315

$$E = \left(\frac{a+b}{n}\right) \cdot \left(\frac{a+c}{n}\right) + \left(\frac{d+b}{n}\right) \cdot \left(\frac{d+c}{n}\right)$$

In [18]: E = (((a + b)/ n) * ((a + c)/ n)) + (((d + b)/ n) * ((d + c)/ n))print(""random luck" part of PC", E)

"random luck" part of PC 0.5085381872771627

$$HSS = \frac{PC - E}{1 - E}$$

In [19]: HSS = (PC - E)/ (1 - E)
print(" Heidke skill score", HSS)

Heidke skill score 0.3589156166475753

$$H = \frac{a}{a+c}$$

In [20]: H = a / (a + c)
print("Hit rate",H)

Hit rate 0.75

$$F = \frac{b}{b+d}$$

In [21]: F = b / (b + d)
print ("false-alarm rate", F)

false-alarm rate 0.3939393939393939

$$FAR = \frac{b}{a+b}$$

In [22]: FAR = b / (a +b)
print("false-alarm ratio", FAR)

false-alarm ratio 0.3023255813953488

$$TSS = H - F$$

In [23]: | TSS = H - F
 print("true skill score", TSS)

true skill score 0.3560606060606061

$$CSI = \frac{a}{a+b+c}$$

In [24]: CSI = a / (a + b +c)
 print("critical success index", CSI)

critical success index 0.5660377358490566

$$a_r = \frac{(a+b)\cdot(a+c)}{n}$$

In [25]: ar = ((a + b) * (a + c))/ n
 print("Hits that might have occurred by random chance", ar)

Hits that might have occurred by random chance 117.8082191780822

$$GSS = \frac{a - a_r}{a - a_r + b + c}$$

In [26]: GSS = (a - ar)/ (a - ar + b + c)
print("Gilbert's skill score", GSS)

Gilbert's skill score 0.21870637505816656

A21. Given forecasts having the contingency table of exercise N20. Protective cost is 5k dollars to avoid a loss of 50k dollars. Climatological frequency of the event is 50%.

• a) Find the value of the forecast.

$$E_{\text{climate}} = \min(C, o \cdot L)$$

E_climate 5k dollars

$$E_{\text{forecast}} = \frac{a}{n}C + \frac{b}{n}C + \frac{c}{n}L$$

```
In [28]: E_forecast = (a/n)*C + (b/n)*C + (c/n)*L
    print(f"E_forecast {round(E_forecast,3)}k dollars")
```

E forecast 9.795k dollars

$$E_{perfect} = o \cdot C$$

```
In [29]: E_perfect = o*C
print(f"E_perfect {round(E_perfect,3)}k dollars")
```

E perfect 2.5k dollars

$$V = \frac{E_{\text{climate}} - E_{\text{forecast}}}{E_{\text{climate}} - E_{\text{perfect}}}$$

```
In [30]: V = (E_climate - E_forecast)/ (E_climate - E_perfect)
    print(f"Economic value {round(V,3)}")
```

Economic value -1.918

• a) cont. The economic value is negative which, indicates your forecast is worst than the climatology.... Time to rethink your job.

$$r_{CL} = C/L$$

• (b) If you can get probabilistic forecasts, then what probability would you want in order to decide to take protective action?

You should take protective action whenever the forecast probability p of the event exceeds your cost/loss ratio. In this case. Where p > 0.1

A22. Given the table below of k = 1 to 20 forecasts of probability pk that 24-h accumulated precipitation will be above 25 mm, and the verification ok = 1 if the observed precipitation was indeed above this threshold.

k	p_k	o_k	k	p_k	o_k
1	0.9	1	11	0.4	0
2	0.85	1	12	0.35	0
3	0.8	0	13	0.3	1
4	0.75	1	14	0.25	0
5	0.7	1	15	0.2	0
6	0.65	1	16	0.15	1
7	0.6	0	17	0.1	0
8	0.55	1	18	0.05	0
9	0.5	0	19	0.02	0
10	0.45	1	20	0	0

• (a) Find the Brier skill score.

$$BSS = 1 - \frac{\sum_{k=1}^{N} (p_k - o_k)^2}{\left(\sum_{k=1}^{N} o_k\right) \cdot \left(N - \sum_{k=1}^{N} o_k\right)}$$

```
In [32]: k = np.arange(1,21,1)
    pk = np.array([0.9, 0.85, 0.8, 0.75, 0.7, 0.65, 0.6, 0.55, 0.5, 0.45, 0.4, 0.35, 0.3, 0.25, 0.2, 0.15, 0.1, 0.05, 0.02, 0.0])
    ok = np.array([1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0])
    N = float(len(k))
    k1 = np.sum((pk-ok)**2)
    k2 = (np.sum(ok) * (N - np.sum(ok)))

BSS = 1 - (k1/k2)

print(f"Brier skill score {BSS}")
```

Brier skill score 0.9629

• (b) For probability bins of width $\Delta p = 0.2$, plot a reliability diagram, and $j = round(p_k/\Delta p, 0)$

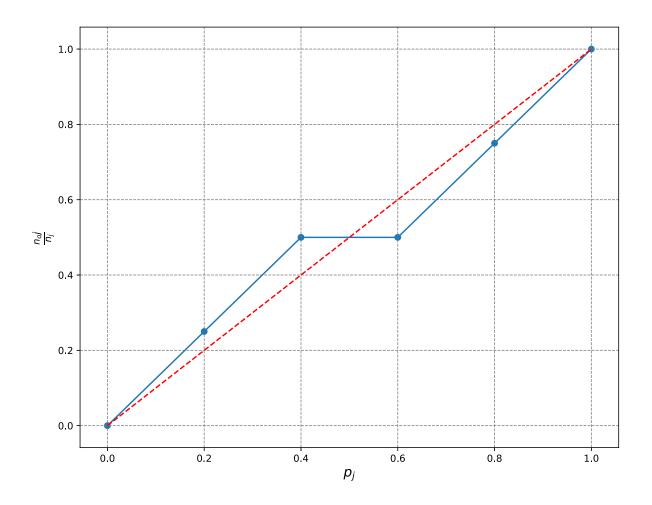
```
In [33]:
         dp = 0.2
         pk_dp = pk/dp
         def rounder(x):
             x = x + 0.01 ### This is required....took so long to figure that out
             if (x-int(x) >= 0.5):
                  j = np.ceil(x)
                 return int(j)
             else:
                  j = np.floor(x)
                 return int(j)
         j = []
         for value in pk_dp:
                  ji = rounder(value)
                  j.append(ji)
         j = np.array(j)
         J = np.arange(np.min(j), np.max(j)+1,1)
         print(f"Number of bins {len(J)}")
         pj = np.linspace(0, 1, len(J), endpoint=True)
         oj = j[ok==1]
         unique_j, nj = np.unique(j, return_counts=True)
         unique_oj, n_oj = np.unique(oj, return_counts=True)
         noj = np.append(0, n oj)
```

Number of bins 6

```
In [35]: fig, ax = plt.subplots(1,1, figsize=(10,8))
    fig.suptitle('Reliability diagram', fontsize= plt_set.title_size, fontwe
    ight="bold")
    ax.set_ylabel(r'$ \frac{n_oj}{n_j}$', fontsize = plt_set.label)
    ax.set_xlabel('$p_j$', fontsize = plt_set.label)
    ax.xaxis.grid(color='gray', linestyle='dashed')
    ax.yaxis.grid(color='gray', linestyle='dashed')
    ax.plot(pj, (noj/nj))
    ax.scatter(pj, (noj/nj))
    x = np.linspace(0,1,100)
    ax.plot(x, x, "--r")

plt.show()
```

Reliability diagram



• (c) find the reliability Brier skill score.

$$BSS_{\text{reliability}} = \frac{\sum_{j=0}^{J} \left[\left(n_j \cdot p_j \right) - n_{oj} \right]^2}{\left(\sum_{k=1}^{N} o_k \right) \cdot \left(N - \sum_{k=1}^{N} o_k \right)}$$

```
In [34]: k1 = np.sum(((nj * pj) - noj)**2)
k2 = (np.sum(ok)) * (N - np.sum(ok))

BSSR = k1/k2

print(f"Reliability Brier skill score {round(BSSR,8)}")
```

Reliability Brier skill score 0.0040404

A23. For any one part of this exercise (Ex a to Ex d) of this problem, a 10-member ensemble forecast system forecasts probabilities that 24-h accumulated rainfall will exceed 5 mm. The observation flags (o) and forecast probabilities (p) are given in the table (in the next column) for a 30-day period. Calculate the hit rate and false-alarm rate for the full range of allowed probability thresholds, and plot the result as a ROC diagram. Also find the area under the ROC curve and find the ROC skill score

Day	o	Ex a: <i>p</i> (%)	<i>p</i> (%)	<i>p</i> (%)	<i>p</i> (%)	Day	o	Ex a: <i>p</i> (%)	<i>p</i> (%)	<i>p</i> (%)	<i>p</i> (%)
1	1	50	10	100	0	16	0	60	30	20	40
2	0	20	0	0	10	17	1	70	60	60	50
3	1	20	30	90	20	18	1	90	70	60	60
4	1	60	40	90	30	19	1	80	80	60	70
5	0	50	30	0	40	20	0	70	70	30	80
6	0	20	40	0	50	21	0	10	80	30	90
7	0	30	50	10	60	22	0	10	90	30	100
8	1	90	80	80	70	23	0	0	0	40	10
9	0	40	70	10	80	24	0	0	10	40	20
10	1	30	100	80	90	25	1	80	40	50	30
11	1	100	100	70	100	26	0	0	30	40	40
12	0	10	0	10	0	27	0	0	40	0	50
13	0	0	0	20	10	28	1	100	70	50	60
14	0	10	10	20	20	29	0	10	60	0	70
15	1	80	40	70	30	30	1	90	10	50	0

```
a = \text{count of days with hits} (o_j, f_j) = (1, 1)
```

 $b = \text{count of days with false alarms } (o_i f_i) = (0, 1)$

 $c = \text{count of days with misses } (o_j, f_j) = (1, 0)$

 $d = \text{count of days: correct rejection } (o_j, f_j) = (0, 0)$

In [36]: import pandas as pd table = {"Days": np.arange(1,31,1), "o": np.array([1, 0, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 1 , 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1]), "p": np.array([50, 20, 20, 60, 50, 20, 30, 90, 40, 30, 100, 10, 0, 10, 80, 60, 70, 90, 80, 70, 10, 10, 0, 0, 80, 0, 0, 100, 10, 90])} prect = np.arange(0,110,10)zero = np.zeros_like(table['p']) table['a'], table['b'], table['c'], table['d'] = [], [], [], [] for i in range(len(prect)): flag = np.where(table['p'] < prect[i], zero, 1)</pre> table[prect[i]]= flag a = flag[table['o']==1] table['a'].append(np.sum(a)) b = flag[table['o']==0] table['b'].append(np.sum(b)) d = np.where(flag != table['o'], zero, -1) dd = (np.sum(d) + np.sum(a))*-1table['d'].append(dd) c = np.where((table['o']==0) & (flag==1), zero, -1)table['c'].append((np.sum(a)+np.sum(c) + dd)*-1)df = pd.DataFrame.from dict(table, orient='index') df = df.transpose() print(df)

Days	s o		p	a	b	С	d 0	10	20	30	40	50
0	1.0	1.0	50.0	13.0	17.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0
1.0	2.0	0.0	20.0	13.0	12.0	0.0	5.0	1.0	1.0	1.0	0.0	0.0
0.0 2 0.0	3.0	1.0	20.0	13.0	7.0	0.0	10.0	1.0	1.0	1.0	0.0	0.0
3 1.0	4.0	1.0	60.0	12.0	5.0	1.0	12.0	1.0	1.0	1.0	1.0	1.0
4 1.0	5.0	0.0	50.0	11.0	4.0	2.0	13.0	1.0	1.0	1.0	1.0	1.0
5	6.0	0.0	20.0	11.0	3.0	2.0	14.0	1.0	1.0	1.0	0.0	0.0
6 0.0	7.0	0.0	30.0	10.0	2.0	3.0	15.0	1.0	1.0	1.0	1.0	0.0
7 1.0	8.0	1.0	90.0	9.0	1.0	4.0	16.0	1.0	1.0	1.0	1.0	1.0
8	9.0	0.0	40.0	8.0	0.0	5.0	17.0	1.0	1.0	1.0	1.0	1.0
9 0.0	10.0	1.0	30.0	5.0	0.0	8.0	17.0	1.0	1.0	1.0	1.0	0.0
10 1.0	11.0	1.0	100.0	2.0	0.0	11.0	17.0	1.0	1.0	1.0	1.0	1.0
11 0.0	12.0	0.0	10.0	NaN	NaN	NaN	NaN	1.0	1.0	0.0	0.0	0.0
12 0.0	13.0	0.0	0.0	NaN	NaN	NaN	NaN	1.0	0.0	0.0	0.0	0.0
13 0.0	14.0	0.0	10.0	NaN	NaN	NaN	NaN	1.0	1.0	0.0	0.0	0.0
14 1.0	15.0	1.0	80.0	NaN	NaN	NaN	NaN	1.0	1.0	1.0	1.0	1.0
15 1.0	16.0	0.0	60.0	NaN	NaN	NaN	NaN	1.0	1.0	1.0	1.0	1.0
16 1.0	17.0		70.0	NaN				1.0				1.0
17 1.0	18.0	1.0	90.0	NaN	NaN	NaN	NaN	1.0	1.0	1.0	1.0	1.0
18	19.0	1.0	80.0	NaN	NaN	NaN	NaN	1.0	1.0	1.0	1.0	1.0
19 1.0	20.0	0.0	70.0	NaN	NaN	NaN	NaN	1.0	1.0	1.0	1.0	1.0
20 0.0 21	21.0	0.0	10.0	NaN NaN	NaN	NaN	NaN	1.0	1.0	0.0	0.0	0.0
0.0	22.0	0.0	0.0	Nan	NaN NaN	NaN NaN	NaN NaN	1.0	0.0	0.0	0.0	0.0
0.0	24.0	0.0	0.0	Nan	Nan	Nan	NaN	1.0	0.0	0.0	0.0	0.0
0.0	25.0	1.0	80.0	NaN	Nan	Nan	NaN	1.0	1.0	1.0	1.0	1.0
1.0	26.0	0.0	0.0	NaN	Nan	Nan	NaN	1.0	0.0	0.0	0.0	0.0
0.0	27.0	0.0	0.0	NaN	NaN	NaN	NaN	1.0	0.0	0.0	0.0	0.0
0.0 27	28.0	1.0	100.0	NaN	NaN	NaN	NaN	1.0	1.0	1.0	1.0	1.0

1.0

28 29.0 10.0 0.0 NaN NaN NaN NaN 1.0 1.0 0.0 0.0 0.0 0.0 29 30.0 1.0 90.0 NaN 1.0 1.0 1.0 1.0 1.0 NaN NaN NaN 1.0 60 70 80 90 100 0 0.0 0.0 0.0 0.0 0.0 1 0.0 0.0 0.0 0.0 0.0 2 0.0 0.0 0.0 0.0 0.0 3 1.0 0.0 0.0 0.0 0.0 4 0.0 0.0 0.0 0.0 0.0 5 0.0 0.0 0.0 0.0 0.0 6 0.0 0.0 0.0 0.0 0.0 7 1.0 1.0 1.0 1.0 0.0 8 0.0 0.0 0.0 0.0 0.09 0.0 0.0 0.0 0.0 0.0 10 1.0 1.0 1.0 1.0 1.0 11 0.0 0.0 0.0 0.0 0.0 12 0.0 0.0 0.0 0.0 0.0 13 0.0 0.0 0.0 0.0 0.0 0.0 14 1.0 1.0 1.0 0.0 15 1.0 0.0 0.0 0.0 0.0 16 1.0 0.0 1.0 0.0 0.0 17 1.0 1.0 1.0 1.0 0.0 18 1.0 0.0 1.0 1.0 0.0 19 0.0 1.0 1.0 0.0 0.0 20 0.00.0 0.0 0.0 0.0 21 0.0 0.0 0.0 0.0 0.0 22 0.0 0.0 0.0 0.0 0.0 23 0.0 0.0 0.0 0.0 0.0 24 1.0 0.0 1.0 1.0 0.0 25 0.0 0.0 0.0 0.0 0.0 0.0 26 0.0 0.0 0.0 0.0 27 1.0 1.0 1.0 1.0 1.0 28 0.0 0.0 0.0 0.0 0.0 29 1.0 1.0 1.0 1.0 0.0

$$H = \frac{a}{a+c}$$

```
In [37]: H = df.a / (df.a + df.c)
          H = H[0:11]
          print(f" Hit rate {H}")
         Hit rate 0
                           1.000000
                1.000000
          2
                1.000000
          3
                0.923077
          4
                0.846154
          5
                0.846154
          6
                0.769231
          7
                0.692308
          8
                0.615385
          9
                0.384615
          10
                0.153846
          dtype: float64
```

```
F = \frac{b}{b+d}
```

```
In [38]: F = df.b / (df.b + df.d)
          F = F[0:11]
         print(f"False-alarm rate
                                      {F}")
         False-alarm rate
                              0
                                    1.000000
                0.705882
         1
         2
                0.411765
          3
                0.294118
          4
                0.235294
          5
                0.176471
          6
                0.117647
          7
                0.058824
          8
                0.00000
                0.00000
          9
          10
                0.00000
         dtype: float64
```

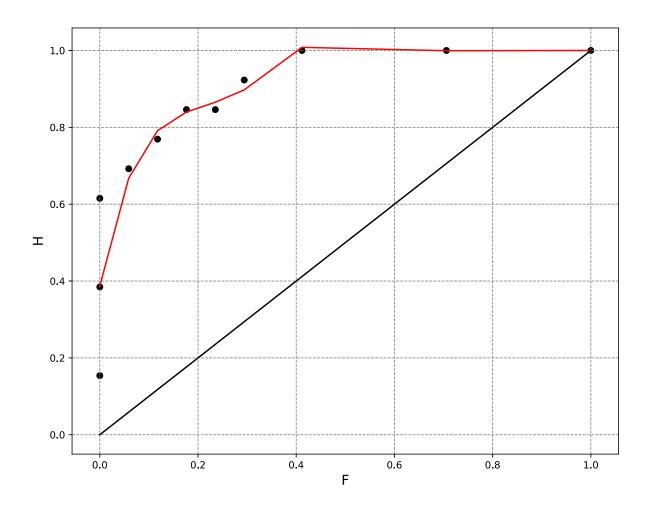
Plot the result as a ROC diagram

```
In [39]: poly = np.polyfit(F,H,5)
    poly_y = np.polyld(poly)(F)

fig, ax = plt.subplots(1,1, figsize=(10,8))
    fig.suptitle('ROC diagram', fontsize= plt_set.title_size, fontweight="bold")
    ax.set_xlabel('F', fontsize = plt_set.label)
    ax.set_ylabel('H', fontsize = plt_set.label)
    ax.xaxis.grid(color='gray', linestyle='dashed')
    ax.yaxis.grid(color='gray', linestyle='dashed')
    ax.plot(F, poly_y, "r")
    ax.scatter(F, H, color = 'k')
    x = np.linspace(0,1,100)
    ax.plot(x, x, color = "k")

plt.show()
```

ROC diagram



Find the area under the ROC curve and find the ROC skill score.

$$SS_{ROC} = (2 \cdot A) - 1$$

```
In [40]: from scipy.integrate import simps

# Compute the area using the composite trapezoidal rule.
area = np.trapz(H[::-1], F[::-1])
area = round(area,3)
SSR = (2*area) - 1
SSR = round(SSR,3)
print(f"Area under the ROC curve: {area} and ROC skill score: {SSR}")
```

Area under the ROC curve: 0.932 and ROC skill score: 0.864