January 6, 2020

Assignment Set for Laboratory 1

ATSC 409: Hand-in answers to questions 1, 2 and 3.
EOSC 511/ATSC 506: Hand-in answers to questions 1, 3 and 4.

All questions should be done by hand (not by computer) and show your steps.

1. Given the following four (x,y) points (-5,-1), (0,0), (5,1), (8,4) find the y-value at x=3 using
   (a) Linear Interpolation
   (b) Quadratic Interpolation
2. Given the equation
$$\frac{\partial y}{\partial t} = y(y+t) \tag{1}$$
   write down
   (a) forward Euler difference formula
   (b) backward Euler difference formula
   (c) centered difference formula
3. The equation
$$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = 0,\ y = \cos(x) \text{ at } t = 0,\ \frac{\partial y}{\partial t} = c\sin(x) \text{ at } t = 0 \tag{2}$$
   has a solution $y = \cos(x - ct)$.
   (a) Expand both derivatives as centred differences.
   (b) Show that the algebraic solution is an exact solution of the difference formula if we choose $\Delta x = c\Delta t$.
4. Given
$$\frac{\partial y}{\partial t} = -\alpha y,\ y = 1 \text{ at } t = 0 \tag{3}$$
   (a) Show that the forward Euler method gets a smaller answer than the backward Euler method for all $t > 0$, provided that $0 < \alpha^2\Delta t^2 < 1$.
   (b) Solve the equation analytically.
   (c) Show that the forward Euler always under-estimates the answer provided that $\alpha\Delta t < 1$ and $\alpha\Delta t \neq 0$.