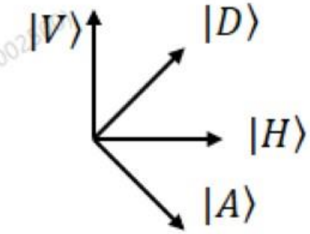
Summary of previous lecture

- Mainly from 2.2.3 of the textbook
- Postulate 3: Copenhagen interpretation
  - Assumption
    - The particle is in a state $|\psi\rangle$ (normalized vector)
    - We want to measure the variable corresponding to Ω (Hermitian operator)
  - Measured value: one of the eigenvalues ω_i (real)
  - Probability of measuring ω_i: $|\langle\omega_i|\psi\rangle|^2$
  - The state right after the measurement: $|\omega_i\rangle$ (collapse)
  - Expected value of measurement: $\langle\psi|\Omega|\psi\rangle$
- Other interpretations
  - Hidden variable theory
  - Multiverse theory

Uncertainty principle

- Polarization measurement
  - Initial state: $|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$
  - Polarization measuring device 1: 2 for $|H\rangle$ and -1 for $|V\rangle$
    ➔ $PMD1 = 2|H\rangle\langle H| - |V\rangle\langle V|$
  - Polarization measuring device 2: 1 for $|H\rangle$ and -1 for $|V\rangle$
    ➔ $PMD2 = |H\rangle\langle H| - |V\rangle\langle V|$



$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$ $\quad |\psi\rangle = |H\rangle$ $\quad |\psi\rangle = |H\rangle$ $\quad |\psi\rangle = |H\rangle$

  - Polarization measuring device 3: 1 for $|D\rangle$ and -1 for $|A\rangle$
    ➔ $PMD3 = |D\rangle\langle D| - |A\rangle\langle A|$

$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$

$|\psi\rangle = |H\rangle = \frac{1}{\sqrt{2}}|D\rangle + \frac{1}{\sqrt{2}}|A\rangle$

$|\psi\rangle = |D\rangle = \frac{1}{\sqrt{2}}|H\rangle + \frac{1}{\sqrt{2}}|V\rangle$

$|\psi\rangle = |V\rangle$

Polarization measurement

- Do we really observe postulate 3 with polarization measurement?

$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$ $\quad$ $|\psi\rangle = |H\rangle = \frac{1}{\sqrt{2}}|D\rangle + \frac{1}{\sqrt{2}}|A\rangle$ $\quad$ $|\psi\rangle = |D\rangle = \frac{1}{\sqrt{2}}|H\rangle + \frac{1}{\sqrt{2}}|V\rangle$ $\quad$ $|\psi\rangle = |V\rangle$

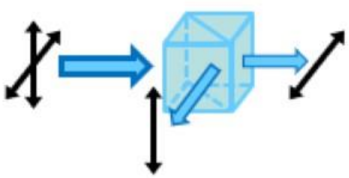
  - With the above measurement, what we usually measure is the followings:
  - Assume that we measure 100 mW of light power at ①.
  - Then at ②, we would measure only 50 mW of light power.
  - At ③, we measure only 25 mW of light power.

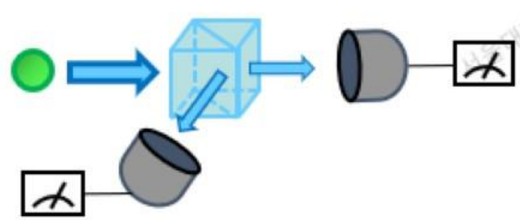
What is the power level?

- Frequency of 532nm: $f = \frac{v}{\lambda} = \frac{c}{532\ nm} = \frac{3\times10^8 m/s}{532\times10^{-9} m} = 5.639 \times 10^{14}\ /s$
- Single photon energy: $\hbar\omega = (1.054 \times 10^{-34}\ m^2kg/s)(2\pi \times 5.639 \times 10^{14}\ /s) = 3.7 \times 10^{-19}\ kg\ m^2 s^{-2}$
- Power: Energy/time=$3.7 \times 10^{-19}\ Joule/sec$ (Watt)

- Where is the collapse of the quantum state?
  - What would happen if we attenuate the light power such that there is only a single photon during 1 second?

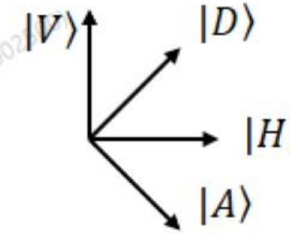
Polarizing beam splitter (PBS)





Uncertainty principle

- Polarization measurement
  - Initial state: $|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$
  - Polarization measuring device 1: 2 for $|H\rangle$ and -1 for $|V\rangle$
    - $P1 = 2|H\rangle\langle H| - |V\rangle\langle V|$
  - Polarization measuring device 2: 1 for $|H\rangle$ and -1 for $|V\rangle$
    - $P2 = |H\rangle\langle H| - |V\rangle\langle V|$
  - Do $P1$ and $P2$ commute? $[P1, P2] = 0$
    - When two operators corresponding to measurements commute with each other, once one of the measurement is over, the subsequent measurement by the other operator won't affect the quantum state. ⇔ Simultaneously diagonalizable with common basis

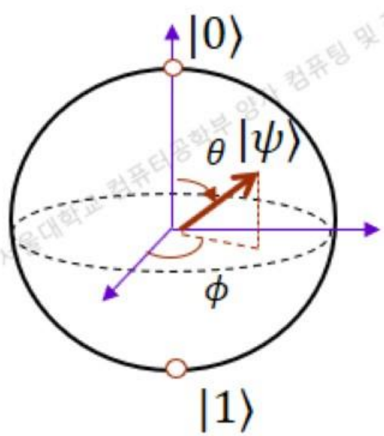


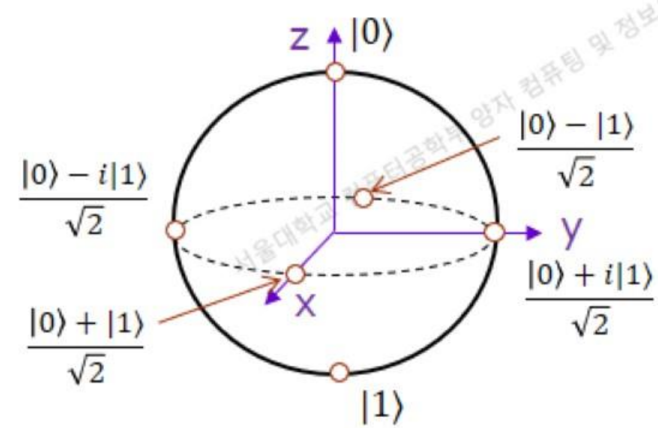
  - Polarization measuring device 3: 1 for $|D\rangle$ and -1 for $|A\rangle$
    - $P3 = |D\rangle\langle D| - |A\rangle\langle A| = \frac{|H\rangle+|V\rangle}{\sqrt{2}}\frac{\langle H|+\langle V|}{\sqrt{2}} - \frac{|H\rangle-|V\rangle}{\sqrt{2}}\frac{\langle H|-\langle V|}{\sqrt{2}}$
      $= \frac{1}{2}(|H\rangle\langle H| + |H\rangle\langle V| + |V\rangle\langle H| + |V\rangle\langle V|) - \frac{1}{2}(|H\rangle\langle H| - |H\rangle\langle V| - |V\rangle\langle H| + |V\rangle\langle V|)$
      $= \frac{1}{2}(2|H\rangle\langle V| + 2|V\rangle\langle H|) = |H\rangle\langle V| + |V\rangle\langle H|$
  - Do $P3$ and $P2$ commute? $[P3, P2] \neq 0$
    - $[P3, P2] = P3P2 - P2P3$
    - $P3P2 = (|H\rangle\langle V| + |V\rangle\langle H|)(|H\rangle\langle H| - |V\rangle\langle V|) = |H\rangle\langle V||H\rangle\langle H| - |H\rangle\langle V||V\rangle\langle V| + |V\rangle\langle H||H\rangle\langle H| - |V\rangle\langle H||V\rangle\langle V| = -|H\rangle\langle V| + |V\rangle\langle H|$
    - $P2P3 = (|H\rangle\langle H| - |V\rangle\langle V|)(|H\rangle\langle V| + |V\rangle\langle H|) = |H\rangle\langle H||H\rangle\langle V| + |H\rangle\langle H||V\rangle\langle H| - |V\rangle\langle V||H\rangle\langle V| - |V\rangle\langle V||V\rangle\langle H| = |H\rangle\langle V| - |V\rangle\langle H|$
    - $[P3, P2] = P3P2 - P2P3 = -2|H\rangle\langle V| + 2|V\rangle\langle H| \neq 0$
    - When the two measurement operators do not commute with each other, generally the measurement of one operator will affect the resulting quantum states from the previous measurement by the other operator.

Quantum bits

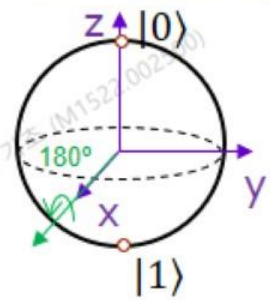
- Section 1.2 (Read section 1.1 for the historical perspective)
- Qubit
  - Abstract mathematical objects: a vector in two-dimensional complex vector space composed of basis $|0\rangle$ and $|1\rangle$ (called computational basis)
  - Physical system
- Arbitrary qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
  - $|\alpha|^2 + |\beta|^2 = 1$
  - Measurement gives 0 or 1 ➔ we cannot directly get α and β with single measurement. Why? Then how to measure α and β?
- Bloch sphere
  - Visualization of an arbitrary qubit
    - $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = e^{i\gamma}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle\right)$

    ➔ Global phase cannot be measured: $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$

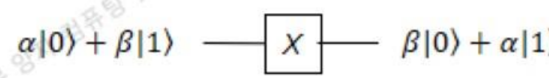




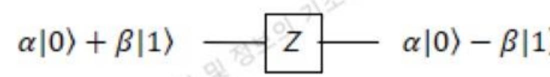
Single-qubit gates



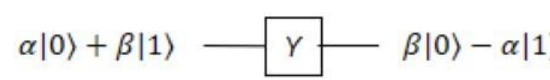
- $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$, $|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$
- X gate
  - Similar to classical NOT gate
  - $|0\rangle \Rightarrow |1\rangle$ and $|1\rangle \Rightarrow |0\rangle$: $X|0\rangle = |1\rangle$, $X|1\rangle = |0\rangle$ ➔ $X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
  - $X(\alpha|0\rangle + \beta|1\rangle) = \alpha|1\rangle + \beta|0\rangle$
  - Rotation in Bloch sphere



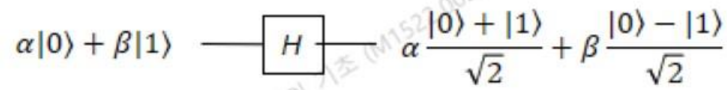
- Z gate
  - $Z|0\rangle = |0\rangle$, $Z|1\rangle = -|1\rangle$ ➔ $Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$



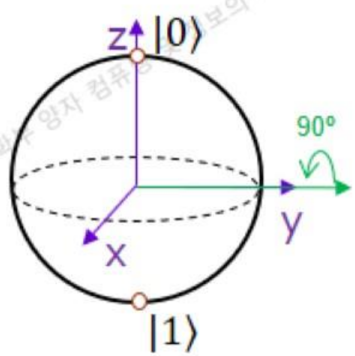
- Y gate: $Y = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$



Single-qubit gates



- Hadamard gate
  - $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
  - How to measure the phase of $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$?
  - $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $H^2 = ?$
  - Which rotation corresponds to Hadamard gate?



vs