

Summary of previous lecture

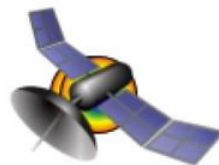
- Quantum circuits
 - Quantum SWAP gate
 - No loop, No FANOUT
 - Controlled-U gate
 - Measurement symbol
- No cloning theorem
 - We cannot copy the same quantum state when an arbitrary quantum state is given
- Measurement in other bases
 - Basis transformation (unitary transform) before the measurement
 - Bell basis
 - Generation of Bell basis
 - Measurement in Bell basis

Quantum entanglement

- Recently Chinese quantum satellite succeeded in distributing quantum entangled state:



Station A



$$(|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B) / \sqrt{2}$$



Station B

- Case 1) What happens if station A and station B measure respective photons in H-V basis?
 - Their measurement will be always opposite to each other → anti-correlation
- Case 2) What happens if station A and station B measure respective photons in D-A basis?
 - Their measurement will be always opposite to each other → anti-correlation !!!

Quantum entanglement

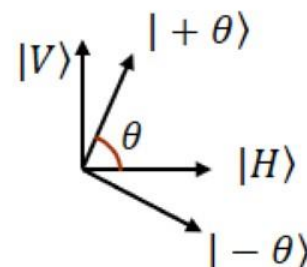
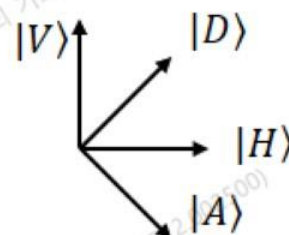
- Case 2) What happens if station A and station B measure respective photons in D-A basis?

- We know that $\begin{cases} |D\rangle = (|H\rangle + |V\rangle)/\sqrt{2} \\ |A\rangle = (|H\rangle - |V\rangle)/\sqrt{2} \end{cases}$

- Also we know that $\begin{cases} |H\rangle = (|D\rangle + |A\rangle)/\sqrt{2} \\ |V\rangle = (|D\rangle - |A\rangle)/\sqrt{2} \end{cases}$

- $$\begin{aligned} |\psi^-\rangle &= [|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B]/\sqrt{2} \\ &= [(|D\rangle + |A\rangle)_A (|D\rangle - |A\rangle)_B - (|D\rangle - |A\rangle)_A (|D\rangle + |A\rangle)_B]/2\sqrt{2} \\ &= [|D\rangle_A |D\rangle_B - |D\rangle_A |A\rangle_B + |A\rangle_A |D\rangle_B - |A\rangle_A |A\rangle_B]/2\sqrt{2} \\ &\quad - [|D\rangle_A |D\rangle_B + |D\rangle_A |A\rangle_B - |A\rangle_A |D\rangle_B - |A\rangle_A |A\rangle_B]/2\sqrt{2} \\ &= [\quad \quad \quad -|D\rangle_A |A\rangle_B + |A\rangle_A |D\rangle_B \quad \quad \quad]/\sqrt{2} \end{aligned}$$

- Homework: prove that the same argument is valid even for any arbitrary angle.



Quantum entanglement

- Can we reproduce the same results using a classical device?
 - We can reproduce the same result as case 1) by making a device which generates random number 0 or 1, and when that random number is 0, send out H-polarized photon to A and V-polarized photon to B. When the random number is 1, send out V-polarized photon to A and H-polarized photon to B.
 - Can such device reproduce the same result as case 2)? No
- How can we distinguish such kind of fake device and the real quantum entanglement device?
 - Randomize the choice of measurement basis and check the measured correlation

Quantum entanglement

- Can we implement communication faster than the speed of light by using EPR pair (i.e. Bell basis)?
 - NO!!!

기업 · 과학·기술

[F] 아인슈타인도 "귀신 같은 현상"이라던 양자통신, 현실이 되다

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기사 100자평(32)

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1200km 떨어진 곳서 정보 동시 수신... 중국, 위성 이용해 성공

양자역학에서는 서로 얽힘 상태인 두 입자는 아무리 멀리 떨어져 있더라도 그 관계가 깨지지 않는다. 이 때문에 수 광년(빛이 1년 동안 이동하는 거리) 떨어져 있어도 전자의 상태가 업에서 다운으로 바뀌면 동시에 양전자의 상태도 반대로 바뀐다. 각 입자의 상태를 0 혹은 1과 같은 하나의 정보라고 간주하면 입자의 상태 변화를 정보로 볼 수 있다. **빛보다 빠른 정보 전달이 가능해지는 것이다.** 하지만 양자 얽힘은 현실에선 구현하기 쉽지 않다. 양자물리학의 토대를 마련한 알베르트 아인슈타인도 생전 양자 얽힘에 대해 "귀신 같은(spooky) 현상"이라며 쉽게 받아들이지 않았다.

댓글 (1)

김재완 (johnja****) 2017.07.15 21:32:03 신고

이 기사 필자로 표시된 김재완 교수입니다. 제가 보낸 원고의 일부분과 제가 기자와 만나 알려준 내용을 기반으로 작성되었을텐데, 내용 중 중요한 부분에 잘못이 있습니다. 중요한 몇 가지는, (1) 빛보다 빠른 정보전달이 가능해진다 (2) 화성 지구 사이 통신시간차 20분이 사라진다 등입니다. 물리적으로 정보전달이 빛보다 빨라질 수는 없습니다.

댓글 (5)

Quantum entanglement

- Schrödinger's cat state

- $|\psi\rangle = (|\text{excited}\rangle_{\text{atom}}|\text{Alive}\rangle_{\text{cat}} + |\text{ground}\rangle_{\text{atom}}|\text{Dead}\rangle_{\text{cat}})/\sqrt{2}$

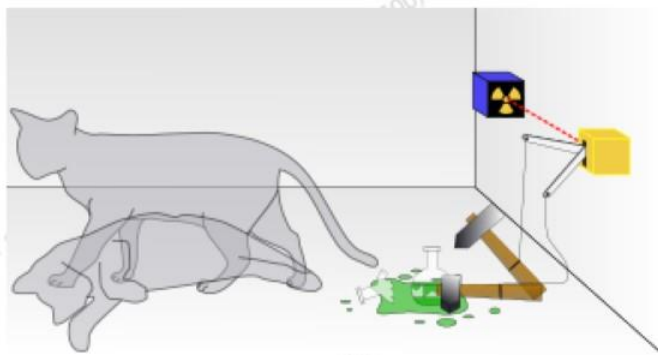
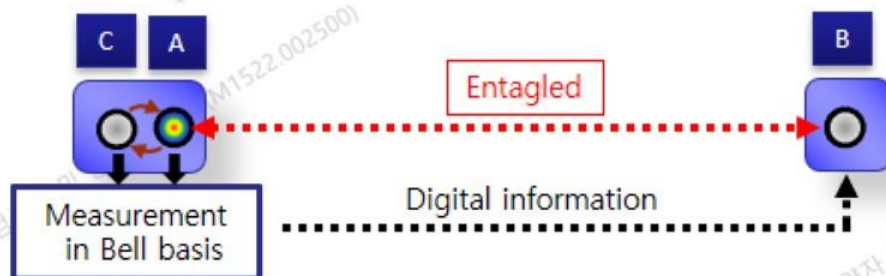


Image from

https://en.wikipedia.org/wiki/Schr%C3%B6dinger%27s_cat#/media/File:Schrodingers_cat.svg

- "Is the moon there when nobody looks?"

Quantum teleportation



- $|\Psi\rangle_C = (\alpha|0\rangle_C + \beta|1\rangle_C)$: arbitrary quantum state to teleport

- $|\Psi\rangle_C |\psi^-\rangle_{AB} = (\alpha|0\rangle_C + \beta|1\rangle_C)(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)/\sqrt{2}$

$$= [\alpha|0\rangle_C|0\rangle_A|1\rangle_B - \alpha|0\rangle_C|1\rangle_A|0\rangle_B + \beta|1\rangle_C|0\rangle_A|1\rangle_B - \beta|1\rangle_C|1\rangle_A|0\rangle_B]/\sqrt{2}$$

$$= [\alpha(|\phi^+\rangle_{CA} + |\phi^-\rangle_{CA})|1\rangle_B - \alpha(|\psi^+\rangle_{CA} + |\psi^-\rangle_{CA})|0\rangle_B + \beta(|\psi^+\rangle_{CA} - |\psi^-\rangle_{CA})|1\rangle_B - \beta(|\phi^+\rangle_{CA} - |\phi^-\rangle_{CA})|0\rangle_B]/2$$

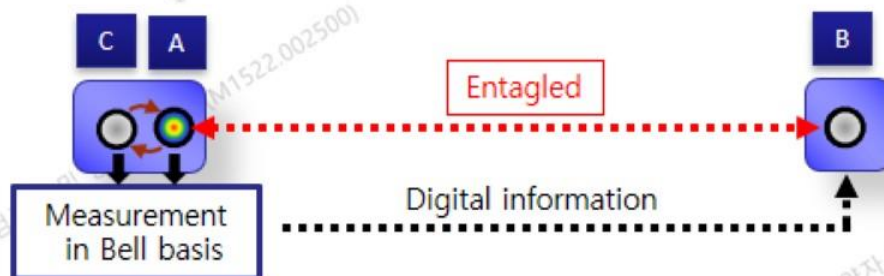
$$= [|\psi^+\rangle_{CA}(-\alpha|0\rangle_B + \beta|1\rangle_B) - |\psi^-\rangle_{CA}(\alpha|0\rangle_B + \beta|1\rangle_B) + |\phi^+\rangle_{CA}(\alpha|1\rangle_B - \beta|0\rangle_B) + |\phi^-\rangle_{CA}(\alpha|1\rangle_B + \beta|0\rangle_B)]/2$$

$$\begin{cases} |\psi^+\rangle_{CA} = [|0\rangle_C|1\rangle_A + |1\rangle_C|0\rangle_A]/\sqrt{2} \\ |\psi^-\rangle_{CA} = [|0\rangle_C|1\rangle_A - |1\rangle_C|0\rangle_A]/\sqrt{2} \\ |\phi^+\rangle_{CA} = [|0\rangle_C|0\rangle_A + |1\rangle_C|1\rangle_A]/\sqrt{2} \\ |\phi^-\rangle_{CA} = [|0\rangle_C|0\rangle_A - |1\rangle_C|1\rangle_A]/\sqrt{2} \end{cases}$$



$$\begin{cases} |0\rangle_C|1\rangle_A = [|\psi^+\rangle_{CA} + |\psi^-\rangle_{CA}]/\sqrt{2} \\ |1\rangle_C|0\rangle_A = [|\psi^+\rangle_{CA} - |\psi^-\rangle_{CA}]/\sqrt{2} \\ |0\rangle_C|0\rangle_A = [|\phi^+\rangle_{CA} + |\phi^-\rangle_{CA}]/\sqrt{2} \\ |1\rangle_C|1\rangle_A = [|\phi^+\rangle_{CA} - |\phi^-\rangle_{CA}]/\sqrt{2} \end{cases}$$

Quantum teleportation



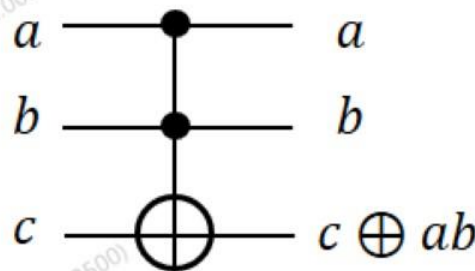
- $|\Psi\rangle_C = (\alpha|0\rangle_C + \beta|1\rangle_C)$: arbitrary quantum state to teleport
- $|\Psi\rangle_C |\psi^-\rangle_{AB} = [|\psi^+\rangle_{CA}(-\alpha|0\rangle_B + \beta|1\rangle_B) - |\psi^-\rangle_{CA}(\alpha|0\rangle_B + \beta|1\rangle_B) + |\phi^+\rangle_{CA}(\alpha|1\rangle_B - \beta|0\rangle_B) + |\phi^-\rangle_{CA}(\alpha|1\rangle_B + \beta|0\rangle_B)]/2$
- If we can measure $|\psi^-\rangle_{CA}$, original state $|\Psi\rangle_C$ seems to be reproduced at qubit B.
- What happens if we measure other Bell basis such as $|\psi^+\rangle_{CA}$ or $|\phi^-\rangle_{CA}$?
 - We need to apply different unitary operations on qubit B depending on the measurement result of qubit C and qubit A.
 - For example, if we measure $|\phi^+\rangle_{CA}$, X gate should be applied to qubit B.
- How can we measure qubit C and qubit A in Bell basis?
- Can we claim that we can implement communication faster than the speed of light using quantum teleportation?

Reversible gate

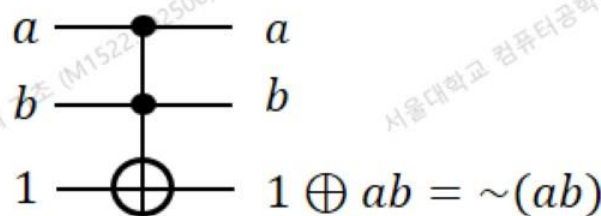
- Section 1.4.1
- Classical AND gate: irreversible \Leftrightarrow We cannot tell what the input was, when only the output is given
- However, unitary operation is reversible
 - \Leftrightarrow Preservation of orthogonality
 - Unitary operator maps different inputs to distinguishable outputs
 - Proof) If not, there exist $|x\rangle$ and $|y\rangle$ such that $|x\rangle \neq |y\rangle$ and $U|x\rangle = U|y\rangle$. Apply U^\dagger to both sides, $U^\dagger U|x\rangle = U^\dagger U|y\rangle$. Therefore the assumption is contradictory!
- Can we simulate a classical logic circuit using a quantum circuit?
- ➔ Replace the digital gate with reversible gate.
- Toffoli gate, Fredkin gate are reversible gate.

Toffoli gate

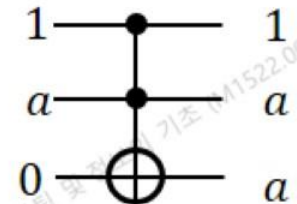
Input			Output		
a	b	c	a'	b'	c'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0



NAND



FANOUT

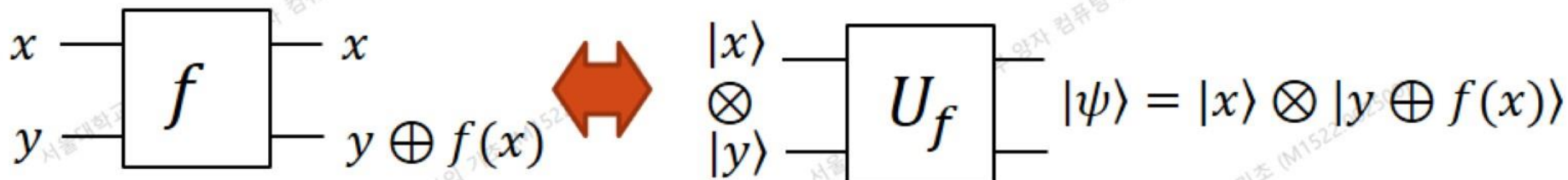


An arbitrary classical circuit can be simulated by an equivalent reversible circuit.

Toffoli gate can be implemented by unitary gate.

Quantum parallelism

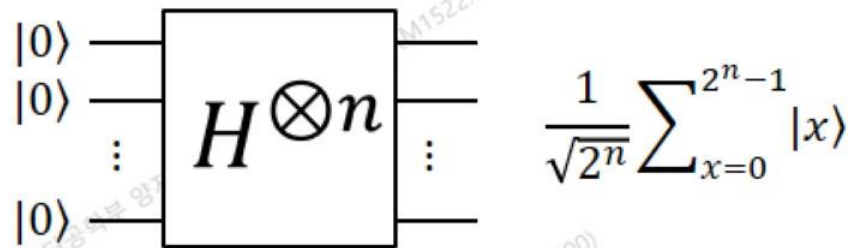
- Section 1.4.2
- Suppose $f(x): \{0,1\} \rightarrow \{0,1\}$
- Assume we built the following circuit:



- Instead of $|x\rangle = |0\rangle$ or $|1\rangle$, try $|x\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|y\rangle = |0\rangle$
- $U_f \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle \right) = U_f \left(\frac{|0\rangle \otimes |0\rangle}{\sqrt{2}} \right) + U_f \left(\frac{|1\rangle \otimes |0\rangle}{\sqrt{2}} \right) = \frac{|0\rangle \otimes |f(0)\rangle + |1\rangle \otimes |f(1)\rangle}{\sqrt{2}}$
- It looks like we obtained the multiple results with a single processing.

$$\begin{aligned}
 &|0\rangle \xrightarrow{H} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\
 &|0\rangle \xrightarrow{H} \frac{|0\rangle + |1\rangle}{\sqrt{2}}
 \end{aligned}
 = \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

Quantum parallelism



- With such kind of parallel input, even if we obtain $\frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x, f(x)\rangle$, it is not efficient by itself due to the quantum state collapse during the measurement process.
→ We need an algorithm to utilize such kind of superposition.