Weekly Problem Set #8 – Week 9

PLEASE SHOW ALL YOUR WORK FOR FULL CREDIT. Include intermediate steps in ways that others can understand or writing sentences that help to communicate your assumptions and logic. If you utilize any software tools or apps (e.g. Mathematica, Desmos, ChatGTP, etc.), you must transparently acknowledge your use of them in your HW submission. A subset of these problems will be graded for correctness. The rest of the problems will be graded for effort.

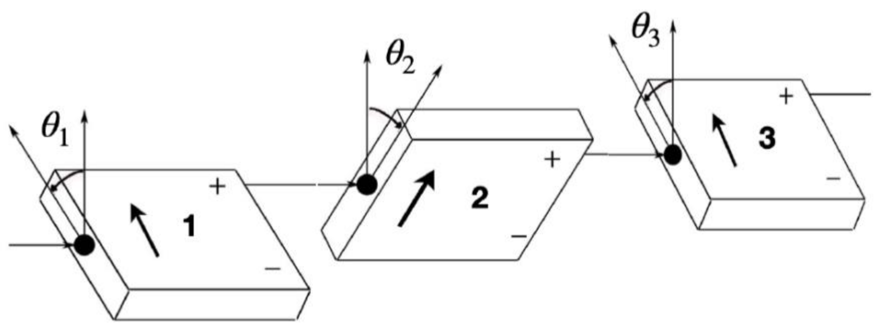
1. **(6 pts) Normalization and probabilities with a discrete spin-1/2 system.**
   Consider the state vector: $|\psi_{in}\rangle = 6|\uparrow_z\rangle - i|\downarrow_z\rangle$
   a. Normalize this state vector by finding the normalization constant and re-write an expression for the normalized state vector.

   b. Find the probabilities of measuring the spin component along the z-axis to be $S_z = +\frac{\hbar}{2}$ and $S_z = -\frac{\hbar}{2}$.

   c. Verify that the probability of spin up (in the z-basis) and the probability of spin down (in the z-basis) add up to one.

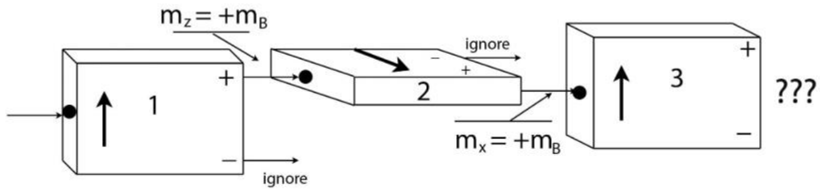
2. **(4 pts) Big Picture Findings of Stern-Gerlach Experiments.**
   a. Describe in words what the classical prediction (or classically expected results) of the Stern-Gerlach experiment were.
   b. Describe the empirical results of the Stern-Gerlach experiment and how they violated classical expectations.

3. **(6 pts) Stern-Gerlach.** We have three Stern-Gerlach analyzers all set up in the following orientation:



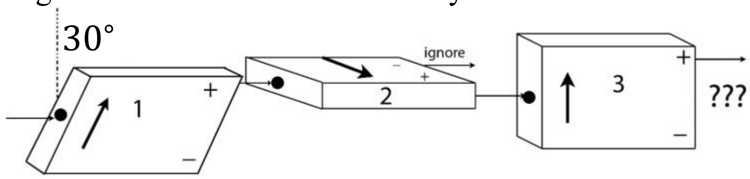
Analyzer 1 is at an angle $\theta_1 = -15°$ from the vertical, Analyzer 2 is at an angle $\theta_2 = 35°$ from the vertical, and Analyzer 3 is at an angle le $\theta_3 = -20°$ from the vertical.
   a. What is the probability that atoms leaving the + channel of Analyzer 1 leave the + channel of Analyzer 3?
   b. What is the probability that atoms leaving the + channel of Analyzer 1 leave the − channel of Analyzer 3?
   c. Explain why the probabilities in (a) and (b) do not add up to 1.

4. **(5 pts). Stern-Gerlach.** In class we considered the following set-up:



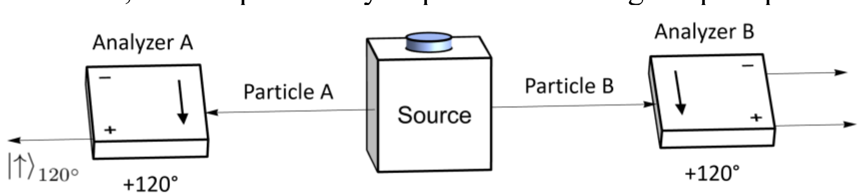
For the experimental set-up above, we concluded of the atoms entering the third apparatus, approximately half of the atoms leave from the plus-channel of analyzer 3 and approximately half of the atoms leave from the minus-channel of analyzer 3.

Let's consider a modification to this set up which is that the first analyzer is tilted 30 degrees from the vertical and Analyzers 2 & 3 are left unchanged.

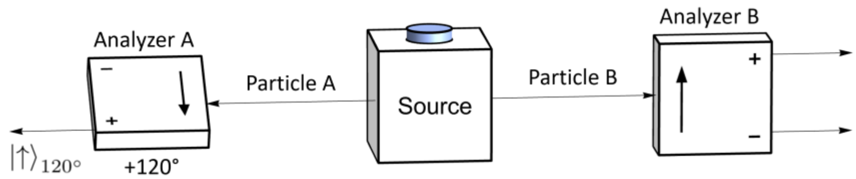


What is the probability for an atom leaving the plus-channel of Analyzer 1 to exit from the plus-channel of Analyzer 3?

5. **(6 pts). Entanglement.** Imagine you have a set up like the one from Thursday's lecture, which produces pairs of entangled particles from a black box source. One of the entangled particles, particle A, goes left into Stern-Gerlach analyzer A. The other particle of the entangled pair, particle B, goes right into Stern-Gerlach analyzer B. The pair is represented by the quantum state $|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow_z\uparrow_z\rangle + |\downarrow_z\downarrow_z\rangle]$. In this entangled state, recall that conducting a measurement on one particle's spin determines the other particle's spin state to be the same as the state of the one you measured.
   a. Using a SG Analyzer A oriented 120 degrees from z-axis, you measure particle A to be $|\uparrow_{120°}\rangle$. After this measurement, what is the quantum state of particle B? If you measure particle B's spin with **SG Analyzer B oriented 120 degrees** from the z-axis, what is probability of particle B exiting the plus-port of Analyzer B?



   b. Using SG Analyzer A oriented in 120 degrees from z-axis, you measure the particle A to be $|\uparrow_{120°}\rangle$. After this measurement, what is the quantum state of particle B? If you measure particle B's spin with **SG analyzer B oriented in the +z orientation**, what is probability of measuring spin-up?



c. Are your answers consistent with the observations from experiments with these devices that we made in class? Particularly, is your answer to part b consistent with Observation 1, and is your answer to part c consistent with Observation 2? (See slides from class.) Explain in a couple of sentences.