Weekly Problem Set #4 – Week 4

PLEASE SHOW ALL YOUR WORK FOR FULL CREDIT. Showing your work includes writing out intermediate steps in ways that others can understand or writing sentences that help to communicate your assumptions and logic. If you utilize any software tools or apps (e.g. Mathematica, Desmos, ChatGTP, etc.), you must transparently acknowledge your use of them in your HW submission. A subset of these problems will be graded for correctness. The rest of the problems will be graded for effort.

Part 0: Quantum Circuits

Question 1 (6 pts). (Wong E2.36). Practice using "Quirk," an online simulator for quantum circuits (https://algassert.com/quirk). Explain your reasoning and justify your answers.

Part 1: Linear Algebra

Note: If you find yourself need more of a refresher on linear algebra, please work through exercises (3.1-3.9). These problems will not be turned in and graded, but you will be expected to be fluent with these linear algebra operations.

Question 2 (6 pts). (Wong E3.10, on Change of basis). Show your work.

Question 3 (6 pts). (Wong E3.11, on Change of basis). Show your work.

Question 4 (4 pts). (Wong E3.15, on Gates as Matrices). Calculate by hand, using matrix notation. Show your work.

Question 5 (4 pts). (Wong E3.18, on Unitary Matrices). Show your work.

Question 6 (4 pts). (Wong E3.21, on Outer Products). Show your work.

Question 7 (4 pts). (Wong E3.23, on Completeness). Show your work.

Part 2: Measurement Examples

Question 8 (5 pts). A qubit is in the state $|0\rangle$. If you measure it in the Y-basis $\{|i\rangle, |-i\rangle\}$ and then measure it again in the Z-basis $\{|0\rangle, |1\rangle\}$, what is the probability of getting

(a) $|0\rangle$?

(b) $|1\rangle$?

Question 9 (9 pts).

(a) Imagine the following sequence of gates acting on the |0> qubit: HZH|0>. If you send 1000 qubits in the |0> state through this system, what are the possible z-measurement outcomes and their associated probabilities after the second Hadamard gate?

(b) Next, imagine that HZ[z-measurement]H|0>. If you send 1000 qubits in the |0> state through the first Hadamard gate, (bi) what are the possible z-measurement outcomes and their associated probabilities after the first Hadamard gate? (bii) What are the two possible resultant qubit states that could then enter the Z-gate (following your z-measurements)? (biii) Approximately how many of each resultant qubit state would you have? (biv) Are any superposition states entering the z-gate? (Yes or No) (bv) If all 1000 qubits (following the z-measurement) are then passed through the next HZ gates, what are the possible z-measurement outcomes and their associated probabilities after the second Hadamard gate?

(c) Why do these two examples give different results?