Weekly Problem Set #2 – Week 2

PLEASE SHOW ALL YOUR WORK FOR FULL CREDIT. Showing your work includes writing out intermediate steps in ways that others can understand or writing sentences that help to communicate your assumptions and logic. If you utilize any software tools or apps (e.g. Mathematica, Desmos, ChatGTP, etc.), you must transparently acknowledge your use of them in your HW submission. A subset of these problems will be graded for correctness. The rest of the problems will be graded for effort.

Part 0: Practice with Complex Number. As we move into the next quantum unit, we will expect you to have some fluency with complex numbers. Take this opportunity to review complex numbers using the resource we have provided.

Question 0 (3 pts). Practice using complex numbers to solve these problems:

a. What is the complex conjugate of this expression: Ce^{ikx}? What is the modulus of this expression: Ce^{ikx}? (where $k, C,$ and x are real)
b. Rewrite the following expression e^{ikx} in terms of sines and cosines (where k and x are real).
c. What is the real part of the expression in part (b)? What is the imaginary part of the expression in part (b)?
d. What is the complex conjugate of the expression in part (b)?
e. What is the complex conjugate of this expression: $-7i + 5$?
f. What is the modulus of this expression: $-7i + 5$?

Question 1 (5 pts): Demonstrate your understanding of complex numbers by solving the following problems with 2-D vectors. In each of the following problems, a and b are positive real numbers.

a. Consider this vector: $|\alpha\rangle = a + ib$. If I want to reflect this vector about the y-axis (or the imaginary axis) without changing its length to give me $|\alpha'\rangle$, what would I need to change? Tell me $|\alpha'\rangle$ in terms of the variables you've been given.

b. Consider this vector beta: $|\beta\rangle = a + ib$. If I want to reflect this vector about the x-axis (or the real axis) without changing its length to give me $|\beta'\rangle$, what would I need to change? Tell me $|\beta'\rangle$ in terms of the variables you've been given.

c. Consider this vector gamma: $|\gamma\rangle = a + ib$. Then I multiply the vector $|\gamma\rangle$ by $e^{i\pi}$ to get $|\gamma'\rangle$. What is the result of this transformation graphically? Draw $|\gamma\rangle$ and $|\gamma'\rangle$ on the complex plane.

d. Consider this vector delta: $|\delta\rangle = a + ib$. Then I multiply the vector $|\delta\rangle$ by $e^{\frac{i\pi}{2}}$ to get $|\delta'\rangle$.What is the result of this transformation graphically? Draw $|\delta\rangle$ and $|\delta'\rangle$ on the

complex plane.

e. Is the action in part (d) equivalent to any of the earlier actions in parts (a) - (c)? If so, which one? Explain your reasoning.

Part 1: Superposition & Bloch Sphere

Question 2 (5 pts). (Wong, E2.3). Draw a Bloch sphere and label the following locations:

(a) Where a qubit is exactly $|0\rangle$.
(b) Where a qubit is exactly $|1\rangle$.
(c) Where a qubit is half $|0\rangle$ and half $|1\rangle$.
(d) Where a qubit is more $|0\rangle$ than $|1\rangle$.
(e) Where a qubit is more $|1\rangle$ than $|0\rangle$.

Question 3 (3 pts). Play with the Bloch Sphere Simulation (https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/blochsphere/blochsphere.html) from QuViz. In this simulation, "spin-up" ($|\uparrow\rangle$) is equivalent to $|0\rangle$ and "spin-down" ($|\downarrow\rangle$) is equivalent to $|1\rangle$.

a) Give an example of a polar angle and azimuthal angle that maximizes the probability of measuring "spin up."
b) Give an example of a polar angle and azimuthal angle that minimizes the probability of measuring "spin up."
c) Give an example of a polar angle and azimuthal angle that results in spin up and spin down being equally likely.

Part 2. Measurement & State Normalization

Question 4 (6 pts). (Wong E2.6). Explain your reasoning and justify your answers.

Question 5 (4 pts). (Wong E2.7). Explain your reasoning and justify your answers.

Question 6 (4 pts) (Wong E2.8). Explain your reasoning and justify your answers.

Question 7 (6 pts) (Wong E2.9). Explain your reasoning and justify your answers.

Question 8 (6 pts) (Wong E2.10). Explain your reasoning and justify your answers.

Question 9 (6 pts) (Wong E2.11). Explain your reasoning and justify your answers.

Question 10 (4 pts) (Wong E2.12). Explain your reasoning and justify your answers.

Question 11 (4 pts) (Wong E2.13). Explain your reasoning and justify your answers.