PS 7

1. $|\psi_{00}\rangle = \alpha|000\rangle + \beta|100\rangle$

after first CNOT,

$$\alpha|000\rangle + \beta|110\rangle$$

after second CNOT,

$$\underline{\alpha|000\rangle + \beta|111\rangle}$$

$$= \alpha|0_L\rangle + \beta|1_L\rangle$$

2.

(a) don't need to apply one

(b) q_0 is likely to be flipped.

$\rightarrow$ X gate on q_0

(c) q_2 is likely to be flipped

$\rightarrow$ X gate on q_2

(d) q_1 is likely to be flipped

$\rightarrow$ X gate on q_1

3. $|\psi 00\rangle = \alpha|000\rangle + \beta|100\rangle$

i) 1st CNOT

$\rightarrow \alpha|000\rangle + \beta|110\rangle$

ii) 2nd CNOT

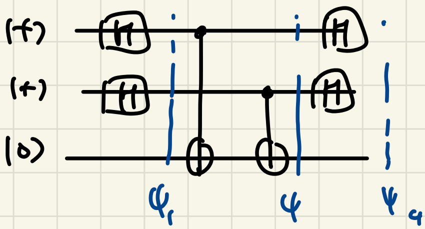
$\rightarrow (\alpha|000\rangle + \beta|111\rangle)_H$

iii) Hadamard

$\rightarrow \alpha|{+}{+}{+}\rangle + \beta|{-}{-}{-}\rangle$

4.

(a)



$|\varphi_1\rangle = |000\rangle$
$|\varphi_2\rangle = |000\rangle$
$|\varphi_3\rangle = |{+}{+}0\rangle$

$\therefore |{+}{+}0\rangle$

(b)

$a = |+\rangle,\ b = |-\rangle$

$|\varphi_1\rangle = |010\rangle,\ |\varphi_2\rangle = |011\rangle,\ |\varphi_3\rangle = |{+}{-}1\rangle$

$\therefore |{+}{-}1\rangle$

(c) $a = |-\rangle,\ b = |+\rangle$

$|\varphi_1\rangle = |100\rangle,\ |\varphi_2\rangle = |101\rangle,\ |\varphi_3\rangle = |{-}{+}1\rangle$

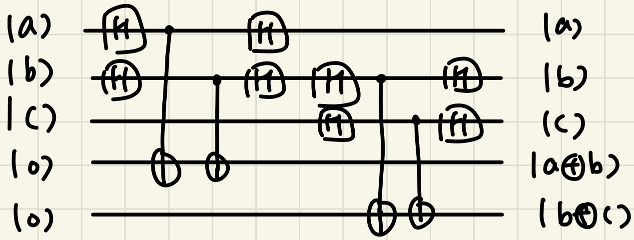
$\therefore |{-}{+}1\rangle$

(d) $a = |-\rangle,\ b = |-\rangle$

$|\varphi_1\rangle = |110\rangle,\ |\varphi_2\rangle = |110\rangle,\ |\varphi_3\rangle = |{-}{-}0\rangle$

$\therefore |{-}{-}0\rangle$

5.

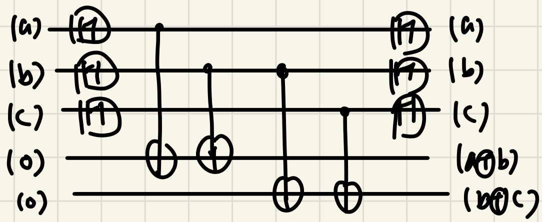


from 4.39

$H^2 = I$.

So for $|a\rangle, |b\rangle, |c\rangle$ in both circuits they will remain $|a\rangle, |b\rangle, |c\rangle$ are output, as Hadamard gates will be canceled.

for CNOTs, they are operating with $|a\rangle, |b\rangle, |c\rangle$ after Hadamard in both circuits, so in sum two circuits are basically identical



6.

$$\alpha\left(i\sqrt{1-\varepsilon^2}\,|{+}{+}{+}\rangle + \varepsilon\,|{-}{+}{+}\rangle\right) + \beta\left(i\sqrt{1-\varepsilon^2}\,|{-}{-}{-}\rangle + \varepsilon\,|{+}{-}{-}\rangle\right)$$

(a) probability: $1-\varepsilon^2$

$\rightarrow \alpha|{+}{+}{+}\rangle + \beta|{-}{-}{-}\rangle$

No gate needed

(b) ε^2

$\rightarrow \alpha|{-}{+}{+}\rangle + \beta|{+}{-}{-}\rangle$

Gate: $Z \otimes I \otimes I$

↪ on the leftmost qubit

(c) probability : 0

→ if were not 0, then it would be

$$\alpha|++-\rangle + \beta|--+\rangle$$

(d) probability : 0

→ if possible, then would be

$$\alpha|+-+\rangle + \beta|-+-\rangle$$

7.

(a)

$$|\psi 00\ 000\ 000\rangle \Rightarrow (\alpha|0\rangle + \beta|1\rangle)|00\ 000\ 000\rangle$$

CNOT → $\alpha|000\rangle|000\rangle|000\rangle + \beta|100\rangle|100\rangle|100\rangle$

(b)

Hadamard operates independently to each block on the first qubit of them (1, 4, 7 th)

Hadamard → $\frac{\alpha}{2}(|000\rangle + |100\rangle)(|000\rangle + |100\rangle)(|000\rangle + |100\rangle)$

$+ \frac{\beta}{2}(|000\rangle - |100\rangle)(|000\rangle - |100\rangle)(|000\rangle - |100\rangle)$

(c) $|000\rangle \to |000\rangle$, $|100\rangle \to |111\rangle$: first qubit is control and the other two are target

$$\Rightarrow \frac{\alpha}{2^{3/2}}(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) + \frac{\beta}{2^{3/2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$