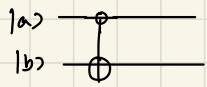
PS 6

1.

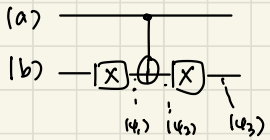
i) original CNOT



$\|a,b\rangle$	$\|a, a\oplus b\rangle$
0 0	0 0
0 1	0 1
1 0	1 1
1 1	1 0

$\therefore |a, a\oplus b\rangle = |a, \varphi_3\rangle$

ii) anci - CNOT with X gates.



$\|a,b\rangle$	$\|a,\varphi_1\rangle$	$\|a,\varphi_2\rangle$	$\|a,\varphi_3\rangle$
0 0	0 1	0 1	0 0
0 1	0 0	0 0	0 1
1 0	1 1	1 0	1 1
1 1	1 0	1 1	1 0

2.

$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$

$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$

$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}$

$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}$

$|\phi^+\rangle\langle\phi^+| + |\phi^-\rangle\langle\phi^-| + |\psi^+\rangle\langle\psi^+| + |\psi^-\rangle\langle\psi^-|$

$= \frac{1}{2}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}\begin{bmatrix}1&0&0&1\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}\begin{bmatrix}1&0&0&-1\end{bmatrix}$

$+ \frac{1}{2}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}\begin{bmatrix}0&1&1&0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}\begin{bmatrix}0&1&-1&0\end{bmatrix}$

$= \frac{1}{2}\left(\begin{bmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{bmatrix} + \begin{bmatrix}1&0&0&-1\\0&0&0&0\\0&0&0&0\\-1&0&0&1\end{bmatrix}\right.$

$\left. + \begin{bmatrix}0&0&0&0\\0&1&1&0\\0&1&1&0\\0&0&0&0\end{bmatrix} + \begin{bmatrix}0&0&0&0\\0&1&-1&0\\0&-1&1&0\\0&0&0&0\end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix}2&0&0&0\\0&2&0&0\\0&0&2&0\\0&0&0&2\end{bmatrix} = I.$

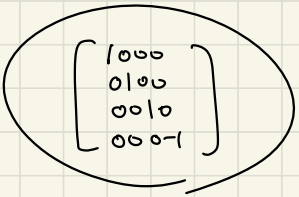
3.

$|00\rangle \rightarrow |00\rangle$

$|01\rangle \rightarrow |01\rangle$

$|10\rangle \rightarrow |11\rangle$

$|11\rangle \rightarrow -|11\rangle$



4.

(a) $|000\rangle \rightarrow |001\rangle$

$|001\rangle \rightarrow |000\rangle$

$|010\rangle \rightarrow |010\rangle$

$|011\rangle \rightarrow |011\rangle$

$|100\rangle \rightarrow |100\rangle$

$|101\rangle \rightarrow |101\rangle$

$|110\rangle \rightarrow |110\rangle$

$|111\rangle \rightarrow |111\rangle$

(b)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

5.

(a) $\langle +|-\rangle = 0 \Rightarrow$ possible

(b) $\langle i|-i\rangle = 0 \Rightarrow$ possible

(c) $\langle 0|+\rangle = \langle 0|\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)$

$= \frac{1}{\sqrt{2}}(\langle 0|0\rangle + \langle 0|1\rangle)$

$= \frac{1}{\sqrt{2}} \Rightarrow$ impossible

6.

(a) Toffoli → basically same with CNOT → Encouglement

⇒ "complex amplitude" is missing

(b) superposition is missing

(c) No encouglement