Problem Set 2

Q0.

(a) $\underline{Ce^{-ikx}}$, $|Ce^{ikx}| = |c| \cdot |e^{ikx}| = |c| = \underline{C}$

$= |\cos(kx) + i\sin(kx)| = \sqrt{\cos^2(kx) + \sin^2(kx)} = 1$

(b) $e^{ikx} = \cos(kx) + i\sin(kx)$

(c) $Re(e^{ikx}) = \cos(kx)$, $Im(e^{ikx}) = i\sin(kx)$

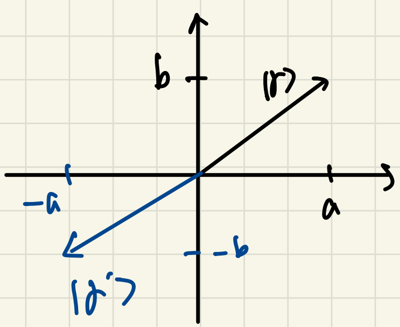
(d) $\cos(kx)$, $-i\sin(kx)$

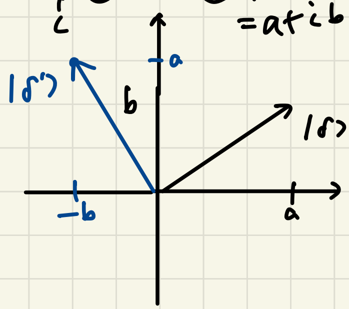
(e) $7i+5$ (f) $\sqrt{49+25} = \sqrt{74}$

Q1.

(a) $|\alpha'\rangle = -a+ib$ (b) $|\beta'\rangle = a-ib$

(c) $|\gamma'\rangle = \underbrace{e^{i\pi}}_{-1} \times \underbrace{|\gamma\rangle}_{=a+ib}$ (d) $|\delta'\rangle = \underbrace{e^{\frac{i\pi}{2}}}_{i} \cdot \underbrace{|\delta\rangle}_{=a+ib}$





(e) No. (a), (b) and (c) are all reflections, about y-axis, x-axis and the origin, respectively. But (d) is 90° rotation.

Q2.

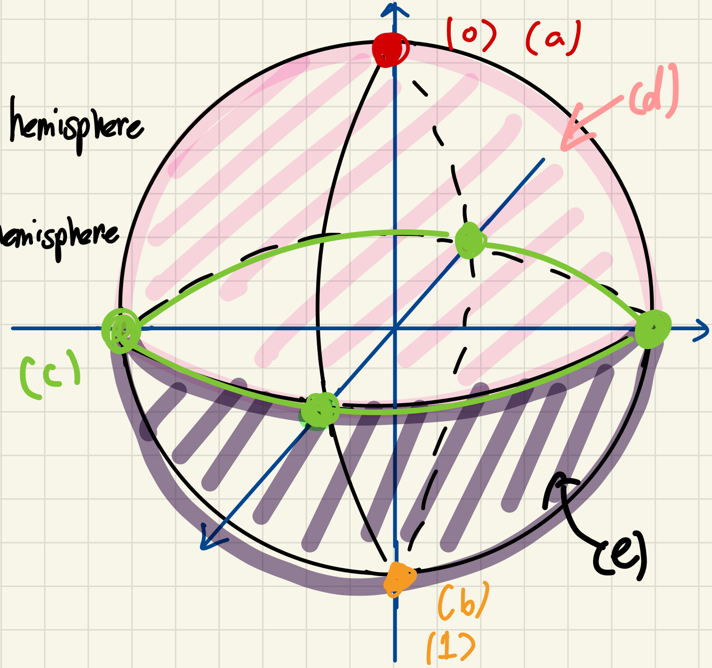
(a)

(b)

(c)

(d) : upper hemisphere

(e) : lower hemisphere



Q3

(a) polar angle : 0, azimuthal angle : 0 (does not matter)

(b) polar angle : π, azimuthal angle : 0 (does not matter)

(c) polar angle : $\pi/2$, azimuthal angle : 0 (does not matter)

Q4. (Exercise 2.6)

(a) $\left|\frac{1+i\sqrt{3}}{3}\right|^2 = \left(\frac{1+i\sqrt{3}}{3}\right)\left(\frac{1-i\sqrt{3}}{3}\right) = \frac{1+3}{9} = \frac{4}{9}$

(b) $\left|\frac{2-i}{3}\right| = \frac{5}{9}$

Q5. (E.2.7)

(a) 1 (b) 0 , already collapsed.

Q6. (E.2.8)

$$\left| \frac{e^{i\pi/8}}{\sqrt{5}} \right|^2 = \frac{1}{5}, \quad \beta^2 = \frac{4}{5}. \quad \therefore \beta = \frac{2}{\sqrt{5}}$$

or $\frac{2}{\sqrt{5}} e^{i\phi}$

Q7. (E.2.9)

(a) $|2e^{i\pi/6}|^2 + |-3|^2 = 4 + 9 = 13$

$\therefore A = \frac{1}{\sqrt{13}}$

(b) $\left|\frac{2e^{i\pi/6}}{\sqrt{13}}\right|^2 = \frac{4}{13}$ (c) $\left|\frac{-3}{\sqrt{13}}\right|^2 = \frac{9}{13}$

Q8. (E.2.10)

(a) $|0\rangle$ and $|1\rangle$, with probability of $\frac{1}{4}$ and $\frac{3}{4}$, respectively.

(b) $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

$|+\rangle + |-\rangle = \underset{\sqrt{2}}{\cancel{\frac{2}{\sqrt{2}}}}|0\rangle$, $|+\rangle - |-\rangle = \underset{\sqrt{2}}{\cancel{\frac{2}{\sqrt{2}}}}|1\rangle$

$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$, $|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$

$$\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle = \frac{1}{2}\left(\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)\right) - \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)\right)$$

$$= \frac{1}{2\sqrt{2}}(|+\rangle + |-\rangle) - \frac{\sqrt{3}}{2\sqrt{2}}(|+\rangle - |-\rangle)$$

$$= \underline{\frac{1-\sqrt{3}}{2\sqrt{2}}|+\rangle + \frac{1+\sqrt{3}}{2\sqrt{2}}|-\rangle}$$

(c) $|+\rangle : \frac{\cancel{4}-2\sqrt{3}}{\cancel{8}}$, $|-\rangle : \frac{4+2\sqrt{3}}{\cancel{16}}$

$= \frac{2-\sqrt{3}}{4}$ $\quad = \frac{1+\sqrt{3}}{4}$

Q9. $\underbrace{\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle}_{|\psi\rangle} = \alpha|a\rangle + \beta|b\rangle$

$\langle a|\psi\rangle = \alpha$, $\langle b|\psi\rangle = \beta$.

i) $\left(\frac{\sqrt{3}}{2}\langle 0| - \frac{i}{2}\langle 1|\right)\left(\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle\right)$

$= \frac{\sqrt{3}}{4}\langle 0|0\rangle - \cancel{\frac{3}{4}\langle 0|1\rangle} + \cancel{\frac{i}{4}\langle 1|0\rangle} - \frac{\sqrt{3}}{4}i\langle 1|1\rangle$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}i$

ii) $\left(-\frac{i}{2}\langle 0| + \frac{\sqrt{3}}{2}\langle 1|\right)\left(\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle\right)$

$= -\frac{i}{4}\langle 0|0\rangle + \cancel{\frac{\sqrt{3}}{4}i\langle 0|1\rangle} + \cancel{\frac{\sqrt{3}}{4}\langle 1|0\rangle} - \frac{3}{4}\langle 1|1\rangle$

$= -\frac{1}{4}(3+i)$

(a) $|\psi\rangle = \frac{\sqrt{3}}{4}(1+i)|a\rangle - \frac{1}{4}(3+i)|b\rangle$

(b) $|a\rangle$ with $\frac{\cancel{6}}{\cancel{16}} = \frac{3}{8}$, $|b\rangle$ with $\frac{\cancel{10}}{\cancel{16}} = \frac{5}{8}$

Q10.

(a) $\frac{1}{2}$ (b) $\frac{1}{2}$

$|0\rangle \xrightarrow{\text{measure 1}} |+\rangle$ or $|-\rangle$

$\Big\downarrow$ measure 2

$|0\rangle$ or $|1\rangle$,

previous measurement does not matter

Q11.

(a) No, global phase does not affect measurement

(b) Yes, when measured by X basis.

(c) No, global phase does not affect measurement