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Research Article

Impact of High-Frequency Trading with an Order Book Imbalance Strategy on Agent-Based Stock Markets

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It is known that there is a positive correlation between order book imbalance and future returns. Although some previous studies using actual trading data have suggested that high-frequency trading (HFT) may take this characteristic into account, HFT firms have not disclosed their specific strategies. Furthermore, there has been a long-standing debate in the empirical research field as to whether HFT is the cause of flash crashes, but no final conclusion has been reached. In the present study, we analysed the impacts of HFT taking into account the correlation between order book imbalance and future returns on a stable market and on a market with a flash crash, using agent-based simulations, which are said to be capable of analysing events in their essence. We also analysed how HFT investment performance differs between those two market conditions. The results showed that HFT has the effect of further stabilizing the market when the market is stable but does not take place during flash crashes and so is unable to affect the market either for the good or the bad. The results also suggest that the proposed HFT's performance is more sensitive to market price fluctuations than conventional HFT (i.e., HFT following a position market-making strategy) and tends to have high risk and high returns.

1. Introduction

In recent years, high-frequency trading (HFT), which uses powerful computers to trade at high speed, has had a significant impact on the financial markets. For example, Ohyama and Suzuki [1] reported that the proportion of HFT orders among all orders was about 70%, whereas the proportion of the trading value of HFT was about 40% in the Tokyo Securities Exchange (TSE). Accordingly, there have been numerous studies on the impact of HFT on a market. Some studies have reported that HFT provides market liquidity [2–5]. Jarnecic et al. [3] compared HFT orders with the remainder of trading orders in the limit order book and found that the former were submitted at multiple prices to the limit order book, concentrated around the quote. This finding indicated HFT improves market liquidity. Hosaka [4] has put forward the suggestion that the HFT transactions have increased in depth, which means the number of orders whose prices are around the best price in the order book. Using NASDAQ trading data, Carrion [5] found that spreads widened in trades where HFT provided liquidity and narrowed in trades where HFT took away liquidity. From these results, the authors conclude that HFT provides liquidity when liquidity is scarce and consumes liquidity when liquidity is plentiful.

On the other hand, researchers are divided on whether HFT provides liquidity during a financial crisis. For example, Ohyama and Suzuki [1] claimed HFT provided liquidity in terms of spread, i.e., it narrowed bid/ask spreads, during the COVID-19 crisis. However, Weill [6] studied the optimal liquidity provision of HFT during financial crashes and revealed that HFT provided the optimal amount of liquidity if HFT had sufficient capital at the time of the disruption. This result showed that HFT does not always provide buy orders at the time of a crash.

There have been many studies about the impact of HFT on the market during a flash crash, such as the flash crash of May 6, 2010, in particular [7–11]. For example, the CFTC-SEC reported that HFT initially provided liquidity to the market, but between 2:41 and 2:44 p.m., HFT traders aggressively sold about 2,000 E-Mini contracts in order to reduce their temporary long positions [7, 8]. Sornette and Von der Becke [9] stated that it was an almost certain that HFT would lead to a higher frequency of crashes if there was much aggressive high-frequency selling that would frequently clear out all 10 levels of depth before the offer price could adjust downward during the flash crash on May 6, 2010, as reported by Nanex (Nanex report on May 6th flash crash) https://www.nanex.net/20100506/FlashCrashAnalysis_CompleteText.html). Golub et al. [10] concluded that it appears likely that mini flash crashes are caused by HFT activity, given the speed and the magnitude of the crashes.

Various HFT strategies have also been reported [4, 12, 13]. ASIC [12] categorized strategies into three types: electronic liquidity provision strategies, statistical arbitrage strategies, and liquidity detection strategies. A market-making strategy is a type of electronic liquidity provision strategy in which the majority of HFT trading volume and more than 80% of HFT limit order submissions are associated with market-making [13].

There have also been a number of studies on the market impact of order imbalance, which means the imbalance between sell and buy order submissions. One type of such studies looks at order book imbalance (OBI). OBI is the difference between the number of buy orders and that of sell orders in the order book around the best quote. It is known that OBI is correlated with future returns, i.e., positive returns are obtained when there are more buy orders than sell orders around the best quote, and negative returns are obtained when there are more sell orders than buy orders [14, 15]. Hereinafter, we call this property the OBI property. Empirical analysis reveals that HFT may be based on the correlation between OBI and future returns [16–18]. Cartea et al. [15] investigated the correlation between high-frequency price changes and OBI (they referred to OBI as the order flow imbalance) and proposed a linear model of the relationship between them. Goldstein et al. [18] claimed that the relative level of stock prices in the future increases as the number of buy orders relative to sell orders in the limit order book increases. Although few reports have been published on specific strategies, Stoikov [19] proposed a micro-price that is a mid-price adjustment that incorporates OBIs and bid-ask spreads. He found that the micro-price was a better predictor for short-term movements of mid-prices than mid-prices and volume-weighted mid-prices.

In recent years, although many complex forecasting methods based on deep learning have been developed for stock price prediction [20], HFT is often conducted using relatively simple algorithms for speed. However, such algorithms can cause large losses if unexpected events such as flash crashes occur. Nevertheless, when trying to analyze the impact of an unexpected event such as a flash crash on HFT, it is impractical and difficult to cause such an event to occur in an actual market. Even if a flash crash can be made to occur, it is difficult to extract only its direct impact on the market because various other external factors exist simultaneously in the market.

This study attempts to circumvent these barriers and analyze the intrinsic impact of a flash crash on a market by using an artificial market (AM). An AM is a financial market multiagent system constructed virtually on a computer [21–25]. In an AM, each agent, which is assumed to be an investor, is given a unique trading strategy and is allowed to trade financial assets based on its own strategy. We can then see how the market is affected by the behaviors of many agents and how the agents in turn are affected by changes in the market. There have been previous studies on market liquidity using AMs [26–29], and the relationship between HFT and flash crashes has also been investigated with AMs [30, 31]. Karvik et al. [30] used AMs to find that the frequency of flash crashes may be proportional to the frequency of HFT transactions. This finding suggests that HFT is a cause of flash crashes, as also suggested by the empirical studies mentioned above. Vuorenmaa and Wang [31] found using an AM that the probability of a flash crash increases as the number of HFT agents increases or their positions decrease.

However, there has been no discussion of the impacts of HFT with an OBI strategy on stable markets and markets with flash crashes, based on the results of analyses with AMs; furthermore, as mentioned above, empirical studies have not clarified whether HFT behaviour is the cause of flash crashes. Therefore, the contributions in our paper are as follows.

(i) A stable market and an unstable market with a flash crash as AMs were built to investigate how three types of HFT agents affect the markets. The three types of HFT agents were as follows. One was an HFT agent with a market-making strategy, which is common among HFT strategies. Another was an HFT agent with an OBI strategy that combines a market-making strategy with one looking at the positive correlation between OBI and future returns. The third was an HFT agent with the reverse OBI strategy that combines a market-making strategy with one looking at the negative correlation between OBI and future returns.

(ii) The finding that HFT agents' transactions contribute to more stable prices in the stable market, whereas their transactions are not expected to be effective in stabilizing price in the market with the flash crash, was obtained.

(iii) It was investigated how the performance of each HFT agent is affected by each of the two markets and it was found that the HFT strategy taking into account the OBI tends to be high-risk and high-return.

The rest of this paper is organized as follows. Section 2 explains our proposed AM. Section 3 reports on simulations that we conducted using our AM. Section 4 presents the simulation results. Section 5 gives our conclusions.

2. AM Model

We built a new AM with three types of HFT agents based on the model constructed by Yagi et al. [29], as their model is able to reproduce the statistical characteristics of the long-term price fluctuations found in previous empirical studies. In our market model, normal agents (Section 2.1) and three types of HFT agents (Section 2.2) are available. The position-based market maker (PMM) adopts a position market-making strategy, the position-based OBI market maker (POMM) combines the position market-making strategy with an OBI strategy, and a position-based reverse OBI market maker (PrOMM) combines the position market-making strategy with the reverse OBI strategy. Note that PrOMM may not be following a realistic strategy, but it is provided for comparison with PMM and POMM. The pricing mechanism in the model is a continuous double auction. This means that if there are buy (sell) order prices in the order book that are higher (lower) than the sell (buy) order price of the agent, then an agent's order is immediately matched to the highest buy order (lowest sell order) in the order book. On the other hand, if there are no orders in the order book, then the order does not match any other order and remains in the order book. We call the former type of order placed by an agent a market order and the latter type a limit order. The orders in the order book are canceled at time t_c (i.e., the order effective period) after the order was placed. The tick size ΔP is the minimum unit for the price. When orders are sell (buy) orders, fractional values smaller than ΔP are rounded up (down).

Normal agent (NA) $j=0$ to NA $j=n-1$ submit orders for one share at a time in sequence, where after the order by NA $n-1$ is submitted, NA 0 submits the next order. The time t is incremented by 1 each time an NA places an order. Therefore, the process goes forward one step even if an order placed by an NA becomes a limit order, that is, if the order does not match to any other orders in the order book. An HFT agent places both a buy order and a sell order just before the NA submission. The HFT agent cancels their previous buy and sells in the order book if any remains there and places new buy and sell limit orders. The process does not move forward one step after an HFT order submission. All agents trade only one type of risk asset. All agents can trade assets indefinitely because the quantity of cash of each agent is set indefinitely, and they can short sell.

2.1. Normal Agents (NAs). NAs are assumed to be general investors in the real world and designed to be able to replicate the characteristic of the real markets. An NA determines the order price by combining the following three trading strategies: the fundamental strategy, the technical strategy, and the noise strategy. The fundamental strategy refers to the fundamental price to make investment decisions. The technical strategy uses past price movements to make investment decisions. The noise strategy represents trial-and-error investment decisions. The weights of the fundamental and technical strategies are changed by learning as market conditions change.

We now explain the order process and the learning process of NA. In the order process, NA j decides the order prices as follows. The rate of the expected price of NA j at time t, i.e., the expected return, is given by the following equation.

$$r_{ej}^{t} = \frac{1}{w_{1j}^{t} + w_{2j}^{t} + u_j}\left(w_{1j}^{t} r_{1j}^{t} + w_{2j}^{t} r_{2j}^{t} + u_j \epsilon_j^{t}\right). \tag{1}$$

The initial term in equation (1) normalizes the impact of the three trading strategies. r_{ij}^{t} is the i-th expected return of NA j at time t. r_{1j}^{t} denotes the expected return of the fundamental strategy and is calculated as $\ln\left(P_f/P^{t-1}\right)$, which means that NA j expects a positive (negative) return when the fundamental price is higher (lower) than the previous market price. Note that P^t is the market price at time t, and P_f is the fundamental price, which is constant over time. The initial market price P^0 is set to P_f, and the market price is set to the most recent price when no trades have been made. r_{2j}^{t} denotes the expected return of the technical strategy and is calculated as $\ln\left(P^{t-1}/P^{t-1-\tau_t}\right)$, which means that NA j expects a positive (negative) return when the historical return is positive (negative). Note that τ_t is taken from the uniform distribution between 1 and $\tau_{\max}$ at the start of the simulation for NA j. ϵ_j^t is set as a normally distributed random error with a mean of 0 and standard deviation σ_ϵ.

w_{ij}^{t} is the strategy weight of NA j at time t, w_{1j}^{t} denotes the fundamental strategy weight, and w_{2j}^{t} denotes the technical strategy weight. Both strategy weights are decided according to the uniform distribution between 0 and $w_{i,\max}$ at the beginning of the simulation. Each weight is changed by using the learning process described later. u_j is the noise strategy weight and is decided according to the uniform distribution between 0 and $u_{\max}$ at the beginning of the simulation. We model the strategy weights as random variables chosen independently.

The expected price of NA j at time t, P_{ej}^{t}, is calculated according to the following equation.

$$P_{ej}^{t} = P^{t-1}\exp\left(r_{ej}^{t}\right). \tag{2}$$

The order price of NA j at time t, P_{oj}^{t}, is determined by the uniform distribution between $P_{ej}^{t} - P_d$ and $P_{ej}^{t} + P_d$, where P_d is constant. If P_{oj}^{t} is less than P_{ej}^{t}, then, NA j submits a buy order whose price is P_{oj}^{t} for one share. If P_{oj}^{t} is greater than P_{ej}^{t}, then NA j submits a sell order whose price is P_{oj}^{t} for one share.

The learning process is performed before the order process at each time. Comparing the sign of r_{ij}^{t} $(i=1,2)$ with that of $r_l^t = \ln\left(P^{t-n}/P^{t-1-t_l}\right)$, if both signs are the same, $w_{i,j}^{t}$ is updated as follows:

$$w_{i,j}^{t} \leftarrow w_{i,j}^{t} + k_l\left|r_l^t\right| q_j^t\left(w_{i,\max} - w_{i,j}^{t}\right), \tag{3}$$

where k_l is a constant, and q_j^t is set according to the uniform distribution between 0 and 1. If r_{ij}^{t} and r_l^t have opposite signs, $w_{i,j}^{t}$ is updated as follows:

$$w_{i,j}^{t} \leftarrow w_{i,j}^{t} - k_l\left|r_l^t\right| q_j^t\, w_{i,j}^{t}. \tag{4}$$

These equations mean that the weights of strategies whose predicted direction of price change coincides with the actual direction of price change are raised, while the weights of strategies that are out of line are lowered. Furthermore, $w^t_{i,j}$ is set according to the uniform distribution between 0 and $w_{i,\max}$ with probability δ_l. This is an objective model of the search for a better strategy by trial and error to fundamentally reevaluate the previous strategy.

2.2. HFT Agents. There are three types of HFT agents, i.e., PMM, POMM, and PrOMM, in our market model. PMM adopts a position market-making strategy. This strategy is as follows. First, PMM calculates the basic order price as the mid-price. The mid-price is the average of the best-bid price and the best-ask price plus a value depending on its own position, which means the amount of assets held by PMM. Next, PMM simultaneously submits a buy order whose price is the value of the basic order price minus the spread, which is equal to the amount of its own expected return per transaction, and a sell order whose price is the value of the basic order price plus the spread [29]. Generally, PMM tries to keep its position neutral to avoid the price fluctuation risk of its own asset. Thus, the more PMM holds an asset, the lower PMM tends to set its buy and sell order prices so that its sell order has a better chance of matching an NA's buy market order. Otherwise, the more PMM short-sells the asset, the higher it tends to set its buy and sell order prices so that its buy order has a better chance of matching an NA's sell market order [32, 33].

POMM and PrOMM determine their basic order prices using not only their own positions but also the difference between the market buy depth and the market sell depth. Note that depth means the amount of orders around the highest range of buy orders and the lowest range of sell orders. In this paper, the market buy depth is the amount of buy orders from the best-bid price to Dp (=50) ticks lower than the best-bid price, and the market sell depth is the amount of sell orders from the best-ask price to Dp ticks higher than the best-ask price.

Let the best-ask and best-bid prices, the mid-price, the base spread of the HFT agent, the HFT agent's position, and the coefficient of its position be $P^{t,\text{sell}}$, $P^{t,\text{buy}}$, $P^{t,\text{mid}} = (P^{t,\text{sell}} + P^{t,\text{buy}})/2$, θ_{pm}, s^t_{pm}, and w_{pm}, respectively. Then, the HFT agent's basic order price $P^t_{fv,pm}$, buy order price $P^{t,\text{buy}}_{o,\text{pm}}$, and sell order price $P^{t,\text{sell}}_{o,\text{pm}}$ are as determined by the following equations:

$$
\begin{aligned}
P^t_{fv,\text{pm}} &= \left(1 - w_{\text{pm}}\left(s^t_{\text{pm}}\right)^3\right)P^{t,\text{mid}},\\
P^{t,\text{buy}}_{o,\text{pm}} &= P^t_{fv,\text{pm}} - \frac{1}{2}P_f\theta_{\text{pm}},\\
P^{t,\text{sell}}_{o,\text{pm}} &= P^t_{fv,\text{pm}} + \frac{1}{2}P_f\theta_{\text{pm}}.
\end{aligned}
\tag{5}
$$

As mentioned above, PMM trades without taking large positions to avoid price fluctuation risk. To model this, $P^t_{fv,\text{pm}}$ is calculated nonlinearly from $P^{t,\text{mid}}$ in equation (5). The power exponent of the weighted portion of the position is set to an odd number (=3) to take into account the direction of the position. As a result, the larger the HFT agent's buy (sell) position is, the easier its sell (buy) order finds a match.

OBI at time t, o^t_{pm}, is defined by the following equation:

$$
o^t_{pm} = \frac{Dp^{t,\text{buy}} - Dp^{t,\text{sell}}}{Bp^{t,\text{buy}} + Bp^{t,\text{sell}}}, \tag{6}
$$

where $Dp^{t,\text{buy}}$, $Dp^{t,\text{sell}}$, $Bp^{t,\text{buy}}$, and $Bp^{t,\text{sell}}$ are the buy depth at time t, the sell depth at time t, the amount of buy orders, and the amount of sell orders in the order book, respectively.

When the coefficient of OBI is w_{om}, the basic order price of POMM $P^t_{fv,\text{pom}}$ and that of PrOMM $P^t_{fv,\text{prom}}$, the buy order price of POMM $P^{t,\text{buy}}_{o,\text{pom}}$ and that of PrOMM $P^{t,\text{buy}}_{o,\text{prom}}$, and the sell order price of POMM $P^{t,\text{sell}}_{o,\text{pom}}$ and that of PrOMM $P^{t,\text{sell}}_{o,\text{prom}}$ are given by equations (7)–(12), respectively.

$$
P^t_{fv,\text{pom}} = \left(1 - w_{\text{pm}}\left(s^t_{\text{pm}}\right)^3 + w_{\text{om}}\left(o^t_{\text{pm}}\right)^2\right)P^{t,\text{mid}}, \tag{7}
$$

$$
P^t_{fv,\text{prom}} = \left(1 - w_{\text{pm}}\left(s^t_{\text{pm}}\right)^3 - w_{\text{om}}\left(o^t_{\text{om}}\right)^2\right)P^{t,\text{mid}}, \tag{8}
$$

$$
P^{t,\text{buy}}_{o,\text{pom}} = P^t_{fv,\text{pom}} - \frac{1}{2}P_f\theta_{\text{pm}}, \tag{9}
$$

$$
P^{t,\text{sell}}_{o,\text{pom}} = P^t_{fv,\text{pom}} + \frac{1}{2}P_f\theta_{\text{pm}}, \tag{10}
$$

$$
P^{t,\text{buy}}_{o,\text{prom}} = P^t_{fv,\text{prom}} - \frac{1}{2}P_f\theta_{\text{pm}}, \tag{11}
$$

$$
P^{t,\text{sell}}_{o,\text{prom}} = P^t_{fv,\text{prom}} + \frac{1}{2}P_f\theta_{\text{pm}}. \tag{12}
$$

Note that the larger the buy (sell) depth is than the sell (buy) depth, the more likely POMM's buy (sell) limit orders are to be executed with sell (buy) market orders relative to NAs' buy (sell) limit orders. On the other hand, the larger the buy (sell) depth is than the sell (buy) depth, the more likely PrOMM's sell (buy) limit orders are to be executed with buy (sell) market orders of NAs. Furthermore, when OBI is small, POMM (PrOMM) does not want to have much influence on its own order strategy because OBI may be coincidental, but when it is large, POMM (PrOMM) wants to make a large correction because of the possibility of some large buying (selling) pressure. To model these characteristics, the weighted portions of OBI in equations (7) and (8) are made nonlinear.

Here, the order prices of HFT agents are reset according to equations (13) and (14) and do not become market orders of the HFT agents if either or both of $P^{t,\text{buy}}_{o,\text{pm}} \geq P^{t,\text{sell}}$ and $P^{t,\text{sell}}_{o,\text{pm}} \leq P^{t,\text{buy}}$ are satisfied.

$$\text{If } P_{o,\text{pm}}^{t,\text{buy}} \geq P^{t,\text{sell}}, \text{ then}$$

$$\begin{aligned} P_{o,\text{pm}}^{t,\text{buy}} &= P^{t,\text{sell}} - \Delta P, \\ P_{o,\text{pm}}^{t,\text{sell}} &= \left(P^{t,\text{sell}} - \Delta P\right) + P_f \theta_{\text{pm}}. \end{aligned} \tag{13}$$

$$\text{If } P_{o,\text{pm}}^{t,\text{sell}} \leq P^{t,\text{buy}}, \text{ then}$$

$$\begin{aligned} P_{o,\text{pm}}^{t,\text{buy}} &= \left(P^{t,\text{buy}} + \Delta P\right) - P_f \theta_{\text{pm}}, \\ P_{o,\text{pm}}^{t,\text{sell}} &= P^{t,\text{buy}} + \Delta P. \end{aligned} \tag{14}$$

3. Simulation

3.1. Overview. In this study, we built two market simulation environments. In one, the market price transition is stable; that is, market volatility is low. Hereinafter, we call this the stable market. In the other, a flash crash occurs, the market price transition becomes unstable, and market volatility is high. We call this the unstable market. Letting each of the three types of HFT agents participate in each market, we investigate how the HFT agent affects market price, volatility, and the performance of the HFT agent.

The stable market is achieved by keeping a fundamental price P_f constant throughout the simulation. The unstable market is realized by having a flash crash occur, i.e., having NAs submit sell orders with order price 1 with a probability 20% during the 30,000 time units from time 100,001 to time 130,000. HFT agents place their orders according to the proposed strategies in the simulations (including during the plunge).

Each simulation ends at the time $t_e = 800{,}000$. The values of the other parameters are shown in Table 1.

Table 1: Parameters.

Parameter	Initial value
t_e	800,000
N	1,000
$w_{1,\max}$	1
$w_{2,\max}$	10
$u_{\max}$	1
$\tau_{\max}$	10,000
σ_ε	0.06
P_d	1,000
t_c	10,000
ΔP	1.0
P_f	10,000
t_l	10,000
k_l	4
δ_l	0.01
θ_{pm}	0.003
w_{pm}	5.0×10^{-8}
w_{om}	10.0

Table 2: Stylized facts.

Kurtosis		4.089
Autocorrelation coefficients for squared returns	Lag	
	1	0.158
	2	0.123
	3	0.090
	4	0.074
	5	0.064
Proportion of the OBI property		2.396%

3.2. Validation of Proposed AM. Real financial markets have particular statistical properties such as volatility clustering and a fat tail, as empirical studies have indicated [34–36]. We confirmed whether our proposed AM reproduces the statistical features of the price transitions [37]. Although early AMs were not able to reproduce the ubiquitous scaling laws of returns, recent AMs have been able to replicate volatility clustering and a fat tail [29, 38–42]. Thus, we set the model parameters as listed in Table 1 so as to reproduce these features (these are also the settings used in Yagi et al. [29]). Table 2 shows that both the autocorrelation coefficients for squared returns with several lags and kurtosis are positive, which means that all runs replicated volatility clustering and a fat tail. Thus, the model reproduces long-term statistical characteristics observed in actual financial markets. The statistics for stylized facts in Table 2 are averages over 20 simulation runs, for which we calculated price returns at intervals of 100 time units.

As mentioned in Section 1, it is known that OBI is correlated with future returns (OBI property) in actual financial markets, i.e., positive returns are obtained when there are more buy orders than sell orders around the best quote, and negative returns when there are more sell orders than buy orders [14, 15]. When the buy depth $Dp^{t,\text{buy}}$ is greater than (less than) the sell depth $Dp^{t,\text{sell}}$ at the most recently executed time t, we calculate the value obtained by subtracting the number of times the return ($\ln (P^{t'}/P^t)$) is negative (positive) from the number of times the return is positive (negative) at the next execution time t', and then divide the result by the number of transactions (volume). This value is positive if the OBI property is satisfied and is called here the proportion of OBI property.

The proportion of OBI property in Table 2, which is the average of 20 simulation runs, is indeed positive in this model and is statistically significant at the 0.05 level (since previous studies [14, 15] focused on the correlation between the magnitude of OBI and the magnitude of returns, the correlation was analysed quantitatively using, for example, regression lines. On the other hand, since we focused on the qualitative impact of the OBI property on HFT agents' behaviours, it is sufficient to confirm that the OBI property holds in our AM). Therefore, it can be confirmed that the proposed model is valid.

4. Results and Discussion

4.1. Price Transitions. In this section, we compare market price transitions in a stable market with those in an unstable market with flash crashes when each of the three types of HFT agents (PMM, POMM, and PrOMM) participates. In the stable markets, price changes in the market where an HFT agent participates are smaller than those in the market where HFT agents do not participate (refer to Figure 1). This result is consistent with the result of Yagi et al. [29] that when HFT agents trade in the stable market, prices converge approximately between the buy and sell order prices of HFT agents (the HFT agents' spread), resulting in price stability. On the other hand, in the unstable market, price changes are similar between whether or not HFT agents participate (refer to Figure 2). If the orders of HFT agents played the same role as in the stable market, the price declines would not be as large as those in the market without HFT agents because the buy orders of HFT agents would support the price when the flash crash occurred. Thus, we checked the extent to which buy orders of HFT agents were executed during each phase in the unstable market.

Tables 3–5 summarize the numbers of orders in the order book for each agent type when a new order is placed and a trade is executed during each of the four phases. That is, Tables 3–5 show the number of limit orders, which market orders match for each agent type by phase. These values were calculated as averages of 20 simulation runs. Phase 1 is the period before the flash crash begins. Phase 2 is from the time NAs' market sell orders at price 1 begin to be placed until the market price reaches its lowest point. Phase 3 is during the rebound of the price from the lowest point. In this experiment, the rebound is considered to have ended when prices recover to $P_f - d_{p_f}$ ($d_{p_f} = 50$). Phase 4 is the period from the end of Phase 3 to the end of the simulation.

In Phase 1, most of the buy (sell) orders in the order book, which matched market sell (buy) orders were placed by HFT agents. For example, the proportion of buy (sell) orders of PMM to the total limit orders executed is 83.5% (82.6%) in Table 3. Therefore, we can see that most of the market orders matched the limit orders of HFT agents.

In Phase 2, the proportion of buy limit orders of HFT agents, which matched market orders is significantly lower than that in Phase 1 (e.g., that of PMM is 2.0%), while the proportion of buy limit orders of NAs is much higher (e.g., that of NA is 98.0%). On the other hand, the amount of sell limit orders of HFT agents (e.g., that of PMM is 136.0) is larger than that of NAs (that of NAs is 6.8); however, the total amount of sell limit orders, which matched market buy orders (e.g., 7,033.4 (=6,893.3 + 140.1) in Table 3), is much more than that of buy limit orders, which matched market sell orders (e.g., 142.8).

From the abovementioned statements, it can be seen that HFT agents rarely trade, while NAs trade with each other in Phase 2. This result suggests that HFT agents' trades have little effect on market price declines.

Finally, we also check the behaviours of HFT agents and NAs. In Phase 3, the proportions of buy limit orders of HFT agents are higher than those of sell limit orders of HFT agents (e.g., those of PMM are 80.6% and 21.3%, respectively). However, as the total amount of sell limit orders that matched market buy orders (those of PMM and NAs are 5437.8.) is more than that of buy limit orders that matched market sell orders (those of PMM and NAs are 1420.5), HFT agents' trades have not had as great of an impact on price formation as they did during the plunge; i.e., HFT does not seem to have contributed much to the price rebound either. Therefore, in Phase 3, many buy orders are submitted, most of them become market orders, and the price rebounds. In Phase 4, the proportions of both buy and sell limit orders of HFT agents that matched market orders are higher than those of NAs, indicating that the orders of HFT agents match many of the market orders.

As a result, although there are some differences between HFT agents' transactions, the above explanation roughly accounts for why HFT agents do not participate much in trading when prices change widely, and why HFT is not expected to be effective in stabilizing volatility.

4.2. HFT Agents' Performances. For both the stable and unstable markets, the performances of the three types of HFT agents at the end of the simulation are shown in Table 6. These performances are the averages over 20 simulation runs. Figures 3 and 4 show the performance transitions of each HFT agent in the stable and unstable markets, respectively.

The performance of POMM is the best among the three types of HFT agents in the stable market; however, it is the worst among them in the unstable market.

In the stable market, the position market-making strategies of all HFT agents appear to work properly, although there are some differences between their positions. The reasons for the differences in performance between the HFT agents may be as follows.

First, the reason that the performance of POMM is better than that of PMM seems to be that the profit from the OBI strategy is added to the profit from the position market-making strategy, as there is a positive correlation between OBI and future returns. On the other hand, the performance of PrOMM is worse than that of PMM, as PrOMM's strategy expected a negative correlation between OBI and future returns.

Next, we attempt to explain the mechanisms of HFT agents' performances in the unstable market.

When the market price declines sharply, PMM's buy position becomes large (refer to Figure 5) because PMM's buy limit orders match the sell orders of NAs at price 1 during the flash crash. Therefore, PMM's performance deteriorates rapidly as the price falls and the buy limit orders of PMM are executed one after another (see Figure 4).

When the market price reaches the bottom, the downward trend of PMM's performance moderates. This is because PMM changed from a buy position to a sell position around the bottom price. When the price falls significantly, the divergence between it and the fundamental price increases, so the influence of the fundamental strategy of NAs becomes stronger. As a result, many NAs begin to place buy orders, and the market price stops falling and rebounds. PMM's performance deteriorates because the price rises while PMM maintains a sell position, but the deterioration of PMM's performance is slow because the price rises slowly.

Later, as the price rebound progresses and the market price begins to converge with the fundamental price, PMM's performance begins to improve. The reason for this is as

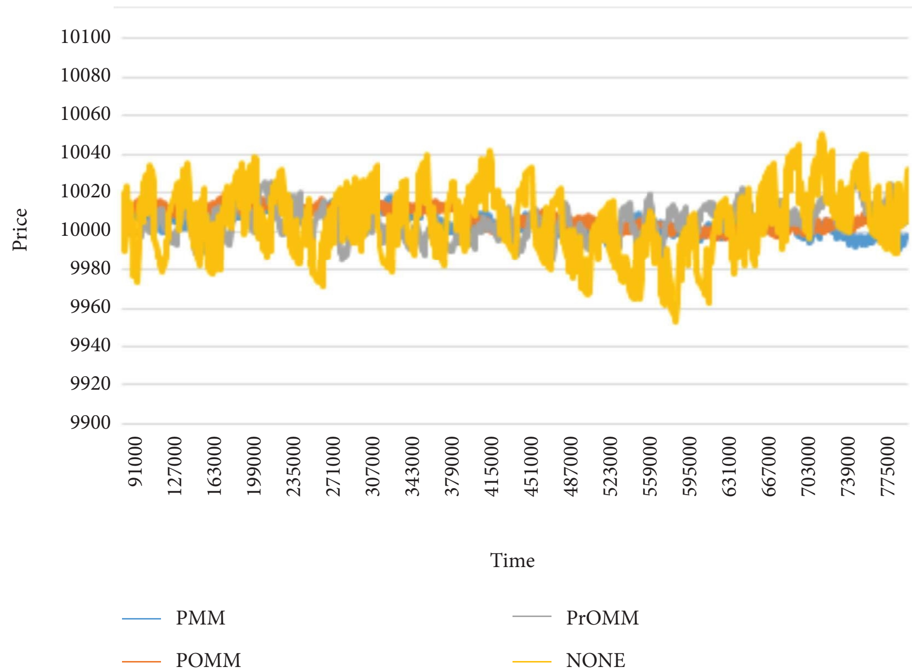

Figure 1: Price transitions of the stable market.

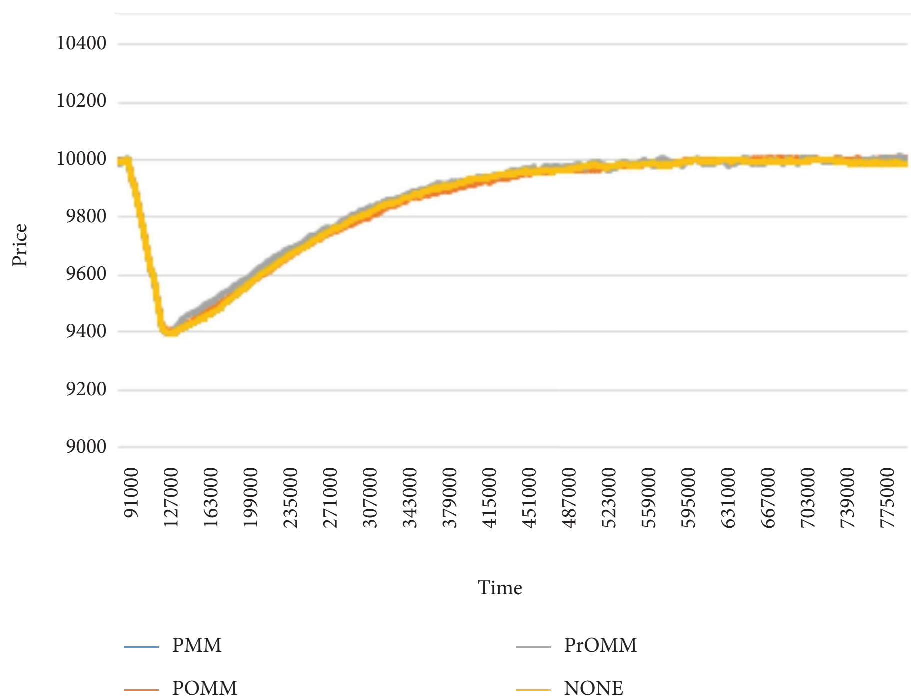

Figure 2: Price transitions of the unstable market.

follows. When the price becomes as high as the fundamental price, NAs place buy orders at a price equal to the fundamental price, but PMM, wishing to unwind its sell position, places buy orders at higher prices, which leads to the unwinding of PMM's buy position. This is because the position market-making strategy begins to function when the excess sell position is eliminated.

Since POMM takes the position of market-making strategy, its performance transition is basically the same as that of PMM, but the influence of the OBI strategy is further reflected in its strategy. Thus, during the flash crash when the buy depth is greater than the sell depth in the order book (refer to Figures 5 and 6), as POMM takes a larger buy position than PMM, POMM's performance is more affected by the flash crash than

Table 3: Numbers and proportions of NA's and PMM's orders in the order book that matched a new order.

		NA		PMM	
Phase 1	Buy orders	190.9	(16.5%)	965.6	(83.5%)
	Sell orders	202.5	(17.4%)	963.9	(82.6%)
Phase 2	Buy orders	6,893.3	(98.0%)	140.1	(2.0%)
	Sell orders	6.8	(4.8%)	136.0	(95.2%)
Phase 3	Buy orders	275.5	(19.4%)	1145.0	(80.6%)
	Sell orders	4277.7	(78.7%)	1160.1	(21.3%)
Phase 4	Buy orders	592.5	(15.0%)	3,357.8	(85.0%)
	Sell orders	986.2	(22.7%)	3,350.6	(77.3%)

Table 4: Numbers and proportions of NA's orders and POMM's orders in the order book that matched a new order.

		NA		PMM	
Phase 1	Buy orders	178.3	(15.2%)	993.0	(84.8%)
	Sell orders	192.7	(16.3%)	990.3	(83.7%)
Phase 2	Buy orders	6905.5	(97.7%)	163.3	(2.3%)
	Sell orders	9.4	(6.5%)	134.2	(93.5%)
Phase 3	Buy orders	329.3	(24.2%)	1,031.3	(75.8%)
	Sell orders	4,327.8	(80.1%)	1,077.5	(19.9%)
Phase 4	Buy orders	567.8	(14.2%)	3,439.8	(85.8%)
	Sell orders	982.0	(22.3%)	3,424.1	(77.7%)

Table 5: Numbers and proportions of NA's orders and PrOMM's orders in the order book that matched a new order.

		NA		PMM	
Phase 1	Buy orders	365.5	(32.7%)	753.4	(67.3%)
	Sell orders	369.5	(32.9%)	755.4	(67.1%)
Phase 2	Buy orders	6,987.4	(98.9%)	77.3	(1.1%)
	Sell orders	7.9	(6.7%)	109.2	(93.3%)
Phase 3	Buy orders	297.2	(21.2%)	1,105.7	(78.8%)
	Sell orders	4,308.9	(80.2%)	1,063.8	(19.8%)
Phase 4	Buy orders	1,177.8	(32.0%)	2,498.7	(68.0%)
	Sell orders	1,575.9	(38.6%)	2,510.2	(61.4%)

Table 6: Performances of HFT agents.

	PMM	POMM	PrOMM
Stable market	29,241.9	30,950.1	6,485.9
Unstable market	−647.2	−23,927.0	32,834.8

that of PMM. Therefore, its performance is worse than that of PMM. Note that the reason why the sell depth is extremely small during the price sharp decline is that the price falls so sharply that NAs' sell orders do not have time to accumulate in the price range between the market price and Dp ticks above it. During the rebound period, because the sell depth is larger than the buy depth, POMM's sell position becomes larger than PMM's sell position. Then, POMM's performance deteriorates further because POMM tries to decrease its sell position even if the market price rises. After that, when the difference between the sell depth and the buy depth converges to some extent, the impact of the OBI strategy becomes smaller, and that of the position market-making strategy becomes larger, POMM's performance recovers.

PrOMM's performance is more strongly affected by the OBI strategy than by the position market-making strategy, because the difference between the sell depth and buy depth during the price rebound period after a sharp decline is so large. Note that the transitions of the sell and buy depths in the market with PrOMM are almost the same as those in the market with POMM (refer to Figure 6). Since PrOMM takes a strategy that expects OBI and future returns to be negatively correlated, after reaching a buy position due to the position market-making strategy at the time of the sharp decline, it quickly changes to a sell position (refer to Figure 5). Thus, PrOMM's performance improves during the subsequent sharp decline. Moreover, when the price

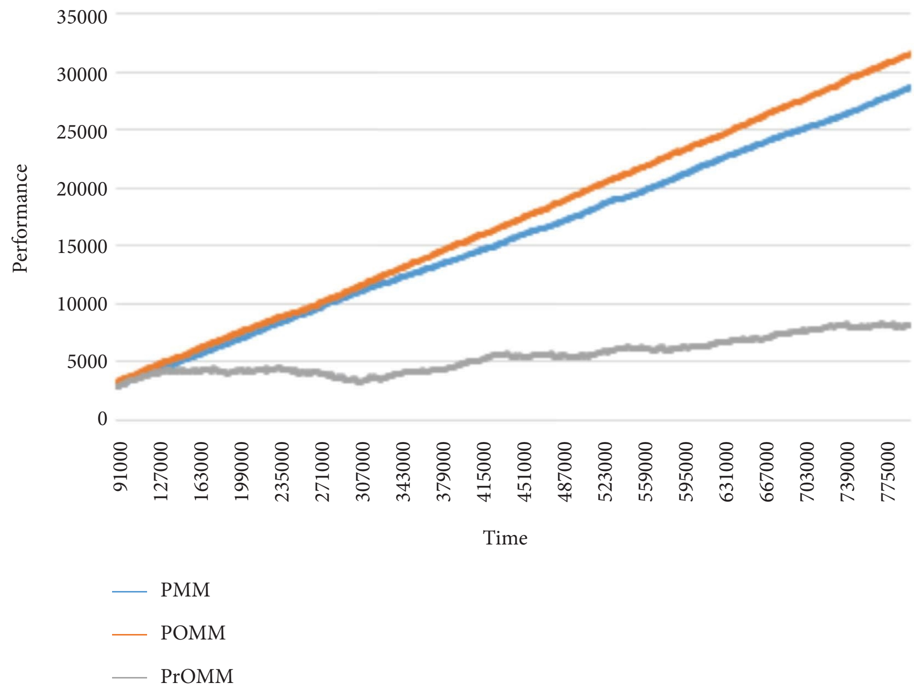

Figure 3: HFT agents' performances in the stable market.

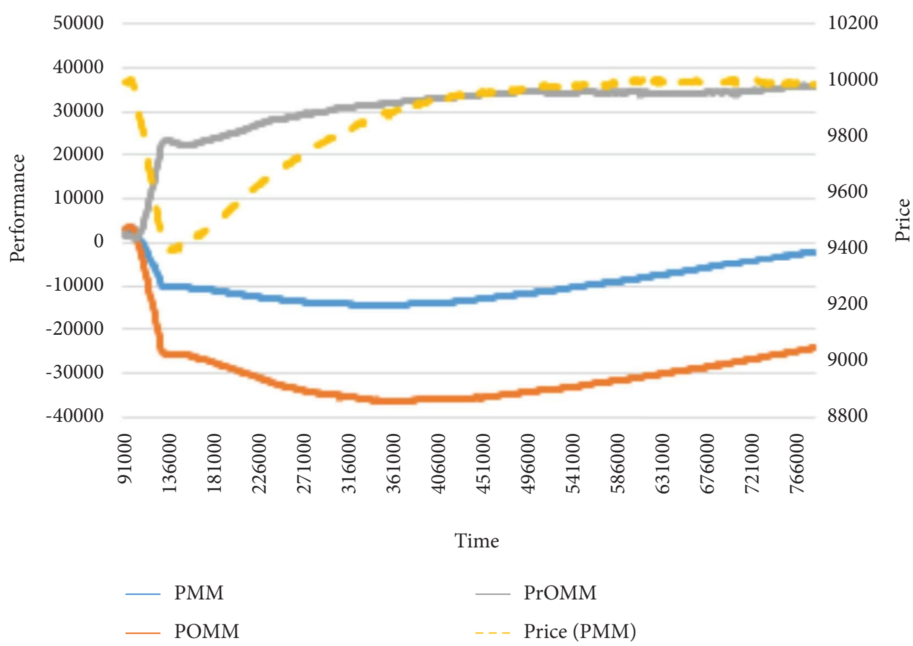

Figure 4: HFT agents' performances in the unstable market.

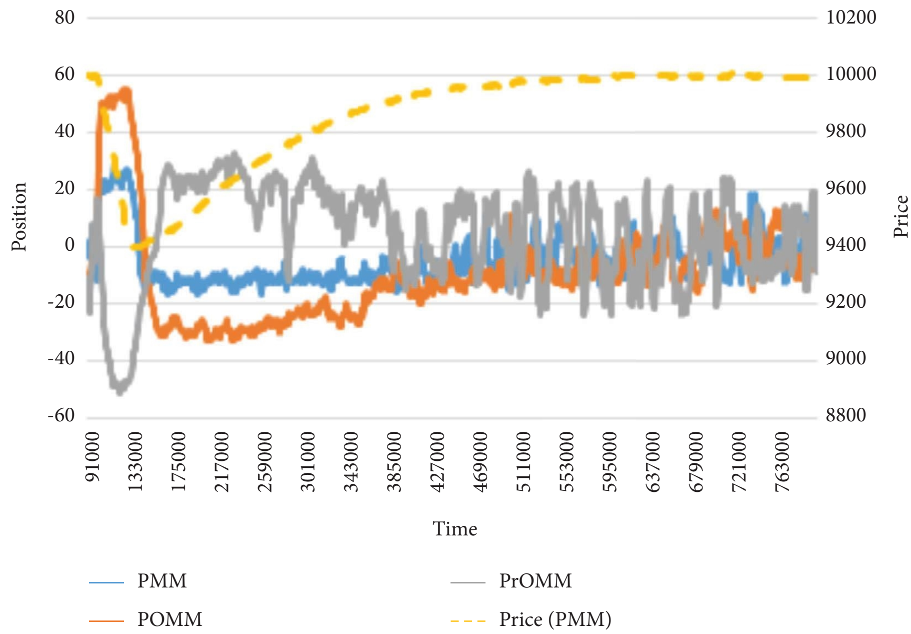

Figure 5: Positions of HFT agents and price transitions in the unstable market.

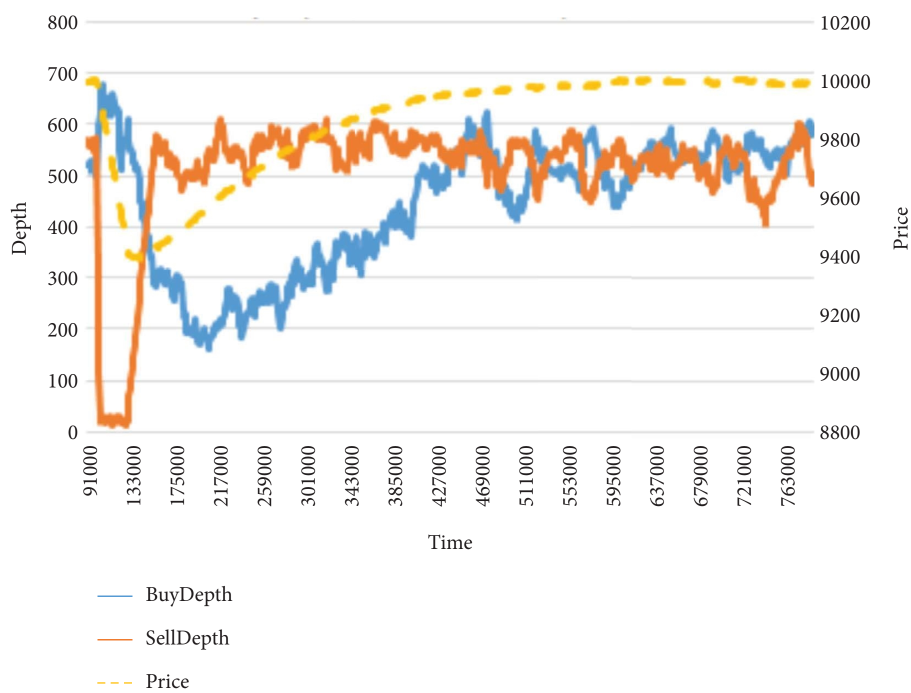

Figure 6: Price and depth in the unstable market.

begins to rebound, it achieves a buy position immediately, and the performance is further improved as the price rebounds.

These results suggest that when flash crashes occur, PrOMM may have the best performance, while the performances of PMM and POMM may deteriorate.

5. Conclusion

In this study, we prepared a stable market and an unstable market with a flash crash as AMs and investigated how three types of HFT agents affect the markets and how the performance of each HFT agent is affected by each of these markets. The three types of HFT agents in our AM were as follows. One was an HFT agent with a position market-making strategy (PMM), which is common among HFT strategies. Another was a PMM with the OBI strategy (POMM), which it combines with the positive correlation between OBI and future returns. The third was a PMM with the reverse OBI strategy (PrOMM), which it combines with the negative correlation between OBI and future returns.

As a result, we found that price changes in the market where HFT agents participate were smaller than those in the market where HFT agents did not participate, whereas price changes were similar regardless of whether HFT agents participated in the market where a flash crash occurred. These results suggest that HFT agents' transactions contribute to more stable prices in the stable market, whereas their transactions are not expected to be effective in stabilizing price in the market with the flash crash.

Regarding the performances of HFT agents, it was found that PMM accumulated solid profits in stable markets and recovered relatively quickly in the flash crash market, even though its performance temporarily deteriorated. On the other hand, POMM achieved high performance in the stable market. This result is consistent with a finding of Goldstein et al. [18], which claimed that the relative level of stock prices in the future increases as the number of buy orders relative to sell orders in the limit order book increases. Stoikov [19] proposed a HFT strategy, which took into account a micro-price that is a mid-price adjustment that incorporates OBIs and bid-ask spreads. He reported that the micro-price was a better predictor for short-term movements of mid-prices than mid-prices and volume-weighted mid-prices. This finding of Stoikov [19] also seems to be consistent with our result. However, POMM performed worse than PMM in the flash crash market. It seems difficult to compare the result with the findings of Goldstein et al. [18] and Stoikov [19], as their findings were based on market data from a relatively stable period. Finally, PrOMM's performances were the opposite of those of POMM. These results may suggest that a trading strategy that takes into account the OBI property tends to be high-risk and high-return.

As mentioned above, there have been previous studies that empirically analysed HFT investment strategies that take into account OBI, but the periods examined in those studies were limited to when the market was relatively stable. Therefore, our first future work is to analyse how an HFT agent with the OBI strategy behaves during periods of market instability, based on empirical data. Although all HFT agents in this study set their order prices using the mid-price as the basic order price, HFT investment behaviours may change if the basic order price differs. For example, Peña [43] proposed pure price as a different reference price from the mid-price and used pure price when proposing algorithmic trading strategies to construct portfolios that balance profitability and default risk. Thus, the pure price deserves consideration as one of the candidates for the basic order price. On the other hand, the differences in investment behaviours among PMM, POMM, and PrOMM may depend only on the OBI strategy if the basic order prices of these agents are calculated by the same method, regardless of what the base price is calculated by. Therefore, our second future work is to investigate HFT behaviour and its impact on the market when the basic order price is different from the mid-price, as in the case of pure price. Finally, as we did not compare our results with realistic data, we intend to research the relationship between our results and realistic data as our third future work.

Data Availability

The data used to support the findings of this study are available at https://onl.bz/rHUULVT.

Disclosure

The opinions expressed herein are solely those of the authors and do not necessarily reflect those of SPARX Asset Management Co., Ltd. The work presented here in is original and has not been previously published, but the initial idea was presented at the 28th JSAI Special Interest Group on Financial Informatics Workshop (https://sigfin.org/028/, in Japanese) in part.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] Y. F. Ohyama and K. Suzuki, "Characterization of high-speed trading," FSA Staff Reports, Financial Services Agency, Chiyoda, Japan, 2021.

[2] A. J. Menkveld, "High frequency trading and the new market makers," *Journal of Financial Markets*, vol. 16, no. 4, pp. 712–740, 2013.

[3] E. Jarnecic, M. Snape, and M. Snape, "The provision of liquidity by high-frequency participants," *Financial Review*, vol. 49, no. 2, pp. 371–394, 2014.

[4] G. Hosaka, "Analysis of high-frequency trading at tokyo stock exchange," Japan Exchange Group, Chuo, Japan, JPX Working Papers, 2014.

[5] A. Carrion, "Very fast money: high-frequency trading on the NASDAQ," *Journal of Financial Markets*, vol. 16, no. 4, pp. 680–711, 2013.

[6] P.-O. Weill, "Leaning against the wind," *The Review of Economic Studies*, vol. 74, no. 4, pp. 1329–1354, 2007.

[7] S. E. C. Cftc, "Recommendations regarding regulatory responses to the market events of May 6, 2010," Technical Report, CFTC-SEC Joint Advisory Committee, Washington, DC, USA, 2011.

[8] D. Easley, M. M. López de Prado, and M. Ohara, "The microstructure of the flash crash: flow toxicity, liquidity crashes, and the probability of informed trading," *Journal of Portfolio Management*, vol. 37, no. 2, pp. 118–128, 2011.

[9] D. Sornette and S. Von der Becke, *Crashes and High Frequency Trading*, Swiss Finance Institute Research Paper, Zürich, Switzerland, 2011.

[10] A. Golub, J. Keane, and S.-H. Poon, *High Frequency Trading and Mini Flash Crashes*, SSRN, Pamplemousses, Mauritius, 2012.

[11] A. Kirilenko, A. S. Kyle, M. Samadi, and T. Tuzun, "The flash crash: high-frequency trading in an electronic market," *The Journal of Finance*, vol. 72, no. 3, pp. 967–998, 2017.

[12] A. Securities and I. Commission, "Report 215: Australian equity market structure," Regulatory Report, ASIC, Kobe, Port island, 2010.

[13] B. Hagströmer and L. Nordén, "The diversity of high-frequency traders," *Journal of Financial Markets*, vol. 16, no. 4, pp. 741–770, 2013.

[14] Á. Cartea, S. Jaimungal, and J. Penalva, *Algorithmic and Highfrequency Trading*, Cambridge University Press, Cambridge, UK, 2015.

[15] Á. Cartea, R. Donnelly, and S. Jaimungal, "Enhancing trading strategies with order book signals," *Applied Mathematical Finance*, vol. 25, no. 1, pp. 1–35, 2018.

[16] C. Cao, O. Hansch, and X. Wang, "The information content of an open limit-order book," *Journal of Futures Markets*, vol. 29, no. 1, pp. 16–41, 2009.

[17] R. Cont, A. Kukanov, and S. Stoikov, "The price impact of order book events," *Journal of Financial Econometrics*, vol. 12, no. 1, pp. 47–88, 2014.

[18] M. A. Goldstein, A. Kwan, and R. Philip, "High-frequency trading strategies," *Management Science*, 2022.

[19] S. Stoikov, *The micro-price: A High Frequency Estimator of Future Prices*, SSRN, Pamplemousses, Mauritius.

[20] A. Thakkar and K. Chaudhari, "A comprehensive survey on deep neural networks for stock market: the need, challenges, and future directions," *Expert Systems with Applications*, vol. 177, Article ID 114800, 2021.

[21] R. G. Palmer, W. Brian Arthur, J. H. Holland, B. LeBaron, and P. Tayler, "Artificial economic life: a simple model of a stockmarket," *Physica D: Nonlinear Phenomena*, vol. 75, no. 1-3, pp. 264–274, 1994.

[22] W. Arthur, J. Holland, B. Lebaron, R. Palmer, and P. Tayler, "Asset pricing under endogenous expectations in an artificial stock market," in *The Economy as an Evolving Complex System II*, pp. 15–44, Addison-Wesley, Boston, 1997.

[23] C. Chiarella, G. Iori, and J. Perelló, "The impact of heterogeneous trading rules on the limit order book and order flows," *Journal of Economic Dynamics and Control*, vol. 33, no. 3, pp. 525–537, 2009.

[24] S.-H. Chen, C.-L. Chang, and Y.-R. Du, "Agent-based economic models and econometrics," *The Knowledge Engineering Review*, vol. 27, no. 2, pp. 187–219, 2012.

[25] C.-H. Yeh and C.-Y. Yang, "Do price limits hurt the market?" *Journal of Economic Interaction and Coordination*, vol. 8, no. 1, pp. 125–153, 2013.

[26] G. Iori, "A microsimulation of traders activity in the stock market: the role of heterogeneity, agents interactions and trade frictions," *Journal of Economic Behavior and Organization*, vol. 49, no. 2, pp. 269–285, 2002.

[27] C. Chiarella and X.-Z. He, "Heterogeneous beliefs, risk, and learning in a simple asset-pricing model with a market maker," *Macroeconomic Dynamics*, vol. 7, no. 04, pp. 503–536, 2003.

[28] S. Ganesh, N. Vadori, M. Xu, H. Zheng, P. Reddy, and M. Veloso, "Reinforcement learning for market making in a multi-agent dealer market," 2019.

[29] I. Yagi, Y. Masuda, and T. Mizuta, "Analysis of the impact of high frequency trading on artificial market liquidity," *IEEE Transactions on Computational Social Systems*, vol. 7, no. 6, pp. 1324–1334, 2020.

[30] G.-A. Karvik, J. Noss, J. Worlidge, and D. Beale, "The deeds of speed: an agent-based model of market liquidity and flash episodes," Technical Report, Bank of England, London, UK, 2018.

[31] T. A. Vuorenmaa and L. Wang, "An agent-based model of the flash crash of May 6, 2010, with policy implications," Social Science Research Network, Pamplemousses, Mauritius, 2014.

[32] Y. Nakajima and Y. Shiozawa, "Usefulness and feasibility of market maker in a thin market," in *Proceedings of the International Conference Experiments in Economic Sciences: New Approaches to Solving Real world Problems*, pp. 1000–1003, Kyoto, Japan, December 2004.

[33] Y. Kusada, T. Mizuta, S. Hayakawa, and K. Izumi, "Impacts of position-based market makers on markets' shares of trading volumes -an artificial market approach," *Social Modeling and Simulations+ Econophysics Colloquium 2014*, vol. 30, 2014.

[34] M. Sewell, "Characterization of financial time series," 2011, http://www.cs.ucl.ac.uk/fileadmin/UCL-CS/research/Research_Notes/RN_11_01.pdf.

[35] R. Cont, "Empirical properties of asset returns: stylized facts and statistical issues," *Quantitative Finance*, vol. 1, no. 2, pp. 223–236, 2001.

[36] R. Tsay, "Analysis of financial time series," in *Wiley Series in Probability and Statistics*, Wiley-Interscience, Hoboken, NJ, USA, 2nd edition, 2005.

[37] H. Levy, M. Levy, and S. Solomon, *Microscopic Simulation of Financial Markets: From Investor Behavior to Market Phenomena*, Elsevier, Amsterdam, Netherland, 2000.

[38] B. LeBaron, "Agent-based computational finance: suggested readings and early research," *Journal of Economic Dynamics and Control*, vol. 24, no. 5-7, pp. 679–702, 2000.

[39] M. Raberto, S. Cincotti, S. M. Focardi, and M. Marchesi, "Agent based simulation of a financial market," *Physica A: Statistical Mechanics and Its Applications*, vol. 299, no. 1-2, pp. 319–327, 2001.

[40] C. Chiarella and G. Iori, "A simulation analysis of the microstructure of double auction markets," *Quantitative Finance*, vol. 2, no. 5, pp. 346–353, 2002.

[41] C. H. Hommes, "Heterogeneous agent models in economics and finance," *Handbook of Computational Economics*, vol. 2, pp. 1109–1186, 2006.

[42] I. Yagi, S. Maruyama, and T. Mizuta, "Trading strategies of a leveraged ETF in a continuous double auction market using an agent-based simulation," *Complexity*, vol. 2020, Article ID 3497689, 7 pages, 2020.

[43] G. Peña, *An Algorithmic Trading Strategy to Balance Profitability and Risk," Big Data in Finance*, Palgrave Macmillan, London, UK, 2022.