

Writing “Prime Numbers and the Riemann Hypothesis”

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Overview

What this is about

Barry Mazur and I spent over a decade writing a popular math book *Prime Numbers and the Riemann Hypothesis*, which will be published by Cambridge University Press in 2016. This talk is meant to provide a glimpse into the writing process and also content of the book.

This is a talk about making research math a little more accessible, about math education, and about technology.

Intended Audience: Research mathematicians! Though there is no mathematics at all in this talk.

Content of the book

Download it now

It's here: <http://wstein.org/rh/>

Download a copy before we have to remove it from the web!

Goal

The goal of our book is simply to explain what the Riemann Hypothesis is really about. It is a book about mathematics by two mathematicians. The mathematics is front and center; we barely touch on people, history, or culture, since there are already numerous books that address the non-mathematical aspects of RH.

Target Audience of Book

Our target audience is math-loving high school students, retired electrical engineers, and you.

Clay Mathematics Institute Lectures: 2005

CMI Public Lectures

The book started in May 2005 when the Clay Math Institute asked Barry Mazur to give a large lecture to a popular audience at MIT and he chose to talk about RH, with me helping with preparations. His talk was entitled "Are there still unsolved problems about the numbers 1, 2, 3, 4, ... ?"

See the notes.



Picture of Barry Mazur...

"Let's write a book!" (10 years ago)

From Talk to Book

Barry's talk went well, and we decided to try to expand on it in the form of a book. We had a long summer working session in a vacation house near an Atlantic beach, in which we greatly refined our presentation.^a

^aI finally switched from Linux to OS X when Ubuntu made a huge mistake pushing out a standard update that hosed X11 for everybody!

Classical Fourier Transform

Going beyond the original Clay Lecture, I kept pushing Barry to see if he could describe RH as much as possible in terms of the classical Fourier transform applied to a function that could be derived via a very simple process from the prime counting function $\pi(x)$. Of course, he could. This led to more questions than it answered, and interesting numerical observations that are more precise than analytic number theorists typically consider.

Our Approach

Fourier Series!

Our approach to writing the book was to try to reverse engineer how Riemann might have been inspired to come up with RH in the first place, given how Fourier analysis of periodic functions was in the air. This led us to some surprisingly subtle mathematical questions, some of which we plan to investigate in research papers. They also indirectly play a role in Simon Spicer's recent UW Ph.D. thesis.^a

^aThe expert analytic number theorist Andrew Granville helped us out of many confusing thickets.

Distributions

In order to use Fourier series we naturally have to rely heavily on Dirac/Schwartz distributions.

Modular Symbols, L -functions

We met again many times, in Boston and Berkeley, for further similar working sessions, which were now entirely done via collaborative editing in SageMathCloud (I wrote the SMC LaTeX editor in order to work on this book). These resulted in side projects:

- ① Our work-in-progress on “(not-so) random walks” associated to modular symbols, which may be useful in understanding certain behavior of L -functions,
- ② a project we have on making the explicit formula for elliptic curve L -functions more explicit.

SIMUW is...

SIMUW is the “Summer Institute for Mathematics at Univ of Washington.” It’s for high school; admission is free and based on student merit, not rich parents, thanks to an anonymous wealthy donor! Get involved at some point, if you have a chance. I taught a SIMUW course one summer from the RH book. I spent one very intense week on the RH book, and another on the Birch and Swinnerton-Dyer conjecture.

SIMUW

Our book is perfect for high school students. For example, we interactively worked with prime races, multiplicative parity, prime counting, etc., using Sage interacts. The students could also prove facts in number theory. They also looked at misleading data and tried to come up with conjectures. *In algebraic number theory, usually the first few examples are a pretty good indication of what is true. In analytic number theory, in contrast, looking at the first few million examples is usually deeply misleading.*

Reader feedback

I dare you to find a typo!

In early 2015, we posted drafts on Google+ daring anybody to find typos. We got massive feedback. I couldn't believe the typos people found. One person would find a subtle issue with half of a bibliography reference in German, and somebody else would find another subtle mistake in the same reference. Best of all, highly critical and careful non-mathematicians read straight through the book and found a large number of typos and minor issues that were just plain confusing to them, but could be easily clarified.

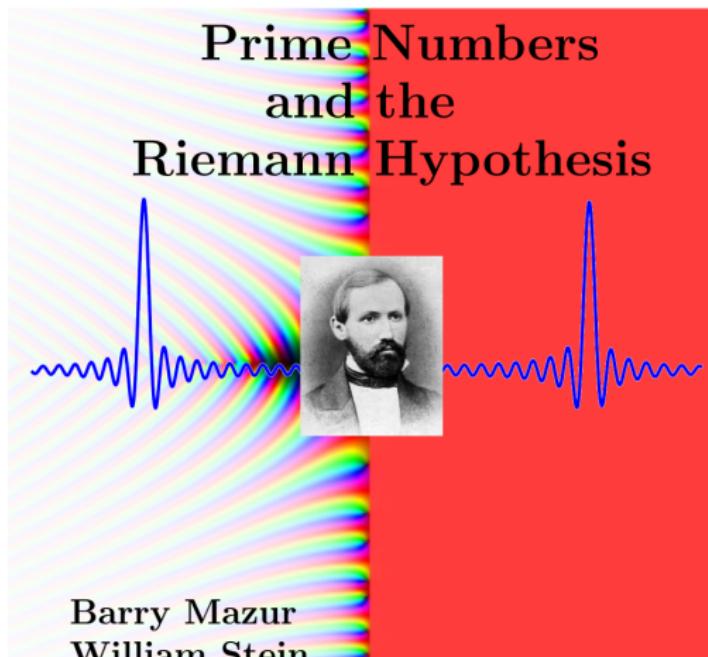
Now it's not riddled with errors

Thanks entirely to the amazingly generous feedback of these readers, when you flip to a random page of our book (go ahead and try), you are now unlikely to see a typo or, what's worse, some corrupted mathematics, e.g., a formula with an undefined symbol.

Designing the cover

Our Cover

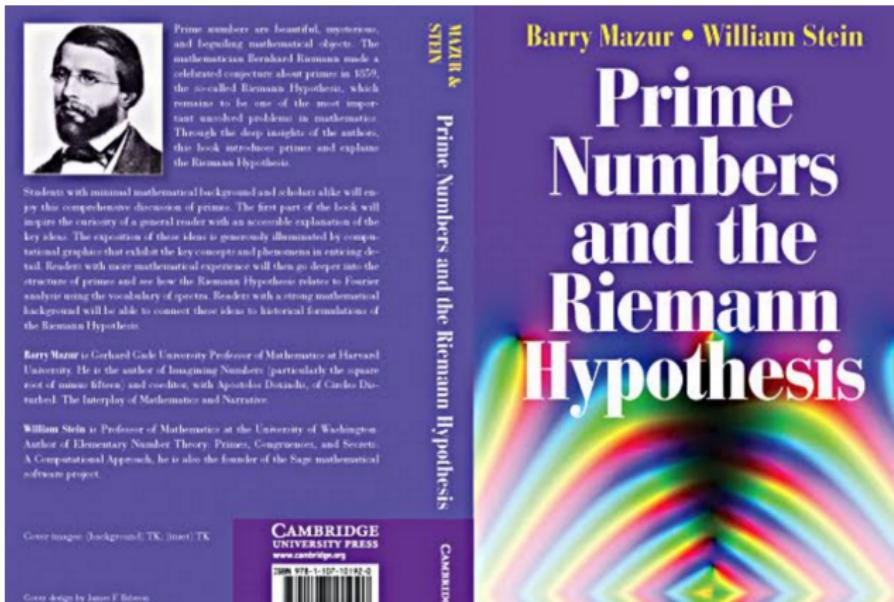
Barry and Gretchen Mazur, Will Hearst, and I designed a cover design that combined the main elements of the book: title, Riemann, zeta:



CUP-ing the cover

Their Cover

Then designers at CUP made our rough design more attractive according their tastes. As non-mathematician designers, they made it look prettier by messing with the Riemann Zeta function...



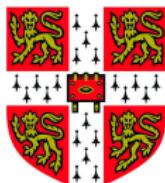
CUP-ing the cover (2)

Their Cover?

I wanted to show you the final cover, but I can't find a copy in email! Basically, they agreed to fix the cover on the previous slide in various ways, I think. Hopefully.

Choosing a publisher

Over years, we talked with people from AMS, Springer-Verlag and Princeton Univ Press about publishing our book. I met CUP editor Kaitlin Leach at the Joint Mathematics Meetings in Baltimore, since the Cambridge University Press (CUP) booth was directly opposite the SageMath booth, which I was running. We decided, due to their enthusiasm, which lasted more than for the few minutes while talking to them (!), past good experience, and general frustration with other publishers, to publish with CUP.



CAMBRIDGE
UNIVERSITY PRESS

Working with CUP

What is was like for us working with CUP

The actual process with CUP has had its ups and downs, and the production process has been frustrating at times, being in some ways not quite professional enough and in other ways extremely professional. Traditional book publication is currently in a state of rapid change. Working with CUP has been unlike my experiences with other publishers.

The Future?

I'm particularly excited to see if we can produce an electronic (Kindle) version of the book later in 2016, and eventually a fully interactive complete for-pay SageMathCloud version of the book, which could be a foundation for something much broader with publishers, which addresses the shortcoming of the Kindle format for interactive computational books. Things like electronic versions of books are the sort of things that AMS is frustratingly slow to get their heads around...

More about working with CUP

Images and Illustrations

For example, CUP was extremely diligent putting huge effort into tracking down permissions for every one of the XXX images in our book. And they weren't satisfy with a statement on Wikipedia that "this image is public domain", if the link didn't work. They tracked down alternatives for all images for which they could get permissions (or in some cases have us partly pay for them). This is in sharp contrast to my experience with Springer-Verlag, which spent about one second on images, just making sure I signed a statement that all possible copyright infringement was my fault (not their's).

Working with CUP: Images and Illustrations

Email I received while writing this talk...

Dirac Image



Inbox x



Adam Kratoska <akratoska@camb

9:06 AM (55 minutes ago)



to mazur, me, Kaitlin

Dear Professor Mazur and Professor Stein,

We have been running into surprising complications with the purchase of the image of Paul Adrien Maurice Dirac. Due to the way that the National Portrait Gallery is set up, it makes it quite complicated for us as a corporation to purchase the image. However, the process should be quite easy for one of you to purchase as an individual through their online system.

Even more about working with CUP

Copyediting...

The CUP copyediting and typesetting appeared to all be outsourced to India, organized by people who seemed far more comfortable with Word than LaTeX. Communication with people that were being contracted out about our book's copyediting was surprisingly difficult, a problem that I haven't experienced before with Springer and AMS. That said, everything seems to have worked out fine so far.



Yet more about working with CUP

Marketing

On the other hand, our marketing contact at CUP mysteriously vanished for a long time; evidently, they had left to another job, and CUP was recruiting somebody else to take over.

However, now there are new people and they seem extremely passionate!

Working with CUP: Marketing

Another email I received while writing this talk (!)...

Prime Numbers



Inbox x



Chris Burrows <cburrows@cambridge.org>



8:10 AM (1 hour ago)



to mazur, me ▼

Dear Professor Mazur and Professor Stein,

I hope you are both well.

My name is Chris Burrows and I'm a publicist in the Academic Books division at Cambridge University Press.

As you know, your new book, *Prime Numbers and the Riemann Hypothesis*, is soon to be published so I'm producing an infographic to be used online and in our email marketing campaigns.

Infographics are aimed at a general audience so what I'd like to do is explain what the Riemann Hypothesis is and why it is so important.

If this sounds agreeable what I need from you is your help identifying short and punchy nuggets of information and illustrations from the book to help do this.

Conclusions

General summary

- ① Publishing a high quality book is a long and involved process.
- ② Working with CUP has been frustrating at times; however, they have recruited a very strong team this year that addresses most issues.
- ③ I hope mathematicians will put more effort into making mathematics accessible to non-mathematicians.
- ④ Hopefully, this talk will provide a more glimpse into the book writing process and encourage others (and also suggest things to think about when choosing a publisher and before signing a book contract!)