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1 Introduction

We did examples from the 2.4 section and Quiz 4.

2 Problem 1

Excercise 3, section 2.4. Solution is on the page A89.

3 Problem 2

Excercise 5:

Sketch the graph of a function that is continuous everywhere except at $x = 3$ and is continuous from the left at 3.

Solution: one such function is given in the solution at the page A89.

4 Problem 3

Excercise 15. The graph is sketched on the page A90. The function needs to be rewritten by factoring out the nominator:

$$\frac{x^2 - x - 12}{x + 3} = \frac{(x - 4)(x + 3)}{x + 3} = x - 4$$

See the solution to Quiz 4 for more information.

5 Problem 4

Excercise 21. Given the function $G(t) = \ln(t^4 - 1)$, find the domain, why the function is continuous at every number in its domain?

Solution: The logarithm $\ln x$ is defined only for $x > 0$. We want the logarithm to be real, e.g. $\ln(-1)$ is not defined. Otherwise, if you remember complex numbers, logarithm can be defined for any (complex) number $z = \rho e^{i\theta}$ as $\ln \rho e^{i\theta} = \ln \rho + i\theta$ and it depends on the branch cut (you can add any number of 2π to θ). Anyway, so we need to determine for which t the expression $t^4 - 1 > 0$. There are many ways to do that, for example we can factor it out: $t^4 - 1 = (t^2 + 1)(t^2 - 1) = (t^2 + 1)(t + 1)(t - 1) > 0$, but $t^2 + 1$ is always positive, so we get:

$$(t + 1)(t - 1) = t^2 - 1 > 0$$

We can either determine t from the left hand side, or just draw a graph of $t^2 - 1$, which is just a parabola shifted down by 1. In any case, we get

$$t \in (-\infty, -1) \cup (1, \infty)$$

Which is also the domain of $G(t)$.

How about the continuity? Every polynomial is continuous and $\ln x$ is continuous for all $x > 0$, so $G(t)$ which is a composition of a logarithm and a polynomial, is continuous for all t from its domain, e.g. for all $t \in (-\infty, -1) \cup (1, \infty)$.

6 Problem 5

Excercise 31. In order for the function to be continuous, $cx+1$ must be equal to cx^2-1 for $x=3$, e.g.:

$$c \cdot 3 + 1 = c \cdot 3^3 - 1$$

$$3c + 2 = 9c$$

$$2 = 6c$$

$$c = \frac{1}{3}$$

7 Problem 6

Excercise 33. $f(x) = x^3 - x^2 + x$ attains all values between $-\infty$ and ∞, so from the Intermediate Value Theorem, it also attains the value 10 for some c, e.g. $f(c) = 10$.

8 Problem 7

Excercise 45. Is there a number that is exactly 1 more than its cube?

Solution: The number x should be exactly 1 more than its cube x^3, e.g. $x = 1 + x^3$, e.g. we need to solve

$$x^3 - x + 1 = 0$$

The function $x^3 - x + 1$ attains all values from $-\infty$ to ∞, so it has at least one real solution.

Optional: In fact, it turns out that it has one real and two complex solutions:

$$x_1 = \frac{-66^{\frac{2}{3}} - 36^{\frac{1}{3}}\left(18 + 2\sqrt{69}\right)^{\frac{2}{3}}}{18\left(18 + 2\sqrt{69}\right)^{\frac{1}{3}}}$$

$$x_2 = \frac{26^{\frac{1}{3}}\sqrt{69} - 6i6^{\frac{1}{3}}\sqrt{23} + 186^{\frac{1}{3}} + 26^{\frac{2}{3}}\left(18 + 2\sqrt{69}\right)^{\frac{1}{3}} - 18i\sqrt{3}6^{\frac{1}{3}} + 2i\sqrt{3}6^{\frac{2}{3}}\left(18 + 2\sqrt{69}\right)^{\frac{1}{3}}}{12\left(18 + 2\sqrt{69}\right)^{\frac{2}{3}}}$$

$$x_3 = \frac{26^{\frac{1}{3}}\sqrt{69} + 6i6^{\frac{1}{3}}\sqrt{23} + 186^{\frac{1}{3}} + 26^{\frac{2}{3}}\left(18 + 2\sqrt{69}\right)^{\frac{1}{3}} + 18i\sqrt{3}6^{\frac{1}{3}} - 2i\sqrt{3}6^{\frac{2}{3}}\left(18 + 2\sqrt{69}\right)^{\frac{1}{3}}}{12\left(18 + 2\sqrt{69}\right)^{\frac{2}{3}}}$$

So let's stick to the real solution x_1:

$$x = \frac{-66^{\frac{2}{3}} - 36^{\frac{1}{3}}\left(18 + 2\sqrt{69}\right)^{\frac{2}{3}}}{18\left(18 + 2\sqrt{69}\right)^{\frac{1}{3}}} \doteq -1.32471795724475$$

Indeed, $x^3 \doteq -2.32471795724475$, which is one less than x.

9 Problem 8

Given a function $f(x) = \sqrt{x}$, what is the slope at a point $x = a$?

This is a very important problem. We need to choose two points on the function, so for example $P(a, f(a))$ and $Q(a+h, f(a+h))$ for some h. The slope is defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In our case $y_2 = f(a+h)$, $y_1 = f(a)$, $x_2 = a+h$ and $x_1 = a$ so we get:

$$m_{PQ} = \frac{f(a+h) - f(a)}{a+h-a} = \frac{f(a+h) - f(a)}{h}$$

The slope m at the point a is then equal to

$$m = \lim_{h \to 0} m_{PQ} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

For our particular case $f(x) = \sqrt{x}$ we get:

$$m = \lim_{h \to 0} \frac{\sqrt{a+h} - \sqrt{a}}{h} = \lim_{h \to 0} \frac{\sqrt{a+h} - \sqrt{a}}{h} \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} = \lim_{h \to 0} \frac{(a+h) - a}{h(\sqrt{a+h} + \sqrt{a})} =$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{a+h} + \sqrt{a})} = \lim_{h \to 0} \frac{1}{\sqrt{a+h} + \sqrt{a}} = \frac{1}{\sqrt{a+0} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$$

10 Quizzes

We did Quiz 4.