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Quiz 5 & 6

Problem 1

Calculate the limit:

$$\lim_{x \rightarrow \infty} (\sqrt{4x^2 + x} - 2x)$$

Problem 2

$$y = x^2$$

calculate the slope of the tangent line at the point $x = a$.

Solution to the Problem 1

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{4x^2 + x} - 2x) &= \lim_{x \rightarrow \infty} (\sqrt{4x^2 + x} - 2x) \frac{\sqrt{4x^2 + x} + 2x}{\sqrt{4x^2 + x} + 2x} = \lim_{x \rightarrow \infty} \frac{(4x^2 + x) - 4x^2}{\sqrt{4x^2 + x} + 2x} = \\ &= \lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + x} + 2x} = \lim_{x \rightarrow \infty} \frac{1}{\frac{\sqrt{4x^2 + x}}{x} + 2} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{4 + \frac{1}{x}} + 2} = \frac{1}{\sqrt{4 + 2}} = \frac{1}{4} \end{aligned}$$

Solution to the Problem 2

$$m = \lim_{h \rightarrow 0} \frac{y(a+h) - y(a)}{h} = \lim_{h \rightarrow 0} \frac{(a+h)^2 - a^2}{h} = \lim_{h \rightarrow 0} \frac{2ah + O(h^2)}{h} = \lim_{h \rightarrow 0} (2a + O(h)) = 2a$$

Grading

Problem 1: you got 4 points for arriving at:

$$\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + x} + 2x}$$

4 points for arriving at:

$$\lim_{x \rightarrow \infty} \frac{1}{\sqrt{4 + \frac{1}{x}} + 2}$$

and 2 points for getting the correct answer $\frac{1}{4}$.

Problem 2: you got 2 points for:

$$m = \lim_{h \rightarrow 0} \frac{y(a+h) - y(a)}{h}$$

Two points for:

$$\lim_{h \rightarrow 0} \frac{(a+h)^2 - a^2}{h}$$

Two points for

$$\lim_{h \rightarrow 0} \frac{2ah + O(h^2)}{h}$$

Two points for:

$$\lim_{h \rightarrow 0} (2a + O(h))$$

and two points for getting the correct answer $2a$.