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Quiz 1

Problem

The position of a car is given by the values in the table.

t (seconds)	0	1	2	3	4	5
s (meters)	0	5	20	45	80	125

(a) Find the average velocity for the time period beginning at $t_1 = 2\,\mathrm{s}$ and ending at

  (i) $t_2 = 5\,\mathrm{s}$

  (ii) $t_2 = 4\,\mathrm{s}$

  (iii) $t_2 = 3\,\mathrm{s}$

(b) Use the numbers in (a) to estimate the instantaneous velocity when $t = 2\,\mathrm{s}$.

Solution

Note: this is a problem 8., section 2.1., just with different numbers.

Average velocity is the defined as the distance traveled, divided by the time period for which we traveled:

$$v = \frac{\Delta s}{\Delta t}$$

More precisely, in our case the average velocity between the times t_1 and t_2 is:

$$v_{t_1 t_2} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Where $s(t)$ is a function, that gives us the dependency of the distance traveled on the time. $s(t_2)$ is then a function $s(t)$ evaluated at a point $t = t_2$, e.g. in particular if $t_2 = 5\,\mathrm{s}$, then $s(t_2)$ is just a number $s(5)$ that can be read from the table above to be $125\,\mathrm{m}$. The function notation (syntax) above is important, so one needs to get used to it. If it sounds a bit confusing though at the beginning, we can rewrite the above formula in couple different equivalent ways:

$$v_{t_1 t_2} = \frac{s_{t_2} - s_{t_1}}{t_2 - t_1}$$

or:

$$v_{t_1 t_2} = \frac{s_2 - s_1}{t_2 - t_1}$$

It's just a notation and all the three formulas are equivalent and they express the same thing, they tell us which numbers to take and how to plug them in the expression, no less, no more.

(a) (i)

$$v_{t_1 t_2} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{125 - 20}{5 - 2} = \frac{105}{3} = 35$$

(ii)

$$v_{t_1 t_2} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{80 - 20}{4 - 2} = \frac{60}{2} = 30$$

(iii)

$$v_{t_1 t_2} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{45 - 20}{3 - 2} = \frac{25}{1} = 25$$

(b) Instantaneous velocity is equal to the average velocity if the time interval goes to zero. Looking at the numbers above, we see that for the time interval 3 s the average velocity is 35 ($\frac{\text{m}}{\text{s}}$), for the interval 2 s it is 30 and for 1 s it is 25. So the best guess is that the instantaneous velocity would be around $v = 20\,\frac{\text{m}}{\text{s}}$.

Better solution to the part b)

The solution above is the best we can do with our current knowledge, but surely we can do better than just guess. One way to do that would be to find a continuous function that passes through all the points in the table, thus allowing us to calculate the instantaneous velocity with any precision we want. One such function is:

$$s(t) = \frac{1}{2} \cdot 10 \cdot t^2$$

(Verify that it indeed generates the table above.) The instantaneous velocity at any time is then given by:

$$v(t) = \frac{\mathrm{d}s}{\mathrm{d}t} = 10t$$

In particular for $t = 2$ we get

$$v(2) = 10 \cdot 2 = 20$$

We'll be learning that in a week or two.