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class: MATH 181
date: March 26, 2009

1 Introduction

Today we did some hard and involved problems from the section 4.1 and then a preview of the section 4.2 plus a home take quiz.

2 Problem 1

It's 4.1, problem 31.

If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of $0.3\,\frac{\Omega}{\mathrm{s}}$ and $0.2\,\frac{\Omega}{\mathrm{s}}$, respectively, how fast is R changing when $R_1 = 80\Omega$ and $R_2 = 100\Omega$?

Answer

We express R:

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

and differentiate with respect to t:

$$R' = \frac{(R_1R_2)'(R_1+R_2) - R_1R_2(R_1+R_2)'}{(R_1+R_2)^2} = \frac{(R_1'R_2 + R_1R_2')(R_1+R_2) - R_1R_2(R_1'+R_2')}{(R_1+R_2)^2}$$

Now we can substitute the values for R_1, R_2, $\frac{\mathrm{d}R_1}{\mathrm{d}t} = 0.3\,\frac{\Omega}{\mathrm{s}}$ and $\frac{\mathrm{d}R_2}{\mathrm{d}t} = 0.2\,\frac{\Omega}{\mathrm{s}}$:

$$R' = \frac{(0.3\cdot 100 + 80\cdot 0.2)(80+100) - 80\cdot 100(0.3+0.2)}{(80+100)^2}\frac{\Omega}{\mathrm{s}} = \frac{107}{810}\frac{\Omega}{\mathrm{s}} \doteq 0.132\frac{\Omega}{\mathrm{s}}$$

3 Problem 2

It's 4.1, problem 37.

A runner sprints around a circular track of radius 100 m at a constant speed of $7\,\frac{\mathrm{m}}{\mathrm{s}}$. The runner's friend is standing at a distance 200 m from the center of the track. How fast is the distance between the friends changing when the distance between them is 200 m?

Answer

Let the position of the runner be $(x, y) = (r\cos\omega t, r\sin\omega t) = (r\cos\frac{v}{r}t, r\sin\frac{v}{r}t)$ where ω is the angular velocity of the runner and so it follows $r\omega = v$. $r = 100\,\mathrm{m}$ is the radius of the circle. Let $R = 200\,\mathrm{m}$ be the distance between the second friend and the center of the track and l the distance between friends. The position of the second friend is then $(0, -R)$ and the distance is:

$$l^2 = x^2 + (y+R)^2$$

We want to calculate $\frac{\mathrm{d}l}{\mathrm{d}t}$ at the point $l = 200\,\mathrm{m}$, so after differentiating:

$$2ll' = 2xx' + 2(y+R)y'$$

thus:

$$l' = \frac{1}{l}(xx' + (y+R)y')$$

Then we need to evaluate x' and y', so we differentiate the position of the runner:

$$x = r\cos\frac{v}{r}t$$

$$y = r\sin\frac{v}{r}t$$

$$x' = -v\sin\frac{v}{r}t = -v\frac{y}{r}$$

$$y' = v\cos\frac{v}{r}t = v\frac{x}{r}$$

and substitute back:

$$l' = \frac{1}{l}(xx' + (y+R)y') = \frac{1}{l}(x(-v\frac{y}{r}) + (y+R)v\frac{x}{r}) = \frac{1}{l}Rv\frac{x}{r}$$

Now let's evaluate this derivative at the point $l = 200\,\mathrm{m} = \mathrm{R}$:

$$l' = \frac{1}{l}Rv\frac{x}{r} = \frac{1}{R}Rv\frac{x}{r} = v\frac{x}{r}$$

Last thing we need to do is to evaluate x. There are two solutions:

$$x = r\cos(\pi+\beta) = r(\cos\pi\cos\beta - \sin\pi\sin\beta) = -r\cos\beta$$

and

$$x = r\cos(-\beta) = r\cos\beta$$

so we can write:

$$x = \pm r\cos\beta$$

where β is the angle between the line (x,y)-$(0,0)$ and the horizontal line. Looking at the triangle (x,y)-$(0,-R)$-$(0,0)$, we can see that it has sides l, l and r and the angle between l and r is $\frac{\pi}{2} - \beta$, so:

$$\cos\left(\frac{\pi}{2} - \beta\right) = \frac{r}{2l}$$

hence:

$$\cos\left(\frac{\pi}{2} - \beta\right) = \cos\frac{\pi}{2}\cos\beta + \sin\frac{\pi}{2}\sin\beta = \sin\beta = \frac{r}{2l}$$

now we can calculate x:

$$x = \pm r\cos\beta = \pm r\sqrt{1-\sin^2\beta} = \pm r\sqrt{1-\left(\frac{r}{2l}\right)^2}$$

So the answer is:

$$l' = v\frac{x}{r} = v\frac{\pm r\sqrt{1-\left(\frac{r}{2l}\right)^2}}{r} = \pm v\sqrt{1-\left(\frac{r}{2l}\right)^2}$$

Putting in numbers:

$$l' = \pm 7\sqrt{1-\left(\frac{100}{2\cdot 200}\right)^2}\,\frac{\mathrm{m}}{\mathrm{s}} = \pm 7\frac{\sqrt{15}}{4}\,\frac{\mathrm{m}}{\mathrm{s}} \doteq \pm 6.78\,\frac{\mathrm{m}}{\mathrm{s}}$$

4 Problem 3

Finally we did the problem 37 from the section 4.2.

5 Quiz

The Quiz 17 is a hometake, due Tuesday.