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1 Introduction

Today we did some hard and involved problems from the section 4.1 and then a preview of the section 4.2 plus a home take quiz.

2 Problem 1

It's 4.1, problem 31.

If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of $0.3\,\frac{\Omega}{\mathrm{s}}$ and $0.2\,\frac{\Omega}{\mathrm{s}}$, respectively, how fast is R changing when $R_1 = 80\Omega$ and $R_2 = 100\Omega$?

Answer

We express R:

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

and differentiate with respect to t:

$$R' = \frac{(R_1R_2)'(R_1+R_2) - R_1R_2(R_1+R_2)'}{(R_1+R_2)^2} = \frac{(R_1'R_2 + R_1R_2')(R_1+R_2) - R_1R_2(R_1'+R_2')}{(R_1+R_2)^2}$$

Now we can substitute the values for R_1, R_2, $\frac{\mathrm{d}R_1}{\mathrm{d}t} = 0.3\,\frac{\Omega}{\mathrm{s}}$ and $\frac{\mathrm{d}R_2}{\mathrm{d}t} = 0.2\,\frac{\Omega}{\mathrm{s}}$:

$$R' = \frac{(0.3\cdot 100 + 80\cdot 0.2)(80+100) - 80\cdot 100(0.3+0.2)}{(80+100)^2}\,\frac{\Omega}{\mathrm{s}} = \frac{107}{810}\,\frac{\Omega}{\mathrm{s}} \doteq 0.132\,\frac{\Omega}{\mathrm{s}}$$

3 Problem 2

It's 4.1, problem 37.

A runner sprints around a circular track of radius $100\,\mathrm{m}$ at a constant speed of $7\,\frac{\mathrm{m}}{\mathrm{s}}$. The runner's friend is standing at a distance $200\,\mathrm{m}$ from the center of the track. How fast is the distance between the friends changing when the distance between them is $200\,\mathrm{m}$?

Answer

Let the position of the runner be $(x, y) = (r\cos\omega t, r\sin\omega t) = (r\cos\frac{v}{r}t, r\sin\frac{v}{r}t)$ where ω is the angular velocity of the runner and so it follows $r\omega = v$. $r = 100\,\mathrm{m}$ is the radius of the circle. Let $R = 200\,\mathrm{m}$ be the distance between the second friend and the center of the track and l the distance between friends. The position of the second friend is then $(0, -R)$ and the distance is:

$$l^2 = x^2 + (y+R)^2$$

We want to calculate $\frac{\mathrm{d}l}{\mathrm{d}t}$ at the point $l = 200\,\mathrm{m}$, so after differentiating:

$$2ll' = 2xx' + 2(y+R)y'$$

thus:

$$l' = \frac{1}{l}(xx' + (y+R)y')$$

Then we need to evaluate x' and y', so we differentiate the position of the runner:

$$x = r\cos\frac{v}{r}t$$

$$y = r\sin\frac{v}{r}t$$

$$x' = -v\sin\frac{v}{r}t = -v\frac{y}{r}$$

$$y' = v\cos\frac{v}{r}t = v\frac{x}{r}$$

and substitute back:

$$l' = \frac{1}{l}(xx' + (y+R)y') = \frac{1}{l}(x(-v\frac{y}{r}) + (y+R)v\frac{x}{r}) = \frac{1}{l}Rv\frac{x}{r}$$

Now let's evaluate this derivative at the point $l = 200\,\mathrm{m} = \mathrm{R}$:

$$l' = \frac{1}{l}Rv\frac{x}{r} = \frac{1}{R}Rv\frac{x}{r} = v\frac{x}{r}$$

Last thing we need to do is to evaluate x. There are two solutions:

$$x = r\cos(\pi + \beta) = r(\cos\pi\cos\beta - \sin\pi\sin\beta) = -r\cos\beta$$

and

$$x = r\cos(-\beta) = r\cos\beta$$

so we can write:

$$x = \pm r\cos\beta$$

where β is the angle between the line (x,y)-$(0,0)$ and the horizontal line. Looking at the triangle (x,y)-$(0,-R)$-$(0,0)$, we can see that it has sides l, l and r and the angle between l and r is $\frac{\pi}{2} - \beta$, so:

$$\cos\left(\frac{\pi}{2} - \beta\right) = \frac{r}{2l}$$

hence:

$$\cos\left(\frac{\pi}{2} - \beta\right) = \cos\frac{\pi}{2}\cos\beta + \sin\frac{\pi}{2}\sin\beta = \sin\beta = \frac{r}{2l}$$

now we can calculate x:

$$x = \pm r\cos\beta = \pm r\sqrt{1 - \sin^2\beta} = \pm r\sqrt{1 - \left(\frac{r}{2l}\right)^2}$$

So the answer is:

$$l' = v\frac{x}{r} = v\frac{\pm r\sqrt{1 - \left(\frac{r}{2l}\right)^2}}{r} = \pm v\sqrt{1 - \left(\frac{r}{2l}\right)^2}$$

Putting in numbers:

$$l' = \pm 7\sqrt{1 - \left(\frac{100}{2\cdot 200}\right)^2}\,\frac{\mathrm{m}}{\mathrm{s}} = \pm 7\frac{\sqrt{15}}{4}\,\frac{\mathrm{m}}{\mathrm{s}} \doteq \pm 6.78\,\frac{\mathrm{m}}{\mathrm{s}}$$

4 Problem 3

Finally we did the problem 37 from the section 4.2.

5 Quiz

The Quiz 17 is a hometake, due Tuesday.