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class: MATH 181
date: January 22, 2009

1 Introduction

2 Problem 1

A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

t(min)	5	10	15	20	25	30
V(gal)	694	444	250	111	28	0

(a) If P is the point (15, 250) on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with $t = 5, 10, 20, 25$, and 30.

(b) Estimate the slope of the tangent line at P by averaging the slopes of two secant lines.

(c) Use a graph of the function to estimate the slope of the tangent line at P. (This slope represents the rate at which the water is flowing from the tank after 15 minutes.)

Solution

(a) For $t = 5$, we get $Q(5, 694)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 694}{15 - 5} = -\frac{444}{10} = -44.4$$

For $t = 10$, we get $Q(10, 444)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 444}{15 - 10} = -38.8$$

For $t = 20$, we get $Q(20, 111)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 111}{15 - 20} = -27.8$$

For $t = 25$, we get $Q(25, 28)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 28}{15 - 25} = -22.2$$

For $t = 30$, we get $Q(30, 0)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 0}{15 - 30} = -16.6$$

(b) We would be averaging the secant lines that are close to the tangent line, in particular the ones for $t = 10$ and $t = 20$, i.e.

$$m = \frac{-38.8 + (-27.8)}{2} = -33.3$$

(c) You just need to plot the graph and the tangent line, then you read the "Δy" and "Δx" from the graph and estimate the slope as

$$m = \frac{\Delta y}{\Delta x}$$

. Ideally you should also get something around -33.