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1 Introduction

This was a review lesson of precalculus things, mainly equation of a line, exponentials and logarithms and trigonometry.

2 Equation of a line

The equation of a line is

$$y = mx + b\,,$$

where m is a slope and b is the y-intercept. Given two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the slope m can be expressed by:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{"rise"}}{\text{"run"}}\,.$$

2.1 Point slope intercept equation

$$y - y_1 = m(x - x_1)$$

2.2 Example 1

A line passes through points $(1, 4)$ and $(2, 10)$ find the equation of a line. Solution:

$$y = 6x - 2$$

2.3 Example 2

A line passes through a point $(1, 4)$ and has a slope 6, find the equation of a line. Solution:

$$y = 6x - 2$$

3 Exponentials and Logarithms

$$a^x a^y = a^{x+y}$$
$$\frac{a^x}{a^y} = a^{x-y}$$
$$(a^x)^y = a^{xy}$$
$$(ab)^x = a^x b^x$$

$$\log_a xy = \log_a x + \log_a y$$
$$\log_a \frac{x}{y} = \log_a x - \log_a y$$
$$\log_a x^y = y \log_a x$$
$$\log_e x = \ln(x)$$
$$e^{\ln x} = x$$
$$\ln e^x = x$$

3.1 Example 3

$$\ln x = 5$$

Express x:

$$e^{\ln x} = e^5$$

$$x = e^5$$

3.2 Example 4

$$2 = e^y$$

Express y:

$$\ln 2 = \ln e^y = y \ln e = y$$

3.3 Example 5

Derive the formula $\log_a x = \frac{\ln x}{\ln a}$:

$$a^y = x$$

Then $y = \log_a x$. Let's take a (natural) logarithm of both sides:

$$\ln a^y = \ln x$$

$$y \ln a = \ln x$$

$$y = \frac{\ln x}{\ln a}$$

E.g. we have

$$y = \frac{\ln x}{\ln a} = \log_a x$$

4 Trigonometry

Definitions of $\sin x$ and $\cos x$ using a unit circle, e.g.:

$$\sin\theta = \frac{\Delta y}{1} = \Delta y$$

$$\cos\theta = \frac{\Delta x}{1} = \Delta x$$

Use the unit circle to derive:

$$\sin 0 = 0$$

$$\cos 0 = 1$$

$$\sin \pi = 0$$

$$\cos \pi = -1$$

$$\sin 2\pi = 0$$

$$\cos 2\pi = 1$$

$$\sin 3\pi = 0$$

$$\cos 3\pi = -1$$

4.1 Trigonometric identities

$$\sin^2 x + \cos^2 x = 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

Use these to derive the other identities, for example:

$$\sin 2x = \sin(x + x) = \sin x \cos x + \cos x \sin x = 2 \sin x \cos x$$

$$\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x = \cdots$$

$$\cos 2x = \cos(x + x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = -1 + 2\cos^2 x$$

Use the last one to show:

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$