TA: Ondřej Čertík
web: http://hpfem.math.unr.edu/~ondrej/
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# 1 Introduction

Today we calculated the quiz 0 correctly on the board, then did the problem 1 (problem 1 in the book) and 2 (problem 5 in the book) below and finally did the quiz 1 (problem 8 in the book just with different numbers).

# 2 Problem 1

That is the problem 1 in the section 2.1 in the book.

A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

| t(min) | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| V(gal) | 694 | 444 | 250 | 111 | 28 | 0 |

(a) If P is the point (15, 250) on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with $t = 5, 10, 20, 25$, and 30.

(b) Estimate the slope of the tangent line at P by averaging the slopes of two secant lines.

(c) Use a graph of the function to estimate the slope of the tangent line at P. (This slope represents the rate at which the water is flowing from the tank after 15 minutes.)

## Solution

(a) For $t = 5$, we get $Q(5, 694)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 694}{15 - 5} = -\frac{444}{10} = -44.4$$

For $t = 10$, we get $Q(10, 444)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 444}{15 - 10} = -38.8$$

For $t = 20$, we get $Q(20, 111)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 111}{15 - 20} = -27.8$$

For $t = 25$, we get $Q(25, 28)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 28}{15 - 25} = -22.2$$

For $t = 30$, we get $Q(30, 0)$ and

$$m_{PQ} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{250 - 0}{15 - 30} = -16.6$$

(b) We would be averaging the secant lines that are close to the tangent line, in particular the ones for $t = 10$ and $t = 20$, i.e.

$$m = \frac{-38.8 + (-27.8)}{2} = -33.3$$

(c) You just need to plot the graph and the tangent line, then you read the "$\Delta y$" and "$\Delta x$" from the graph and estimate the slope as

$$m = \frac{\Delta y}{\Delta x}$$

. Ideally you should also get something around -33.

# 3 Problem 2

This is the problem 5 in the section 2.1 in the book.

If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet after $t$ seconds is given by $y = 40t - 16t^2$.

(a) Find the average velocity for the time period beginning at $t_1 = 2$ and ending at

  (i) $t_2 = 2.5\,\mathrm{s}$
  (ii) $t_2 = 2.1\,\mathrm{s}$
  (iii) $t_2 = 2.05\,\mathrm{s}$
  (iv) $t_2 = 2.01\,\mathrm{s}$

(b) Find the instantaneous velocity when $t = 2$.

## Solution

See the solution to the quiz 1 (the other pdf from the web), where I did a thorough explanation of everything. So let's just calculate:

(a) (i) $v = \frac{y(2.5)-y(2)}{2.5-2} = \frac{(40\cdot 2.5-16\cdot 2.5^2)-(40\cdot 2-16\cdot 2^2)}{0.5} = -32$
  (ii) $v = \frac{y(2.1)-y(2)}{2.1-2} = -25.6$
  (iii) $v = \frac{y(2.05)-y(2)}{2.05-2} = -24.8$
  (iv) $v = \frac{y(2.01)-y(2)}{2.01-2} = -24.16$

(b) A guess would be that the instantaneous velocity is $v = -24$ (see the solution to the Quiz 1 for an explanation). But surely we can do better than guessing, so once we learn how to differentiate in a week or two, then we can calculate the instantaneous velocity right away for any $t$:

$$v(t) = \frac{\mathrm{d}y}{\mathrm{d}t} = 40 - 32t$$

and in particular for $t = 2$ we get:

$$v(2) = 40 - 32 \cdot 2 = -24$$

That is an exact value and it's very quick, no need for the tedious calculations and guessing above, but the price for that is that one needs to learn a bit of calculus.