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Quiz 4

Problem 1

Given a function $f(x)$:

$$f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & \text{if } x \neq 3; \\ 4, & \text{if } x = 3. \end{cases}$$

(i) Sketch the graph of the function

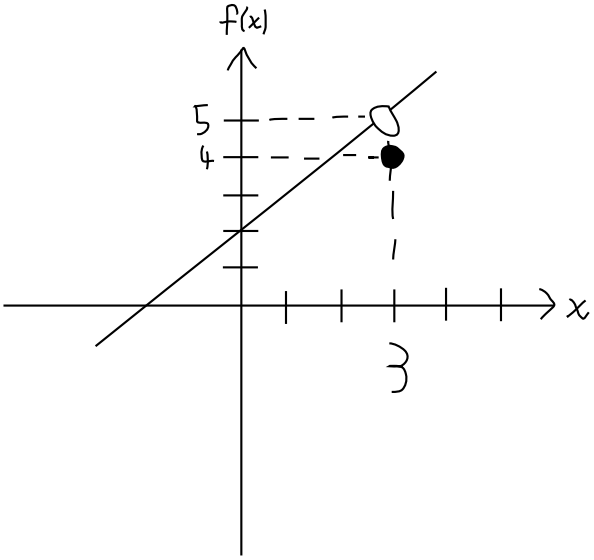
(ii) Is the function continuous at the point $x = 3$? Why?

Solution to Problem 1

(i) We first rewrite the expression for $x \neq 3$ into a simpler form, so that we can sketch the graph:

$$f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & \text{if } x \neq 3; \\ 4, & \text{if } x = 3. \end{cases} = \begin{cases} \frac{(x-3)(x+2)}{x-3}, & \text{if } x \neq 3; \\ 4, & \text{if } x = 3. \end{cases} = \begin{cases} x+2, & \text{if } x \neq 3; \\ 4, & \text{if } x = 3. \end{cases}$$

Now we can sketch the graph:



(ii) The definition of a continuity at the point $x = 3$ is:

$$\lim_{x \to 3} f(x) = f(3)$$

If the equation holds, the function is continuous, if it doesn't, it is discontinuous. In our case $f(3) = 4$ and

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} x + 2 = 5$$

so $\lim_{x \to 3} f(x) \neq f(3)$ and the function is not continuous at the point $x = 3$.

Grading

You got 4 points for rewriting:

$$f(x) = \begin{cases} x+2, & \text{if } x \neq 3; \\ 4, & \text{if } x = 3, \end{cases}$$

2 points for graphing the function and 4 points for correctly determinning the continuity including the reasoning why.