$$v = 0{,}999998\,c$$

(1) $\left(\frac{\vec{r}}{r^3}\right)$

$$\operatorname{div}\left(\frac{\vec{r}}{r^3}\right) = \frac{\partial\left(\frac{r_x}{r^3}\right)}{\partial x} + \frac{\partial\left(\frac{r_y}{r^3}\right)}{\partial y} + \frac{\partial\left(\frac{r_z}{r^3}\right)}{\partial z}$$

$$\frac{\partial\left(\frac{r_x}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right)}{\partial x} = \frac{\frac{\partial r_x}{\partial x}\cdot(x^2+y^2+z^2)^{\frac{3}{2}} - \frac{\partial}{\partial x}\left((x^2+y^2+z^2)^{\frac{3}{2}}\right)\cdot r_x}{(x^2+y^2+z^2)^3} =$$

$$= \frac{0 - \frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}}\cdot(2x)\cdot r_x}{(x^2+y^2+z^2)^3} = \frac{-3\,r\cdot r_x\cdot x}{r^6} = \frac{-3\,r_x x}{r^5}$$

$$\Rightarrow \boxed{\operatorname{div} \frac{\vec{p}}{r^3} = \frac{-3}{r^5} \cdot (p_x x + p_y y + p_z z) = \frac{-3}{r^5} \cdot (\vec{p} \cdot \vec{r})}$$

$$\operatorname{rot}\left(\frac{\vec{p}}{r^3}\right) = \left(\partial_y\left(\frac{p_z}{r^3}\right) - \partial_z\left(\frac{p_y}{r^3}\right); \partial_z\left(\frac{p_x}{r^3}\right) - \partial_x\left(\frac{p_z}{r^3}\right); \partial_x\left(\frac{p_y}{r^3}\right) - \partial_y\left(\frac{p_x}{r^3}\right)\right)$$

$$\partial_y\left(\frac{p_z}{r^3}\right) = \frac{\partial_y(p_z) \cdot r^3 - p_z\, \partial_y\left((x^2+y^2+z^2)^{\frac{3}{2}}\right)}{r^6} =$$

$$= \frac{0 - p_z \frac{3}{2} r \cdot 2y}{r^6} = \frac{-3 p_z y}{r^5} \rightarrow \text{obdobně další členy}$$

$$\boxed{\operatorname{rot} \frac{\vec{p}}{r^3}} = \left(\frac{-3p_z y}{r^5} + \frac{3p_y z}{r^5}; \frac{-3p_x z}{r^5} + \frac{3p_z x}{r^5}; \frac{-3p_y x}{r^5} + \frac{3p_x y}{r^5}\right) =$$

$$= -\frac{3}{r^5} \cdot (p_z y - p_y z; p_x z - p_z x; p_y x - p_x y) =$$

$$= \boxed{+\frac{3}{r^5} \cdot (\vec{p} \times \vec{r})}$$

$$(p_x, p_y, p_z) \times (x, y, z) \qquad \varepsilon_{ijk}$$

$$(p_y z - p_z y; p_z x - p_x z \ldots)$$

Ein. sum. konvence

$$② \ \mathrm{div}\ \mathrm{rot}\ \vec{F} = \vec{\nabla}\cdot(\vec{\nabla}\times\vec{F}) \overset{\downarrow}{=} \frac{\partial}{\partial x_i}\left[\vec{\nabla}\times\vec{F}\right]_i =$$

$$= \frac{\partial}{\partial x_i}\varepsilon_{ijk}\frac{\partial}{\partial x_j}F_k \Rightarrow \partial_i\partial_j\varepsilon_{ijk}F_k =$$

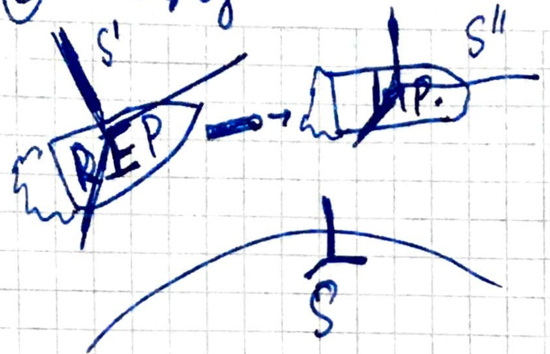
$$= \varepsilon_{123}\partial_x\partial_y F_z + \varepsilon_{132}\partial_x\partial_z F_y + \varepsilon_{231}\partial_y\partial_z F_x +$$

$$+ \varepsilon_{213}\partial_y\partial_x F_z + \varepsilon_{321}\partial_z\partial_y F_x + \varepsilon_{312}\partial_z\partial_x F_y =$$

$$= \cancel{\partial_x\partial_y F_z}\ \boxed{\cancel{-\partial_x\partial_z F_y}} + \cancel{\partial_y\partial_z F_x}\ \cancel{-\partial_y\partial_x F_z}\ \cancel{-\partial_z\partial_y F_x} + \boxed{\cancel{\partial_z\partial_x F_y}} =$$

$$= \underline{\underline{0}}$$

③ v_{proj} vůči lodi? $l_{projekt.} = 1\,m$



v_{proj}^{S} vůči S: $v_{proj}^{S} = \dfrac{v_{rep}^{S} + v_{proj}^{S'}}{1 + \dfrac{v_{rep}^{S} v_{proj}^{S'}}{c^2}}$

$v_{proj}^{S'}$ vůči raketě: $v_{proj}^{S'} = \dfrac{v_{rep}^{S} - v_{proj}^{S}}{1 - \dfrac{v_{rep}^{S} v_{proj}^{S}}{c^2}}$

musí platit: $v_{proj}^{S} > v_{imp}^{S}$

$$\frac{v_{rep}^{S} + v_{proj}^{S'}}{1 + \frac{v_{rep}^{S} v_{proj}^{S'}}{c^2}} > v_{imp}^{S}$$

$$v_{rep}^{S} + v_{proj}^{S'} > v_{imp}^{S} + \frac{v_{rep}^{S} v_{imp}^{S} v_{proj}^{S'}}{c^2}$$

$$v_{proj}^{S'} \cdot \left(1 - \frac{v_{rep}^{S} v_{imp}^{S}}{c^2}\right) > v_{imp}^{S} - v_{rep}^{S}$$

$$v_{proj}^{S'} > \frac{v_{imp}^{S} - v_{rep}^{S}}{1 - \frac{v_{rep}^{S} v_{imp}^{S}}{c^2}} = \frac{0{,}1c}{1 - 0{,}56} = \frac{5}{22}c$$

$$v_{proj}^{S} > \frac{v_{rep}^{S} + v_{proj}^{S'}}{1 + \frac{v_{rep}^{S} v_{proj}^{S'}}{c^2}} = \frac{0{,}7c + \frac{5}{22}c}{1 + \frac{7}{10} \cdot \frac{5}{22}} = \frac{4}{5}c = 0{,}8c$$

$$l_{proj}^{S} = l_{proj}^{S'} \cdot \sqrt{1-\frac{(v_{proj}^{S})^2}{c^2}}$$

$$l_{proj}^{S'} \cdot \sqrt{1-\frac{(v_{proj}^{S})^2}{c^2}} < l_{proj}^{S'} \sqrt{1-\frac{0{,}8^2 c^2}{c^2}} = \sqrt{0{,}36}\ m = 0{,}6\ m$$

$$l_{proj}^{S} < 0{,}6\,m$$