

1. **A** To find the left Riemann sum we'll start evaluating the rectangles at the left-endpoints which are 0, 3, 4, 5 and 8.

$$\int_0^{10} C(t)dt = C(0) \cdot 3 + C(3) \cdot 1 + C(4) \cdot 1 + C(5) \cdot 3 + C(8) \cdot 2 = 510 \text{ thousands of units}$$

B Because the function $C(t)$ is decreasing over the interval then the left Riemann sum is an overestimate.

2. **A** On the interval $[2, -5]$ there's one half of the semi circle and a line segment.

$$\int_{-5}^2 f(x)dx = -\frac{9\pi}{4} - 12 - \frac{2 \cdot 1}{2} + \frac{2 \cdot 1}{2} = -12 - \frac{9}{4}\pi$$

B We'll use the simple integration rules to solve this one

$$\int_0^6 [g(x) + 2]dx = \int_{-5}^6 g(x)dx - \int_{-5}^0 g(x)dx + \int_0^6 2dx = 32$$

C We'll solve this similarly as the two previous problems.

$$\int_0^6 [3g(x) - f(x)]dx = 3 \int_0^6 g(x)dx - \int_0^6 f(x)dx = 60 - 17 - 4\pi = 43 - 4\pi$$

3. **A**

$$\int \tan^3(x) \sec^2(x)dx \text{ let } u = \tan(x) \Rightarrow dx = \frac{du}{\sec^2(x)}$$

$$\int u^3 du = \frac{u^4}{4} + C = \frac{\tan^4(x)}{4} + C$$

B

$$\int e^{\cos(x)} \sin(x)dx \text{ let } u = \cos(x) \Rightarrow dx = -\frac{du}{\sin(x)}$$

$$-\int e^u du = -e^u + C = -e^{\cos(x)} + C$$

C

$$\int \frac{x}{\sqrt{1-16x^2}}dx \text{ let } u = 1-16x^2 \Rightarrow dx = -\frac{du}{32x}$$

$$-\frac{1}{32} \int \frac{1}{\sqrt{u}} du = -\frac{1}{16} \sqrt{u} + C = -\frac{1}{16} \sqrt{1-16x^2} + C$$

4. **A** There are two possible ways of writing this Riemann sum as an definite integral.

$$1. \int_0^4 \sqrt{2+x} dx$$

$$2. \int_2^6 \sqrt{x} dx$$

B The integrals above will give the same value so it doesn't not matter which one we will evaluate.

$$\int_0^6 \sqrt{x} dx = \left[\frac{2}{3} x^{3/2} \right]_0^6 = 4\sqrt{6} - \frac{4\sqrt{2}}{3}$$

5. We'll solve this by following the integration rules.

$$\begin{aligned}\int_0^{\pi/2} \sec^2(x/k) dx &= k \\ k[\tan(x/k)]_0^{\pi/2} &= k \\ [\tan(x/k)]_0^{\pi/2} &= 1 \\ \tan\left(\frac{\pi/2}{k}\right) - \tan\left(\frac{0}{k}\right) &= 1 \\ \tan\left(\frac{\pi/2}{k}\right) &= 1 \\ \frac{\pi/2}{k} &= \pi/4 + n\pi \text{ where } n \in \mathbb{Z} \\ k &= 2\end{aligned}$$