1. a) To find the average rate of change over the interval $[-1, a]$ we will use the average rate formula $m = \frac{\Delta y}{\Delta x}$

$$m = \frac{\Delta y}{\Delta x} = \frac{f(a) - f(-1)}{a - (-1)} = \frac{\sqrt{a^2-1} - \sqrt{(-1)^2-1}}{a+1} = \frac{\sqrt{a^2-1}}{a+1} = \frac{(a+1)^{0.5}(a-1)^{0.5}}{a+1} =$$
$$(a+1)^{-0.5}(a-1)^{0.5} = \frac{\sqrt{a-1}}{\sqrt{a+1}}$$

To find the exact rate of change at $x = -1$ we have to find the left handed limit as a approaches -1 because the function f does not exist on the interval $[-1, 1] \rightarrow$ function is not differentiable on that interval.

$$\lim_{a\to-1^-} \frac{\sqrt{a-1}}{\sqrt{a+1}} = -\infty \text{ Therefore } y' \text{ at } x = -1 \text{ doesn't exist.}$$

b) To find the instanatenous change of rate at $x = 2$ we will use the limit def. of derivative.

$$\lim_{x\to2} \frac{f(x) - f(2)}{x-2} = \lim_{x\to2} \frac{\sqrt{x^2-1} - \sqrt{3}}{x-2} = \lim_{x\to2} \frac{\sqrt{x^2-1} - \sqrt{3}}{x-2} \frac{\sqrt{x^2-1} + \sqrt{3}}{\sqrt{x^2-1} + \sqrt{3}}$$
$$\lim_{x\to2} \frac{x+2}{\sqrt{x^2-1} + \sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

2. a) To estimate the values of derivatives at points we will use the average slope formula $m = \frac{\Delta y}{\Delta x}$.
For $x- = 2$ we will use points $[-2, g(-2)]$ *and* $[-1, g(-1)]$
$g'(2) \approx \frac{3-2}{-2-(-1)} \approx -1$
For $x = 0$ we can say that $g'(0) = 0$ because it's the turning point.
For $x = 1$ we will use points $[1, g(1)]$ *and* $[0, g(0)]$
$g'(1) \approx \frac{3-0}{0-1} \approx -3$
b) To find the equation of tangent line at $x = 1$ we will use the point-slope formula because we know the average slope at that point.

$$y = -3x + 3$$

3. Let's find the derivatives.

$$a)\ f'(0) + 3h'(0) + 0 = -3$$
$$b)\ (\frac{f(x)h(x)}{g(x)} - 2x^{-1})' = -\frac{2}{9}$$

4. We can rewrite the function as $f(x) = sin(x)\frac{1}{sin(x)} - \frac{1}{tan(x)} = 1 - \frac{1}{tan(x)} = 1 - cot(x)$

$$f'(x) = csc^2(x) \rightarrow f'(\pi/4) = 2$$

5. We can see that the limit we have to evaluate is actually the derivative at $x = 1$ written as definition of derivative using limit. To find the limit we just have to find $f'(1)$.

$$f('x) = -4^x \cdot ln(4) \cdot ln(x) - \frac{4^x}{x} \rightarrow f'(1) = 0 - 4 = -4$$

After substituting to the derivative we found $f'(1) = -4$.