

1. We'll use the chain rule to find the derivatives

A) $h'(x) = f'(g(x)) \cdot g'(x)$

B) $h'(x) = 3(f(g(x)))^2 \cdot f'(g(x)) \cdot g'(x)$

C) $h'(x) = f'(g(5x)^4) \cdot 4g(5x)^3 \cdot g'(5x) \cdot 5$

D) $h'(x) = f'(\arcsin(6x)) \cdot \frac{6}{\sqrt{1-36x^2}}$

2. **A)** To find $\frac{dy}{dx}$ we need to use the implicit differentiation.

$$\begin{aligned}\frac{d}{dx}(x^3 + y^3 - 2xy) &= \frac{d}{dx}(2) \\ 3x^2 + 3y^2 \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= \frac{2y - 3x^2}{3y^2 - 2x}\end{aligned}$$

B) Now we just need to find the slope at $(-1, 1)$ and substitute into the point-slope formula.

$$\begin{aligned}\frac{dy}{dx}\bigg|_{y=1} &= \frac{2 - 3}{3 + 2} = -\frac{1}{5} \\ \text{Tangent line} \Rightarrow y - 1 &= -\frac{1}{5}(x + 1)\end{aligned}$$

3. **A)** To find $f'(x)$ we'll once again use the chain rule and inverse trigonometric differentiation.

$$f'(x) = -\frac{\sin(\arcsin(x))}{\sqrt{1-x^2}} = -\frac{x}{\sqrt{1-x^2}}$$

$\sin(\arcsin(x))$ becomes x because if we put the inverse function of x into the function we get x

B) Now we'll find the slope at $x=0.5$ and substitute into the point-slope formula. (And also find $f(0.5)$ which is $\sqrt{3}/2$)

$$\begin{aligned}m = f'\left(\frac{1}{2}\right) &= -\frac{\frac{1}{2}}{\sqrt{\frac{3}{4}}} = -\frac{1}{\sqrt{3}} \\ \text{Tangent line} \Rightarrow y - \frac{\sqrt{3}}{2} &= -\frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)\end{aligned}$$

4. **A)** To find $\frac{dy}{dx}$ we'll just simply use the implicit differentiation.

$$\begin{aligned}\frac{d}{dx}(x^2 + y^2 - 4x - 2y + 1) &= 0 \\ 2x + 2y \frac{dy}{dx} - 4 - 2 \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= \frac{2 - x}{y - 1}\end{aligned}$$

B) To find the coordinates of those points on the curve where the tangent line is vertical we just need to find places where the derivative doesn't exist (the function is not differentiable) therefore we need to find the "limit" of the derivative so that it will

be equal to $\pm\infty$.

$$\lim_{x \rightarrow a} \left(\frac{2-x}{y-1} \right) = \pm\infty$$

We can see that the derivative doesn't exist at $y=1$. If we substitute $y=1$ into the curve formula we find two solutions $\Rightarrow (0, 1)$ and $(4, 1)$

5. **A)** In this part we'll once again use the implicit differentiation.

$$\begin{aligned} k'(x) &= f'(g(x)) \cdot g'(x) \\ k'(-1) &= f'(1) \cdot 1 = -3 \end{aligned}$$

B) With solving the B part will help us the inverse derivative rules and then we'll have to substitute the slope to the point-slope formula.

$$j'(1) = \frac{1}{f'(j(1))} = \frac{1}{2}$$

Now we'll substitute into the point-slope formula at the point $(1, -2)$

$$\Rightarrow y + 2 = \frac{1}{2}(x - 1)$$