1.
$$f(x) = \begin{cases} 4x - 7 & x \leq 2 \\ e^{x-2} & x > 2 \end{cases}$$

To prove that the function $f$ is continous at $x = 2$ we have to look at $\lim_{x\to 2} f(x)$.

$$\text{Left-handed limit: } \lim_{x\to 2^-} f(x) = \lim_{x\to 2} 4x - 2 = 1$$
$$\text{Right handed limit: } \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} e^{x-2} = 1$$
$$\text{Therefore: } \lim_{x\to 2} f(x) = 1$$

In next step of solving this problem we have to show that $f(2)$ exists which it does because $f(2) = 4 \cdot 2 - 7 = 1$.
The last requirement to prove that $f$ is continous at $x = 2$ is that $f(2) = \lim_{x\to 2} f(x)$ which it does $\to f(2) = \lim_{x\to 2} f(x) = 1$.

2. According to the graph $\Rightarrow$

$$\text{Part A: } \lim_{x\to 4^-} f(x) = 5$$
$$\text{Part B: } \lim_{x\to 4^+} f(x) = 2$$
$$\text{Part C: } \lim_{x\to 2} f(x) = 6$$

3. Let's (not) find those limits $\Rightarrow$
**Part A**: We will use our knowledge of simple trigonometric limits therefore we know that $\lim_{x\to 0} \frac{sin(nx)}{nx} = 1$.

$$\lim_{x\to 0} \frac{sin(2x)}{3x} = \lim_{x\to 0} \frac{\frac{2}{3}}{\frac{2}{3}} \cdot \frac{sin(2x)}{3x} = \frac{2}{3} \cdot \lim_{x\to 0} \frac{sin(2x)}{2x} = \frac{2}{3} \cdot 1 = \frac{2}{3}$$

**Part B**: We have to rationalize and simplify the expression to find the limit.

$$\lim_{x\to 4} \frac{\sqrt{x-3} - 1}{x - 4} \cdot \frac{\sqrt{x-3} + 1}{\sqrt{x-3} + 1} = \lim_{x\to 4} \frac{x - 4}{(x-4)(\sqrt{x-3} + 1)} = \lim_{x\to 4} \frac{1}{\sqrt{x-3} + 1} = \frac{1}{2}$$

4. Using limit theorems we know that $\Rightarrow$

$\lim_{x\to 2}[f(x) \cdot g(x)] = \lim_{x\to 2} f(x) \cdot \lim_{x\to 2} g(x) \Rightarrow$ Now we just need to find the values in the graphs
$\lim_{x\to 2} f(x) = DNE \wedge \lim_{x\to 2} g(x) = DNE \Rightarrow DNE \cdot DNE = DNE$ (DNE times DNE is still DNE :))

5. We consider this piecewise function $h(x)$.

$$h(x) = \begin{cases} \dfrac{x-3}{x^2-9} & x \neq -3; 3 \\ k & x = 3 \end{cases}$$

If we want to make $h(x)$ continuous at $(-3, \infty)$ by finding $k$ we have to look at the limit as x approaches 3 of $\frac{x-3}{x^2-9}$.

$$\lim_{x\to 3} \frac{x-3}{x^2-9} = \lim_{x\to 3} \frac{x-3}{(x-3)(x+3)} = \lim_{x\to 3} \frac{1}{x+3} = \frac{1}{6}$$

To prevent a removable discontinuity at $x = 3$, then $h(3) = \lim_{x\to 3} f(x) = k = \frac{1}{6} \Rightarrow k$ must be equal to $\frac{1}{6}$.