0602 Slides

MA 116

June 2025

9.2 Procedure

The only question type for 9.2. Given a quantitative population, we want to estimate its population mean μ by an interval estimator to a confidence level $(1-\alpha)100\%$.

1. Step 1. Determine if the distribution of our new variable t can be approximated by the Student's t-distribution: there are 2 situations.
2. Step 2. If yes, calculate your α. e.g. 95% confidence level $\leftrightarrow$ $\alpha = 0.05$.
3. Step 3. Look at the particular sample we obtained. What are its particular n, s, $\overline{x}$?
4. Step 4. Calculate $E = t_{\alpha/2}\dfrac{s}{\sqrt{n}}$. Be careful that $t_{\alpha/2}$ depends on $df = n-1$.
5. Step 5. Conclude that $\overline{x} \pm E$ is our $(1-\alpha)100\%$ confidence interval estimator.

Quiz 2

Quiz 2 will only cover 9.1, 9.2, 10.1, 10.2.
There will be 6 questions.
Question 1. (9.1) Calculate an interval estimator for a population proportion p.
Question 2. (9.1) Given a margin of error E' we want to achieve, determine the sample size n needed for an interval estimator of p to have at most this error.
Question 3. (9.2) Calculate an interval estimator for a population mean $\overline{x}$.
Question 4. (10.1) Basic concepts of hypothesis testing: H_0, H_1, Type of errors, how to draw conclusion to a hypothesis test.
Question 5. (10.2) Classical method of hypothesis test for a population proportion: assume H_0 holds, calculate test statistics z_0 to see whether z_0 falls into critical region.
Question 6. (10.2) Confidence interval method of hypothesis test for a population proportion.

Definition. (Margin of error–estimating a population mean)

Sample size $=n$. Define $E = t_{\alpha/2}\dfrac{s}{\sqrt{n}}$ where $t_{\alpha/2}$ is with $n-1$ degrees of freedom.

Definition. (Confidence interval–estimating a population mean)

If we obtain a particular sample mean $\overline{x}$, sample standard deviation s, and sample size n. Pick a level of confidence $(1-\alpha)100\%$. We may then calculate $E = t_{\alpha/2}\dfrac{s}{\sqrt{n}}$. A confidence interval of confidence level $(1-\alpha)100\%$ of μ is $[\overline{x}-E, \overline{x}+E]$.

Conditions for those formulas to work: (1) the distribution of the new variable t can be approximated by the Student's t-distribution. (2) Any sampling is random. (3) $n < 0.05N$.

Example. Suppose the underlying population is of a very large size and normally distributed with unknown population mean μ. We want to estimate this μ. We obtained a sample of size 2 $\{2, 4\}$. Let's construct a confidence interval of confidence level 90%.

Steps

1. Can the distribution of the new variable t be approximated by the Student's t-distribution?
2. α?
3. To use the formula $E = t_{\alpha/2}\dfrac{s}{\sqrt{n}}$ we need to calculate s and find out $t_{\alpha/2}$ with $df = n - 1$ from the Student's t-distribution table.
4. Calculate $\overline{x}$ and conclude that the interval estimator of 90% confidence interval is $\overline{x} \pm E$.

What is the other situation in which the distribution of the new variable t can be approximated by the Student's t-distribution?

Steps in Hypothesis Testing

1. Make a statement regarding the nature of the population.
2. Collect evidence (sample data) to test the statement.
3. Analyze the data to assess the plausibility of the statement.

In practice: Steps in Hypothesis Testing

1. Make a statement regarding the nature of the population, i.e. determine the population parameter (μ, p?) we are using, determine H_0 and H_1.
2. Collect evidence (sample data) to test the statement. I.e. Obtain a sample to use. Assume H_0 to be true all the time.
3. Analyze the data to assess the plausibility of the statement. Assume H_0 to be true all the time, i.e. if $H_0: \ p = 0.8$ then we actually take $\mu_{\hat{p}} = p = 0.8$ to construct a distribution of $\hat{p}$, and see where our particular sample's proportion lies on this distribution.

Example.

The packaging on a light bulb states that the bulb will last 500 hours under normal use. A consumer advocate would like to know if the mean lifetime of a bulb is less than 500 hours. Determine H_0, H_1, and the type of this hypothesis test.

		Reality	
		H_0 Is True	H_1 Is True
Conclusion	Do Not Reject H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

The packaging on a light bulb states that the bulb will last 500 hours under normal use. A consumer advocate would like to know if the mean lifetime of a bulb is less than 500 hours. $H_0 : \mu = 500$, $H_1 : \mu < 500$. Left-tailed test.

Suppose in this hypothesis test we make a Type I error. What's happening?

		Reality	
		H_0 Is True	H_1 Is True
Conclusion	Do Not Reject H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

The packaging on a light bulb states that the bulb will last 500 hours under normal use. A consumer advocate would like to know if the mean lifetime of a bulb is less than 500 hours. $H_0 : \mu = 500$, $H_1 : \mu < 500$. Left-tailed test.

Type I error happens if the sample we obtained evidences that we should reject H_0, so we draw the conclusion that the mean lifetime of a bulb is less than 500 hours, but in fact the light bulbs indeed have an average lasting time of 500 hours.

		Reality	
		H_0 Is True	H_1 Is True
Conclusion	Do Not Reject H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

The packaging on a light bulb states that the bulb will last 500 hours under normal use. A consumer advocate would like to know if the mean lifetime of a bulb is less than 500 hours. $H_0 : \mu = 500$, $H_1 : \mu < 500$. Left-tailed test.

Suppose in this hypothesis test we make a Type II error. What's happening?

		Reality	
		H_0 Is True	H_1 Is True
Conclusion	Do Not Reject H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

The packaging on a light bulb states that the bulb will last 500 hours under normal use. A consumer advocate would like to know if the mean lifetime of a bulb is less than 500 hours. $H_0 : \mu = 500$, $H_1 : \mu < 500$.

Type II error happens if the sample we obtained evidences that we should not reject H_0, so we draw the conclusion that [there is not sufficient evidence to conclude that the mean lifetime of a bulb is less than 500 hours], but in fact the light bulbs have an average lasting time of less than 500 hours.

Drawing conclusion

Because any hypothesis test decision is based on incomplete (sample vs. population) information, we never say that we **accept** the null hypothesis. without having access to the entire population, we don't know the exact value of the parameter stated in the null hypothesis. Rather, we say that we **do not reject** the null hypothesis if our sample indicates that the null hypothesis H_0 could be true.

The conclusion to a hypothesis test is ALWAYS as follows: There (is/is not) sufficient evidence to conclude that [insert $\mathbf{H}_1$ statement].

Example. Suppose that the sample we obtained evidences that we should not reject H_0. Our conclusion would be:

Because [some data analysis result of our sample data set], there is not sufficient evidence to conclude that that the mean lifetime of a bulb is less than 500 hours.

The previous few pages are reviewing for Question 4 on Quiz 2.

Question 4. (10.1) Basic concepts of hypothesis testing: H_0, H_1, Type of errors, how to draw conclusion to a hypothesis test.

10.2

Section 10.2: Procedures to conduct a hypothesis test for a population proportion.

We discuss two approaches: classical vs. confidence interval. Those are Question 5, Question 6 on Quiz 2, respectively.

Question 5. (10.2) Classical method of hypothesis test for a population proportion: assume H_0 holds, calculate test statistics z_0 to see whether z_0 falls into critical region.
Question 6. (10.2) Confidence interval method of hypothesis test for a population proportion.

Classical approach

Example.

According to a Gallup poll conducted in 2008, 80% of Americans felt satisfied with the way things were going in their personal lives. A researcher wonders if the percentage of satisfied Americans is different today. The researcher obtains a particular random sample of size 100, in which there are 72 positive answers. Choose a level of significance $\alpha = 0.05$.

1. Step 1. H_0, H_1, type of test?
2. Step 2. Assume that H_0 holds. Is $\hat{p}$ approximately normally distributed?

Classical approach

Example.

According to a Gallup poll conducted in 2008, 80% of Americans felt satisfied with the way things were going in their personal lives. A researcher wonders if the percentage of satisfied Americans is different today. The researcher obtains a particular random sample of size 100, in which there are 72 positive answers. Choose a level of significance $\alpha = 0.05$.

1. Step 1. $H_0 : p = 0.8$; $H_1 : p \neq 0.8$, two-tailed test.
2. Step 2. Assume $p = 0.8$.. $\hat{p}$ is approximately normally distributed because the population size is large and $np(1-p) = 16 \geq 10$.
3. Determine the critical region.
4. Does our sample fall into the critical region?

If the sample falls into the critical region, we say the result is statistically significant and then reject H_0.

Classical approach

1. Step 1. $H_0 : p = 0.8$; $H_1 : p \neq 0.8$, two-tailed test.
2. Step 2. Assume $p = 0.8$.. $\hat{p}$ is approximately normally distributed because the population size is large and $np(1-p) = 16 \geq 10$.
3. Step 3. Critical region, after change of variable, $z < -z_{0.025}$ and $z > z_{0.025}$.
4. Step 4. Test statistic $z_0 = -2$ falls into the critical region, so the result is statistically significant.
5. Conclusion: Because our sample statistic falls into the critical region, there is enough evidence to conclude that the percentage of satisfied Americans is different today.

10.2 Confidence interval approach

This approach is easy as long as you remember to verify that $\hat{p}$ is approximately normally distributed and you can construct $\hat{p} \pm E$.

Confidence interval approach is only for two-tailed test

Specify H_0 and H_1 and make sure the test is two-tailed. Assume H_0 holds! Given a confidence level $(1-\alpha)100\%$ and a particular random sample of some size n, we verify that $\hat{p}$ is approximately normally distributed. We may then calculate $\hat{p} \pm E$ using $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$. By our assumption that H_0 holds, if p does not lie in this interval, we reject H_0. If p lies in this interval, we do not reject H_0.

10.2 Confidence interval approach: Example

Do a hypothesis test with 95% confidence interval,
$H_0: \ p = 0.34$, $H_1: \ p \neq 0.34$. A particular sample has $\hat{p} = \dfrac{353}{1200}$.

1. Check *hatp* is normally distributed.
2. Calculate E.
3. Write out our interval $\hat{p} \pm E$.
4. Draw conclusion.

10.2 Confidence interval approach: Example

Do a hypothesis test with 95% confidence interval,
$H_0: p = 0.34$, $H_1: p \neq 0.34$. A particular sample has $\hat{p} = \dfrac{353}{1200}$.

1. $\hat{p}$ is normally distributed: $0.34 \cdot 0.66 \cdot 1200 \geq 10$.
2. Assume H_0 holds. $\alpha = 0.05$.

$$E = 1.96\sqrt{0.294 \cdot 0.706/1200} = 0.026.$$

3. $\hat{p} \pm E$ is $[0.27, 0.32]$. Since by assumption that $p = 0.34$, which does not lie in this interval, we reject H_0. There is sufficient evidence to conclude that $p \neq 0.34$.

10.3 Hypothesis test for a population mean

We want to know if Generaion Z has a higher average phone screen time than average Americans, given that the average phone screen time of Americans is 5 hours. Let's do a hypothesis test with a level of significance $\alpha = 0.05$.

Suppose we obtain a random sample of size 100 from Generaion Z Americans with a sample mean $\overline{x} = 6.5$, sample variance $s = 1.5$.

What are my H_0 and H_1?

10.3 Hypothesis test for a population mean

We want to know if Generaion Z has a higher average phone screen time than average Americans, given that the average phone screen time of Americans is 5 hours. Let's do a hypothesis test with a level of significance $\alpha = 0.05$.

Suppose we obtain a random sample of size 100 from Generaion Z Americans with a sample mean $\overline{x} = 6.5$, sample variance $s = 1.5$.

$H_0 : \mu = 5$; $H_1 : \mu > 5$. Right-tailed test.

How can we determine the critical region and whether our sample statistic falls into the critical region? We again asssume H_0 holds and change to variable t via

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}}.$$