Formula Sheet

Population mean formula. $\mu = \frac{\sum x_i}{N}$ where the summation is taken over all data points in the population, and N is the population size.

Population variance formula. $\sigma^2 = \frac{\sum(x_i - \mu)^2}{N}$ where the summation is taken over all data points in the population, and N is the population size.

Population standard deviation formula. $\sigma = \sqrt{\sigma^2}$.

Sample mean formula. $\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, where n is the sample size.

Sample variance formula. $s^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}$, where n is the sample size.

Sample standard deviation formula. $s = \sqrt{s^2}$.

Normal distrubution/Bell curve

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-1/2[(x-\mu)/\sigma]^2)$$

Change of Variable formulas.

Given a normal variable x with a mean μ and a standard deviation σ, we may convert it to a standard normal variable z by the formula

$$z = \frac{x-\mu}{\sigma}$$

Then,

$$P(\frac{x-\mu}{\sigma} \leq a) = P(x \leq a\sigma + \mu)$$
$$P(\frac{x-\mu}{\sigma} \geq a) = P(x \geq a\sigma + \mu)$$

for any number a.

$$P(x \leq a) = P(z \leq \frac{a-\mu}{\sigma})$$
$$P(x \geq a) = P(z \geq \frac{a-\mu}{\sigma})$$

for any number a.

---

z_α formulas.

- $P(z \geq z_\alpha) = \alpha$.
- For any $0 \leq \alpha \leq 1$ we have $-z_\alpha = z_{1-\alpha}$.

Sampling distribution (Ch.8) formulas.
Fix a population and a sample size n. Assume all samplings are random.

Theorem		need $n < 0.05N$?
1	$\mu_{\overline{x}} = \mu$ (μ is the mean of the original population)	NO
2	$\sigma_{\overline{x}} = \sigma/\sqrt{n}$ (σ is the standard deviation of the original population)	YES
3	x being approximately normal implies $\overline{x}$ is approximately normal	NO
4	CLT: $n \geq 30$ implies $\overline{x}$ is approximately normal	YES
5	$\mu_{\hat{p}} = p$	NO
6	$\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$	YES
7	$np(1-p) \geq 10$ implies $\hat{p}$ is approximately normal	YES

Confidence Interval (Ch.9) formulas.

Interval estimator for population proportion, margin of error formula to a level of confidence $(1-\alpha)100\%$.

We use $n\hat{p}(1-\hat{p}) \geq 10$, where $\hat{p}$ is the sample proportion of our particular sample, to determine if the variable $\hat{p}$ is approximately normally distributed.

If $\hat{p}$ is approximately normally distributed, we can use

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

to construct an interval estimator of p.

Determine sample size needed, given α and an error E'.

1. **Method 1.**

$$n = \tilde{p}(1-\tilde{p})\left(\frac{z_{\alpha/2}}{E'}\right)^2$$

rounded up to the next integer, where $\tilde{p}$ is a **prior point estimator** of p.

2. **Method 2.** If we do not have a prior point estimator information,

$$n = 0.25 \left(\frac{z_{\alpha/2}}{E'} \right)^2$$

rounded up to the next integer.

Interval estimator for population mean, margin of error formula to a level of confidence $(1 - \alpha)100\%$.
If the variable

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

has a distribution that can be approximated by the Student's t-distribution, we may use

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}}$$

(where $t_{\alpha/2}$ is with $n - 1$ degrees of freedom) to construct a confidence interval of confidence level $(1 - \alpha)100\%$ of μ by $[\overline{x} - E, \, \overline{x} + E]$.

Hypothesis test for a population proportion.

		Reality	
		H_0 Is True	H_1 Is True
Conclusion	Do Not Reject H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

Test statistic formula

$$z_0 = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}},$$

where $\hat{p}$ is the sample mean of the particular sample obtained. This formula can only be used if the variable $\hat{p}$ is normal.

Hypothesis test for a population mean, change of variable to *t*.

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}}.$$

If x is approximately a normal variable, or if $n > 30$, this variable t follows Student's t-distribution with $df = n - 1$.

Hypothesis test for two population proportions, independent samples.
If $n\hat{p}_1(1 - \hat{p}_1) \geq 10$ and $n\hat{p}_2(1 - \hat{p}_2) \geq 10$, then the variable $\hat{p}_1 - \hat{p}_2$ has an approximately normal distribution. We may convert $\hat{p}_1 - \hat{p}_2$ to a standard normal variable via

$$z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\hat{p}(1 - \hat{p})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$.

Hypothesis test for two population means, matched pair data.

$d_i = x_i - y_i$

If d is approximately normally distributed, or if $n > 30$, then the new variable

$$t = \frac{\overline{d} - \mu_d}{s_d/\sqrt{n}}$$

follows Student's t-distribution with $df = n - 1$.

Hypothesis test for two population means, independent samples.

If the two populations are both normally distributed, or if both n_1 and n_2 are > 30, then the new variable

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

approximately follows Student's t-distribution with the smaller of $n_1 - 1$ or $n_2 - 1$ degrees of freedom.

Sample linear correlation coefficient

$$r = \frac{\sum_i \left(\dfrac{(x_i - \overline{x})}{s_x} \dfrac{(y_i - \overline{y})}{s_y} \right)}{n-1} = \frac{\sum_i (x_i - \overline{x})(y_i - \overline{y})}{s_x s_y (n-1)}$$

Least squares regression line of a sample

$$b_1 = r \cdot \frac{s_y}{s_x}$$

$$b_0 = \overline{y} - b_1 \overline{x}$$

$$\hat{y} = b_0 + b_1 x$$

Inference about $\boldsymbol{\beta_1}$

$$s_e = \sqrt{\frac{\sum_i (y_i - \hat{y}_i)^2}{n-2}} = \sqrt{\frac{\sum_i \text{residuals}^2}{n-2}}$$

where $\sum_i \text{residuals}^2$ is the sum of square resuduals of a least squares regression line.

$$s_{b_1} = \frac{s_e}{s_x \sqrt{n-1}}$$

Change of variable in a hypothesis test about $\boldsymbol{\beta_1}$

$$t = \frac{b_1 - \beta_1}{s_{b_1}}$$

follows Student's t-distribution with $n - 2$ degrees of freedom if the residual plot of the sample has no obvious pattern and the distribution of the error ϵ is assumed to be normal.

Convert a categorical variable to dummy variables

Suppose u is a catagorical variable of k levels: **Level 1** to **Level $\boldsymbol{k}$**. Picking **Level 1** to be the base level allows the following dummy variables to be defined:

$$x_1 = \begin{cases} 1, & \text{if } u \text{ is at Level 2} \\ 0, & \text{otherwise} \end{cases} \qquad x_2 = \begin{cases} 1, & \text{if } u \text{ is at Level 3} \\ 0, & \text{otherwise} \end{cases} \qquad \cdots \qquad x_{k-1} = \begin{cases} 1, & \text{if } u \text{ is at Level } k \\ 0, & \text{otherwise} \end{cases}$$

Given a least squares prediction equation of a sample whose data points are of the form $(x_1,\, x_2, ..., x_{k-1},\, y)$, where x_i are dummy variables of a categorical variable u of k levels, and y is a quantitative response variable. The lease squares prediction equation of this sample

$$\hat{y} = b_0 + b_1 x_1 + \cdots + b_{k-1} x_{k-1}$$

is given by the following formulas.

$$b_0 = \overline{y}_1$$

$$b_i = \overline{y}_{i+1} - \overline{y}_i,$$

for any $1 \leq i \leq k-1$, where $\overline{y}_i$ is the mean value of the response variable y when u is at **Level $\boldsymbol{i}$**.

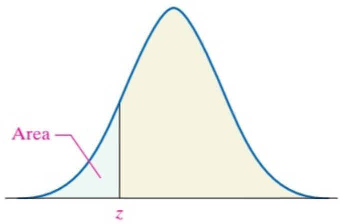

Table V

Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

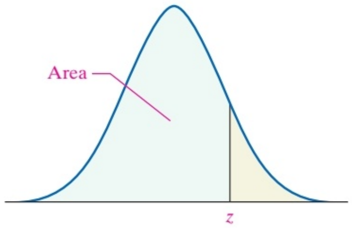

Table V (*continued*)

Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

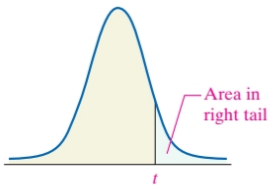

Table VII

***t*-Distribution**
Area in Right Tail

Degrees of Freedom	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.894	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
31	0.682	0.853	1.054	1.309	1.696	2.040	2.144	2.453	2.744	3.022	3.375	3.633
32	0.682	0.853	1.054	1.309	1.694	2.037	2.141	2.449	2.738	3.015	3.365	3.622
33	0.682	0.853	1.053	1.308	1.692	2.035	2.138	2.445	2.733	3.008	3.356	3.611
34	0.682	0.852	1.052	1.307	1.691	2.032	2.136	2.441	2.728	3.002	3.348	3.601
35	0.682	0.852	1.052	1.306	1.690	2.030	2.133	2.438	2.724	2.996	3.340	3.591
36	0.681	0.852	1.052	1.306	1.688	2.028	2.131	2.434	2.719	2.990	3.333	3.582
37	0.681	0.851	1.051	1.305	1.687	2.026	2.129	2.431	2.715	2.985	3.326	3.574
38	0.681	0.851	1.051	1.304	1.686	2.024	2.127	2.429	2.712	2.980	3.319	3.566
39	0.681	0.851	1.050	1.304	1.685	2.023	2.125	2.426	2.708	2.976	3.313	3.558
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
70	0.678	0.847	1.044	1.294	1.667	1.994	2.093	2.381	2.648	2.899	3.211	3.435
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
90	0.677	0.846	1.042	1.291	1.662	1.987	2.084	2.368	2.632	2.878	3.183	3.402
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291