Pre-class Work

Link to Google Doc:

https://docs.google.com/document/d/1LvUFPdh7bKzyjMAjzLQbMt1mwCC6YFu4cPEKxPU3l7A/edit?usp=sharing

Link to Github:

https://github.com/cesar-ca/cs166-modeling-and-analysis/tree/main/CS166%20Session%2021

In today's lesson, we explore the interaction between people's opinion and the strengths of their social connections. The basic idea is that social dynamics are driven by two factors

- People prefer forming social relationships with others who share their opinions or interests.

- People's opinions or interests tend to become similar to those of others in their social circle.

We model these two processes on a small-world network to see how both opinions and relationships change over time.

Think of people's opinions on controversial topics like politics or religion. We represent each person as a node in a network with an opinion attribute associated with it. The opinion attribute can take on values between 0 and 1. People with similar values for their opinion are in agreement on the topic and people with very different values disagree strongly. Here are two nodes with differing opinions (difference = 0.5).

node i
$o_i = 0.3$

node j
$o_j = 0.8$
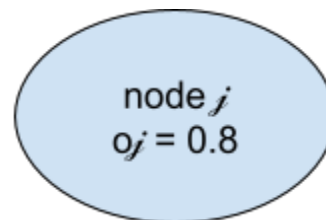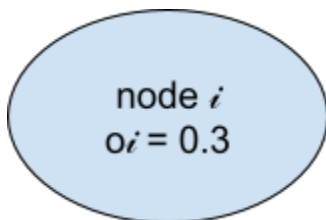
We represent the existence of a social relationship between two people as an edge between two nodes in the network and assign a weight to each edge, with values ranging from 0 to 1. A weight close to 0 means that the relationship between two people is weak and that those two people's opinions do not influence each other much. A weight close to 1 means that two people have a very strong relationship and will tend to adjust their opinions to be closer to each other. Below is a network with a strong relationship between two nodes.

$$w_{ij} = 0.9$$

node i
$o_i = 0.3$

node j
$o_j = 0.8$
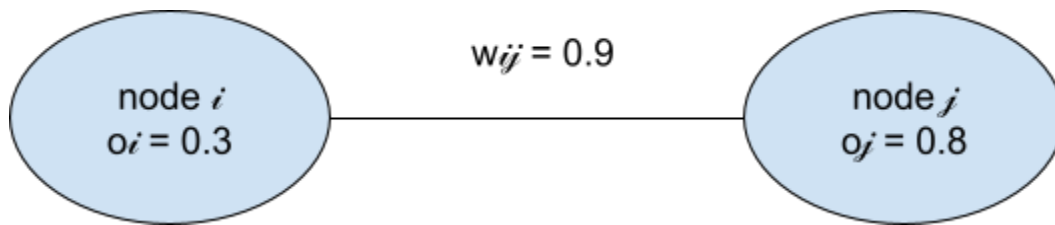
Update Rules

The network dynamics have 3 parts.

1. People change their opinions to more closely match the opinions of people with whom they have a strong relationship. To model this part, we select a random edge from the network and let the people connected by that edge interact. Think of this interaction as two people having a conversation about the topic on which everyone has an opinion. We update the opinion of each person to move a bit closer together. The stronger their relationship, the more they will move their opinions closer to each other.

    The change in opinion of Person i when talking to Person j is

    $$\Delta o_i = \alpha w_{ij} (o_j - o_i)$$

    Person j's opinion also changes, but in the opposite direction to Person i's, thus bringing their opinions closer together. In the equation above $\alpha \in (0, 0.5]$ is a

parameter of the model. The larger α is, the faster people change their opinions to match other people's. The closer α is to 0, the more stubborn (less likely to change their opinions) people are.

2. People strengthen or weaken their relationships depending on whether they agree or disagree, respectively. During the same interaction as in Step 1, the weight of the edge connecting nodes i and j is also changed. The change in weight is

$$\Delta w_{ij} = \beta \, w_{ij} \, (1 - w_{ij}) \, (1 - \gamma \, |o_i - o_j|)$$

Here $\beta \in (0, 1)$ and $\gamma > 0$ are parameters of the model. If $\gamma \leq 1$ then all weights will converge to 1 over time since differing opinions don't matter enough to decrease edge weights. If $\gamma > 1$, the weight between two nodes will decrease if the opinions of the nodes are different enough – if $|o_i - o_j| > \gamma^{-1}$.

As a final step when updating weights, we remove an edge from the network if its weight drops below 0.05. This step models a social relationship that has broken down.

3. Finally, we model new social connections between random people who are not yet connected. This is a relatively rare occurrence, so we do Steps 1 and 2 above 99% of the time and Step 3 only 1% of the time. Think of this process as randomly meeting someone new and forming a friendship with them. The edge weight is initially set to 0.5 in these cases. Over time the weight will increase towards 1 or decrease towards 0 depending on whether the two people have similar or differing opinions, using Steps 1 and 2.

We use the parameter values $\alpha = 0.03, \beta = 0.3, \gamma = 4$ here, but feel free to experiment with different parameter settings.

Example

To demonstrate how the rules work, let's update the opinions and edge weights of this pair of nodes:
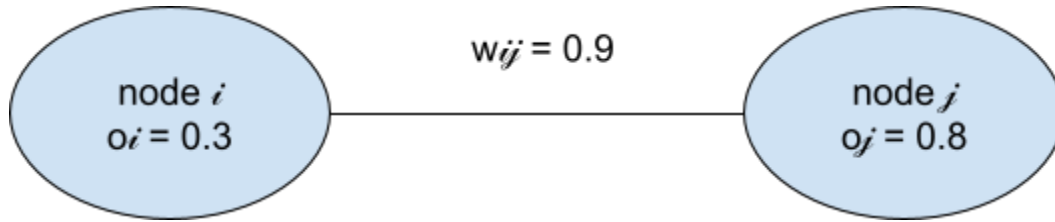
We update the opinions of the nodes. Note that $\Delta o_i =- \Delta o_j$. This is always the case.

- $\Delta o_i = \alpha\, w_{ij}(o_j - o_i) = 0.03 \times 0.9 \times (0.8 - 0.3) = 0.0135$, therefore
  $o_i' = o_i + \Delta o_i = 0.3135$
- $\Delta o_j = \alpha\, w_{ij}(o_i - o_j) = 0.03 \times 0.9 \times (0.3 - 0.8) =- 0.0135$, therefore
  $o_j' = o_j + \Delta o_j = 0.7865$

We then also update the weight of the edge.

- $\Delta w_{ij} = \beta\, w_{ij}(1 - w_{ij})(1 - \gamma\,|o_i - o_j|) = 0.3 \times 0.9 \times (1 - 0.9) \times (1 - 4\,|0.3 - 0.8|) =-$
  , therefore, $w_{ij}' = w_{ij} + \Delta w_{ij} = 0.873$

So these two people's opinions are now a little closer together, but the strength of their relationship is also slightly weaker than before.

Questions about the Model

We use the parameter values $\alpha = 0.03, \beta = 0.3, \gamma = 4$ here, but feel free to experiment with different parameter settings.

1. Use equation (2) to show that if $\gamma \leq 1$, all edge weights will eventually converge to a value of 1. Do this by showing that the change in weight, Δw_{ij}, is always positive and that w_{ij} will therefore always increase towards 1.

The equation,

$$\Delta w_{ij} = \beta\, w_{ij}\, (1 - w_{ij})\, (1 - \gamma\, |o_i - o_j|)$$

We know that a very large difference in opinion $o_i - o_j$ is at max 1, so for a $\gamma \leq 1$, the expression $\gamma\, |o_i - o_j| \leq 1$ because even with a very large difference, the result is less than or equal to 1.

The change in weight depends also on the values of β, and the weight of the edge which are non-negative values. So, the change in weight will be positive and the weight of the edge will increase towards 1.

2. What happens when two nodes with very different opinions interact? Assume a very large difference in opinion $(o_i - o_j \approx 1)$ and calculate the new difference in opinion and the new weight after an update.

When two nodes with very different opinions interact, we can calculate the new difference in opinion with the equation given and we can assume a strong relationship which means $w_{ij} \approx 1$

- $\Delta o_i = \alpha\, w_{ij}(o_j - o_i) = 0.03 \times 1 \times (-1) = -0.03$, therefore
  $o_i' = o_i - 0.03$
- $\Delta o_j = \alpha\, w_{ij}(o_i - o_j) = 0.03 \times 1 \times (1) = 0.03$, therefore $o_j' = o_j + 0.03$

$$o_i' - o_j' = o_i - 0.03 - o_j - 0.03 = o_i - o_j - 0.06 \approx 0.94$$

So, the difference becomes a bit closer together.

The new weight, we calculate with the equation
$\Delta w_{ij} = \beta\, w_{ij}\, (1 - w_{ij})\, (1 - \gamma\, |o_i - o_j|) = 0.3 \times w_{ij} \times (1 - w_{ij}) \times (-3) = -0.9\, w_{ij}\, (1 - w_{ij})$,
therefore, $w_{ij}' = w_{ij}(0.1 + 0.9 w_{ij})$

Depending on the weight, the original w_{ij}, the change will differ, for example for 0.1, 0.4, 0.6, 0.9

- $w_{ij}' = 0.1(0.1 + 0.9 * 0.1) = 0.019$
- $w_{ij}' = 0.4(0.1 + 0.9 * 0.4) = 0.184$

- $w_{ij}' = 0.6(0.1 + 0.9 * 0.6) = 0.384$
- $w_{ij}' = 0.9(0.1 + 0.9 * 0.9) = 0.819$

3. What happens when two nodes with very similar opinions interact?

When two nodes with very similar opinions interact, we can calculate the new difference in opinion with the equation given
- $o_i \approx o_j$ which means that their difference is equal to almost 0
- $\Delta o_i = \alpha w_{ij}(o_j - o_i) \approx 0$, therefore $o_i' = o_i$
- $\Delta o_j = \alpha w_{ij}(o_i - o_j) \approx 0$, therefore $o_j' = o_j$

So, the difference remains unchanged.

The new weight, we calculate with the equation
- $\Delta w_{ij} = \beta w_{ij}(1 - w_{ij})(1 - \gamma |o_i - o_j|) = \beta w_{ij}(1 - w_{ij}) = 0.3 w_{ij}(1 - w_{ij})$, therefore,
  $w_{ij}' = w_{ij}(1.3 - 0.3w_{ij})$

Depending on the weight, the original w_{ij}, the change will differ, for example for 0.1, 0.4, 0.6, 0.9
- $w_{ij}' = 0.1(1.3 - 0.3 * 0.1) = 0.127$
- $w_{ij}' = 0.4(1.3 - 0.3 * 0.4) = 0.472$
- $w_{ij}' = 0.6(1.3 - 0.3 * 0.6) = 0.672$
- $w_{ij}' = 0.9(1.3 - 0.3 * 0.9) = 0.927$

4. What happens when two nodes with somewhat different opinions (differing by 0.4) interact?

When two nodes with somewhat different opinions interact (0.4), we can calculate the new difference in opinion with the equation given and a strong connection
For $o_j - o_i = -0.4$ and $o_i - o_j = 0.4$
- $\Delta o_i = \alpha w_{ij}(o_j - o_i) = 0.03 * 1 * (-0.4) = -0.012$, therefore
  $o_i' = o_i - 0.012$

- $\Delta o_j = \alpha w_{ij}(o_i - o_j) = 0.03 * 1 * (0.4) = 0.012$, therefore
  $o_j' = o_j + 0.012$

$o_i' - o_j' = o_i - 0.012 - o_j - 0.012 = o_i - o_j - 0.024 = 0.4 - 0.024 \approx 0.376$

So, the difference becomes a bit closer together.

The new weight, we calculate with the equation
- $w_{ij}' = w_{ij}(1.3 - 0.3w_{ij})$

Depending on the weight, the original w_{ij}, the change will differ, for example for 0.1, 0.4, 0.6, 0.9
- $w_{ij}' = 0.1(1.3 - 0.3 * 0.1) = 0.127$
- $w_{ij}' = 0.6(1.3 - 0.3 * 0.6) = 0.672$
- $w_{ij}' = 0.9(1.3 - 0.3 * 0.9) = 0.927$