Pre-class work

- Read through the model described below and answer the questions in the *Questions about the model* section.
- Create a Google document and record your work and all exercises. **Make sure the Google document is shared** so that it can be assessed, and **be ready to paste a link to your document into a class poll**.

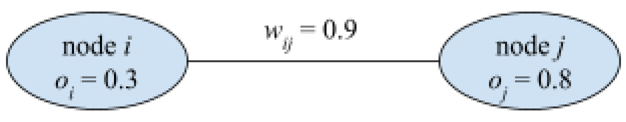
In today's lesson, we explore the interaction between people's opinions and the strengths of their social connections. The basic idea is that social dynamics are driven by two factors

- People prefer forming social relationships with others who share their opinions or interests.
- People's opinions or interests tend to become similar to those of others in their social circle.

We model these two processes on a small-world network to see how both opinions and relationships change over time.

Think of people's opinions on controversial topics like politics or religion. We represent each person as a node in a network with an *opinion* attribute associated with it. The opinion attribute can take on values between 0 and 1. People with similar values for their opinion are in agreement on the topic and people with very different values disagree strongly. Here are two nodes with differing opinions (difference = 0.5).

We represent the existence of a social relationship between two people as an edge between two nodes in the network and assign a weight to each edge, with values ranging from 0 to 1. A weight close to 0 means that the relationship between two people is weak and that those two people's opinions do not influence each other much. A weight close to 1 means that two people have a very strong relationship and will tend to adjust their opinions to be closer to each other. Below is a network with a strong relationship between two nodes.



Update rules

The network dynamics have 3 parts.

1. People change their opinions to more closely match the opinions of people with whom they have a strong relationship. To model this part, we select a random edge from the network and let the people connected by that edge interact. Think of this interaction as two people having a conversation about the topic on which everyone has an opinion. We update the opinion of each person to move a bit closer together. The stronger their relationship, the more they will move their opinions closer to each other.

   The change in opinion of Person *i* when talking to Person *j* is

   $$\Delta o_i = \alpha\, w_{ij}\, (o_j - o_i) \tag{1}$$

   Person *j*'s opinion also changes, but in the opposite direction to Person *i*'s, thus bringing their opinions closer together. In the equation above $\alpha \in (0, 0.5]$ is a parameter of the model. The larger α is, the faster people change their opinions to match other people's. The closer α is to 0, the more stubborn (less likely to change their opinions) people are.

2. People strengthen or weaken their relationships depending on whether they agree or disagree, respectively. During the same interaction as in Step 1, the weight of the edge connecting nodes *i* and *j* is also changed. The change in weight is

   $$\Delta w_{ij} = \beta\, w_{ij}\, (1 - w_{ij})\, (1 - \gamma\, |o_i - o_j|\,) \tag{2}$$

   Here $\beta \in (0, 1)$ and $\gamma > 0$ are parameters of the model. If $\gamma \le 1$ then all weights will converge to 1 over time since differing opinions don't matter enough to decrease edge weights. If $\gamma > 1$, the weight between two nodes will decrease if the opinions of the nodes are different enough – if $|o_i - o_j| > \gamma^{-1}$.

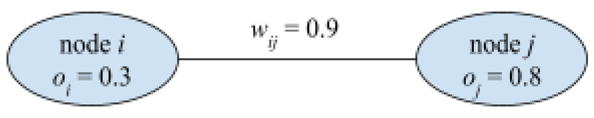
   As a final step when updating weights, we remove an edge from the network if its weight drops below 0.05. This step models a social relationship that has broken down.

3. Finally, we model new social connections between random people who are not yet connected. This is a relatively rare occurrence, so we do Steps 1 and 2 above 99% of the time and Step 3 only 1% of the time. Think of this process as randomly meeting someone new and forming a friendship with them. The edge weight is initially set to 0.5 in these cases. Over time the weight will increase towards 1 or decrease towards 0 depending on whether the two people have similar or differing opinions, using Steps 1 and 2.

We use the parameter values $\alpha = 0.03$, $\beta = 0.3$, $\gamma = 4$ here, but feel free to experiment with different parameter settings.

Example

To demonstrate how the rules work, let's update the opinions and edge weight of this pair of nodes:



We update the opinions of the nodes. Note that $\Delta o_i = -\Delta o_j$. This is always the case.

- $\Delta o_i = \alpha\, w_{ij}\, (o_j - o_i) = 0.03 \times 0.9\ \times (0.8 - 0.3) = 0.0135$, therefore $o_i' = o_i + \Delta o_i = 0.3135$
- $\Delta o_j = \alpha\, w_{ij}\, (o_i - o_j) = 0.03 \times 0.9\ \times (0.3 - 0.8) = -0.0135$, therefore $o_j' = o_j + \Delta o_j = 0.7865$

We then also update the weight of the edge.

- $\Delta w_{ij} = \beta\, w_{ij}\, (1 - w_{ij})\, (1 - \gamma\, |o_i - o_j|\,) = 0.3 \times 0.9 \times (1 - 0.9) \times (1 - 4\, |0.3 - 0.8|\,) = -0.027$, therefore, $w_{ij}' = w_{ij} + \Delta w_{ij} = 0.873$

So these two people's opinions are now a little closer together, but the strength of their relationship is also slightly weaker than before.

Questions about the model

- For your pre-class work, **you should answer at least the questions below**, exploring how opinions and edge weights change under the rules described above.
- **Afterward, you should then also attempt to implement this model in Python.** We explore the model implementation and interpretation of results further in class.

These questions are meant to guide you towards better understanding the model rules and their effects, by analyzing how simple interactions between nodes modify opinions and edge weights. This will help you understand the behavior of the model as a whole when simulated on a large network.

1. Use equation (2) to show that if $\gamma \le 1$, all edge weights will eventually converge to a value of 1. Do this by showing that the change in weight, Δw_{ij}, is always positive and that w_{ij} will therefore always increase towards 1.
2. (Optional) Determine an upper bound on γ for the model to still make sense. We need to avoid the possibility that edge weights can change too much – drop below 0 or grow above 1 in a step.
3. What happens when two nodes with very different opinions interact? Assume a very large difference in opinion $(o_i - o_j \approx 1)$ and calculate the new difference in opinion and the new weight after an update.
4. What happens when two nodes with very similar opinions interact?
5. What happens when two nodes with somewhat different opinions (differing by 0.4) interact?
6. (Optional) Draw a [vector field plot] or a [stream plot] showing how the edge weight and difference in opinion values change in a single 2-person interaction like the ones you explored above, but for all values of $w_{ij} \in [0, 1]$ and $|o_i - o_j| \in [0, 1]$. Use the `quiver()` function in Matplotlib for vector field plots and the `streamplot()` function for stream plots.