The trial wavefunction for the Helium atom is:

$$\Psi_T(\vec{r}_1, \vec{r}_2) = \exp\left[-2r_1 - 2r_2 + \frac{r_{12}}{2(1+\alpha r_{12})}\right], \tag{1}$$

where $\vec{r}_1$ and $\vec{r}_2$ denote the vectorial positions of the two electrons, $r_{12} = \|\vec{r}_1 - \vec{r}_2\|$, and α the variational parameter. The local energy is given by the expression:

$$E_L(\vec{r}_1, \vec{r}_2) = -4 + (\hat{r}_1 - \hat{r}_2)(\vec{r}_1 - \vec{r}_2)\frac{1}{r_{12}(1+\alpha r_{12})^2} - \frac{1}{r_{12}(1+\alpha r_{12})^3} - \frac{1}{4(1+\alpha r_{12})^4} + \frac{1}{r_{12}}, \tag{2}$$

where $\hat{r}_1$ and $\hat{r}_2$ are the normalised unit vectors in the respective directions of $\vec{r}_1$ and $\vec{r}_2$.

To update the variational parameter α, we need to compute:

$$\ln(\Psi_T) = -2r_1 - 2r_2 + \frac{r_{12}}{2(1+\alpha r_{12})} \implies \frac{d\ln(\Psi_T)}{d\alpha} = -\frac{r_{12}^2}{2(1+\alpha r_{12})^2}. \tag{3}$$

We then get that α is updated accordingly to the expression $\alpha_{new} = \alpha_{old} - \gamma \left(\frac{dE}{d\alpha}\right)_{old}$, where

$$\frac{dE}{d\alpha} = 2\left(\langle E_L \rangle \left\langle \frac{r_{12}^2}{2(1+\alpha r_{12})^2} \right\rangle - \left\langle E_L \cdot \frac{r_{12}^2}{2(1+\alpha r_{12})^2} \right\rangle\right). \tag{4}$$

Jos Thijssen found in his book the values shown in the table 1.

Figure 1: $\langle E \rangle$ and $\mathrm{Var}(\langle E \rangle)$ in terms of the variational parameter α.

Helium atom		
α	$\langle E \rangle$	var($\langle E \rangle$)
0.05	−2.8713(4)	0.1749(2)
0.075	−2.8753(4)	0.1531(2)
0.10	−2.8770(3)	0.1360(2)
0.125	−2.8780(4)	0.1223(2)
0.15	−2.8778(3)	0.1114(2)
0.175	−2.8781(3)	0.1028(2)
0.20	−2.8767(4)	0.0968(2)
0.25	−2.8746(10)	0.0883(2)