Equilibration of barotropic turbulence by stimulated generation of near-inertial waves

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1 The model

The balanced dynamics satisfies

$$q_t + J(\psi, q) = \xi_q - \mu\zeta + \mathcal{D}_q \,, \tag{1}$$

`balanced_dy`

where the quasigeostrophic potential vorticity is

$$q = \underbrace{\triangle\psi}_{\stackrel{\text{def}}{=}\zeta} + \underbrace{\tfrac{1}{f_0}\left[\tfrac{1}{4}\triangle|\phi|^2 + \tfrac{\mathrm{i}}{2}J(\phi^\star, \phi)\right]}_{\stackrel{\text{def}}{=}q^w} . \tag{2}$$

`qgpv`

The back-rotated near-inertial velocity ϕ satisfies the YBJ equation

$$\phi_t + J(\psi, \phi) + \phi\tfrac{\mathrm{i}}{2}\zeta - \tfrac{\mathrm{i}}{2}\eta\triangle\phi = \xi_\phi - \gamma\phi + \mathcal{D}_\phi \,. \tag{3}$$

`ybj_dynami`

In (1), μ is the linear drag, ξ_q is a white-noise forcing with random horizontal structure with a ring-like isotropic spectrum peaking at $|\mathbf{k}_f|$ and variance σ_q^2. In (3), ξ_ϕ is a white-noise spatially uniform forcing with variance σ_ϕ^2, and γ is a linear damping. In (1) and (3), the $\mathcal{D}$ terms represent small-scale dissipation.

1.1 Power integrals

The balanced kinetic energy equation is

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\tfrac{1}{2}\langle|\nabla\psi|^2\rangle}_{\stackrel{\text{def}}{=}\mathcal{K}} = -\langle\Gamma_r + \Gamma_a\rangle + \Xi + -\langle\psi\xi_q\rangle - \;\mu\langle|\nabla\psi|^2\rangle - \langle\psi\mathcal{D}_q\rangle \,, \tag{4}$$

`Ke`

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where Γ_r and Γ_a are energy conversion terms and Ξ is a source of balanced kinetic energy due to wave dissipation (cf. RWY); $\langle\rangle$ represents spatial average. If the were no waves, i.e., $\phi(t=0)=0$ and $\sigma_\phi^2=0$, then $\Gamma_r=\Gamma_a=0$, and the expectation of the work due to the white-noise forcing is $-\langle\psi\xi_q\rangle=\sigma_q^2$. Ignoring small-scale dissipation, the expectation of equilibrated energy level is $\mathcal{K}=\sigma_q^2/2\mu$.

The wave action equation is

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\tfrac{1}{2f_0}\langle|\phi|^2\rangle}_{\stackrel{\mathrm{def}}{=}\mathcal{A}}=\tfrac{1}{2f_0}\langle\phi^\star\xi_\phi+\phi\xi_\phi^\star\rangle-\tfrac{1}{f_0}\gamma\langle|\phi|^2\rangle+\tfrac{1}{2f_0}\langle\phi^\star\mathcal{D}_\phi+\phi\mathcal{D}_\phi^\star\rangle\,. \tag{5}$$ $\boxed{\mathtt{A}}$

The expectation of the work due to the white-noise forcing is $\frac{1}{2}\langle\phi^\star\xi_\phi+\phi\xi_\phi^\star\rangle=\sigma_\phi^2$, and the expectation of the equilibrated action is $\mathcal{A}=\sigma_\phi^2/2f_0\gamma$.

The potential energy equation is

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\tfrac{\lambda^2}{4}\langle|\nabla\phi|^2\rangle}_{\stackrel{\mathrm{def}}{=}\mathcal{P}}=\Gamma_r+\Gamma_a-\tfrac{\lambda^2}{2}\gamma\langle|\nabla\phi|^2\rangle-\tfrac{\lambda^2}{2}\langle\triangle\phi^\star\mathcal{D}_\phi+\triangle\phi\mathcal{D}_\phi^\star\rangle\,. \tag{6}$$ $\boxed{\mathtt{P}}$

2 A reference solution

Figure 1 shows snapshots of potential vorticity and wave action density for a solution with $\sigma_w^2=2\sigma_q^2$ and $\gamma=4\mu$; table 1 describes in detail the parameters of this reference solution. The equilibrated potential vorticity in figure 1a resembles the vorticity field of waveless two-dimensional turbulence with its ubiquitous eddies, filaments, and coherent structures. A main difference is that the the potential vorticity of this wave-modified turbulence is more fine-grained (see spectrum?).

Table 1: Details of the reference solution.

parameters_reference

Parameter	Description	Value
N	Number of modes	512
L_d	Domain size	$2\pi \times 200$ km
σ_q^2	Balanced-forcing variance	$1.45\ 10^{-8}$ m^2 s^{-3}
σ_w^2	Wave-forcing variance	$5.78\ 10^{-8}$ m^2 s^{-3}
$k_f L_d/2\pi$	Balanced-forcing wavenumber	8
$dk_f L_d/2\pi$	Balanced-forcing width	1
μ	Linear bottom drag coefficient	$5.78\ 10^{-8}$ s^{-1}
γ	Linear wave damping coefficient	$2.31\ 10^{-7}$ s^{-1}
N	Buoyancy frequency	$5\ 10^{-3}$ s^{-1}
f_0	Coriolis frequency	$1\ 10^{-4}$ s^{-1}
$\mathcal{D}_\phi$	Exponential spectral filter	—
$\mathcal{D}_q$	Exponential spectral filter	—

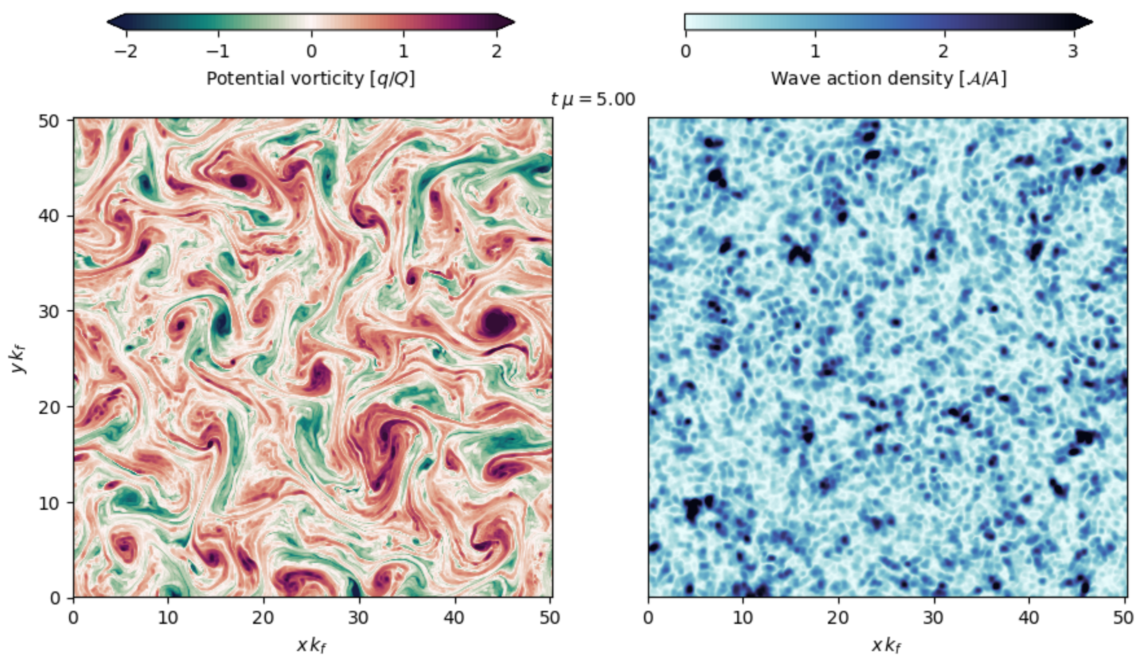

Figure 1: Snapshot of potential vorticity and wave action density from the solution with parameters in table 1. The scale of potential vorticity is $Q = \sigma_q/\mu^{1/2}$ and the scale of wave action density is $A = \sigma_w^2/f_0\gamma$.

snapshots_

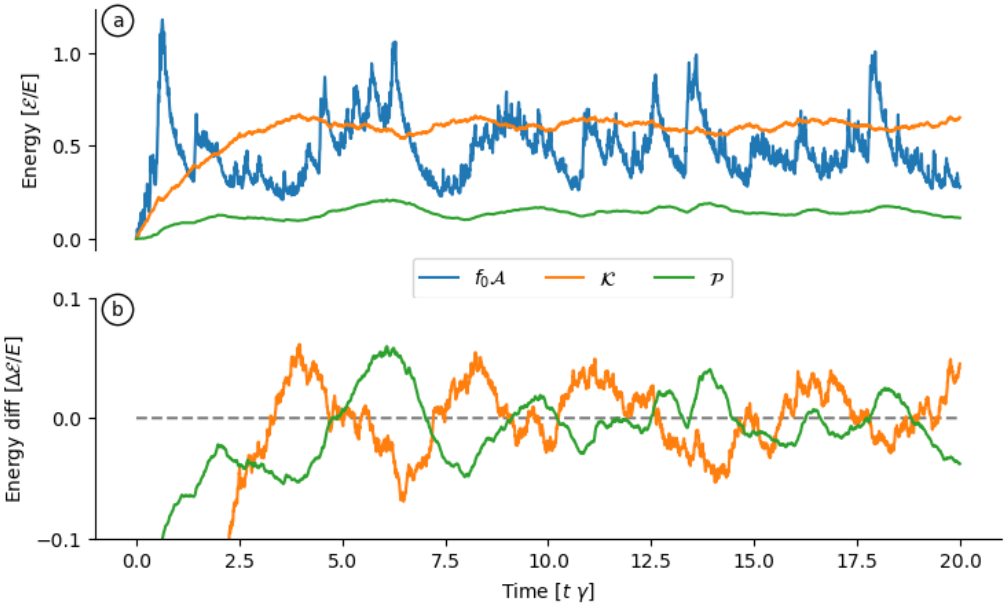

Figure 2: (a) Balanced kinetic energy ($\mathcal{K}$) and wave potential energy ($\mathcal{P}$) and wave kinetic energy ($f_0\mathcal{A}$) from the solution with parameters in table 1. The energy difference, $\Delta\mathcal{K}$ and $\Delta\mathcal{P}$, about a time average after equilibration ($t\,\gamma \geq 5$). energies_r