Equilibration of barotropic turbulence by stimulated generation of near-inertial waves

CR & WRY *

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1 The model

The balanced dynamics satisfies

$$q_t + J(\psi, q) = \xi_q - \mu\zeta + \mathcal{D}_q\,, \tag{1}$$

`balanced_dy`

where the quasigeostrophic potential vorticity is

$$q = \underbrace{\triangle\psi}_{\stackrel{\text{def}}{=}\zeta} + \underbrace{\tfrac{1}{f_0}\left[\tfrac{1}{4}\triangle|\phi|^2 + \tfrac{\mathrm{i}}{2}J(\phi^\star, \phi)\right]}_{\stackrel{\text{def}}{=}q^w}\,. \tag{2}$$

`qgpv`

The back-rotated near-inertial velocity ϕ satisfies the YBJ equation

$$\phi_t + J(\psi, \phi) + \phi\tfrac{\mathrm{i}}{2}\zeta - \tfrac{\mathrm{i}}{2}\eta\triangle\phi = \xi_\phi - \gamma\phi + \mathcal{D}_\phi\,. \tag{3}$$

`ybj_dynami`

In (1), μ is the linear drag, ξ_q is a white-noise forcing with random horizontal structure with a ring-like isotropic spectrum peaking at $|\mathbf{k}_f|$ and variance σ_q^2. In (3), ξ_ϕ is a white-noise spatially uniform forcing with variance σ_ϕ^2, and γ is a linear damping. In (1) and (3), the $\mathcal{D}$ terms represent small-scale dissipation.

1.1 Power integrals

The balanced kinetic energy equation is

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\tfrac{1}{2}\langle|\nabla\psi|^2\rangle}_{\stackrel{\text{def}}{=}\mathcal{K}} = -\langle\Gamma_r + \Gamma_a\rangle + \Xi + -\langle\psi\xi_q\rangle - \ \mu\langle|\nabla\psi|^2\rangle - \langle\psi\mathcal{D}_q\rangle\,, \tag{4}$$

`Ke`

*Scripps Institution of Oceanography, University of California at San Diego, La Jolla, CA 92093–0230, USA.

Table 1: Details of the reference solution.

parameters_reference

Parameter	Description	Value
N	Number of modes	512
L_d	Domain size	$2\pi \times 200$ km
σ_q^2	Balanced-forcing variance	$1.45\ 10^{-8}\ \mathrm{m}^2\ \mathrm{s}^{-3}$
σ_w^2	Wave-forcing variance	$5.78\ 10^{-8}\ \mathrm{m}^2\ \mathrm{s}^{-3}$
$k_f L_d/2\pi$	Balanced-forcing wavenumber	8
$dk_f L_d/2\pi$	Balanced-forcing width	1
μ	Linear bottom drag coefficient	$5.78\ 10^{-8}\ \mathrm{s}^{-1}$
γ	Linear wave damping coefficient	$2.31\ 10^{-7}\ \mathrm{s}^{-1}$
N	Buoyancy frequency	$5\ 10^{-3}\ \mathrm{s}^{-1}$
f_0	Coriolis frequency	$1\ 10^{-4}\ \mathrm{s}^{-1}$
$\mathcal{D}_\phi$	Exponential spectral filter	—
$\mathcal{D}_q$	Exponential spectral filter	—

where Γ_r and Γ_a are energy conversion terms and Ξ is a source of balanced kinetic energy due to wave dissipation (cf. RWY); $\langle\rangle$ represents spatial average. If the were no waves, i.e., $\phi(t=0)=0$ and $\sigma_\phi^2=0$, then $\Gamma_r=\Gamma_a=0$, and the expectation of the work due to the white-noise forcing is $-\langle\psi\xi_q\rangle=\sigma_q^2$. Ignoring small-scale dissipation, the expectation of equilibrated energy level is $\mathcal{K}=\sigma_q^2/2\mu$.

The wave action equation is

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\tfrac{1}{2f_0}\langle|\phi|^2\rangle}_{\stackrel{\mathrm{def}}{=}\mathcal{A}} = \tfrac{1}{2f_0}\langle\phi^\star\xi_\phi+\phi\xi_\phi^\star\rangle - \tfrac{1}{f_0}\gamma\langle|\phi|^2\rangle + \tfrac{1}{2f_0}\langle\phi^\star\mathcal{D}_\phi+\phi\mathcal{D}_\phi^\star\rangle\,. \tag{5}$$ A

The expectation of the work due to the white-noise forcing is $\frac{1}{2}\langle\phi^\star\xi_\phi+\phi\xi_\phi^\star\rangle=\sigma_\phi^2$, and the expectation of the equilibrated action is $\mathcal{A}=\sigma_\phi^2/2f_0\gamma$.

The potential energy equation is

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\tfrac{\lambda^2}{4}\langle|\nabla\phi|^2\rangle}_{\stackrel{\mathrm{def}}{=}\mathcal{P}} = \Gamma_r+\Gamma_a-\tfrac{\lambda^2}{2}\gamma\langle|\nabla\phi|^2\rangle-\tfrac{\lambda^2}{2}\langle\triangle\phi^\star\mathcal{D}_\phi+\triangle\phi\mathcal{D}_\phi^\star\rangle\,. \tag{6}$$ P

2 A reference solution

Figure 1 shows snapshots of potential vorticity and wave action density for a solution with $\sigma_w^2=2\sigma_q^2$ and $\gamma=4\mu$; table 1 describes in detail the parameters of this reference solution. The equilibrated potential vorticity in figure 1a resembles the vorticity field of waveless two-dimensional turbulence with its ubiquitous eddies, filaments, and coherent structures. A main difference is that the the potential vorticity of this wave-modified turbulence is more fine-grained (see spectrum?).

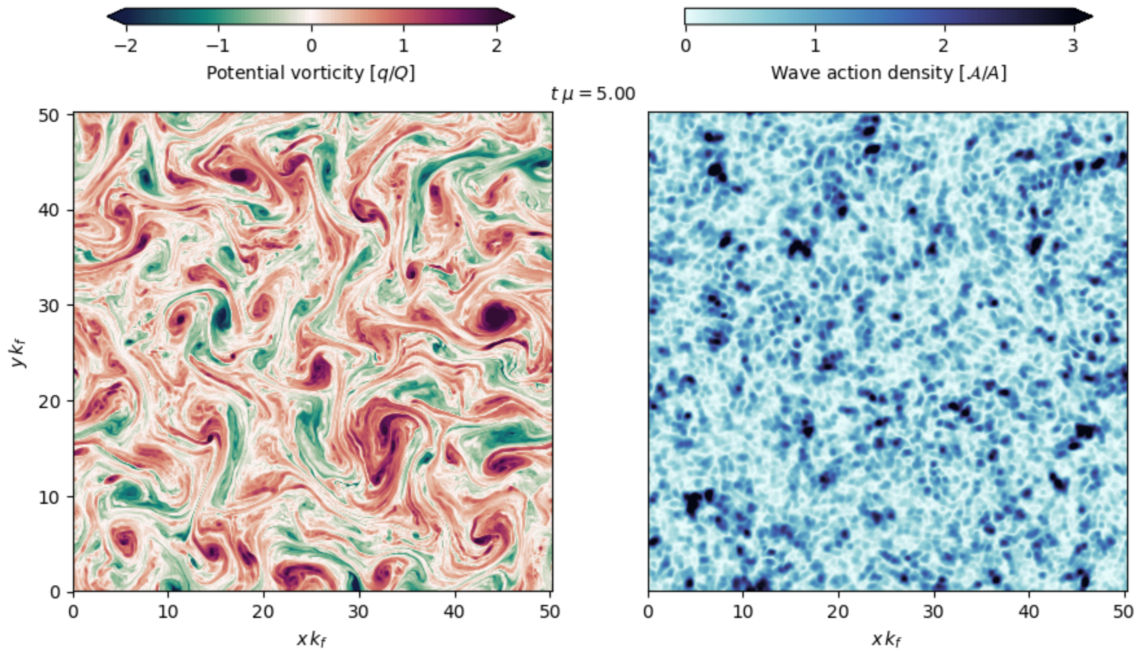

Figure 1: Snapshot of potential vorticity and wave action density for the solution with parameters in table 1. The scale of potential vorticity is $Q = \sigma_q/\mu^{1/2}$ and the scale of wave action density is $A = \sigma_w^2/f_0\gamma$. snapshots_

The snapshot of wave action density depicts the incoherent nature of the equilibrated wave field, which is being scrambled by the turbulent balanced field (figure 1b). The wave field develops scales smaller than the balanced eddies due to straining by the flow and wave interference. This snapshot resembles the wave field in decaying wave-modified two-dimensional turbulence (RWY).

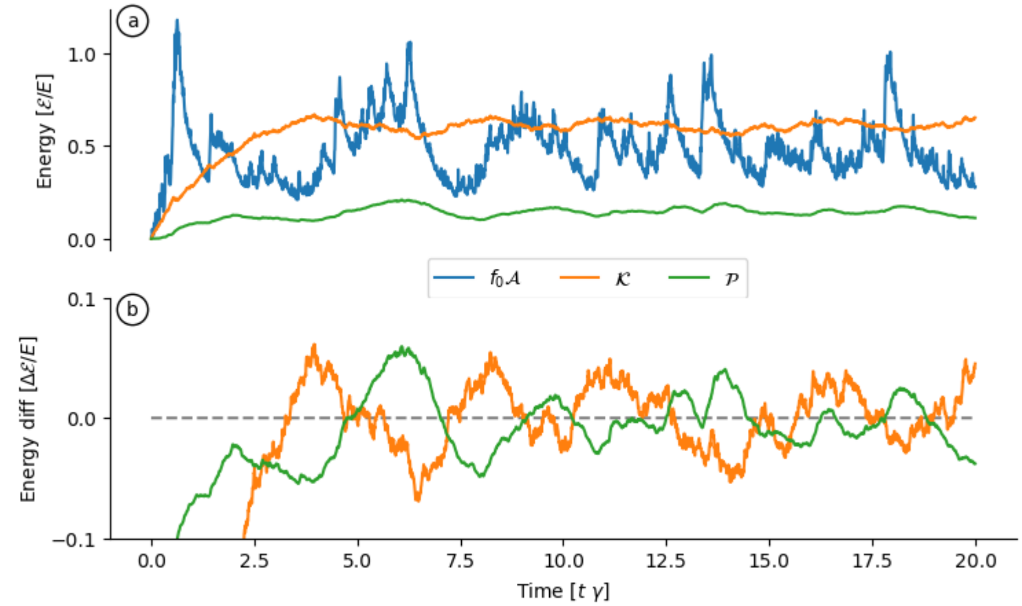

Figure 2: (a) Balanced kinetic energy ($\mathcal{K}$) and wave potential energy ($\mathcal{P}$) and wave kinetic energy ($f_0\mathcal{A}$) for the solution with parameters in table 1. The energy difference, $\Delta\mathcal{K}$ and $\Delta\mathcal{P}$, about a time average after equilibration ($t\,\gamma \geq 5$).

energies_r

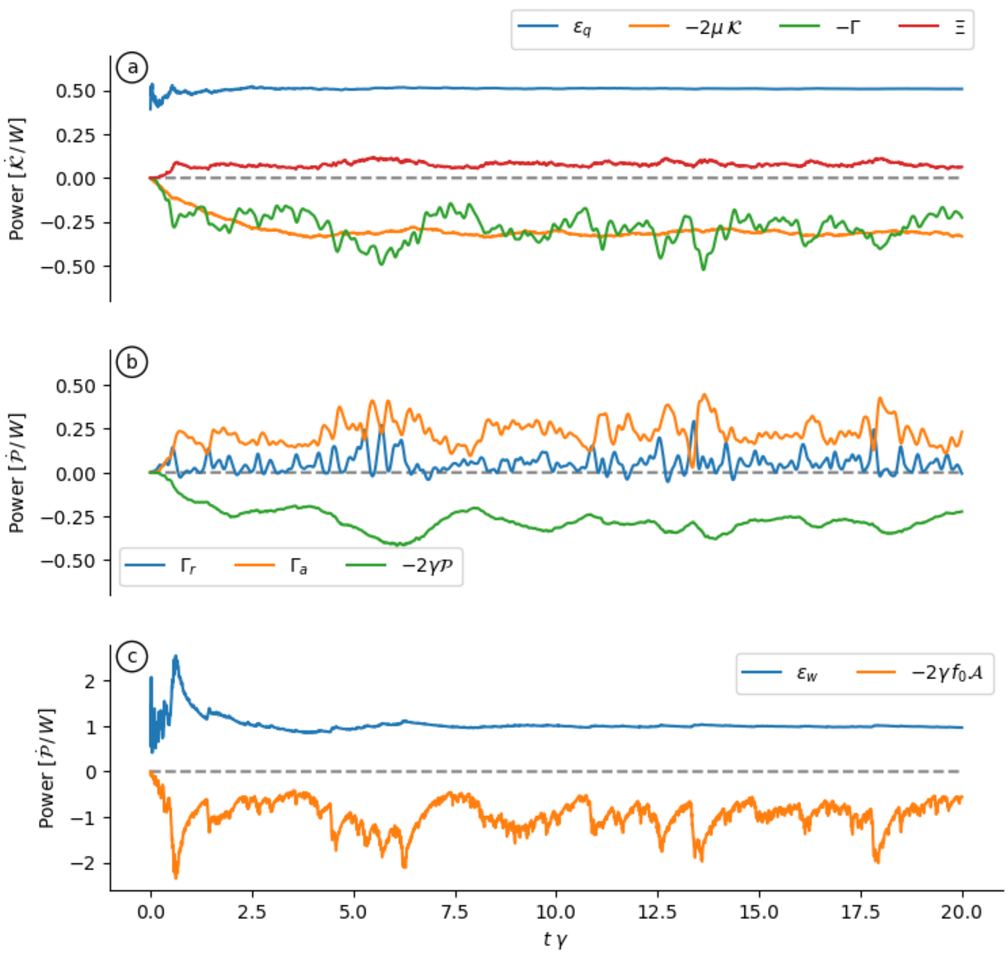

Figure 3: The budget of (a) balanced kinetic energy ($\mathcal{K}$), wave potential energy ($\mathcal{P}$), and (c) wave kinetic energy ($f_0\mathcal{A}$) for the solution with parameters in table 1. The power is scaled by the work due to the wave forcing $W = \sigma_w^2/2$.

energies_r