Basics of Signals and Systems

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Random Variables and Random Processes

Some basics of probability theory are discussed before going to random variables.

Basics of Probability Theory

Probability of an event A represented by $P(A)$ is given by

$$P(A) = \frac{N_A}{N_S}$$

where, N_S is the number of times the experiment is carried and N_A is number of times the event A occured. Probability of any event can be exactly caluculated only when the number of experiments is huge ideally infinity. Hence, we generally go for relative probability which is given above. Clearly, $0 \leq P(A) \leq 1$.

Let $'S'$ be the sample space having N events $A_1, A_2, A_3, \cdots A_N$. Two events are said to be mutually exclusive or statistically independent if $A_i \cap A_j = \phi$ and $\bigcup_{i=1}^{N} A_i = S$ for all i and j.

Joint Probability:

Joint probability of two events A and B which is defined as the probability of the occurence of both the events A and B is given by

$$P(A \cap B) = \frac{N_{A \cap B}}{N_S}$$

Conditional Probability

Conditional probability of two events A and B represented as $P(A/B)$ and defined as the probability of the occurence of event A after the occurence of B is given by

$$P(A/B) = P(A \cap B)/P(B)$$

$$\text{P(B/A) = P(A } \cap B)/P(A)$$

$$\implies P(A/B).P(B) = P(B/A).P(A) = P(A \cap B)$$

Chain rule

Let us consider a chain of events $A_1, A_2, A_3, \cdots A_N$ which are dependent on each other. Then the probability of occurence of the sequence

$$P(A_N, A_{N-1}, A_{N-2}, \cdots A_1) = P(A_N/A_{N-1}, A_{N-2}, \cdots A_1).P(A_{N-1}/A_{N-2}, A_{N-3}, \cdots A_1) \cdots P(A_2/A_1).P$$

Bayes Rule

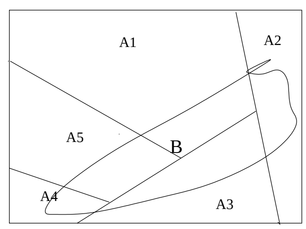

Figure 1: Partition space

In the above figure, if $A_1, A_2, A_3, \cdots A_5$ partition the sample space S, then $A_1 \cap B, A_2 \cap B, A_3 \cap B, A_4 \cap B, and A_5 \cap B$ partition B.

therefore,

$$\begin{aligned} P(B) &= \sum_{i=1}^{n} P(A_i \cap B) \\ &= \sum_{i=1}^{n} P(B/A_i).P(A_i) \end{aligned}$$

$$\begin{aligned} P(A_i/B) &= P(A_i \cap B)/P(B) \\ &= \frac{P(B/A_i).P(A_i)}{\sum_{i=1}^{n} P(B/A_i).P(A_i)} \end{aligned}$$

In the above equation, $P(A_i/B)$ is called posterior probability, $P(B/A_i)$ is called likelihood, $P(A_i)$ is called prior probability and $\sum_{i=1}^{n} P(B/A_i).P(A_i)$ is called evidence.

Random Variable

Random variable is a function whose domain is sample space and whose range is the set of real numbers.

Probabilistic description of a Random Variable

Cummulative Probability Distribution: It is represented as $F_X(x)$ and defined as

$$F_X(x) = P(X \leq x)$$

If $x_1 < x_2$ then $F_X(x_1) < F_X(x_2)$ and $0 \leq F_X(x) \leq 1$.

Probability Density Function: It is represented as $f_X(x)$ and defined as

$$f_X(x) = \frac{d}{dx} F_X(x)$$

$$\implies P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f_X(x)\, dx$$