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HW 5

5.1

1. a. Write pseudocode for a divide-and-conquer algorithm for finding the position of the largest element in an array of n numbers.
   - Divide the array into 2 and find the largest element in both sub arrays and return the index of the highest element out of the two.

   b. What will be your algorithm's output for arrays with several elements of the largest value?

   - This algorithm will return the index of the leftmost largest element

   c. Set up and solve a recurrence relation for the number of key comparison made by your algorithm

   - C(n) = C([n/2]) + C([n/2]) + 1 for n > 1, C (1) = 0

   d. How does this algorithm compare with the brute-force algorithm for this problem?

   - Scanning through the array requires the same number of key comparison but avoids the overhead associated with recursive calls.

3. a. Write pseudocode for divide-and-conquer algorithm for the exponentiation problem of computing a^n where n is a positive integer.

   - Compute a^(n/2) * a^(n/2), meaning divide the set into two and multiply them both together

   b. Set up and solve a recurrence relation for the number of multiplications made by this algorithm

   - The recurrence for the number of multiplications is

   M(n) = M([n/2]) + M([n/2]) + 1 for n > 1, M(1) = 0

   c. How does this algorithm compare with the brute-force algorithm for this problem?

   - The algorithm makes the same number of multiplications as the brute-force method, but uses recursion which makes it better

6. Apply mergesort to sort the list E, X, A, M, P, L, E in alphabetical order

   - E X A M P L E

   E X A   M P L E

E   X A   M P   L E

E X A M P L E

E A X M P E L

A E X E L M P

A E E L M P X

5.2

1. Apply quicksort to sort the list E, X, A, M, P, L, E in alphabetical order. Draw the tree of the recursive calls made

- E X A M P L E

E E A M P L X

A E E M P L X

…

A E E L M P X

6 a. For quicksort with the median-of three pivot selections, are strictly increasing arrays the worst-case input, the best-case input, or neither?

b. Answer the same question for strictly decreasing arrays.

- The best case for both questions. For either an increasing or decreasing subarray, the median of the first, last, and middle values will be the median of the entire subarray. Using it as a pivot will split the subarray in the middle. This will cause the total number of key comparisons be the smallest.

7. a. Estimate how many times faster quicksort will sort an array of one million random numbers than insertion sort.

- log(n) times faster

5.3

1. Design a divide-and-conquer algorithm for computing the number of levels in a binary tree. (In particular, the algorithm must return 0 and 1 for the empty and single-node trees, respectively.) What is the time efficiency class of your algorithm?

```
height(tree: BinaryTree) {
    if (tree==null) {
        return 0;
    }
```

```
        else {

                return max(height(tree.left), height(tree.right)) + 1;

        }

}
```

The time efficiency of this algorithm is O(n)

2 . The following algorithm seeks to compute the number of leaves in a binary tree.
ALGORITHM LeafCounter(T) //Computes recursively the number of leaves in a binary tree //Input: A binary tree T //Output: The number of leaves in T if T = ∅ return 0 else return LeafCounter(Tlef t)+ LeafCounter(Tr ight)
Is this algorithm correct? If it is, prove it; if it is not, make an appropriate correction.

Answer -> it is not correct because it does not count the head of the tree itself all that needs to be done is add +1 after LeaftCounter(Tright).

5 Traverse the following binary tree
a. in preorder.
Answer -> a,b,d,e,c,f
b. in inorder.
Answer -> d,b,e,a,c,f
c. in postorder.
Answer -> d,e,b,f,c,a

7 Which of the three classic traversal algorithms yields a sorted list if applied to a binary search tree? Prove this property.
Answer -> inorder

8. a. Draw a binary tree with 10 nodes labeled 0, 1,..., 9 in such a way that the inorder and postorder traversals of the tree yield the following lists: 9, 3, 1, 0, 4, 2, 7, 6, 8, 5 (inorder) and 9, 1, 4, 0, 3, 6, 7, 5, 8, 2 (postorder)
Answer ->

Exercise 5.4

1. What are the smallest and largest numbers of digits the product of two decimal n-digit integers can have?
Answer-> 1 being the smallest and n/2 being the largest

2. Compute 2101 ∗ 1130 by applying the divide-and-conquer algorithm outlined in the text.
Answer-> 2101 = (2 * 10^3 + 1 * 10^2 + 1 * 10^0)

1103 = (1 * 10^3 + 1 * 10^2 + 3 * 10^0)

2101 * 1103 = (2 * 10^3 + 1 * 10^2 + 1 * 10^0) * (1 * 10^3 + 1 * 10^2 + 3 * 10^0)

= (2 * 1) 10^6 + (2 *3 +1 +1) 10 ^4 + (1 * 3)10^0

= 2317403

5. . How many one-digit additions are made by the pen-and-pencil algorithm in multiplying two n-digit integers? You may disregard potential carries.

Answer -> Four