Chapter 3 Homework

3.1 From $S \dashrightarrow abSc \mid A$

$A \dashrightarrow cAd \mid cd$, derive string *ababccddcc*

Derivation	Rule
$S \Rightarrow abSc$	$S \rightarrow abSc$
$\Rightarrow ababScc$	$S \rightarrow abSc$
$\Rightarrow ababAcc$	$S \rightarrow A$
$\Rightarrow ababcAdcc$	$A \rightarrow cAd$
$\Rightarrow ababccddcc$	$A \rightarrow cd$

$L(G) = \{(ab)^m c^n d^n c^m \mid m, n, > 0\}$

3.2 From $S \dashrightarrow ASB \mid \lambda$

$A \dashrightarrow aAb \mid \lambda$

$B \dashrightarrow bBa \mid ba$, *leftmost* derive string ***aabbba***

Derivation	Rule
$S \Rightarrow ASB$	$S \rightarrow ASB$
$\Rightarrow aAbSB$	$A \rightarrow aAb$
$\Rightarrow aaAbbSB$	$A \rightarrow aAb$
$\Rightarrow aabbSB$	$A \rightarrow \lambda$
$\Rightarrow aabbB$	$S \rightarrow \lambda$
$\Rightarrow aabbba$	$B \rightarrow ba$

rightmost derive ***abaabbbabbaa***

Derivation	Rule
$S \Rightarrow ASB$	$S \rightarrow ASB$
$\Rightarrow ASbBa$	$B \dashrightarrow bBa$
$\Rightarrow ASbbaa$	$B \dashrightarrow ba$
$\Rightarrow AASBbbaa$	$S \rightarrow ASB$
$\Rightarrow AASbabbaa$	$B \dashrightarrow ba$
$\Rightarrow AAbabbaa$	$S \rightarrow \lambda$
$\Rightarrow AaaAbbbabbaa$	$A \dashrightarrow aAb$
$\Rightarrow Aaabbbabbaa$	$A \dashrightarrow \lambda$
$\Rightarrow aAbaabbbabbaa$	$A \dashrightarrow aAb$
$\Rightarrow abaabbbabbaa$	$A \rightarrow \lambda$

hint: $S \Rightarrow ASB \Rightarrow AASBB \Rightarrow AAASBBB$

$L(G) = \{(ab)^{m_i}(ba)^{m_j} \mid 0 \leq i, j \leq n, m_j \geq 0, m_j \geq 1\}$

3.4 a,b) From $S \dashrightarrow AB \mid \lambda$

$A \dashrightarrow aA \mid a$

$B \dashrightarrow AB \mid B \mid b \mid \lambda$, *leftmost/rightmost* derive string ***aaab***

Derivation	Rule
$S \Rightarrow AB$	$S \rightarrow AB$
$\Rightarrow aAB$	$A \dashrightarrow aA$
$\Rightarrow aaB$	$B \dashrightarrow AB$
$\Rightarrow aaAB$	$A \rightarrow a$
$\Rightarrow aaaB$	$B \dashrightarrow b$
$\Rightarrow aaaa$	$S \rightarrow \lambda$

Derivation	*Rule*
$S \Rightarrow AB$	$S \rightarrow AB$
$\Rightarrow AAB$	$B \dashrightarrow AB$
$\Rightarrow AAb$	$B \dashrightarrow b$
$\Rightarrow Aab$	$A \rightarrow a$
$\Rightarrow Aaab$	$A \dashrightarrow Aa$
$\Rightarrow aaab$	$A \rightarrow a$

3.6 a.) $S \rightarrow aaSB \mid \lambda$

$B \rightarrow abB \mid \lambda$

hint: $S \Rightarrow aaSB \Rightarrow aaaaSBB$

$L(G) = \{(aa)^m b^{m+n} \mid m, n \geq 0\}$

b.) $S \rightarrow aSbbb \mid A$

$A \rightarrow cA \mid c$

hint: $S \Rightarrow aSbbb \Rightarrow aaSbbbbbb \Rightarrow aacASbbbbbb$

$L(G) = \{(a)^m c^n (bb)^m \mid m, n \geq 1\}$

c.) $S \rightarrow abSdc \mid A$

$A \rightarrow cdAba \mid \lambda$

hint: $S \Rightarrow abSdc \Rightarrow ababSdcdc \Rightarrow ababAdcdc$

$\Rightarrow ababcdAbadcdc$

$L(G) = \{(ab)^m (cd)^n (ba)^n (dc)^m \mid m, n \geq 1\}$

3.7 $L(G) = \{a^m b^{2m} c^n \mid m, n \geq 0\}$

hint: $S \Rightarrow Sc \Rightarrow Scc \Rightarrow Sccc \Rightarrow Accc \Rightarrow aAbbccc$

$S \rightarrow Sc \mid A \mid \lambda$

$A \rightarrow aAbb \mid \lambda$

3.14 a.) $S \rightarrow aA \mid \lambda$

$A \rightarrow aA \mid bA \mid b$

hint: begins with ***a*** from ***S*** then $A \rightarrow aA \mid bA \mid b$ *alternates* ***a's*** *and* ***b's*** *then terminates with* ***b***

$L(G) = (a \cup b)^* b$

b.) $S \rightarrow aA$

$A \rightarrow aA \mid bB$

$B \rightarrow bB \mid \lambda$

hint: $S \Rightarrow aA \Rightarrow aaA \Rightarrow aabB \Rightarrow aabbB \Rightarrow aabbb$

$L(G) = a^+ b^+$

3.15 $S \rightarrow aA \mid \lambda$
$A \rightarrow aA \mid B \mid \lambda$
$B \rightarrow bB \mid C \mid \lambda$
$C \rightarrow cC \mid \lambda$
$L(G) = a^*b^*c^*$

3.16 $(a \cup b)^*aa(a \cup b)^*bb(a \cup b)^* \cup (a \cup b)^*bb(a \cup b)^*aa(a \cup b)^*$

$G_1 = \{V_1, \sum_1, P_1, S_1\}$
$S_1 \rightarrow aS_1 \mid bS_1 \mid aA$
$A \rightarrow aB$
$B \rightarrow aB \mid bB \mid bC$
$C \rightarrow bD$
$C \rightarrow aD \mid bD \mid \lambda$
$L(G_1) = (a \cup b)^*aa(a \cup b)^*bb(a \cup b)^*$

$G_2 = \{V_2, \sum_2, P_2, S_2\}$
$S_2 \rightarrow aS_2 \mid bS_2 \mid bE$
$E \rightarrow aF$
$F \rightarrow aF \mid bF \mid aD$
$D \rightarrow aE$
$E \rightarrow aE \mid bE \mid \lambda$
$L(G_2) = (a \cup b)^*bb(a \cup b)^*aa(a \cup b)^*$

$L(G) = \{V_1 \cup V_2 \cup S, \sum_1, P_1 \cup P_1 \cup S \rightarrow S_1 \mid S_2, S\}$

$S \rightarrow S_1 \mid S_2$
$L(G) = L(G_1) \cup L(G_2)$

3.20 Strings begin with ***a*** or any number of **b's;** thereafter, any number of ***b's*** are separated by one or two ***a's***

$S \rightarrow aA \mid bB \mid \lambda$
$A \rightarrow bB$
$B \rightarrow$ aS | bS | $bB \mid \lambda$
$L(G) = ((a \cup \lambda)b(a \cup \lambda))$

permutations of ***aba, ab, ba, b*** – $(aba \cup ab \cup ba \cup b)^*$
$(aba \cup ba \cup ab \cup b)^* = ((a \cup \lambda)ba \cup (a \cup \lambda)b)^* =$
$((a \cup \lambda)ba \cup (a \cup \lambda)b\lambda)^* = ((a \cup \lambda)b(a \cup \lambda))^*$

3.33 a) $S \rightarrow aaS \mid aaaaaS \mid \lambda$ ***aaS*** produces even length strings, ***aaaaaS*** adds 5 ***a's*** to the string permissible string in the language are ***a's*** of length 2, 4, 5, 6, 7, . . . 5 ***aaS*** or 2 ***aaaaaS*** produce 10 ***a's*** therefore ambiguous (two distinct leftmost derivations)

$S \rightarrow aa \mid aaaA$
$S \rightarrow aA \mid a$ **is an unambiguous grammar**

b) $S \rightarrow aSA \mid \lambda$
$S \rightarrow bA \mid \lambda$

Derivation	Rule
$S \Rightarrow aSA$	$S \rightarrow aSA$
$\Rightarrow aaSAA$	$S \rightarrow aSA$
$\Rightarrow aa\lambda AA$	$S \rightarrow \lambda$
$\Rightarrow aabAA$	$A \rightarrow bA$
$\Rightarrow aab\lambda A$	$A \dashrightarrow \lambda$
$\Rightarrow aabA$	$A \rightarrow \lambda$
$\Rightarrow aab\lambda$	
$\Rightarrow aab$	

Derivation	Rule
$S \Rightarrow aSA$	$S \rightarrow aSA$
$\Rightarrow aaSAA$	$S \rightarrow aSA$
$\Rightarrow aa\lambda AA$	$S \rightarrow \lambda$
$\Rightarrow aaAA$	$A \rightarrow \lambda$
$\Rightarrow aa\lambda A$	
$\Rightarrow aabA$	$A \rightarrow bA$
$\Rightarrow aab\lambda$	$A \rightarrow \lambda$
$\Rightarrow aab$	

$S \rightarrow aAb \mid \mid \lambda$
$A \rightarrow aAb \mid \lambda$
$L(G) = a^*b^*$ **is an unambiguous grammar**