

## Chapter II HW

### 2.1

1. For each of the following algorithms, indicate (i) a natural size metric for its inputs, (ii) its basic operation, and (iii) whether the basic operation count can be different for inputs of the same size:
  - a. Computing the sum of  $n$  numbers
  - b. Computing  $n!$
  - c. Finding the largest element in a list of  $n$  numbers
  - d. Euclid's algorithm
  - e. Sieve of Eratosthenes
  - f. Pen-and-pencil algorithm for multiplying two  $n$ -digit decimal integers

	i	ii	iii
a	$n$	addition	no
b	the amount of $n$ 's in the problem.	multiplication	no
c	$n$	comparison of two numbers	no
d	Whichever $n$ is largest or smallest	Modulus	Yes
e	The amount of $n$ 's in the problem	Taking out a number from the list	No
f	$N$	Multiplication	no

2. a. Consider the definition-based algorithm for adding two  $n \times n$  matrices. What is its basic operation? How many times is it performed as a function of the matrix order  $n$ ? As a function of the total number of elements in the input matrices?

**Sum of the two corresponding elements of the matrices given. Happens  $n^2$  times.**  
 **$n^2 = N/2$**

b. Answer the same questions for the definition-based algorithm for matrix multiplication.

**Multiplication.  $n^2$  elements in the matrix gets multiplied by  $n$  elements of a vector.**  
 **$n^3 = (N/2)^{3/2}$**

9. For each of the following pairs of functions, indicate whether the first function of each of the following pairs has a lower, same, or higher order of growth (to within a constant multiple) than the second function.

a.  $n(n + 1)$  and  $2000n^2$

**same**

b.  $100n^2$  and  $0.01n^3$

**lower**

c.  $\log_2 n$  and  $\ln(n)$

**same**

d.  $\log_2^{2n}$  and  $\log_2 n^2$

**higher**

e.  $2^{(n-1)}$  and  $2^n$

**same**

f.  $(n-1)!$  And  $n!$

**same**

2.2

2. Use the informal definitions of  $O$ ,  $\theta$ ,  $\Omega$  to indicate the time efficiency class of sequential search

a. in the worst case

**$n$**

b. in the best case

**$1$**

c. in the average case

**$(1 - (p/2)n + (p/2))$  where  $p$  is between 0 and 1**

5. List the following functions according to their order of growth from the lowest to the highest:

$(n-2)!$ ,  $51g(n+100)^{10}$ ,  $2^{2n}$ ,  $0.001n^4 + 3n^3 + 1$ ,  $\ln^2 n$ ,  $\sqrt[3]{n}$ ,  $3^n$

**$51g(n+100)^{10}$ ,  $\ln^2 n$ ,  $\sqrt[3]{n}$ ,  $0.001n^4 + 3n^3 + 1$ ,  $3^n$ ,  $2^{2n}$ ,  $(n-2)!$**

9. We mentioned in this section that one can check whether all elements of an array are distinct by a two-part algorithm based on the array's presorting.

a. If the presorting is done by an algorithm with a time efficiency of  $\theta(n \log n)$ , what will be a time-efficiency class of an entire algorithm?

**$\theta(n \log n)$**

b. If the sorting algorithm used for presorting needs an extra array of size  $n$ , what will be the space-efficiency class of the entire algorithm?

**$\theta(n)$**

2.3

1. a. 250,000

b. 2,046

c.  $n - 1$

d.  $(n^2 + 3n - 4)/2$

e.  $((n^2 - 1)n)/4$

f.  $(3^{n+2} - 9)/2$

g.  $(n^2(n+1)^2)/4$

h.  $n/(n+1)$

2. a.  $n^5$

b.  $n \log n$

c.  $n2^n$

d.  $n^3$

4.
  - a.  $n^2$
  - b. Multiplication
  - c.  $n$
  - d.  $2^b$
  - e. Use  $(n(n+1)(2n+1))/6$  to get  $\theta(1)$

## 2.4

1.
  - a.  $5(n-1)$
  - b.  $4(3^{(n-1)})$
  - c.  $(n(n+1))/2$
  - d.  $2n-1$
  - e.  $1 + \log_3(n)$
3.
  - a.  $2(n-1)$
  - b.  $S \leftarrow 1$   
for  $i \leftarrow 2$  to  $n$  do  
     $S \leftarrow S + i^3$   
return  $S$

## 2.5

2. 144 paris
3.  $F(n+1)$  for  $n \geq 1$

## 2.6

1. add the line  
**If  $j \geq \text{count} \leftarrow \text{count} + 1$**   
right after the while statement's end
4.  $n \log n$  algorithm