Theory of Computation

Tutorial 5 - NFAs

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Plan for today

1. NFAs
2. NFA-to-DFA

NFAs

Introduction to NFAs

Definition. A **nondeterministic finite automaton (NFA)** M is a 5 element tuple $M = (Q, \Sigma, \delta, q_0, F)$ where

Q is the set of all states

Σ is the alphabet

* δ is the transition function $\delta : Q \times \{\Sigma \cup \{\lambda\}\} \to 2^Q$

q_0 is the (unique) initial state

F is the set of final states

A NFA is a machine that reads an input string and decides whether to accept it.
***Unlike a DFA:** The transition function of an NFA can accept λ and **always** returns a set.

Introduction to NFAs

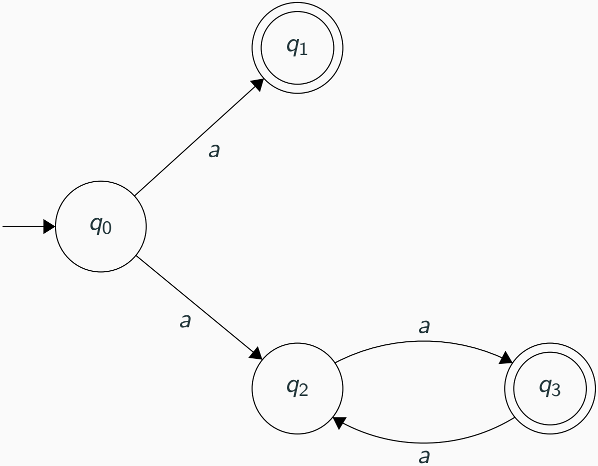
Consider a NFA M
Given a string w, **M** tries all possible walks. If, at the end of the string, ANY of the walks end in a final state the string is accepted. Otherwise, it is rejected.

Definition. The language **L(M)** includes all strings (over the alphabet Σ) that are accepted by **M**.
$L(M) = \{$strings that drive **M** to a final state$\}$

Formally:
$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w)$ contains at least one final state$\}$, where δ^* is the extended transition function $\delta^* : Q \times \Sigma^* \to 2^Q$.

Example

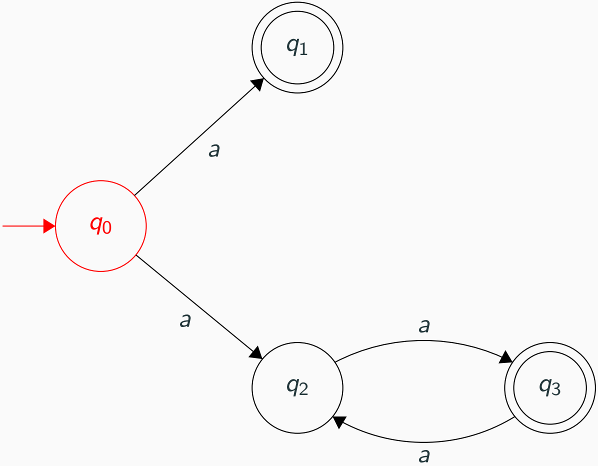
Example. The following is an NFA **M** where
$\mathbf{L(M)} = \{a\} \cup \{a^{2k} : k > 0\}$ $(\Sigma\{a\})$.



Example - Tracing Input

$\mathbf{L(M)} = \{a\} \cup \{a^{2k} : k > 0\}$

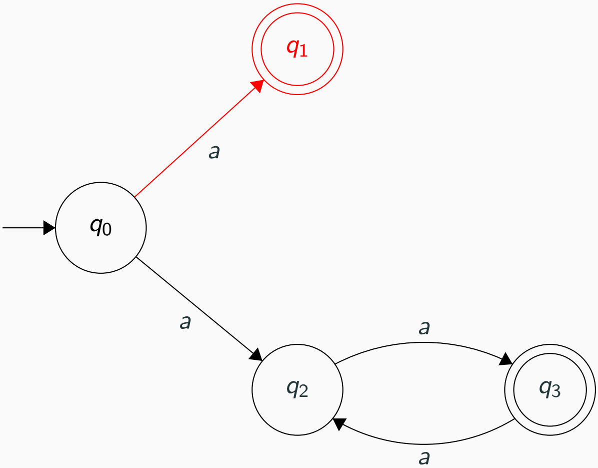
Input String: ^aa



Example - Tracing Input

$\mathbf{L(M)} = \{a\} \cup \{a^{2k} : k > 0\}$

Input String: aa

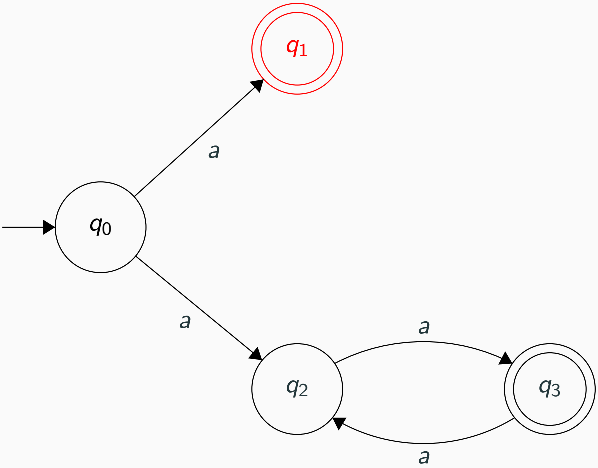


Example - Tracing Input

$\mathbf{L(M)} = \{a\} \cup \{a^{2k} : k > 0\}$

Input String: aa

$\delta(q_1, a) = \emptyset$, no where to go. Are we done?

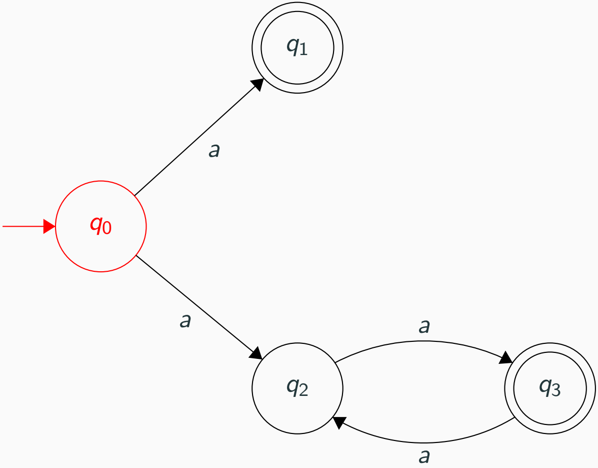


Example - Tracing Input

$\mathbf{L(M)} = \{a\} \cup \{a^{2k} : k > 0\}$

Input String: ˆaa

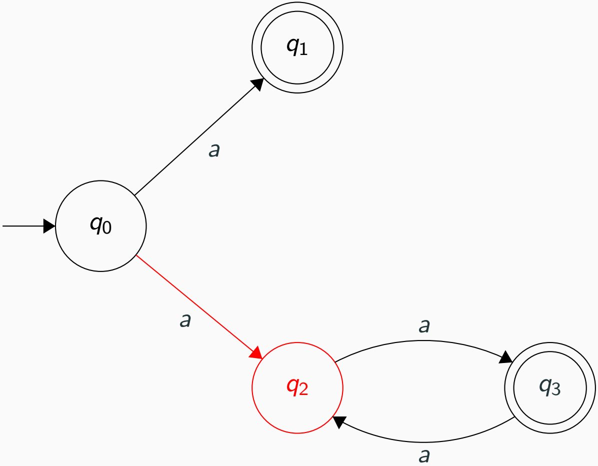
Are we done? No, **M** tries another walk.



Example - Tracing Input

$\mathbf{L(M)} = \{a\} \cup \{a^{2k} : k > 0\}$

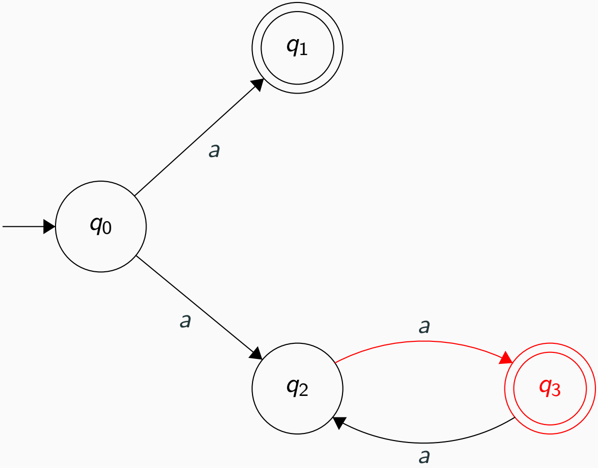
Input String: aa



Example - Tracing Input

$\mathbf{L(M)} = \{a\} \cup \{a^{2k} : k > 0\}$

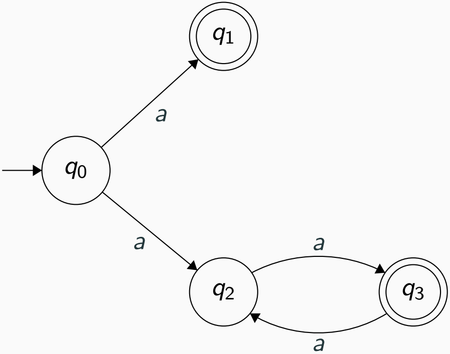
Input String: aa



One of the possible walks ends in a final state, **M** accepts this string.

Example - Tracing Input

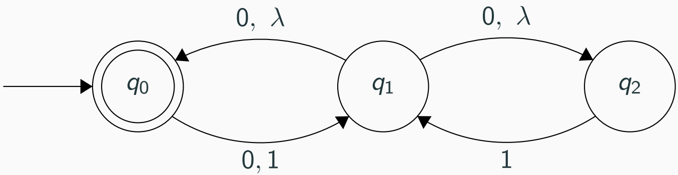
L(M) $= \{a\} \cup \{a^{2k} : k > 0\}$
Input String: aaa



In both traces (top and bottom), do not end up in a final state. **M** rejects this string.

Exercise

Exercise. Given the following NFA **M**,



What is

1. $\delta^*(q_0, 01) =$? Is 01 accepted by this NFA?

NFA-to-DFA

NFA-to-DFA

Given an NFA $N = (Q, \Sigma, \delta, q_0, F)$, how can we convert it to a DFA $M = (Q', \Sigma, \delta', q_0', F')$?

Converting a NFA to DFA is also called **Subset Construction**.

1) Step 1: Start from the initial state S ($S = \{q_0\}$).

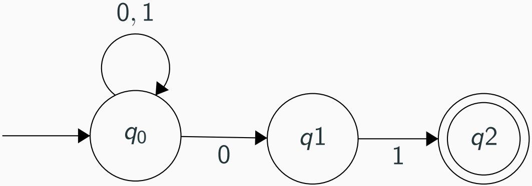
2) Step 2: For each symbol $a \in \Sigma$, find all the states that can be reached from S: $\delta'(S, a) = \bigcup_{p \in S} \delta(p, a)$.

3) Step 3: Repeat Step 2 on every new state that is generated. Repeat until no new states are produced.

4) Step 4: Draw the DFA with states and edges from Step 3.

The initial state for the DFA will be $\{q_0\}$. The final states of the DFA will be all those states S that contain a final state from F. If the original NFA N accepts λ, make $\{q_0\}$ a final state.

Example. Convert the following NFA M to a DFA. M is the NFA accepting all strings (over $\Sigma = \{0, 1\}$) that end in 01.

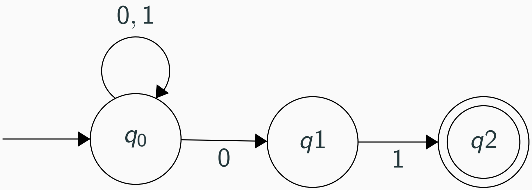


Example

Example. Converting the NFA M to a DFA:

1) Step 1: Start from the start state S. $S = \{q_0\}$

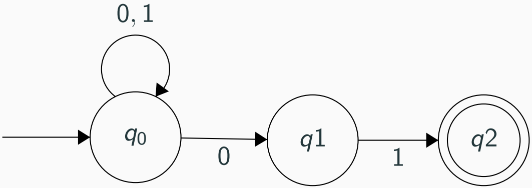
2) Step 2: Find all the states that can be reached from S: $\forall a \in \Sigma$, $\delta'(S, a) = \bigcup_{p \in S} \delta_N(p, a)$. $\delta'(S, 0) = \{q_0, q1\}$, $\delta'(S, 1) = \{q_0\}$



Example

Example.

3) Step 3: Repeat Step 2 on every new state that is generated. Repeat until no new states are produced.

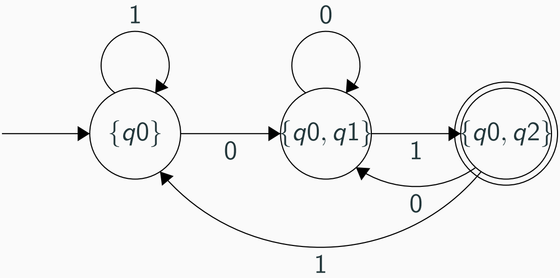


	0	1
→{q0}	{q0, q1}	{q0}
{q0, q1}	{q0, q1}	{q0, q2}
*{q0, q2}	{q0, q1}	{q0}

Example.

4) Step 4: Draw the DFA

	0	1
→{q0}	{q0, q1}	{q0}
{q0, q1}	{q0, q1}	{q0, q2}
*{q0, q2}	{q0, q1}	{q0}



Theorem

Theorem. The set of all languages accepted by NFAs is the same as the set of all languages accepted by DFAs.
Why?

1. Every DFA is an NFA.
2. Every NFA can be converted into a DFA.

Corollary. A language is regular if there is an FA (either DFA or NFA) that accepts it.