

Formal argument showing $L_{HP} \leq_m L_{AP}$ given

$v \in \{0,1\}^*$

$$f(v) = \begin{cases} \langle N, x \rangle & \text{if } v = \langle M, x \rangle \\ \epsilon & \text{otherwise} \end{cases} \rightarrow \text{We already showed that this was computable}$$

W.T.S. $v \in L_{HP} \Leftrightarrow f(v) \in L_{AP}$

Let $v \in \{0,1\}^*$

If v is not a valid encoding then neither is $f(v)$

$\Delta \therefore v \notin L_{HP}, f(v) \notin L_{AP}$

Otherwise, $v = \langle M, x \rangle, f(v) = f(\langle M, x \rangle) = \langle N, x \rangle$

If $\langle M, x \rangle \in L_{HP} \Rightarrow M$ halts on x

By construction N accepts $x \Rightarrow \langle N, x \rangle \in L_{AP}$

If $\langle M, w \rangle \notin L_{HP} \Rightarrow \langle N, w \rangle \notin L_{AP}$

So indeed $L_{HP} \leq_m L_{AP} \Delta \therefore AP$ is undecidable.