

# Comp 330 - Theory of Computation

## Tutorial 3 - DFAs

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# Plan for today

1. Introduction to DFAs
2. Regular Languages

# Introduction to DFAs

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# Formal definition of a DFA

**Definition.** A deterministic finite automaton (DFA)  $M$  is a 5 element tuple  $M = (Q, \Sigma, \delta, q_0, F)$  where

$Q$  is the set of all states

$\Sigma$  is the alphabet

$\delta$  is the transition function  $\delta : Q \times \Sigma \rightarrow Q$

$q_0$  is the (unique) initial state

$F$  is the set of final states

A DFA is a machine that takes as input a string and returns either an accept or a reject.

# DFAs as language accepters

**Consider a DFA  $M$**

**Definition.** The language  $L(M)$  includes all strings (over the alphabet  $\Sigma$ ) accepted by  $M$

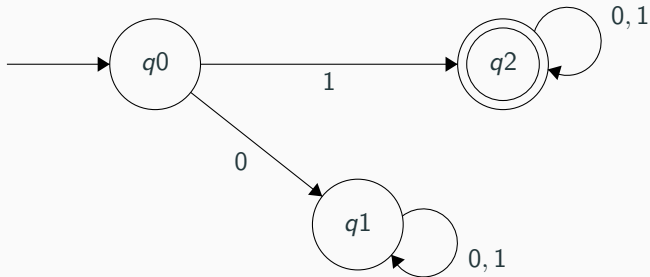
$L(M) = \{\text{strings that drive } M \text{ to a final state}\}$

Which is defined formally as follows:

$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$ , where  $\delta^*$  is extended transition function  $\delta^* : Q \times \Sigma^* \rightarrow Q$ .

## Example

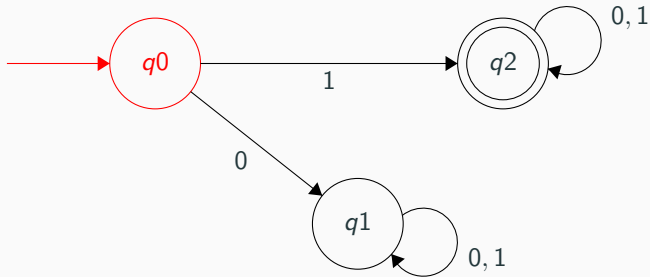
The following is a DFA  $\mathbf{M}$  such that  $\mathbf{L}(\mathbf{M}) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$  for  $\Sigma = \{0,1\}$ .



## Example - Tracing input

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

Input String:  $\hat{1}01$

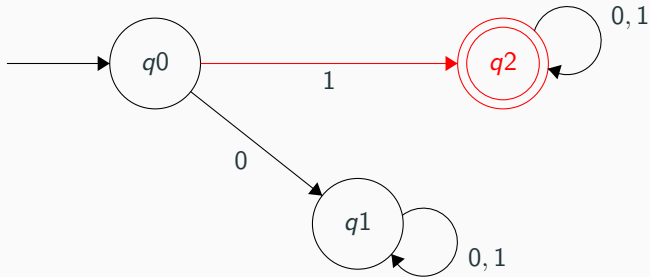




## Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

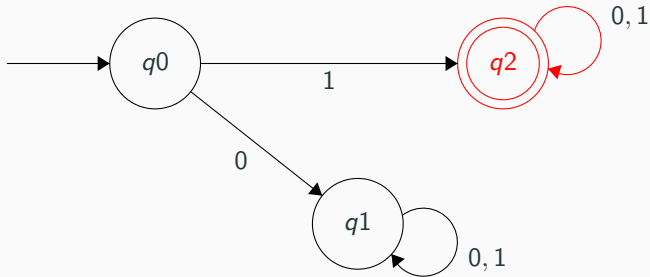
Input String: **1**01



## Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

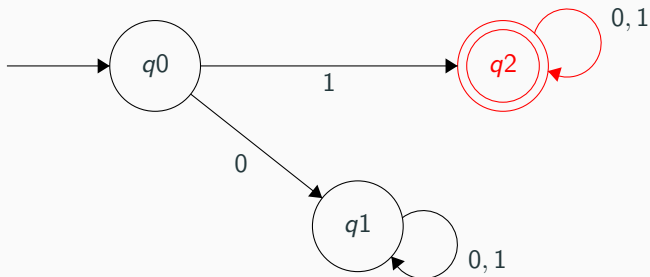
Input String: **1**0**1**



## Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

Input String: 10**1**

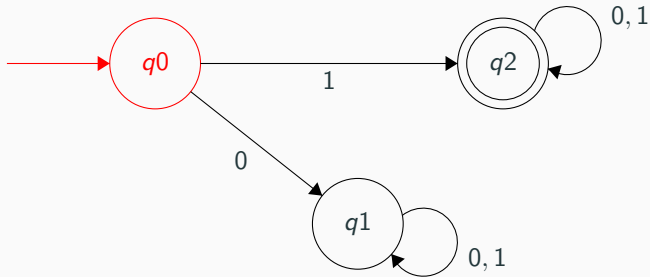


The input finishes in a final state, **M** accepts.

# Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

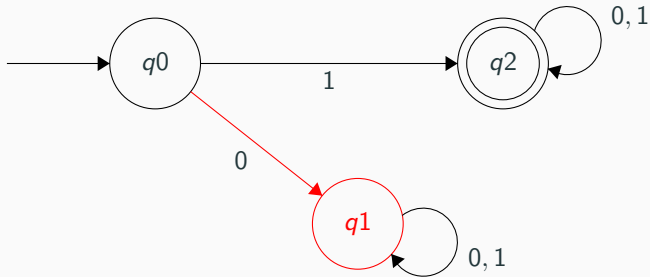
Input String:  $\hat{0}10$



## Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

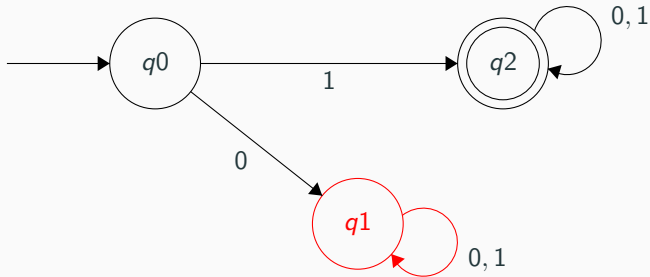
Input String: **0**10



## Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

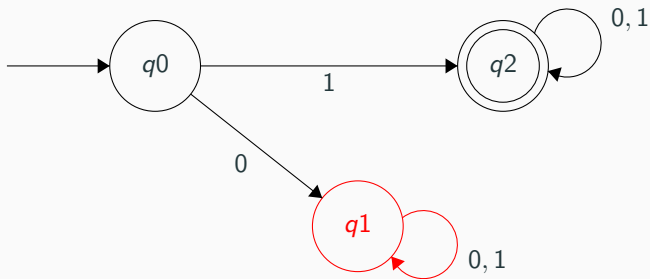
Input String: **0****1****0**



## Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

Input String: 01**0**



The input does not end in a final state, **M** rejects.

## Example

**Example.** Create a DFA that accepts the language  $L = \{w \in \{0,1\}^* : w \text{ contains } 00 \text{ as a substring}\}$ .



# Regular Languages

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**Defintion.** A language  $L$  is regular if there exists a DFA  $M$  such that  $L(M) = L$ . One way to show that a language  $L$  is regular is to show there is a DFA  $M$  that accepts it.

## Example

**Example.** Show that the language

$L = \{a^n : n \text{ is a multiple of 2 but not of 3}\}$  ( $\Sigma = \{a\}$ ) is regular.

To be continued...

Thank you!