Theory of Computation

Tutorial 3 - DFAs

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Plan for today

1. Introduction to DFAs

2. Regular Languages

Introduction to DFAs

DFAs

Formal definition of a DFA

Definition. A **deterministic finite automaton (DFA)** M is a 5 element tuple $M = (Q, \Sigma, \delta, q_0, F)$ where

> Q is the set of all states
>
> Σ is the alphabet
>
> δ is the transition function $\delta : Q \times \Sigma \to Q$
>
> q_0 is the (unique) initial state
>
> F is the set of final states

A DFA is a machine that takes as input a string and returns either an accept or a reject.

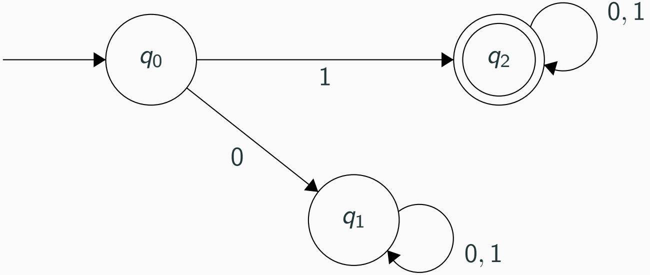
DFAs as language accepters

Definition. Let M be a DFA. The language $L(M)$ includes all strings (over the alphabet Σ) accepted by M. That is, $L(M) = \{$all strings that "drive" M to a final state$\}$.

Formally, we write this as $L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$, where δ^* is the extended transition function $\delta^* : Q \times \Sigma^* \to Q$ which, given a state q and a string w, returns the state that M would be in after reading w starting from q.

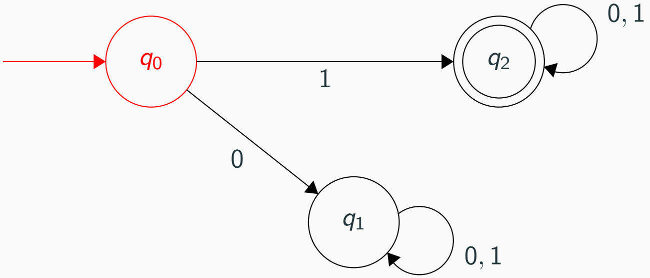
Example

The following is a DFA $\mathbf{M}$ such that $\mathbf{L(M)} = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$ for $\Sigma = \{0,1\}$.



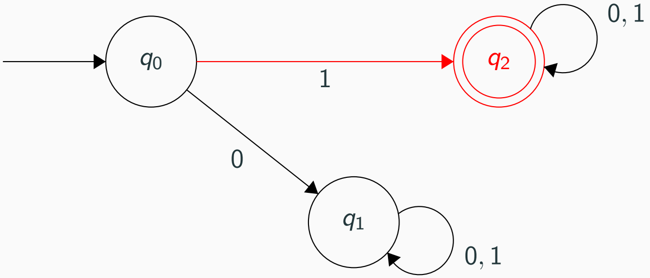
Example - Tracing input

L(M) $= \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$
Input String: ^101



Example

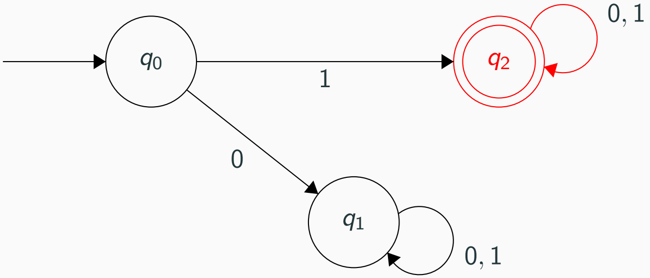
L(M) $= \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$
Input String: 101



Example

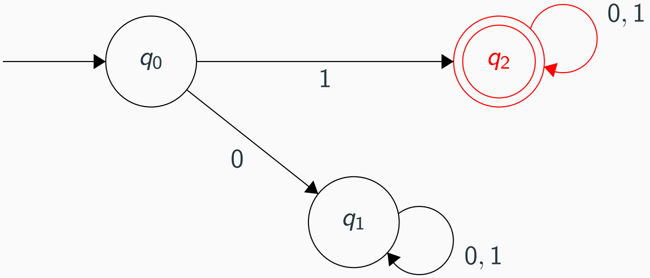
$\mathbf{L(M)} = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

Input String: 101



Example

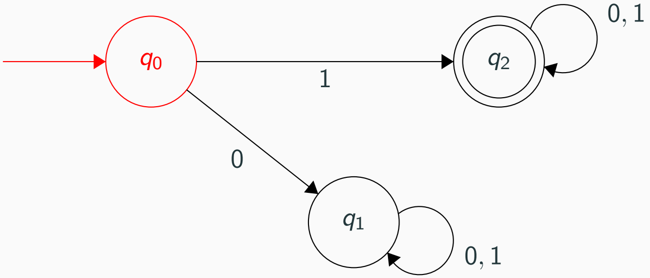
L(M) $= \{w \in \{0,1\}^* : w$ starts with a $1\}$
Input String: 101



The input finishes in a final state, **M** accepts.

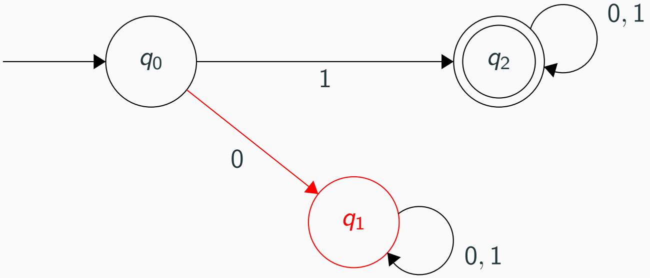
Example

$\mathbf{L(M)} = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$
Input String: ^010



Example

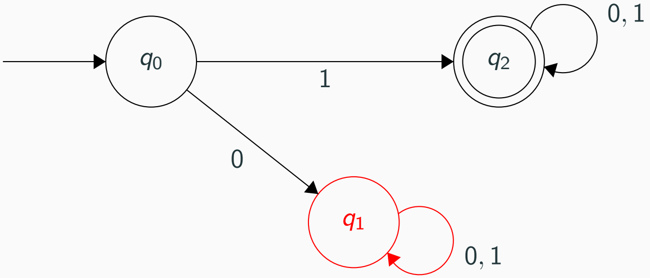
$\mathbf{L(M)} = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$
Input String: 010



Example

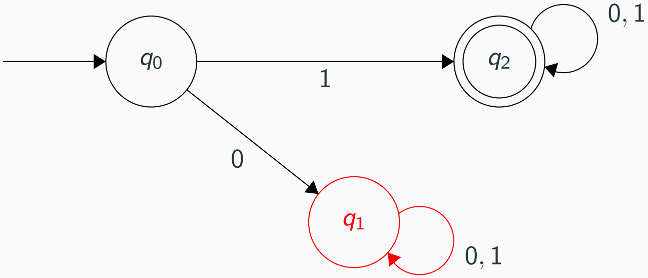
L(M) $= \{w \in \{0,1\}^* : w$ starts with a $1\}$

Input String: 010



Example

$\mathbf{L(M)} = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$
Input String: 010



The input does not end in a final state, **M** rejects.

Example

Example. Create a DFA that accepts the language $L = \{w \in \{0,1\}^* : w$ contains 00 as a substring$\}$.

Regular Languages

Regular Languages

Definition. A language L is regular if there exists a DFA M such that $L(M) = L$. One way to show that a language L is regular is to show there is a DFA M that accepts it.

Example

Example. Show that the language $L = \{a^n : n \text{ is a multiple of 2 but not of 3}\}$ ($\Sigma = \{a\}$) is regular.