# Comp 330 - Lecture 12 - October 12th

IE: In bocca al lupo
In the mouth of the wolf
Good luck!

Today: Midterm review
OHs at 10:30 - Noon → On Zoom
Today 1PM to 3PM - MC 321
Friday: Claude OH on Zoom 9AM - Noon
Cesare 12:30 PM to 2:30 PM - MC 110

Final exam: Dec 12th at 9AM - Noon

## Review Exercises

→ FA Design / REX / Non-REG / T/F

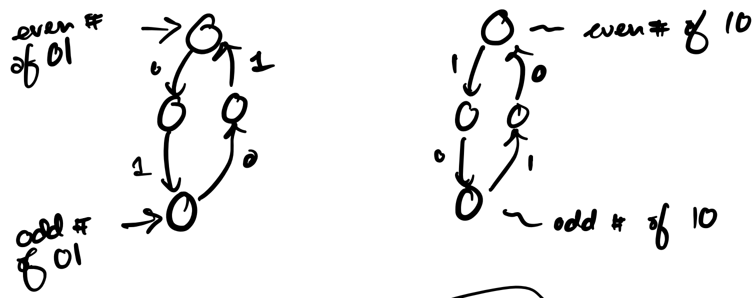
### Exercise 1

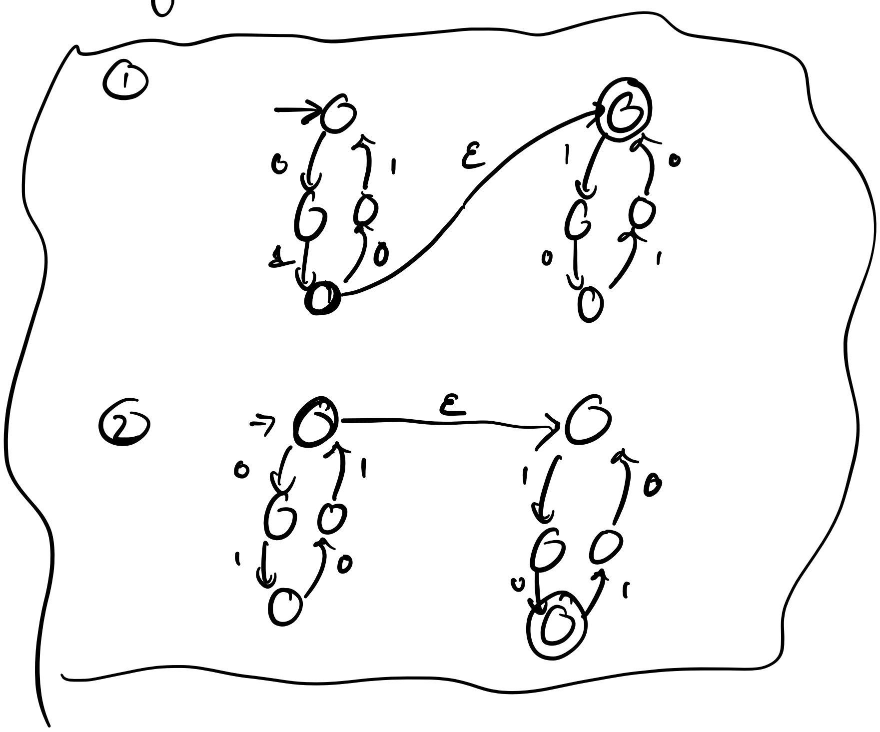
$$L_b = \{ (01)^n (10)^m : n, m \in \mathbb{N},\ n+m \text{ is odd} \}$$

if $n+m$ is odd — ① $n$ is odd, $m$ is even OR
② $m$ is odd, $n$ is even

Create NFA + ε

① $(01)^{\text{odd}}\ (10)^{\text{even}}$ $\qquad$ $\_\ \underline{01}\ \underline{01} \ldots \underline{01}\ \underline{10}\ \underline{10} \ldots$




$L_e = \{ \underline{vwv} : \ v, w \in \{0,1\}^*, |v| = 2 \}$

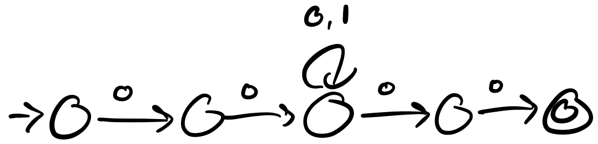
$x = \underline{00}\ \underline{101}\ 00 \in L_e$

$v = 00 \quad w = 101 \quad |v| = 2$

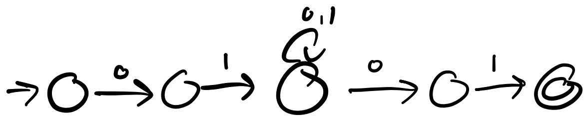
$x = vwv$

$x = 01\ 110\ 10 \notin L_e$

$\Sigma = \{0,1\} \qquad v = 00 \quad 01 \quad 10 \quad 11$



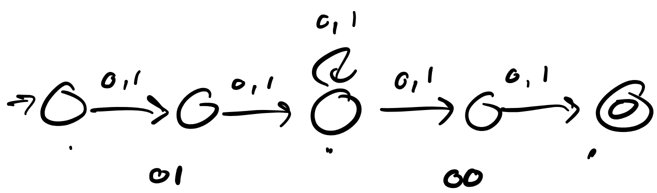
$\leftrightarrow \{00w00 : w \in \{0,1\}^*\}$



$\leftrightarrow \{01w01 : w \in \{0,1\}^*\}$

Same for 10, 11

$x = 00 \notin L_e$




For midterm

- Drawing is sufficient
- Proof of correctness not req

$\Sigma = \{a, b\}$

$L_f = \{a^n b^m : n, m \in \mathbb{N},\ n \bmod 2 \neq m \bmod 2\}$

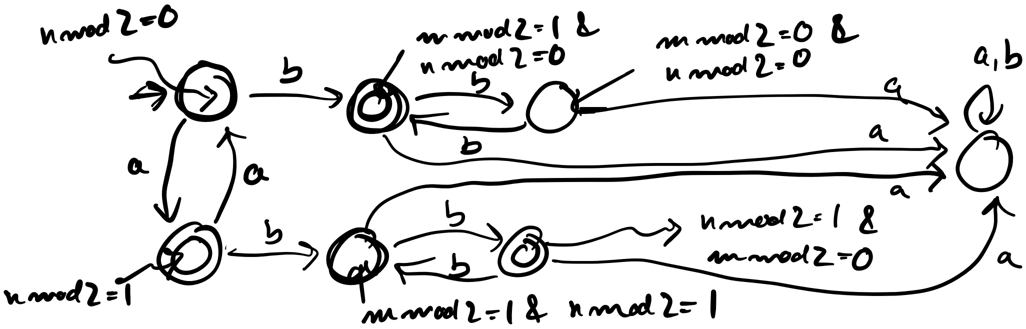
DFA + Minimal

$n \bmod 2 = 0$ & $m \bmod 2 = 1$

OR

$n \bmod 2 = 1$ & $m \bmod 2 = 0$

input string : aaaa..abb....b



Is this minimal ?

## Exercise 2 - Reg Exps

0. $L_a = \{ w \in \{a,b\}^* :$ w starts w/ b & ends w/ ba $\}$

$\downarrow$

$w = b(\ldots\ldots)ba$

$r = b(a+b)^*ba + ba$

4. $\{ w \in \{0,1\}^* :$ The second letter of w is different from the second to last letter of w $\}$

(Assume) ↳ String in this language must have both a second & a second to last letter

$w = \_\ \_$ $\qquad w = \ldots\ldots\ \_\ \_$

Strings by length

length = 0    length = 1    length = 2

$\varepsilon$ ×    0 ×    00 ×   01 ✓

1 ×    11 ×   10 ✓

length = 3

$w = a_1 \underline{\underline{a_2}}\, a_3$ ×

length = 4

$w = \underset{(0+1)}{\_}\ \underset{0}{\_}\ (0+1)^*\ \underset{1}{\_}\ \underset{(0+1)}{\_}$

$\varepsilon, \emptyset$

$\downarrow \quad \downarrow$

$\{\varepsilon\}\ \{\}$

$r = 01 + 10 + (0+1)0(0+1)^*1(0+1) + (0+1)1(0+1)^*0(0+1)$

Exercise 5 Is it REG?

$L_1 = \{ a^q : q \in \mathbb{N}, q \text{ is prime} \}$

Claim $L_1$ is not REG.

Proof P.L.

$\forall$ : Opponent picks $p \in \mathbb{N}$, $p > 0$

$\exists$ : Pick string $w = a^m$, $m$ is the next prime number after $p$. $(m > p)$

$\forall$ : Opponent decomposes the string into

$w = xyz \quad |xy| \le p \quad |y| > 0$

$w = \underbrace{a\,a\ldots}_{x}\underbrace{\ }_{y}\underbrace{a\,a.}_{z} \quad y = a^k \quad 1 \le k \le p$

$\exists$ : Find an $i$ s.t. $w_i = xy^iz \notin L$

$w_i = xy^iz = a^{m+(i-1)\cdot k}$

Goal: Pick $i$ s.t. $m + (i-1)\cdot k$ is not prime

$i - 1 = m \Rightarrow i = m+1$

$\Rightarrow m + m\cdot k = \underline{m(k+1)}$

The # of a's is composite & $\therefore$ $w_i \notin L_1$.

$i \in \mathbb{N}$

$S = \{ a^i : i \text{ is prime} \}$

$x = a^n$ , $n$ is prime $\quad n \neq m$

$y = a^m$ , $m$ is prime

Common error in M-N $\quad S = \{ a^i : i \in \mathbb{N} \}$

$\times \left\{ \begin{array}{l} x, y \in S \\ x = a^n \\ y = a^{n+1} \end{array} \right.$

$\checkmark \left\{ \begin{array}{ll} x, y \in S & , x \neq y, \\ x = a^n & n \neq m \\ y = a^m & \underline{\underline{\text{WLOG}}} \quad n < m \end{array} \right.$

extra special character.

$L_q = \{ x \underline{\underline{c}} x : x \in \{a, b\}^* \}$

abb c abb $\in L_q$

abb c ba $\notin L_q$

Use the M-N Thm to prove non-reg.

1. Pick a set $S = \{ a^n : n \in \mathbb{N} \}$

   $S = \{ a^n \underline{\underline{c}} : n \in \mathbb{N} \}$

   $c \to$ is a special letter.

2. Pick $x, y \in S$ , $x \neq y$

   $x = a^n c \qquad n \neq m$

   $y = a^m c$

3. Pick some distinguishing extension $z$

   $\overset{\in L}{xz} = a^n c \cdot a^n \in L$

   $yz = a^m c \cdot a^n \notin L$

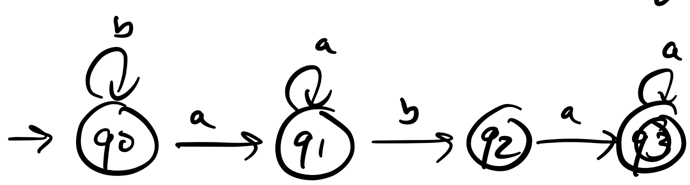
$\notin L$

By M-N The index of $\equiv_L$ is infinite

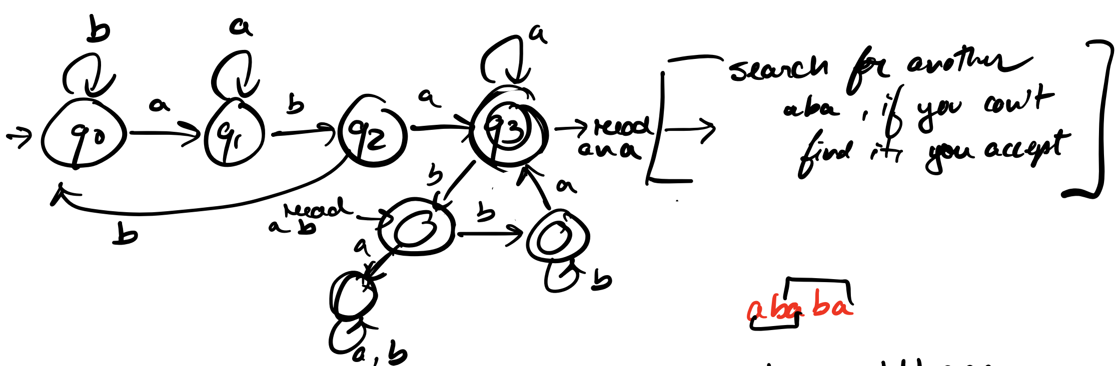
OH MC 321 at 1PM & 10:30 AM on Zoom
Today

## Extra questions

**Q1** $\{w \in \{a,b\}^* :$ w contains exactly one instance of aba $\}$



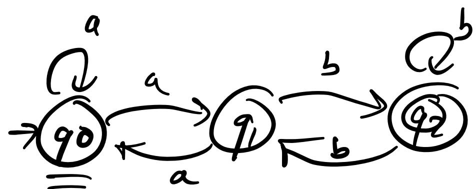

Circle back




ababa

abaaaabbbaaa
aba

**Q4** $M_2$




$\varepsilon$

$\Rightarrow r = (a+b+ab+ba)^*$ denotes $L(M_2)$

$L(r) = L(M_2)$ $\quad L(r) = \Sigma^*$

Q2. 2/5

$\Sigma = \{a, b\}$, $w \in \Sigma^*$,

1) $w$ has at least one pair of consecutive a's

$w$ = a b b a ... aa ... b a a 3

$M = (a+b)^* aa (a+b)^*$

2) $w$ has exactly one pair of consecutive a's.

$(b+ab)^* aa (ba+b)^*$ , babbaa

## C's question

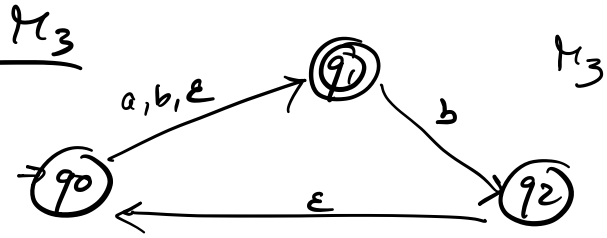
M is a DFA

1. M accepts all strings w/ at least one instance of ba
   → M should accept baba
2. M accepts all strings w/ one instance of ba → M should accept ba

$L_2 = \{ w \in \{a,b\}^* :$ w has exactly one instance of ba$\}$

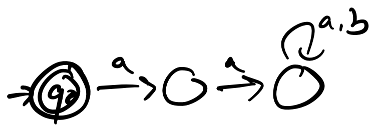
$L_2 \subseteq L(M)$

## Machine $M_3$




c). $r = (a+b+\varepsilon)^* b$ $\quad L(r) = L(M_3)$ $\quad$ F.

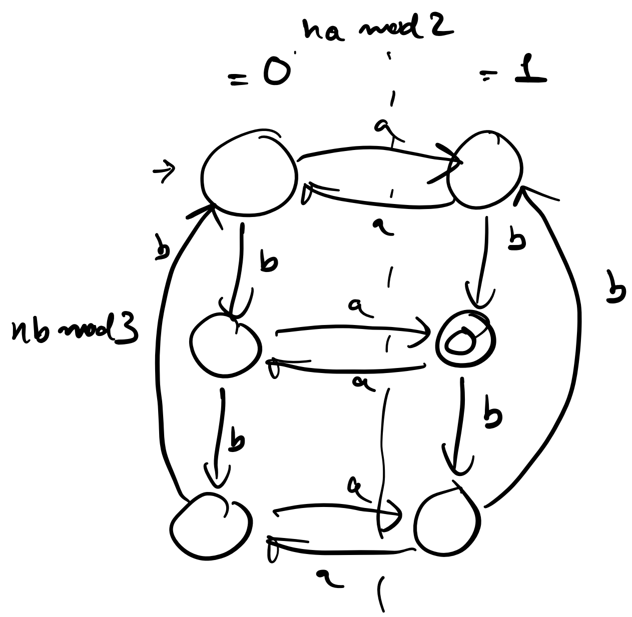
d) An eq DFA will have ~~at least~~ 3 reachable states. $\quad w = aa$




1.g. $L = \{ w \in \{a, b\}^* : n_a(w) \bmod 2 = 1$ & $n_b(w) \bmod 3 = 1 \}$

$n_a \bmod 2 \to 0, 1$

$n_b \bmod 3 \to 0, 1, 2$




$L = \{ a^{i-1} b^i : i \in \mathbb{N} \}$ → Keep track of the count of a's & check that there's one less a than b.

$i = p+1$

$i-1 = p$

# PL proof

$\forall$ : Pick $p \in \mathbb{N}$, $p > 0$

$\exists$ : Pick some string $w \in L$, $|w| \geq p$.

$w = a^{p-1} b^p \in L$, $|w| \geq p$

$\forall$ : $w = \underbrace{\overbrace{aaa \ldots aaa}^{p-1}}_{} \overbrace{bb \ldots b}^{p}$ with $x$ and $y$ marked below

$w = xyz$
$|xy| \leq p$ $|y| > 0$

**Case 1**

$y = a^k$, $k \geq 1$
$k \leq p-1$

Pump up $i = 2$

**Case 2**

$y = a^k b$, $k \geq 0$
$k \leq p-1$

$\overbrace{aaa \ldots aa}^{p-1}\underbrace{ab}_{y=ab} \ldots b$

Pump up $i = 2$

$aa \ldots abab\,b \ldots b$
pattern mismatch

$\exists$ : $w = a^p b^{p+1} \in L$

$\forall$ : $w = \overbrace{a \ldots a}^{p} \overbrace{b \ldots b}^{p+1}$ $|xy| \leq p$

$y = a^k$ $1 \leq k \leq p$

$\exists$ : Pump down $w_0 = xz = a^{p-k} b^{p+1}$

$w_2 = xy^2z = a^{p+k}\, b^{p+1}$

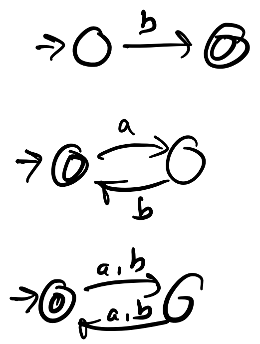
$w_i \notin L$

Union of REG languages

$L_1 \cup L_2 \cup L_3 \cup L_4$

FA FA FA FA

$\to M_1$ $\to M_2$ $\to M_3$ $\to M_4$




Union of $L_1$ w/ $L_2$

REG not-REG

$\overset{R}{L_1} \cup \overset{NR}{L_2} = L_3$

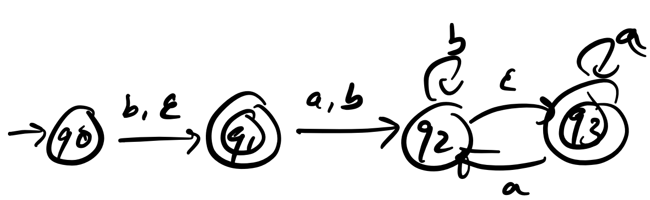
$\Sigma^* \cup \{a^n b^n\} = \Sigma^*$

$w \in \{\varepsilon\} \cup \{a^n b^n : n \geq 1\} = \{a^n b^n : n \in \mathbb{N}\}$

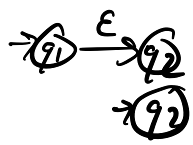
$w = a^n b^n$ , $n \geq 0$

$\underline{M_4}$

$\underline{b} \begin{smallmatrix} a \\ b \end{smallmatrix}$

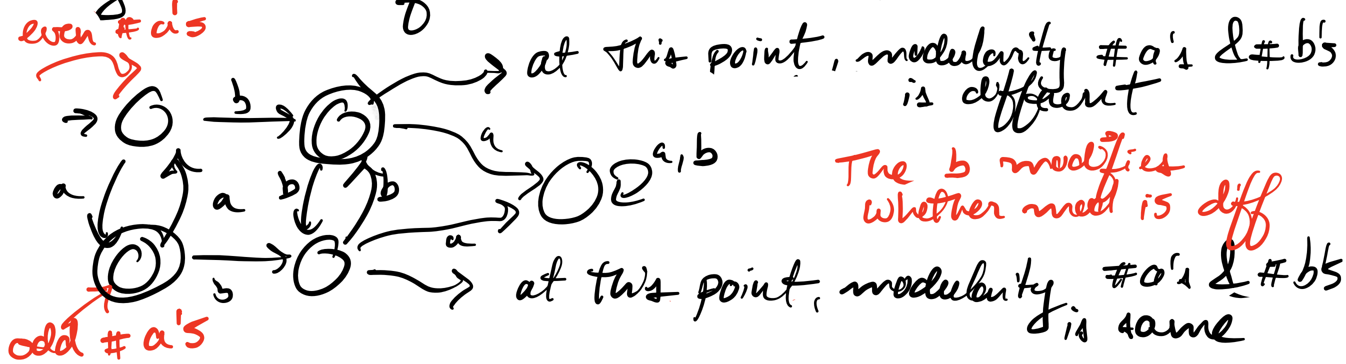



c) ripping out $q_0$ & making



$q_1$ the start state doesn't lead rejecting $\varepsilon$.

Why was $L_f$ not minimal ?




You certainly need to encode even & odd # of a's

But you don't really need to keep track of parity of # b's in both cases.

You only need to consider when modularity of # a's & b's are different