Comp 330 - Lecture 11 - Oct 5th

Italian expression: Sputare il rospo
- To spit the toad
- To speak up

→ A1 graded - post Today/ tomorrow
→ A2 due today
→ A3 released tomorrow
→ 9:45

Using the PL to prove non-regularity

PL : if L is regular then L satisfies the conditions of the PL.

Created a necessary condition for regularity
Do the same : CFL, Strictly local languages

If I can show that L does not satisfy the conditions of the PL then it cannot be reg.

What is the contrapositive of the PL?

The PL for reg languages $\Sigma = \phi$, $L \subseteq \Sigma^*$

if L is regular then (1)

(2)
$$\begin{cases} \exists\, p \in \mathbb{N},\ p > 0. \\ \forall\, w \in L,\ |w| \geq p. \\ \exists\, x, y, z \in \Sigma^* \quad w = xyz \quad |xy| \leq p \quad |y| > 0. \\ \forall\, i \in \mathbb{N},\ w_i = xy^iz \in L \end{cases}$$

[Extra handout : PL is written down using predicate / propositional]

Let's take the contrapositive by negating (1) & (2)

$$(1) \to (2) \equiv \neg (2) \to \neg (1)$$

$$\neg \exists x.\, P(x) \to \forall x.\, \neg P(x)$$

$$\neg \forall x.\, P(x) \to \exists x.\, \neg P(x)$$

$$\boxed{\neg p \to q \equiv p \land \neg q}$$

if
$$\forall\, p \in \mathbb{N},\ p > 0.$$
$$\exists\, w \in L.\ |w| \geq p$$
$$\forall\, x, y, z \in \Sigma^*,\ w = xyz,\ |xy| \leq p,\ |y| > 0$$
$$\exists\, i \in \mathbb{N},\ w_i = xy^iz \notin L.$$

Then L is not regular.

Using the PL to prove non-reg by playing a game:

$\forall\ p \in \mathbb{N},\ p > 0.$

$\exists\ w \in L.\ |w| \geq p$

$\forall\ x, y, z \in \Sigma^*,\ w = xyz,\ |xy| \leq p,\ |y| > 0$

$\exists\ i \in \mathbb{N},\ w_i = xy^iz \notin L.$

Goal of the game: Winning!

You play the $\exists$, Opponent plays the $\forall$

↳ Demon.

<u>Ex</u> Prove, using PL, that $L = \{a^n b^n : n \in \mathbb{N}\}$ is not regular.

$\forall$: Opponent picks $p \in \mathbb{N},\ p > 0$

$\exists$: I pick $w = a^p b^p$ $\quad |w| = 2p \geq p.$

$\forall$: Opponent decomposes w into xyz

$|xy| \leq p,\ |y| > 0$

$$w = \overbrace{aaa \ldots aaa}^{p}\ \overbrace{bbb \ldots bbb}^{p}$$

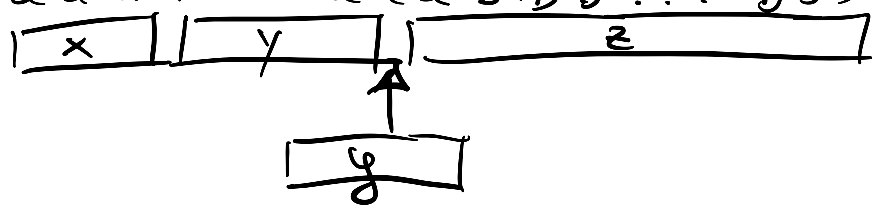
$$w = \underbrace{aaa \ldots\ a}_{\substack{xy \\ |xy| \leq p}}\ \underbrace{aa\, bbb \ldots bbb}_{z}$$

I know that $y = a^k \quad 1 \le k \le p$

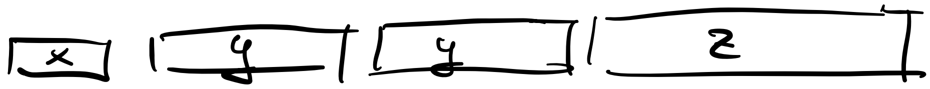
$\exists$: I need to pick an i s.t. $w_i = xy^iz \notin L$.

$i = 2 \qquad w_2 = xy^2z = xyyz$

$$w_1 = \overbrace{aaa \ldots aaa}^{p} b.bb \ldots bbb$$



$w_2 =$



$= a^{p+k} b^p \notin L$

$p + k \geq p + 1 > p$

$k \geq 1$

By the PL, L is not regular. ∎

Closure properties can be helpful in proving non-reg:

<u>Ex</u> $L = \{ w \in \{a, b\}^* : n_a(w) = n_b(w) \}$

Suppose, for contradiction, that L is REG.

by assumption

$$\underbrace{\overset{\text{REG}}{\overline{L}} \cap \overbrace{L(a^*b^*)}^{\text{REG}}}_{\text{REG}} = \underbrace{\{a^n b^n : n \in \mathbb{N}\}}_{\text{REG} \ \times}$$

Ex $L = \{a^n b^m : n, m \in \mathbb{N},\ n \neq m\}$

1. Using M-N Thm
2. Using PL [Exercise, hint: factorial]
3. Using the closure properties.

Assume L is REG, consider

$$\underbrace{\underset{\text{REG}}{\overline{L}} \cap \underset{\text{REG}}{L(a^*b^*)}}_{\text{REG}} = \underbrace{\{a^n b^n : n \in \mathbb{N}\}}_{\text{REG} \ \times}$$

$bbaba \in$ $\quad a^3b^3 \in$

We know that M-N can be used to prove non-reg. But ~~sometimes~~ it's not so easy to find S.

Ex $L = \{a^{n^2} : n \in \mathbb{N}\}$

M-N Thm: $S = \{a^{i^2+i+1} : i \in \mathbb{N}\}$

Use PL

$\forall$: Opponent picks $p \in \mathbb{N}$, $p > 0$

$\exists$: $w = a^{p^2} \in L$, $p^2 \geq p$

$\forall$: $w = \overbrace{a\ a \ldots a\ a}^{p^2}$ $\quad$ $\boxed{x}\ \boxed{y}\ \boxed{z}$ $\qquad |xy| \leq p \quad |y| > 0$

It must be that $y = a^k \quad 1 \leq k \leq p$

$\exists$: $w_1 = \overbrace{a\ a \ldots a\ a}^{p^2}$ $\quad$ $\boxed{x}\ \boxed{y}\ \boxed{z}$ $= xy^1z = a^{p^2}$

$w_i = a\ a \ldots a\ a = a^{\cancel{p^2 + i \cdot k}} = a^{p^2 + (i-1)k}$ Common error

$\boxed{x}\ \underbrace{\boxed{y}\ \boxed{y}\ \boxed{y} \ldots \boxed{y}}_{i}\ \boxed{z}$

$i = 2 \quad w_2 = a^{p^2 + (2-1)k} = a^{p^2 + k} \notin L$

$p^2 < p^2 + k < (p+1)^2$

$k \geq 1 \qquad\qquad k \leq p$

$p^2 + k \geq p^2 + 1 > p^2 \qquad p^2 + k \leq p^2 + p \leq p^2 + 2p$

$< p^2 + 2p + 1$

$= (p+1)^2$

$\therefore$ $p^2 + k$ is not a perfect square

By PL, L is not regular. $\blacksquare$

<u>Ex</u> $L = \{ a^i b^{i^3} : i \in \mathbb{N},\ i \text{ is even} \}$

Is L REG? If Y, design an FA
If N, use PL to prove your answer.

<u>No!</u> $\forall$: Pick $p \in \mathbb{N}$, $p > 0$ (A)

$\exists$: $w = a^p b^{p^3}$ ✗, p is not necessarily
$w = a^{2p} b^{p^3}$ ✗, $(2p)^3 \neq p^3$
$w = a^{2p} b^{(2p)^3}$ ✓ :)

$\forall$: Decompose string $w = xyz$
$y = a^K \quad 1 \leq K \leq p$

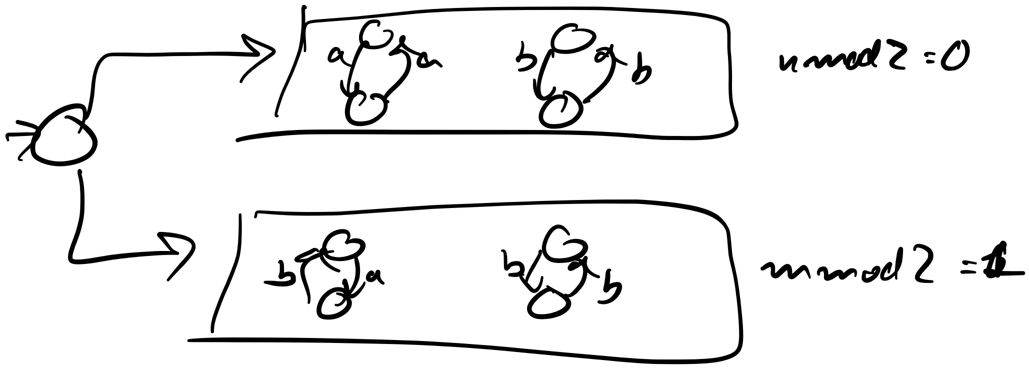
$\exists$: Pump down $i = 0$, $w_0 = xz$
$w_0 = a^{2p-K} b^{(2p)^3} \notin L$

$(2p-K)^3 \leq (2p-1)^3 < (2p)^3$

By PL, L is not REG.

<u>Ex</u> $L = \{ a^n b^m : n \bmod 2 = m \bmod 2,\ n, m \in \mathbb{N} \}$
Is L REG? Yes!

$n \bmod 2 = m \bmod 2 = 0$ OR 1



<u>Ex</u> (for you) $\{a^n b^m : n = m \bmod 2,\ n,m \in \mathbb{N}\}$

<u>Ex</u> Which of the following are REG?

(For you) $L_1 = \{a^n b^m : n < m \le 330\}$

$L_2 = \{a^n b^m : n \le 330 < m\}$

$L_3 = \{a^n b^m : 330 \le n < m\}$

So far, when discussing language operators applied to REG, we've seen they all preserve regularity. Consider the following language operator:

$\Sigma = \{0,1\}$ $\quad 0 < 1$

$x \in \Sigma^*$ $\quad$ sort(x) := rearranges x s.t. letters in x are sorted

sort$(10100) = 00011$

Ex T/F. If L is regular, then so is $sort(L) = \{ sort(x) : x \in L \}$.

F. Counter-example.

$L = L((01)^*) = \{ 01, 010101,$

$sort(010101) = 000111$

$sort(L((01)^*)) = \{ 0^n 1^n : n \in \mathbb{N} \}$

$a^n b^n$.

Midterm - Fri October 13^{th} 6:05PM - 7:25PM

{ LEA 219 -
ADAMS AUDIT

- Bring ID
- Allowed 1 page (one-side) cheat sheet - 12 point

↳ Post Ed announcement for who goes where

hand-written

This is 12 point

this is not 12 point

- I will check!

5 questions

long-answer
{
1 Design FA → DFA + Min (10)
→ NFA (use non-det)
2 Rex $\leq$ a b c d → NFA + ε (10)
3 Use M-N / PL to prove non-reg. (10)
4 Decide whether L is REG → FA → PL (10)
}
5 T/F (5) Lec 2 - Lec 11 (10)

I will post a midterm review guide
(Today / Tmrw)

1. What to Study
2. What AQs to look at
3. Extra practice problems

Fri – Wed Reading Week => No OHs TAs
and from me

Extra OHs – Thur TBD
Fri TBD ~ Polls!