

Theory of Computation

Tutorial 3 - DFAs

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Plan for today

1. Introduction to DFAs
2. Regular Languages

Introduction to DFAs

Formal definition of a DFA

Defintion. A deterministic finite automaton (DFA) M is a 5 element tuple $M = (Q, \Sigma, \delta, q_0, F)$ where

Q is the set of all states

Σ is the alphabet

δ is the transition function $\delta : Q \times \Sigma \rightarrow Q$

q_0 is the (unique) initial state

F is the set of final states

A DFA is a machine that takes as input a string and returns either an accept or a reject.

DFAs as language accepters

Consider a DFA M

Definition. The language $L(M)$ includes all strings (over the alphabet Σ) accepted by M

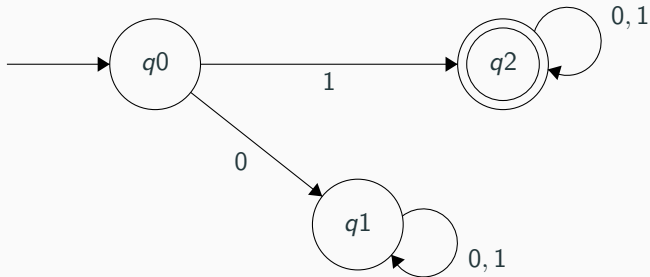
$L(M) = \{\text{strings that drive } M \text{ to a final state}\}$

Which is defined formally as follows:

$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$, where δ^* is extended transition function $\delta^* : Q \times \Sigma^* \rightarrow Q$.

Example

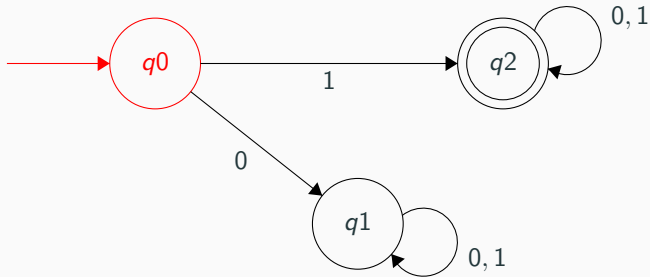
The following is a DFA \mathbf{M} such that $\mathbf{L}(\mathbf{M}) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$ for $\Sigma = \{0,1\}$.



Example - Tracing input

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

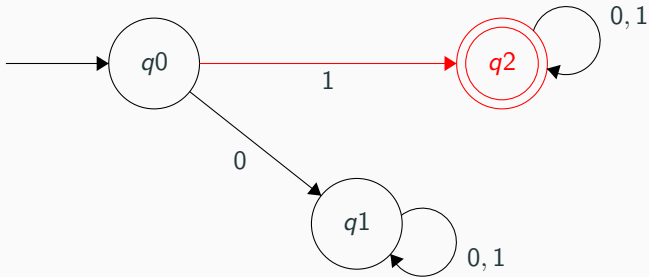
Input String: $\hat{1}01$



Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

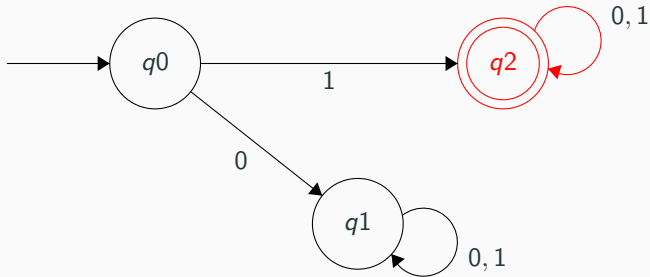
Input String: **1**01



Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

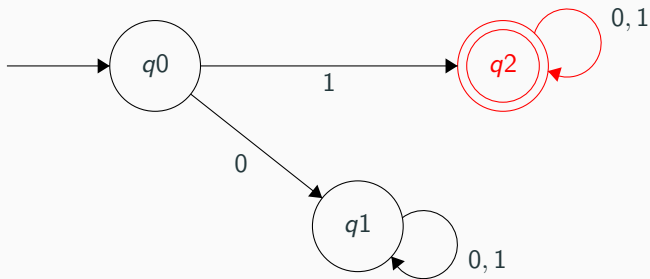
Input String: **1**0**1**



Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

Input String: 10**1**

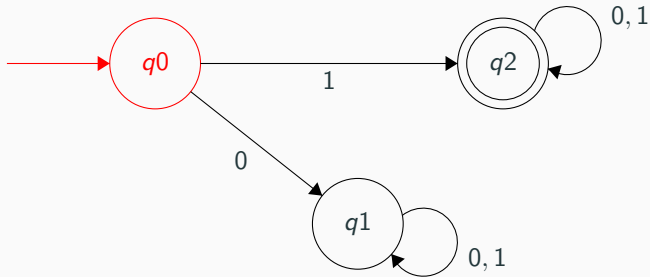


The input finishes in a final state, **M** accepts.

Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

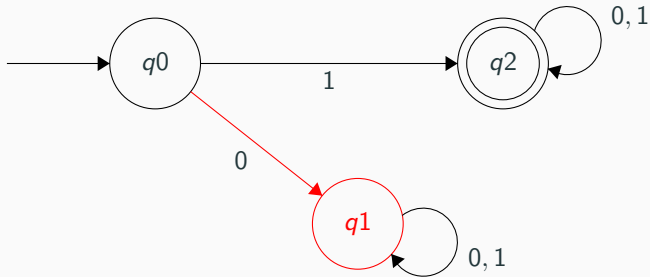
Input String: $\hat{0}10$



Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

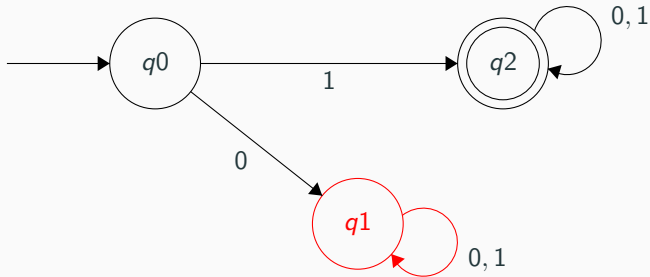
Input String: **0**10



Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

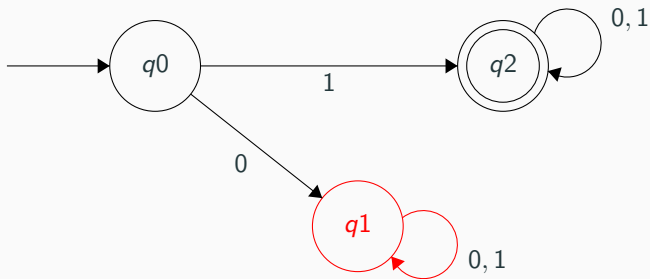
Input String: **0**1**0**



Example

$L(M) = \{w \in \{0,1\}^* : w \text{ starts with a } 1\}$

Input String: 01**0**



The input does not end in a final state, **M** rejects.

Example

Example. Create a DFA that accepts the language $L = \{w \in \{0,1\}^* : w \text{ contains } 00 \text{ as a substring}\}$.

Regular Languages

Defintion. A language L is regular if there exists a DFA M such that $L(M) = L$. One way to show that a language L is regular is to show there is a DFA M that accepts it.

Example

Example. Show that the language

$L = \{a^n : n \text{ is a multiple of 2 but not of 3}\}$ ($\Sigma = \{a\}$) is regular.

To be continued...

Thank you!