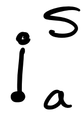
Proof of the PT Lemma

BC $n = 1$



$w = a$

$|w| = 1 = 2^0 \leq 2^{1-1}$ ✓

IH For some $n \in \mathbb{N}$, $\forall K$ $1 \leq K \leq n$, if the depth of $T_{G,w}$ is K then $|w| \leq 2^{K-1}$

IS Show for $n+1$.

$T_{G,w}$

← Both must be variables

Consider $T_{G,w}$ rooted at A & B. Then the depth of each will be at most $(n+1)-1$ i.e

$T_{G,w}(A)$ will have depth between 1 & n

$T_{G,w}(B)$ ——————— " ——— 1 & n

By IH, yield of $T_{G,w}(A) \to w_A$ will have $|w_A| \leq 2^{n-1}$ & similarly $|w_B| \leq 2^{n-1}$

Since $w = w_A \cdot w_B$ $\Rightarrow |w| = |w_A| + |w_B|$

$$\leq 2^{n-1} + 2^{n-1}$$

$$= 2^n = 2^{(n+1)-1}$$ ✓