Decision Tree Assignment

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Contents

Introduction

DM Systems has agreed to supply 500,000 Voip Phones to DISCO Stores in 90 days at fixed price. A key component in the phones is a programmable array logic integrated circuit chip ("PAL chip"), one of which is required in each phone.

DM Systems has bought these chips in the past from an Italian chip manufacturer, IM Chips. However, DM Systems has been approached by a Korean manufacturer, KR Electronics, which is offering a lower price on the chips. This offer is open for only 10 days, and DM Systems must decide whether to buy some or all of the PAL chips from KR. Any chips that DM does not buy from KR will be bought from IM. IM Chips will sell PAL chips to DM for $3.00 per chip in any quantity. KR will accept orders only in multiples of 250,000 PAL chips, and is offering to sell the chips for $2.00 per chip for 250,000 chips, and for $1.50 per chip in quantities of 500,000 or more chips.

The dumping charge

The situation is complicated by a dumping charge that has been filed by IM Chips against KR. If this charge is upheld by the Italian government, then the KR chips will be subject to an antidumping tax. This case will not be resolved until after the point in time when DM must make the purchase decision. If DM buys the KR chips, these will not be shipped until after the antidumping tax would go into effect and the chips would be subject to the tax. Under the terms offered by KR, DM would have to pay any antidumping tax that is imposed. DM believes there is a 60% chance the antidumping tax will be imposed. If it is imposed, then it is equally likely that the tax will be 50%, 100%, or 200% of the sale price for each PAL chip.

The decision tree

Draw a decision tree for this decision.

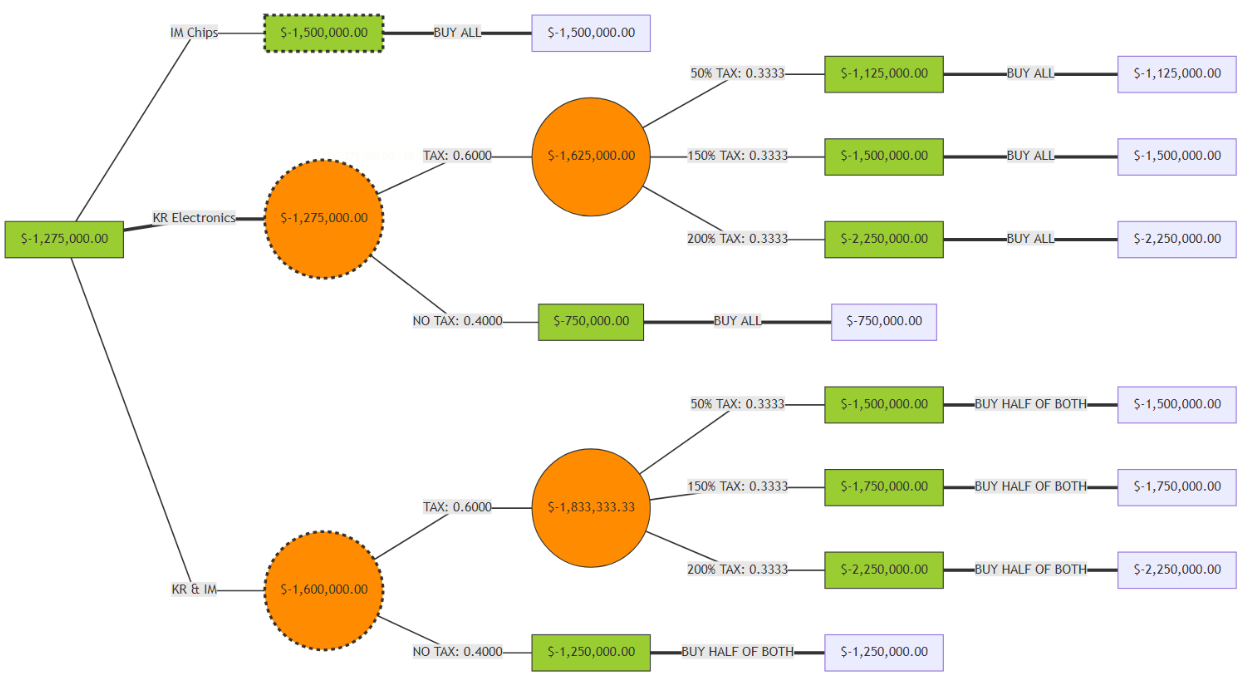
Solution

A decision tree is a tool used to understand how to behave in uncertain context and how to represent different alternative decisions.

Let's import the needed library and then represent the decision tree.

```r
library(yaml)
library(radiant)
library(radiant.model)
library(pander)
```

```r
tree <- yaml.load_file(input = 'dtree.yaml')
results <- dtree(yl=tree)
```



The first node on the left represent the **root node**, all the other green nodes represent the **decision nodes** where all their branches represent all the possible alternative. It's important to notice that from all alternative only one will be selected. The last type of node, the orange circolar one, represent the **chance node** and the number shown in each branch represent the probability that the outcome will occur. The right end of each path through the tree is called endpoint which represents the final outcome of that branch (from the root node and his own endpoint).

Expected value

Using expected value as the decision criterion, determine DM's preferred ordering alternative for the PAL chips.

Solution

After having drawn a decision three a decision criterion is needed to evaluate which alternative should be selected. The decision criterion will takes into account every possible outcome of the tree. In this problem will be used the **expected value**. Since the goal is to minimize the costs the sign will be set to negative. The expected value is computed by multiplying each possible outcome of the uncertain alternative by its probability and then summing the results. To make things clearer let's consider the case where all the chips are bought from KR Electronics in the case where the antiump tax is imposed:
The expected value will be:

$$EV(KR|TAX) = -1125000 * \frac{1}{3} - 1500000 * \frac{1}{3} - 2250000 * \frac{1}{3} = -1625000$$

As can be seen all the expected values are shown into each node of the tree plot, and after having compute all the expected value for each branch the one with the lower value (cost) should be selected and so, in this case, the best decision will be to buy all chips from KR electronics.
The expected value is a good measure over the long run since it represents the average amount that is expected to pay by selecting the alternative.

With the following code is presented how the expeceted value for each branch is manually computed:

```
p_tax <- 0.6
p_each_tax <- 1/3
taxes <- c(1.5, 2, 3)
n <- 500000

im <- n*3

KR_no_tax <- n*1.5
EV_tax_KR<-0
cost_KR <- c()
for(i in 1:3){
  cost_KR[i] <- n*taxes[i]*1.5
  EV_tax_KR <- EV_tax_KR + cost_KR[i]*p_each_tax
}

krim_no_tax <- n/2*2+n/2*3
EV_tax_krim <- 0
cost_krim<-c()
for(i in 1:3){
  cost_krim[i] <- n/2*taxes[i]*2+im/2
  EV_tax_krim <- EV_tax_krim + cost_krim[i]*p_each_tax
}

krim <- (EV_tax_krim*p_tax+krim_no_tax*(1-p_tax))
kr <- (EV_tax_KR*p_tax+KR_no_tax*(1-p_tax))

pander(c('KR & IM'=krim,'KR'=kr,'IM'=im))
```

KR & IM	KR	IM
1600000	1275000	1500000

As can be seen from the three alternatives the one taht should be selected is the one with the lowest value. The process of successively calculating expected values from the endpoints of the decision tree to the root node, as demonstrated in this section, is called a decision tree rollback.

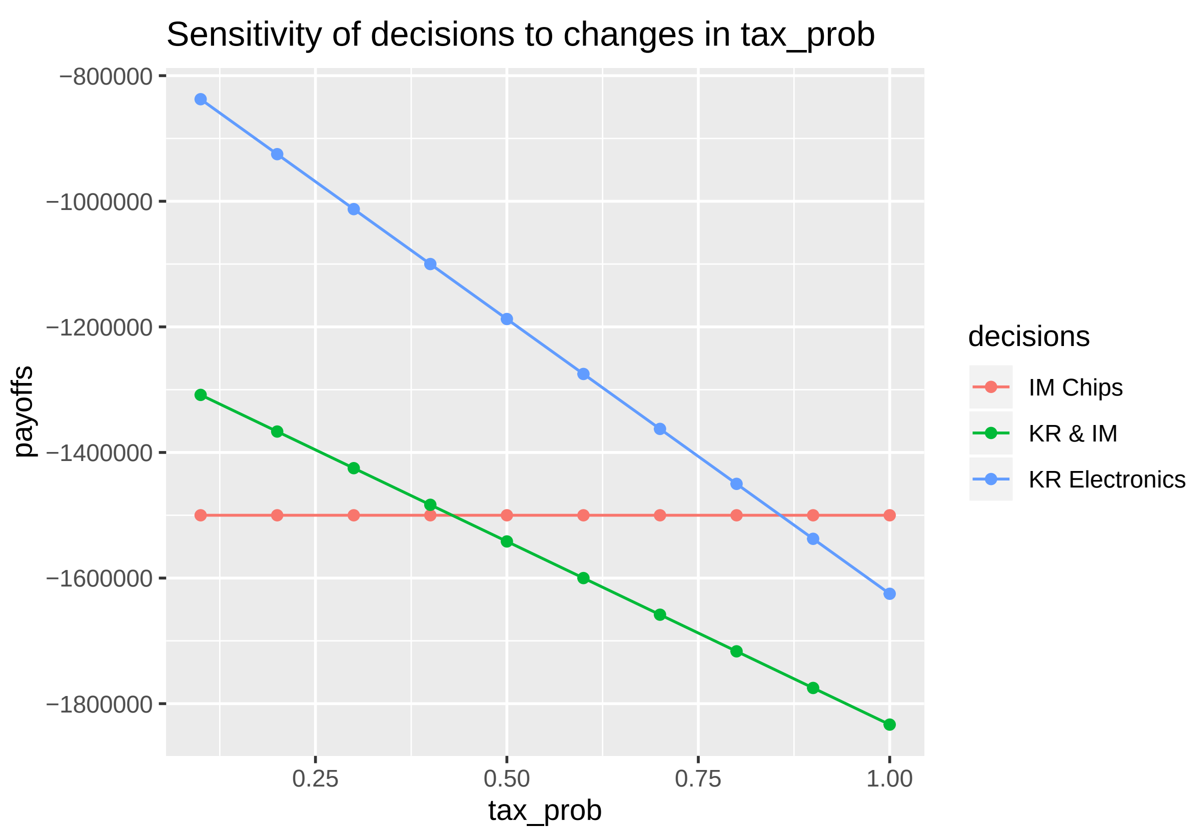
Sensitivity analysis

Perform a sensitivity analysis to evaluate the effect of uncertainties in the process of developing the Decision Tree. In particular consider at least the effect of mistakenly assessing the antidumping tax likelihood.

Solution

In reality, however, it is extremely difficult to define precisely the probability of a certain event. For this reason a sensitivity analysis must be conducted to understand how the decision would vary with one or more uncertain parameters. In this case the goal is to calculate how the expected costs can vary with respect to changes in the probability regarding if the antidump tax will be imposed. let's consider when this probability ranges from 0.1 to 1 as presented in the below image.

```
sensitivity(
  results,
  vars ="tax_prob 0.1 1 0.1;",
  decs=c("IM Chips","KR Electronics","KR & IM"),
  custom = F)
```



Indeed it appear clear that The expected costs for the IM company is constant and is indipendet from the tax while, as expected, an higher probability that the tax will be imposed drive to an augment of costs for both alternatives. Clearly the value where the tax probability is set to 60% are the same presented in the tree.

Utility Function and Certainty Equivalent

Assume that all the information in that exercise still holds, and that DM has an exponential utility function with a risk tolerance of $750,000. Determine DM's preferred ordering alternative using this utility function.

Solution

The utility function can be used as a decision criterion: convert the possible outcomes in a decision problem to utilities using the utility function, and then calculate the expected value of these utilities for each alternative.
For risk averse decision makers is often appropriate to use the following exponential utility function:

$$U(x) = 1 - e^{\frac{x}{R}}$$

In this exercise x represent the costs seen in the tree above and R represents the degree of risk aversion. As R becomes larger the utility function displays less risk aversion.

```
exp_utility <- function(x, R){
  res <- 1-exp(x/R)
  return(res)}
```

Let's now create a data frame containing all possible costs for each branch and then, thanks to some function provided during the exercise session, the expected utilities and the certainty equivalents will be computed.

```
profit <- data.frame(IM=c(1500000, 0, 0, 0),
                     KR=c(1125000,1500000,2250000,750000),
                     'IM|KR'=c(1500000,1750000,2250000,1250000),
                     'KR|TAX'=c(1125000,1500000,2250000, 0),
                     'IM&KR|TAX'=c(1500000,1750000,2250000, 0))
pander(profit)
```

IM	KR	IM.KR	KR.TAX	IM.KR.TAX
1500000	1125000	1500000	1125000	1500000
0	1500000	1750000	1500000	1750000
0	2250000	2250000	2250000	2250000
0	750000	1250000	0	0

```
CalcExpectedUtilityFunction <- function(profit, R){
  UF1 <- exp_utility(profit$IM, R)[1]
  #####
  UF2 <- exp_utility(profit$KR, R)
  UF2_A <- 1/3*(UF2[1]+UF2[2]+UF2[3])
  UF2_B <- min(UF2_A, UF2[4])
  UF2_C <- 0.6*UF2_B+0.4*UF2[4]
  #####
  UF3 <- exp_utility(profit$IM.KR, R)
  UF3_A <- 1/3*(UF3[1]+UF3[2]+UF3[3])
  UF3_B <- min(UF3_A,UF3[4])
  UF3_C <- 0.6*UF3_B+0.4*UF3[4]
  #####
  UF4 <- exp_utility(profit$KR.TAX,R)
```

```
  UF4_A <- 1/3*(UF4[1]+UF4[2]+UF4[3])
  #####
  UF5 <- exp_utility(profit$IM.KR.TAX,R)
  UF5_A <- 1/3*(UF5[1]+UF5[2]+UF5[3])
  return (c(IM=UF1, KR=UF2_C, 'IM&KR'=UF3_C,'KR|TAX'=UF4_A,'IM&KR|TAX'=UF5_A))
}
```

```
pander(CalcExpectedUtilityFunction(profit, 750000))
```

IM	KR	IM&KR	KR|TAX	IM&KR|TAX
-6.389	-6.479	-8.675	-9.652	-11.6

Since the goal is to minimize the costs lower utility values are better: a risk averse decision maker will now chose to buy all chips from IM Chips, even if the utility values for KR electronics and IM Chips are very close.

The Certainty Equivalent funtion used with this utility function is:

$$CE = R - ln(1 - E[U])$$

```
CertEquivalent = function(EU, R){
  CE = R*ln(1-EU)
  return(CE)
}

CalcBranchCE = function(profit, R){
  CE_vett = CertEquivalent(CalcExpectedUtilityFunction(profit, R), R)
  return(CE_vett[1:3])
}
```

```
pander(CalcBranchCE(profit,750000))
```

IM	KR	IM&KR
1500000	1509031	1702172

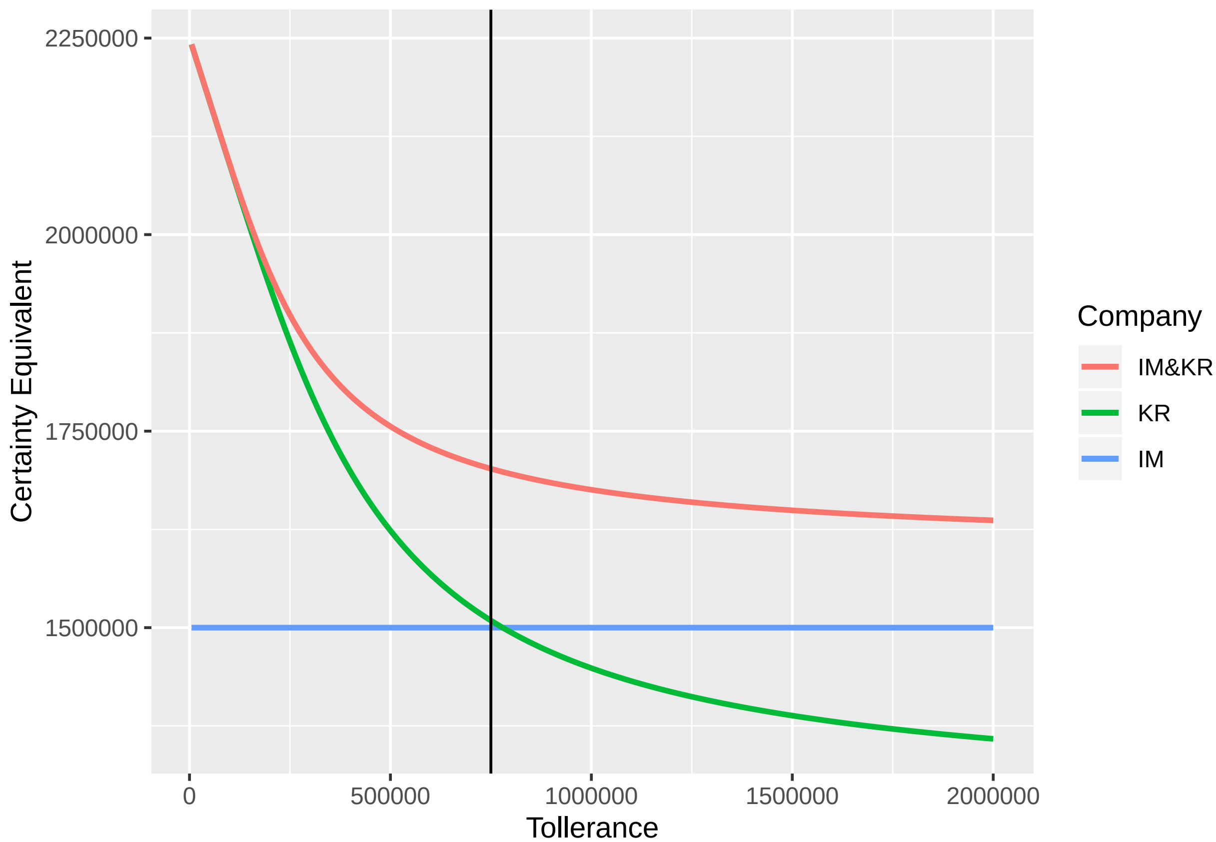
Alternatives with smaller certainty equivalents are more preferred. In this case, since a certainty equivalent is the certain amount that is equally preferred to an alternative then, with respect of the expected utility, the decision maker will prefer to buy all chips from IM Chips.

```
R <- seq(0,2000000,5000)
res1 = c()
res2 = c()
res3 = c()
for (i in 1:length(R)) {
  vRes = CalcBranchCE(profit, R[i])
  res1[i] = vRes[1]
  res2[i] = vRes[2]
  res3[i] = vRes[3]
}
```

In the below plot is shown how the certainty equivalent can vary with respect of the tollerance level R: considering the case where $R = 750000$ as seen above the values of KR electronics and IM Chips are very close and by increasing R those will diverge and the certainty equivalent with respect of KR Electronics will converge around the value $1.250.000$$ but with an higher level of risk. Considering the case where half chips are bought from each company is easy to see that the certainty equivalent will always be highest between all alternatives.

```
library(ggplot2)
prova <- data.frame(res1,res2,res3,R)[-1,]

ggplot(data=prova,aes(x=R)) + geom_line(aes(y=res1, colour='red'), size=1) +
  geom_line(aes(y=res2, colour='green'), size=1) +
  geom_line(aes(y=res3, colour='blue'), size=1) +
  scale_color_discrete(name = "Company", labels = c("IM&KR", "KR",'IM'))+
  xlab("Tollerance") +
  ylab("Certainty Equivalent")+ geom_vline(xintercept=750000)
```



KR revises the offer

In an effort to attract DM's order, KR Electronics has revised its offer as follows: At no increase in price, KR will now provide DM with the right to cancel its entire order for a 10% fee after the outcome of the antidumping suit is known. However, KR will not be able to accept any additional orders from DM once the outcome of the suit is known. Thus, for example, if DM has agreed to purchase 250,000 PAL chips from KR at $2.00 per chip, DM can cancel the order by paying $50,000. This ability to cancel the order is potentially of interest to DM because it knows that IM Chips would be able to supply PAL chips after the outcome of the antidumping suit is known in time for DM to fill the DISCO order.

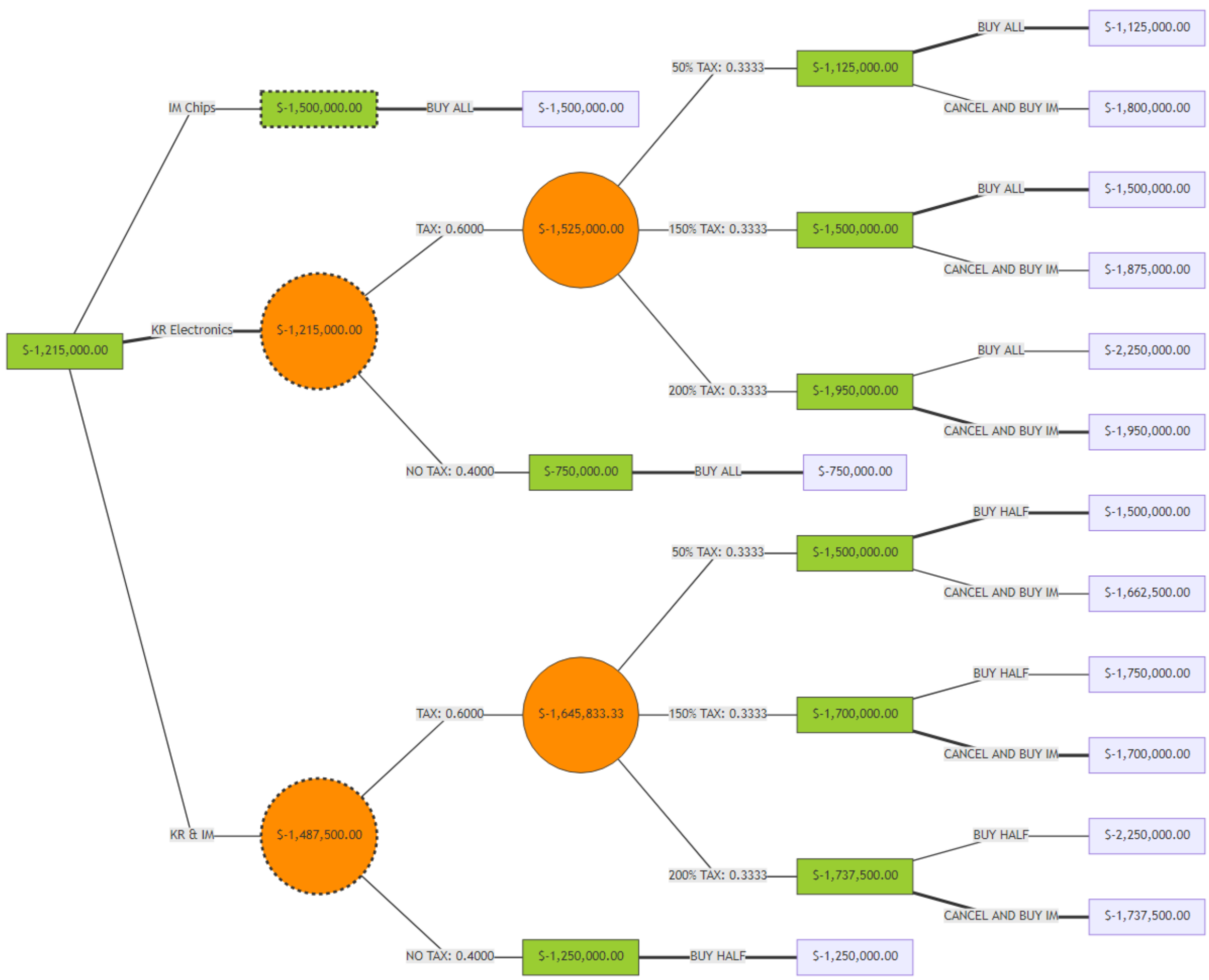
However, DM knows that IM will increase the price of its chips if an antidumping tax is imposed. In particular, if a 50% tax is imposed, then IM will increase its chip price by 15%. If a 100% tax is imposed, then IM will increase its chip price by 20%. Finally, if a 200% tax is imposed, then IM will increase its chip price by 25%.

Assuming that all other information is still valid, determine DM's preferred alternative for the initial order of PAL chips as well as what DM should do if the antidumping tax is imposed.

Solution

Let's introduce those changes into the previous decision tree.

```
tree = yaml.load_file(input = 'dtree2.yaml')
results = dtree(yl=tree)
```



According to the expected value the DM will still prefer to buy all chips from KR electronics. It's easy to see that the possibility of cancelling the order from KR electronincs by paying a small fee lead the DM to a decreased expected cost since, for example, is now more convenient to cancel the order if all chips are bought from KR electronics and a 200% tax will be imposed: in this case there is a save of \$300.000\$.

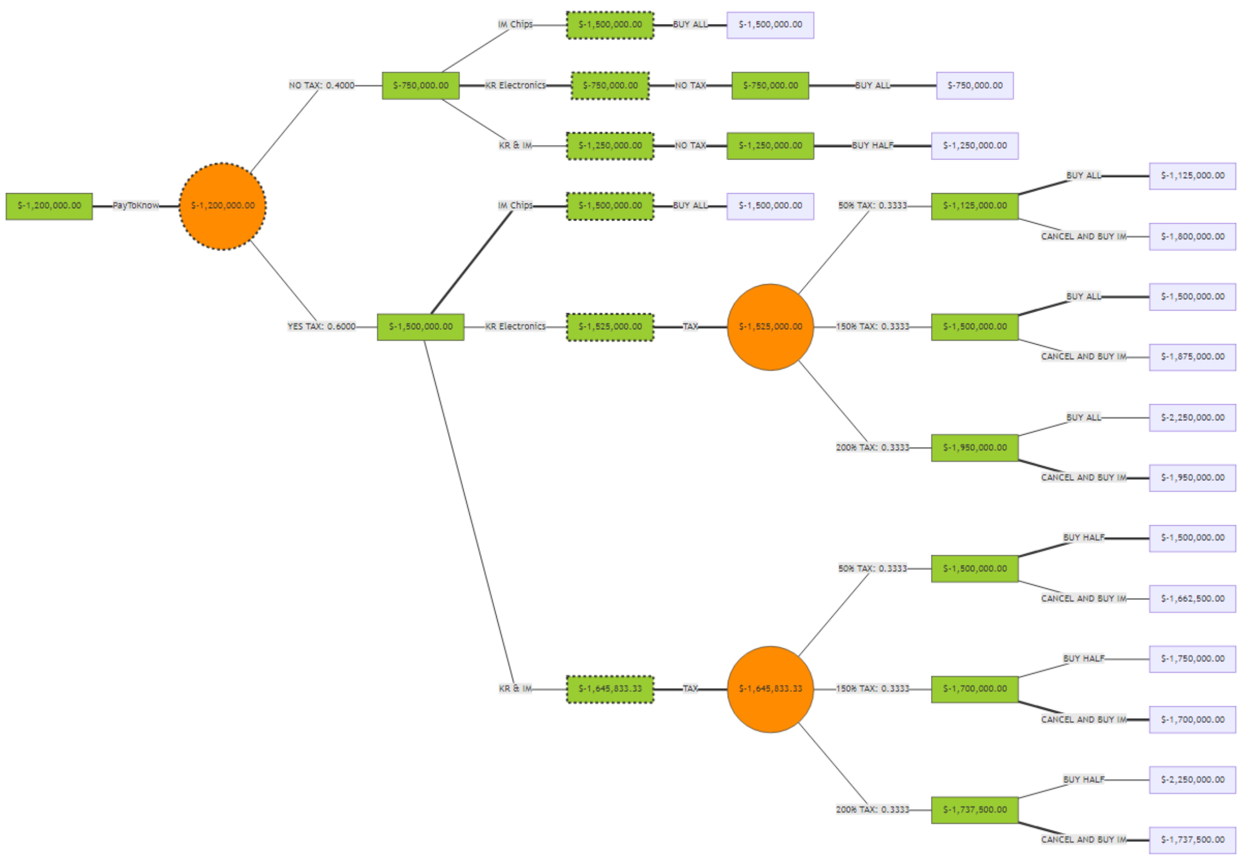
Value of Information

Assume that all the information presented still holds. Using the expected value as the decision criterion, determine the maximum amount that DM should pay for information about whether the antidumping tax will be imposed. Suppose this information can be obtained prior to making the ordering decision.

Solution

A new decision tree is needed to analyze the cost of perfect information: the root node is the **Pay to Know** node representing the expected cost computed whether the tax will be imposed or not.

```
tree = yaml.load_file(input = 'dtree3.yaml')
results = dtree(yl=tree)
```



It's easy to see that, if the tax will not be imposed, the best option will be to buy all chips from KR electronics, while if the tax will be imposed the best option will be to buy all chips from IM Chips. Those are the only options that will be taken into account: indeed the expected value will be computed between those two values $[1.500.000;\ 750.000]$.

$$EV = 0.6 * 1500000 + 0.4 * 750000 = 1200000$$

So the expected cost when the perfect information is available is $1.200.000$\$, in fact the cost of the perfect information will be:

- $1.275.000 - 1.200.000 = 75.000$\$ in the case where KR electronics will not introduce the possibility of cancelling the order.
- $1.215.000 - 1.200.000 = 15.000$\$ in the case where KR electronics will introduce the possibility of cancelling the order.