Code documentation

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1 New approach (v2):

1.1 Counting parameters

$$\Delta\overline{\ln S^e_{ijet}} = \frac{\sigma_j}{\sigma_j - 1}\left(\sum_k \theta_{ke}\overline{S_{kejt}}\Delta \ln A_{kjt} - \Delta \ln A_{ijt}\right) + \varepsilon_{iej}$$

We have $12JT$ observations, and we need to identify $4JT$ π parameters, and 12θ. So, my take on this is that we have the right number of observations, but it is unclear to me what is identifying what.

1.2 Model equations

At the job level it holds that:

$$\Delta\overline{\ln S^e_{ijet}} = \frac{\sigma_j}{\sigma_j - 1}\left(\sum_k \theta_{ke}\overline{S_{kejt}}\Delta \ln A_{kjt} - \Delta \ln A_{ijt}\right) + \varepsilon_{iej} \quad (1)$$

$$1 = \sum_k \theta^e_k \overline{S_{keJt}} \quad (2)$$

$$(3)$$

1.3 Building GMM

In our data we have 3 education levels, 4 skills, J jobs and T periods. We normalize $\Delta \ln A_{4jt} = 0, \forall j, t$

- **Equation (1)**: instruments for this equation are skills by job matrices. More specifically, let x_{ejt} be the row vector $\begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} \end{pmatrix}$. Define the matrix X_{ejt} as the 4×3

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matrix containing the skill indexes by education. This matrix excludes the reference skill. For simplicity, I have ignored the bars in the notation.

$$X_{ejt} = \begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} \end{pmatrix} = \begin{pmatrix} x_{ejt} \\ x_{ejt} \\ x_{ejt} \\ x_{ejt} \end{pmatrix}$$

Let X_e be the block-diagonal matrix with X_{ejt} in the diagonal.

$$X_e = \begin{pmatrix} X_{e11} & \mathbf{0} & \dots & 0 \\ \mathbf{0} & X_{e12} & \dots & 0 \\ \vdots & & & \\ \mathbf{0} & \mathbf{0} & \dots & X_{eJT} \end{pmatrix}$$

The set of instruments for equation 1 is then:

$$Z_1 = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \tag{4}$$

- **Equation** (2): instruments for the sum to restriction are the skill indexes by education level. Let $\tilde{x}_{ejt}$ be the row vector $\begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \end{pmatrix}$ and let $\tilde{X}_e$ be the $N_e \times 4$ matrix stacking all the $\tilde{x}_{ejt}$ for education level e. The instruments for equation 2 are the block diagonal matrix with elements $\tilde{X}_e$:

$$Z_2 = \begin{pmatrix} \tilde{X}_1 & 0 & 0 \\ 0 & \tilde{X}_2 & 0 \\ 0 & 0 & \tilde{X}_3 \end{pmatrix}$$

- **Putting it all together:** let u_l be the stacked vector of equation l errors. Define:

$$Z = \begin{pmatrix} Z_1 & 0 \\ 0 & Z_2 \end{pmatrix}$$

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

the moment equations form the model are given by $\mathbb{E}\left(Z'u\right) = 0$.

I choose parameters θ to minimize the quadratic form:

$$\min_{\theta} \frac{1}{N} \left(u'Z'(Z'Z)^{-1}Zu\right) \tag{5}$$