Code documentation

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1 New approach (v2):

1.1 Model equations

At the job level it holds that:

$$\Delta\overline{\ln S^e_{iJet}} = \frac{\sigma_J}{\sigma_J - 1}\left(\sum_k \theta^e_k \overline{S_{keJt}} \Delta \ln A_{kJt} - \Delta \ln A_{iJt}\right) \tag{1}$$

$$1 = \sum_k \theta^e_k \overline{S_{keJt}} \tag{2}$$

$$\Delta\left[\ln \frac{q_{eJt}}{q_{e'Jt}}\right] = \sum_k \left(\theta^e_k \overline{S_{kJet}} - \theta^{e'}_k \overline{S_{kJe't}}\right) \Delta \ln A_{kjt} + const_{e,e't} \tag{3}$$

1.2 Building GMM

In our data we have 3 education levels, 4 skills, J jobs and T periods.

- **Equation 1**: Let x_{ejt} be the row vector $\begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt}\end{pmatrix}$. Define the matrix X_{ejt} as the 4×4 matrix containing the skill indexes by education. For simplicity, I have ignored the bars in the notation.

$$X_{ejt} = \begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \end{pmatrix} = \begin{pmatrix} x_{ejt} \\ x_{ejt} \\ x_{ejt} \\ x_{ejt} \end{pmatrix}$$

  Let X be the matrix stacking all the X_{ejt} in some order that for now is irrelevant. The only thing that matters is that I use the same order across all definitions. This matrix is of dimension $N \times 4$.

$$X = \begin{pmatrix} X_{111} \\ \vdots \\ X_{3JT} \end{pmatrix}$$

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Let I_e be the $N \times 4$ matrix with columns given by education level e dummies:

$$I_e = \begin{pmatrix} \iota_e & \iota_e & \iota_e & \iota_e \end{pmatrix}$$

Finally let D be the a matrix made of job by year dummies. This matrix has N rows and $J \times T$ columns. The set of instruments for equation 1 is then:

$$Z_1 = \begin{pmatrix} I_1 \odot X & I_2 \odot X & I_3 \odot X & D \end{pmatrix} \tag{4}$$

- **Equation 2**: let $\tilde{X}_e$ be the $N_e \times 4$ matrix stacking all the x_{ejt} for education level e. The instruments for equation 2 are the block diagonal matrix with elements $\tilde{X}_e$:

$$Z_2 = \begin{pmatrix} \tilde{X}_1 & 0 & 0 \\ 0 & \tilde{X}_2 & 0 \\ 0 & 0 & \tilde{X}_3 \end{pmatrix}$$

- **Equation 3:** let $\bar{X}$ be the matrix stacking the row vectors $\begin{pmatrix} x_{1jt} & x_{2jt} & x_{3jt} \end{pmatrix}$ you ok with this?. Let C be the matrix containing education-pair×T dummies. Instruments for equation 3 are:

$$Z_3 = \begin{pmatrix} \bar{X} & C \end{pmatrix} \tag{5}$$

- **Putting it all together:** let u_l be the stacked (in the appropriate order) vector of equation l errors. Define:

$$Z' = \begin{pmatrix} Z_1' & 0 & 0 \\ 0 & Z_2' & 0 \\ 0 & 0 & Z_3' \end{pmatrix}$$

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

the moment equations form the model are given by $\mathbb{E}\left(Z'u\right) = 0$.