Code documentation

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1 New approach (v2):

1.1 Counting parameters

$$\Delta\overline{\ln S^{e}_{ijet}} = \frac{\sigma_j}{\sigma_j - 1}\left(\sum_k \theta_{ke}\overline{S_{kejt}}\Delta \ln A_{kjt} - \Delta \ln A_{ijt}\right) + \varepsilon_{iej}$$

We have $12JT$ observations, and we need to identify $3JT$ π parameters, and 12θ.

1.2 Model equations

Let $\pi_{kjt} = \frac{\sigma_j}{\sigma_j-1}\Delta \ln A_{kjt}$ and $\beta_j = \frac{\sigma_j-1}{\sigma_j}$. At the job level it holds that:

$$\begin{aligned}
\Delta\overline{\ln S^{e}_{ijet}} &= \sum_k \theta_{ke}\overline{S_{kejt}}\pi_{kjt} - \pi_{ijt} \\
1 &= \sum_k \theta^{e}_{k}\overline{S_{keJt}} \\
\Delta\left[\ln \frac{q_{eJt}}{q_{e'Jt}}\right] &= \beta_j\left[\sum_k \left(\theta^{e}_{k}\overline{S_{kJet}} - \theta^{e'}_{k}\overline{S_{kJe't}}\right)\pi_{kjt}\right] + const_{e,e't}
\end{aligned}$$

1.3 Building GMM

In our data we have 3 education levels, 4 skills, J jobs and T periods. We normalize $\Delta \ln A_{4jt} = 0, \forall j, t$

- **Equation (1)**: instruments for this equation are the average skill of the other education group in that job, and skill-job-time dummies.

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- **Putting it all together:** let u_l be the stacked vector of equation l errors. Define:

$$
\begin{aligned}
Z &= \begin{pmatrix} Z_1 & 0 \\ 0 & Z_2 \end{pmatrix} \\
u &= \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}
\end{aligned}
$$

the moment equations form the model are given by $\mathbb{E}\left(Z'u\right) = 0$.

I choose parameters θ to minimize the quadratic form:

$$\min_{\theta} \frac{1}{N}\left(u'Z(Z'Z)^{-1}Z'u\right) \tag{1}$$

1.4 Calculating the gradient

Reminder:

$$\frac{\partial Q}{\partial x} = x'(A + A')$$

I can write the quadratic from as:

$$Q(\gamma) = \frac{1}{N}\left(u(\gamma)'Z(Z'Z)^{-1}Z'u(\gamma)\right)$$

then,

$$\frac{\partial Q(\gamma)}{\partial \gamma} = \frac{2}{N}\left(\frac{\partial u(\gamma)}{\partial \gamma}\right)' Z(Z'Z)^{-1}Zu(\gamma)$$

now, I start element by element:

$$
\begin{aligned}
\frac{\partial \Delta \ln S_{iejt}}{\partial \ln A_{kjt}} &= \theta_{ke}S_{kejt} \\
\frac{\partial \Delta \ln S_{iejt}}{\partial \ln A_{ijt}} &= \theta_{ie}S_{iejt} - 1 \\
\frac{\partial \Delta \ln S_{iejt}}{\partial \ln \theta_{ke}} &= S_{kjt}\Delta \ln A_{kjt}
\end{aligned}
$$

for the sum to 1 restrictions we have:

$$\frac{\partial g_{ejt}(\gamma)}{\partial \theta_{ke}} = -S_{kejt}$$

next thing to do: write the gradient