

# Code documentation

César Garro-Marín\*

June 27, 2022

## 1 New approach (v2):

### 1.1 Model equations

At the job level it holds that:

$$\Delta \overline{\ln S_{iJt}^e} - \Delta \overline{\ln S_{kJt}^e} = \frac{\sigma_J}{\sigma_j - 1} (\Delta \ln A_{kJt} - \Delta \ln A_{iJt}) \quad (1)$$

$$1 = \sum_k \theta_k^e \overline{S_{kJt}^e} \quad (2)$$

$$\Delta \left[ \ln \frac{q_{eJt}}{q_{e'Jt}} \right] = \sum_k \left( \theta_k^e \overline{S_{kJt}^e} - \theta_k^{e'} \overline{S_{kJt}^{e'}} \right) \Delta \ln A_{kJt} + \text{const}_{e,e't} \quad (3)$$

**Note:** in equation (3) there is always a redundant pairwise comparison. Note that  $\Delta \left[ \ln \frac{q_{eJt}}{q_{e'Jt}} \right] - \Delta \left[ \ln \frac{q_{e'Jt}}{q_{e^*Jt}} \right] = \Delta \left[ \ln \frac{q_{eJt}}{q_{e^*Jt}} \right]$

### 1.2 Building GMM

In our data we have 3 education levels, 4 skills,  $J$  jobs and  $T$  periods.

- **Equation (1):** Let  $x_{ejt}$  be the row vector  $(S_{1ejt} \ S_{2ejt} \ S_{3ejt} \ S_{4ejt})$ . Define the matrix  $X_{ejt}$  as the  $4 \times 4$  matrix containing the skill indexes by education. For simplicity, I have ignored the bars in the notation.

$$X_{ejt} = \begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \end{pmatrix} = \begin{pmatrix} x_{ejt} \\ x_{ejt} \\ x_{ejt} \\ x_{ejt} \end{pmatrix}$$

Let  $X$  be the matrix stacking all the  $X_{ejt}$  in some order that for now is irrelevant. The only thing that matters is that I use the same order across all definitions. This matrix

---

\*Boston University, email: [cesarlgm@bu.edu](mailto:cesarlgm@bu.edu)

is of dimension  $N \times 4$ .

$$X = \begin{pmatrix} X_{111} \\ \vdots \\ X_{3JT} \end{pmatrix}$$

Let  $I_e$  be the  $N \times 4$  matrix with columns given by education level  $e$  dummies:

$$I_e = (\iota_e \ \iota_e \ \iota_e \ \iota_e)$$

The set of instruments for equation 1 is then:

$$Z_1 = (I_1 \odot X \ I_2 \odot X \ I_3 \odot X) \quad (4)$$

- **Equation (1):** set of instruments are a set of occupation by year dummies  $Z_2$ .
- **Equation (2):** let  $\tilde{X}_e$  be the  $N_e \times 4$  matrix stacking all the  $x_{ejt}$  for education level  $e$ . The instruments for equation 2 are the block diagonal matrix with elements  $\tilde{X}_e$ :

$$Z_3 = \begin{pmatrix} \tilde{X}_1 & 0 & 0 \\ 0 & \tilde{X}_2 & 0 \\ 0 & 0 & \tilde{X}_3 \end{pmatrix}$$

- **Equation (3)** let  $\bar{X}$  be the matrix stacking the row vectors  $(x_{1jt} \ x_{2jt} \ x_{3jt})$ . Let  $C$  be the matrix containing education-pair $\times$ T dummies. Instruments for equation 3 are:

$$Z_4 = (\bar{X} \ C) \quad (5)$$

- **Putting it all together:** let  $u_l$  be the stacked (in the appropriate order) vector of equation  $l$  errors. Define:

$$Z' = \begin{pmatrix} Z_1' & 0 & 0 & 0 \\ 0 & Z_2' & 0 & 0 \\ 0 & 0 & Z_3' & 0 \\ 0 & 0 & 0 & Z_4' \end{pmatrix}$$

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}$$

the moment equations from the model are given by  $\mathbb{E}(Z'u) = 0$ .

## 2 How did I write the code?

### 2.1 Basic definitions

$$\Lambda = \begin{pmatrix} \Delta \ln A_{111} \\ \Delta \ln A_{121} \\ \Delta \ln A_{131} \\ \vdots \\ \Delta \ln A_{4JT-2} \\ \Delta \ln A_{4JT-1} \\ \Delta \ln A_{4JT} \end{pmatrix} \quad \Theta = \begin{pmatrix} \theta_1^1 \\ \theta_2^1 \\ \theta_3^1 \\ \theta_4^1 \\ \vdots \\ \theta_1^3 \\ \theta_2^3 \\ \theta_3^3 \\ \theta_4^3 \end{pmatrix} \quad \Sigma = \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_J \end{pmatrix} \quad \Xi = \begin{pmatrix} \xi_{11} \\ \vdots \\ \xi_{3T} \end{pmatrix}$$

I defined the model parameter vector as

$$\Pi = \begin{pmatrix} \Lambda \\ \Theta \\ \Sigma \\ \Xi \end{pmatrix}$$

### 2.2 Equation (1)

1. Order observations by skill, education, occupation, and year.
2. I reorder  $\Lambda$  by skill, as follows:

$$\Lambda_1 = \begin{pmatrix} \Delta \ln A_{111} \\ \Delta \ln A_{121} \\ \vdots \\ \Delta \ln A_{1JT} \end{pmatrix}$$

indexes loop over year, job and education. The reordered  $\Lambda$  is given by:

$$\Lambda^r = \begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{pmatrix}$$

3. Create vector  $\tilde{\Sigma}$

$$\tilde{\Sigma} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_J \end{pmatrix}$$

this vector of is has the same dimensions as  $\Lambda^r$ . It places all elements of  $\Sigma$  in the “right” order **I should say what the right order is here**.

4. Define  $D$  as a matrix of dimensions  $(E \times J \times T) \times (J \times T)$ . Define  $Z_1$  as:

$$Z_1 = \begin{pmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{pmatrix}$$

This matrix contains occupation by year dummies. Equation (1) can be written as:

$$y_1 = Z_1 \times (\Lambda^r \odot \tilde{\Sigma})$$

where  $y_1$  was defined in the appropriate order.