Code documentation

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September 20, 2022

1 New approach (v2):

1.1 Counting parameters

$$\Delta\overline{\ln S^e_{ijet}} = \frac{\sigma_j}{\sigma_j - 1}\left(\sum_k \theta_{ke}\overline{S_{kejt}}\Delta\ln A_{kjt} - \Delta\ln A_{ijt}\right) + \varepsilon_{iej}$$

We have $12JT$ observations, and we need to identify $4JT$ π parameters, and 12θ. So, my take on this is that we have the right number of observations, but it is unclear to me what is identifying what.

1.2 Model equations

At the job level it holds that:

$$\Delta\overline{\ln S^e_{iJet}} - \Delta\overline{\ln S^e_{kJet}} = \frac{\sigma_J}{\sigma_j - 1}\left(\Delta\ln A_{kJt} - \Delta\ln A_{iJt}\right) \tag{1}$$

$$1 = \sum_k \theta^e_k\overline{S_{keJt}} \tag{2}$$

$$\Delta\left[\ln\frac{q_{eJt}}{q_{e'Jt}}\right] = \sum_k\left(\theta^e_k\overline{S_{kJet}} - \theta^{e'}_k\overline{S_{kJe't}}\right)\Delta\ln A_{kjt} + const_{e,e't} \tag{3}$$

Note: in equation (3) there is always a redundant pairwise comparison. Note that $\Delta\left[\ln\frac{q_{eJt}}{q_{e'Jt}}\right] - \Delta\left[\ln\frac{q_{e'Jt}}{q_{e^\star Jt}}\right] = \Delta\left[\ln\frac{q_{eJt}}{q_{e^\star Jt}}\right]$

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1.3 Building GMM

In our data we have 3 education levels, 4 skills, J jobs and T periods.

- **Equation (1)**: Let x_{ejt} be the row vector $\begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} \end{pmatrix}$. Define the matrix X_{ejt} as the 3×3 matrix containing the skill indexes by education. This matrix excludes the reference skill. For simplicity, I have ignored the bars in the notation.

$$X_{ejt} = \begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} \end{pmatrix} = \begin{pmatrix} x_{ejt} \\ x_{ejt} \\ x_{ejt} \end{pmatrix}$$

Let X be the matrix stacking all the X_{ejt} in some order that for now is irrelevant. The only thing that matters is that I use the same order across all definitions. This matrix is of dimension $N \times 4$.

$$X = \begin{pmatrix} X_{111} \\ \vdots \\ X_{3JT} \end{pmatrix}$$

Let I_e be the $N \times 4$ matrix with columns given by education level e dummies:

$$I_e = \begin{pmatrix} \iota_e & \iota_e & \iota_e & \iota_e \end{pmatrix}$$

The set of instruments for equation 1 is then:

$$Z_1 = \begin{pmatrix} I_1 \odot X & I_2 \odot X & I_3 \odot X \end{pmatrix} \tag{4}$$

- **Equation** (1): set of instruments are a set of occupation by year doummies Z_2.

- **Equation** (2): let $\tilde{X}_e$ be the $N_e \times 4$ matrix stacking all the x_{ejt} for education level e. The instruments for equation 2 are the block diagonal matrix with elements $\tilde{X}_e$:

$$Z_3 = \begin{pmatrix} \tilde{X}_1 & 0 & 0 \\ 0 & \tilde{X}_2 & 0 \\ 0 & 0 & \tilde{X}_3 \end{pmatrix}$$

- **Equation** (3) let $\bar{X}$ be the matrix stacking the row vectors $\begin{pmatrix} x_{1jt} & x_{2jt} & x_{3jt} \end{pmatrix}$. Let C be the matrix containing education-pair×T dummies. Instruments for equation 3 are:

$$Z_4 = \begin{pmatrix} \bar{X} & C \end{pmatrix} \tag{5}$$

- **Putting it all together:** let u_l be the stacked (in the appropriate order) vector of equation l errors. Define:

$$Z' = \begin{pmatrix} Z_1' & 0 & 0 & 0 \\ 0 & Z_2' & 0 & 0 \\ 0 & 0 & Z_3' & 0 \\ 0 & 0 & 0 & Z_4' \end{pmatrix}$$

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}$$

the moment equations form the model are given by $\mathbb{E}\left(Z'u\right) = 0$.

2 How did I write the code?

2.1 Basic definitions

$$\Lambda = \begin{pmatrix} \Delta \ln A_{111} \\ \Delta \ln A_{121} \\ \Delta \ln A_{131} \\ \vdots \\ \Delta \ln A_{3JT-2} \\ \Delta \ln A_{3JT-1} \\ \Delta \ln A_{3JT} \end{pmatrix} \Theta = \begin{pmatrix} \theta_1^1 \\ \theta_2^1 \\ \theta_3^1 \\ \theta_4^1 \\ \vdots \\ \theta_1^3 \\ \theta_2^3 \\ \theta_3^3 \\ \theta_4^3 \end{pmatrix} \Sigma = \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_J \end{pmatrix} \Xi = \begin{pmatrix} \xi_{11} \\ \vdots \\ \xi_{3T} \end{pmatrix}$$

I defined the model parameter vector as

$$\Pi = \begin{pmatrix} \Lambda \\ \Theta \\ \Sigma \\ \Xi \end{pmatrix}$$

2.2 Equation (1)

1. Order observations by skill, education, occupation, and year.
2. Create vector $\tilde{\Sigma}$

$$\tilde{\Sigma} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_J \end{pmatrix}$$

this vector of is has the same dimensions as Λ. It places all elements of Σ in the "right" order I should say what the right order is here.

3. Define D as a matrix of dimensions $(E \times J \times T) \times (J \times T)$. Define Z_1 as:

$$X_1 = \begin{pmatrix} -D & 0 & 0 \\ 0 & -D & 0 \\ 0 & 0 & -D \end{pmatrix}$$

This matrix contains occupation by year dummies. Equation (1) can be written as:

$$y_1 = X_1 \times (\Lambda \odot \tilde{\Sigma})$$

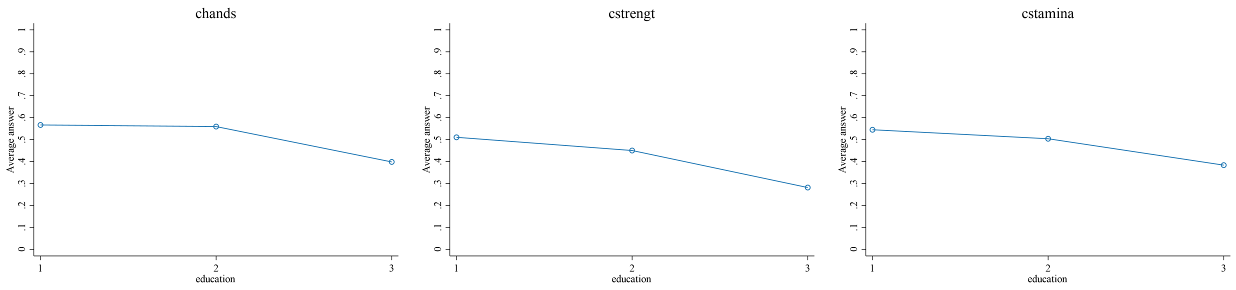
where y_1 was defined in the appropriate order.

3 Summary stats

3.1 Averages

Figure 1: Manual index variables

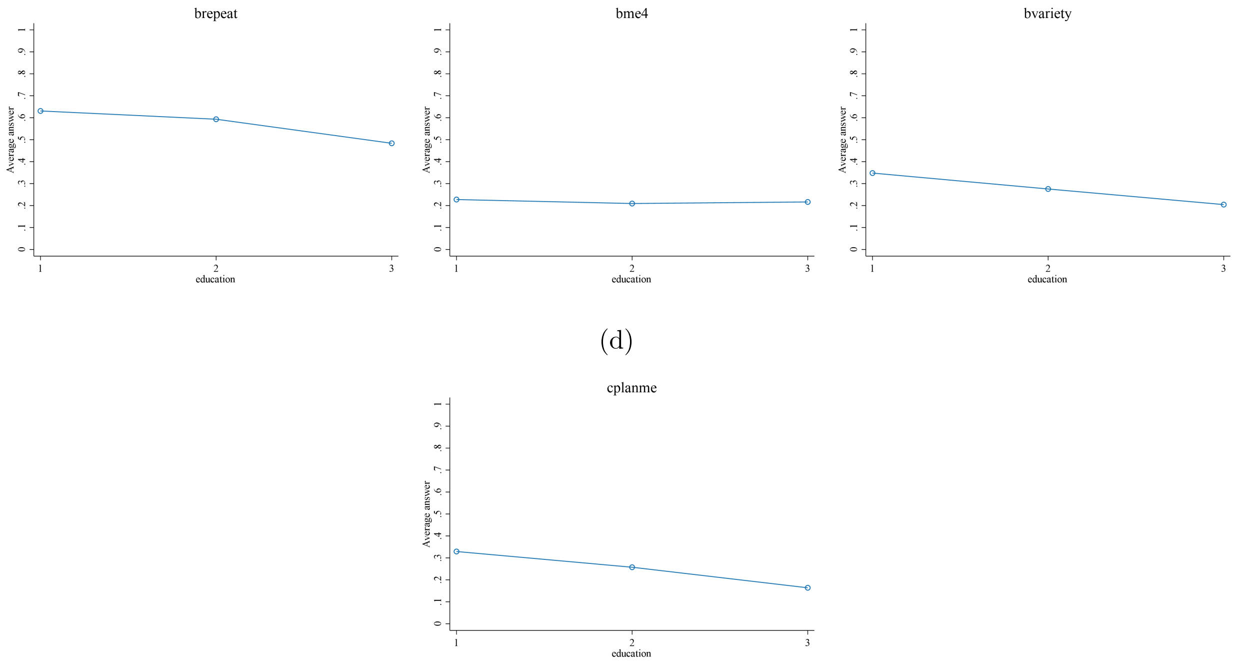
(a) (b) (c)



Note: figure note. Figure generated on 28 Jul 2022 at 10:26:01.

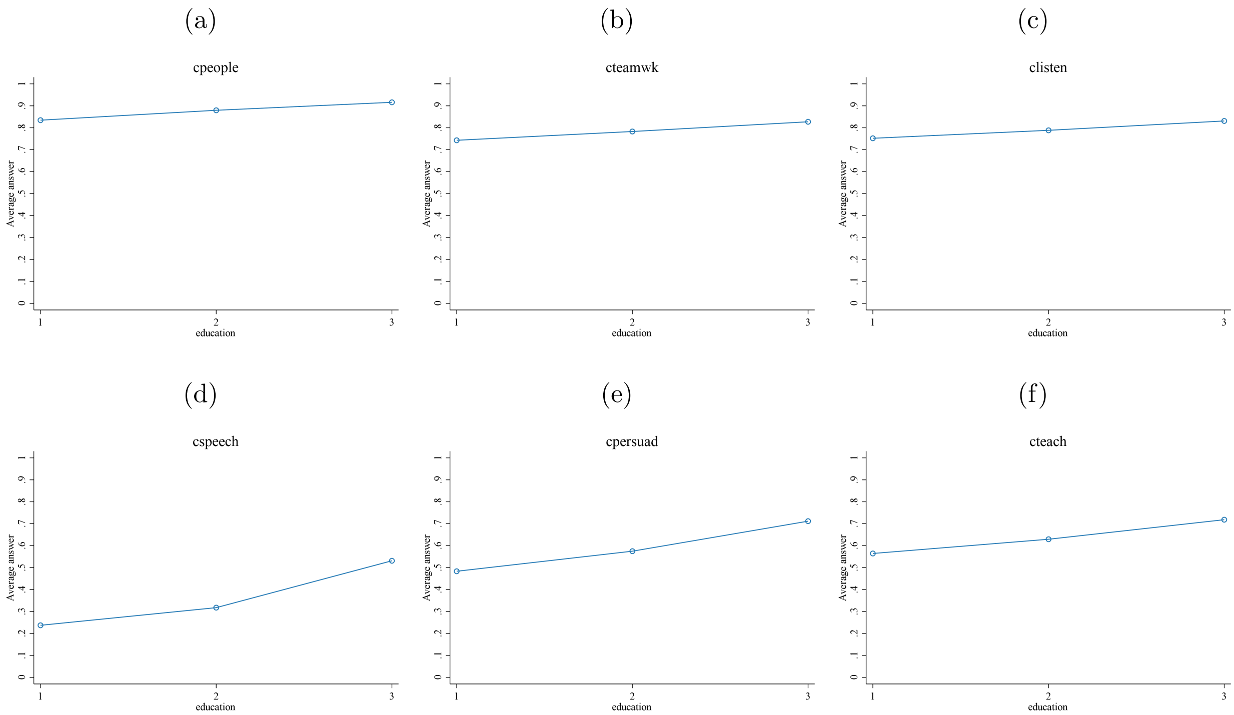
Figure 2: Routine index variables

(a) (b) (c) (d)



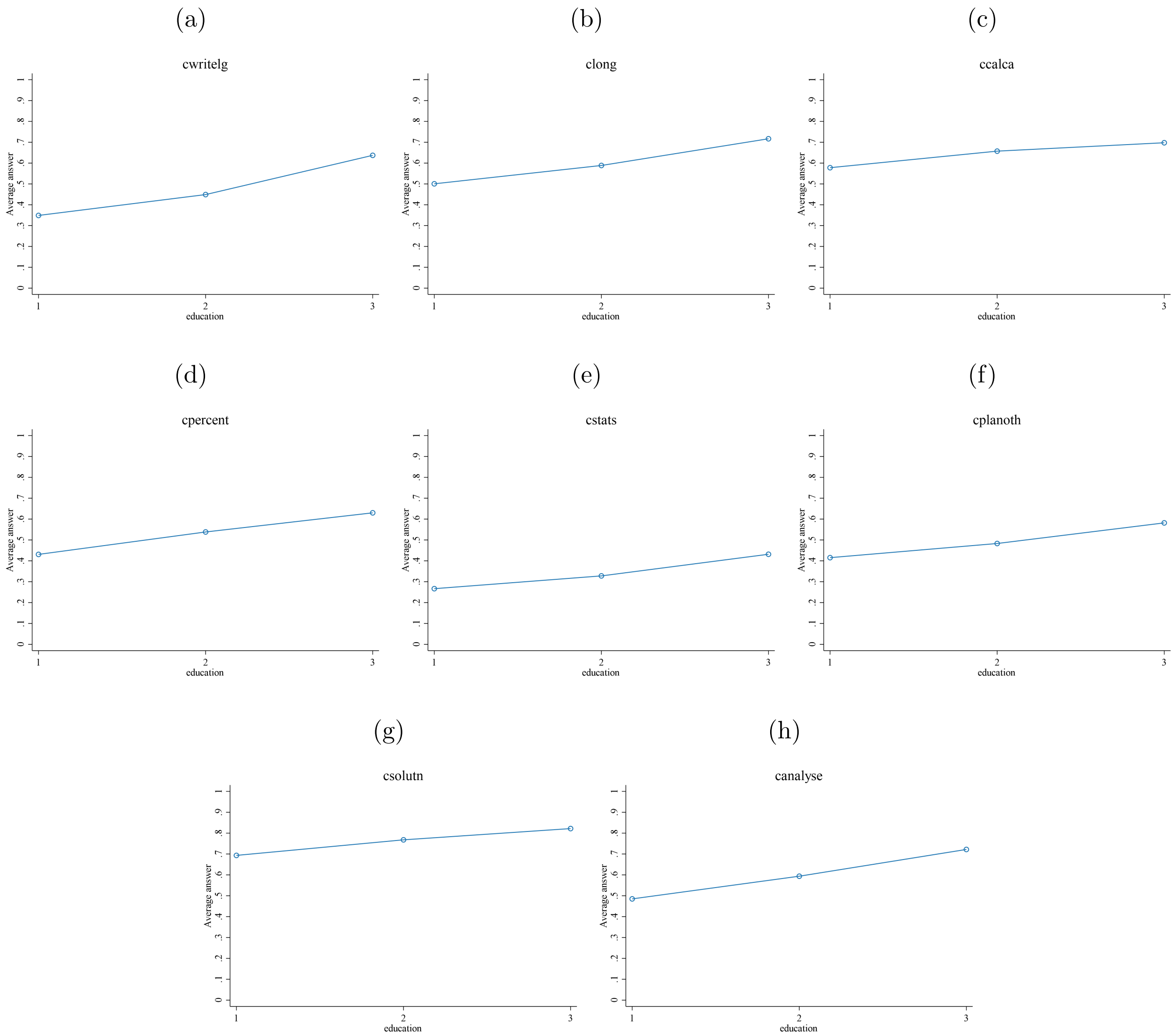
Note: figure note. Figure generated on 28 Jul 2022 at 10:26:01.

Figure 3: Social index variables



Note: figure note. Figure generated on 28 Jul 2022 at 10:29:48.

Figure 4: Abstract index variables

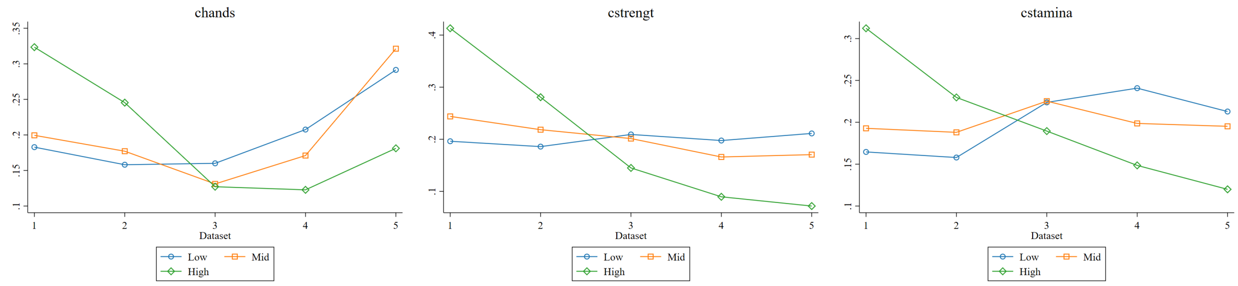


Note: figure note. Figure generated on 28 Jul 2022 at 10:29:48.

3.2 Distributions

Figure 5: Manual index variables

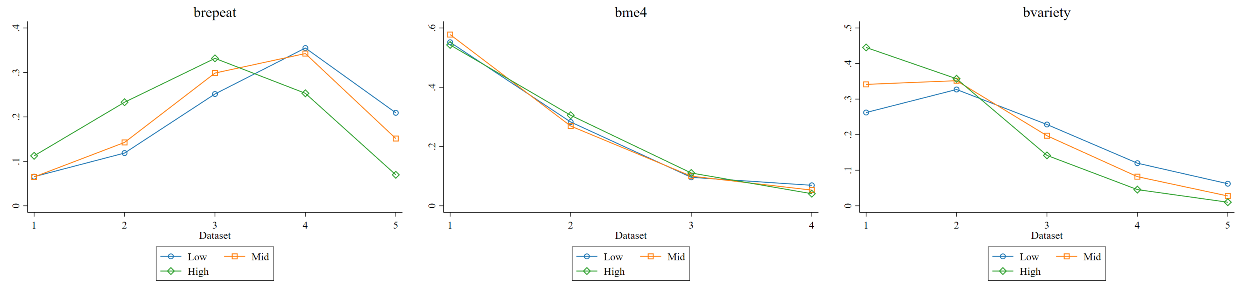
(a) (b) (c)



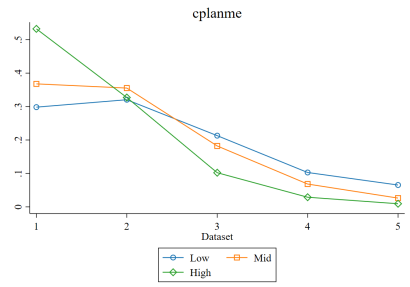
Note: figure note. Figure generated on 28 Jul 2022 at 10:41:45.

Figure 6: Routine index variables

(a) (b) (c)

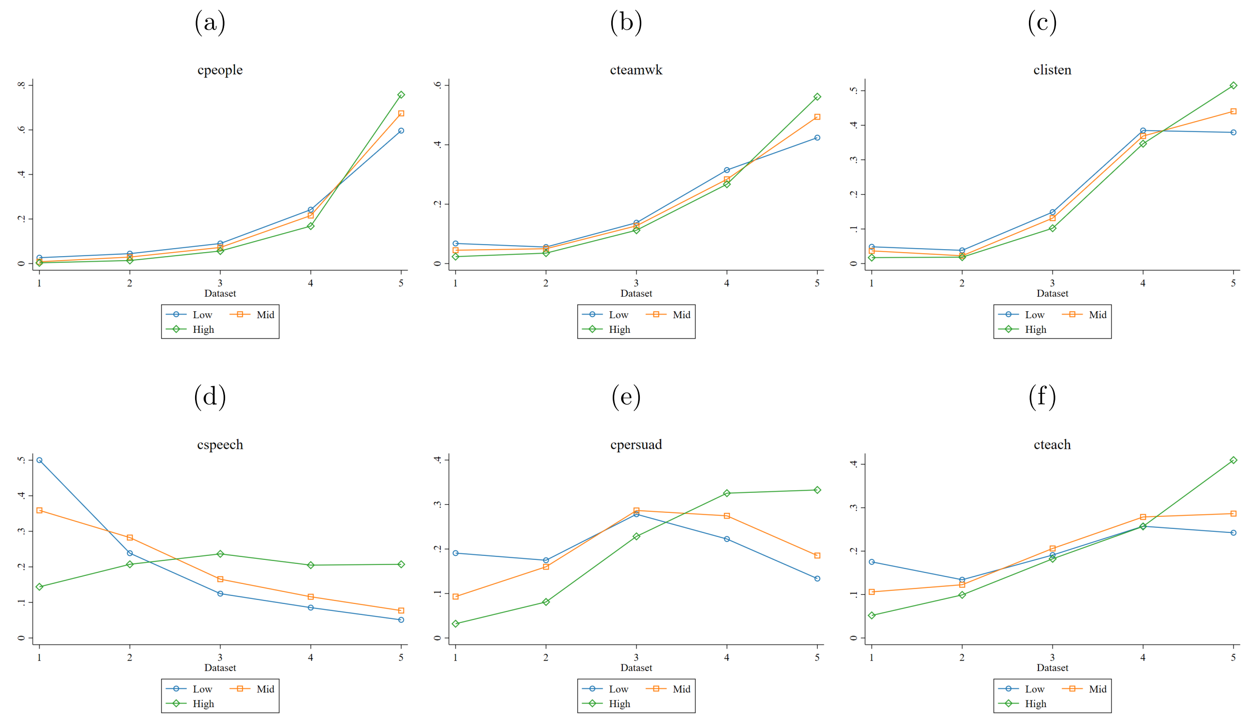


(d)



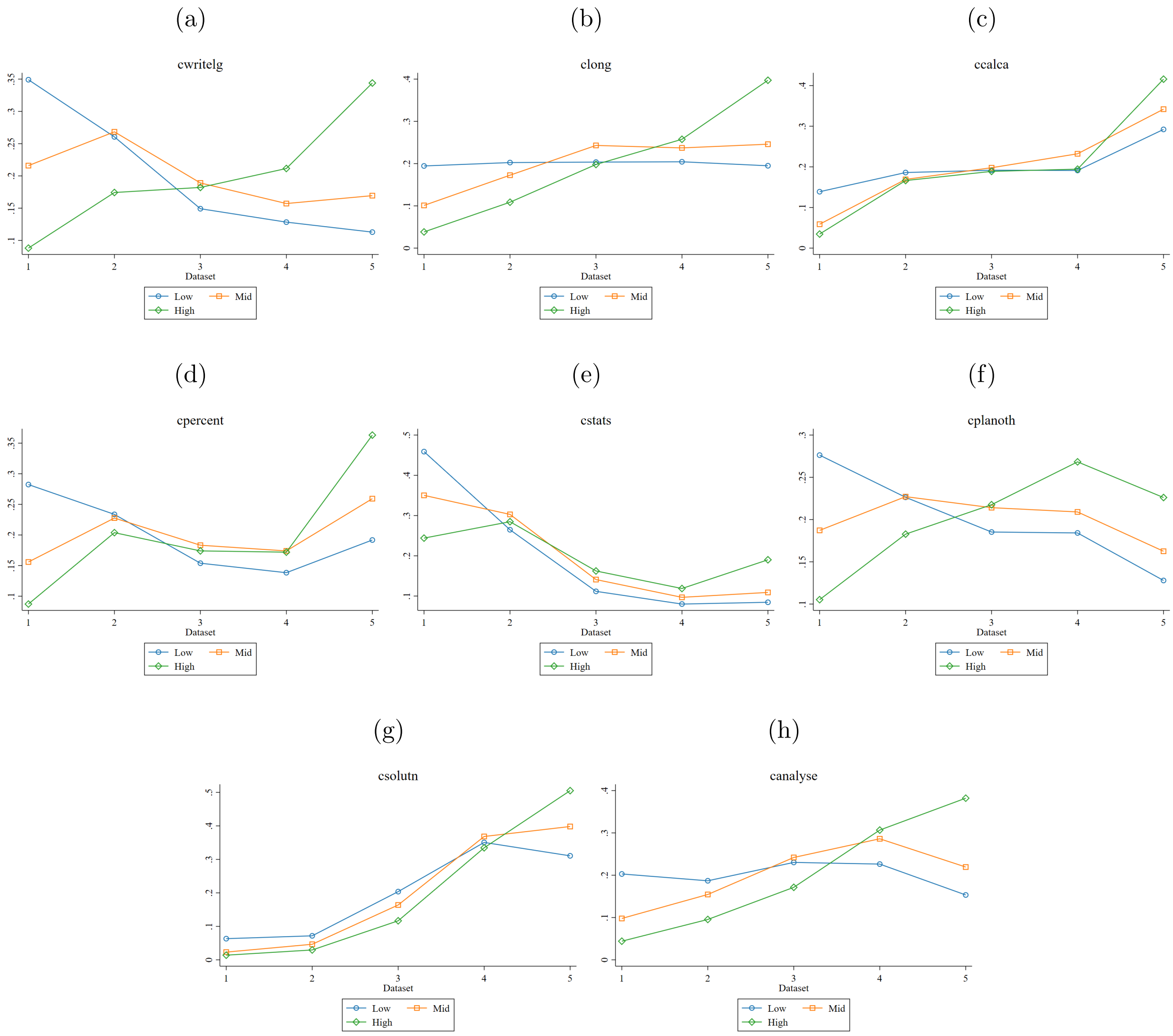
Note: figure note. Figure generated on 28 Jul 2022 at 10:41:45.

Figure 7: Social index variables



Note: figure note. Figure generated on 28 Jul 2022 at 10:41:45.

Figure 8: Abstract index variables



Note: figure note. Figure generated on 28 Jul 2022 at 10:41:45.