Code documentation

César Garro-Marín[*]

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1 New approach (v2):

1.1 Model equations

At the job level it holds that:

$$\begin{aligned}
\Delta\overline{\ln S^e_{iJet}} - \Delta\overline{\ln S^e_{kJet}} &= \frac{\sigma_J}{\sigma_j - 1}\left(\Delta \ln A_{kJt} - \Delta \ln A_{iJt}\right) && (1)\\
1 &= \sum_k \theta^e_k \overline{S_{keJt}} && (2)\\
\Delta\left[\ln \frac{q_{eJt}}{q_{e'Jt}}\right] &= \sum_k \left(\theta^e_k \overline{S_{kJet}} - \theta^{e'}_k \overline{S_{kJe't}}\right)\Delta \ln A_{kjt} + const_{e,e't} && (3)
\end{aligned}$$

Note: in equation (3) there is always a redundant pairwise comparison. Note that $\Delta\left[\ln \frac{q_{eJt}}{q_{e'Jt}}\right] - \Delta\left[\ln \frac{q_{e'Jt}}{q_{e^\star Jt}}\right] = \Delta\left[\ln \frac{q_{eJt}}{q_{e^\star Jt}}\right]$

1.2 Building GMM

In our data we have 3 education levels, 4 skills, J jobs and T periods.

- **Equation (1)**: Let x_{ejt} be the row vector $\begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt}\end{pmatrix}$. Define the matrix X_{ejt} as the 4×4 matrix containing the skill indexes by education. For simplicity, I have ignored the bars in the notation.

$$X_{ejt} = \begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt}\\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt}\\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt}\\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \end{pmatrix} = \begin{pmatrix} x_{ejt}\\ x_{ejt}\\ x_{ejt}\\ x_{ejt}\end{pmatrix}$$

  Let X be the matrix stacking all the X_{ejt} in some order that for now is irrelevant. The only thing that matters is that I use the same order across all definitions. This matrix

[*]Boston University, email: cesarlgm@bu.edu

is of dimension $N \times 4$.

$$X = \begin{pmatrix} X_{111} \\ \vdots \\ X_{3JT} \end{pmatrix}$$

Let I_e be the $N \times 4$ matrix with columns given by education level e dummies:

$$I_e = \begin{pmatrix} \iota_e & \iota_e & \iota_e & \iota_e \end{pmatrix}$$

The set of instruments for equation 1 is then:

$$Z_1 = \begin{pmatrix} I_1 \odot X & I_2 \odot X & I_3 \odot X \end{pmatrix} \tag{4}$$

- **Equation** (1): set of instruments are a set of occupation by year doummies Z_2.
- **Equation** (2): let $\tilde{X}_e$ be the $N_e \times 4$ matrix stacking all the x_{ejt} for education level e. The instruments for equation 2 are the block diagonal matrix with elements $\tilde{X}_e$:

$$Z_3 = \begin{pmatrix} \tilde{X}_1 & 0 & 0 \\ 0 & \tilde{X}_2 & 0 \\ 0 & 0 & \tilde{X}_3 \end{pmatrix}$$

- **Equation** (3) let $\bar{X}$ be the matrix stacking the row vectors $\begin{pmatrix} x_{1jt} & x_{2jt} & x_{3jt} \end{pmatrix}$. Let C be the matrix containing education-pair×T dummies. Instruments for equation 3 are:

$$Z_4 = \begin{pmatrix} \bar{X} & C \end{pmatrix} \tag{5}$$

- **Putting it all together:** let u_l be the stacked (in the appropriate order) vector of equation l errors. Define:

$$Z' = \begin{pmatrix} Z_1' & 0 & 0 & 0 \\ 0 & Z_2' & 0 & 0 \\ 0 & 0 & Z_3' & 0 \\ 0 & 0 & 0 & Z_4' \end{pmatrix}$$

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}$$

the moment equations form the model are given by $\mathbb{E}\left(Z'u\right) = 0$.

2 How did I write the code?

2.1 Basic definitions

$$\Lambda = \begin{pmatrix} \Delta \ln A_{111} \\ \Delta \ln A_{121} \\ \Delta \ln A_{131} \\ \vdots \\ \Delta \ln A_{3JT-2} \\ \Delta \ln A_{3JT-1} \\ \Delta \ln A_{3JT} \end{pmatrix} \Theta = \begin{pmatrix} \theta_1^1 \\ \theta_2^1 \\ \theta_3^1 \\ \theta_4^1 \\ \vdots \\ \theta_1^3 \\ \theta_2^3 \\ \theta_3^3 \\ \theta_4^3 \end{pmatrix} \Sigma = \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_J \end{pmatrix} \Xi = \begin{pmatrix} \xi_{11} \\ \vdots \\ \xi_{3T} \end{pmatrix}$$

I defined the model parameter vector as

$$\Pi = \begin{pmatrix} \Lambda \\ \Theta \\ \Sigma \\ \Xi \end{pmatrix}$$

2.2 Equation (1)

1. Order observations by skill, education, occupation, and year.
2. Create vector $\tilde{\Sigma}$

   $$\tilde{\Sigma} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_J \end{pmatrix}$$

   this vector of is has the same dimensions as Λ. It places all elements of Σ in the "right" order I should say what the right order is here.
3. Define D as a matrix of dimensions $(E \times J \times T) \times (J \times T)$. Define Z_1 as:

   $$X_1 = \begin{pmatrix} -D & 0 & 0 \\ 0 & -D & 0 \\ 0 & 0 & -D \end{pmatrix}$$

   This matrix contains occupation by year dummies. Equation (1) can be written as:

   $$y_1 = X_1 \times (\Lambda \odot \tilde{\Sigma})$$

   where y_1 was defined in the appropriate order.