

Code documentation

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1 New approach (v2):

1.1 Model equations

At the job level it holds that:

$$\Delta \overline{\ln S_{iJt}^e} - \Delta \overline{\ln S_{kJt}^e} = \frac{\sigma_J}{\sigma_j - 1} (\Delta \ln A_{kJt} - \Delta \ln A_{iJt}) \quad (1)$$

$$1 = \sum_k \theta_k^e \overline{S_{kJt}^e} \quad (2)$$

$$\Delta \left[\ln \frac{q_{eJt}}{q_{e'Jt}} \right] = \sum_k \left(\theta_k^e \overline{S_{kJt}^e} - \theta_k^{e'} \overline{S_{kJt}^{e'}} \right) \Delta \ln A_{kJt} + \text{const}_{e,e't} \quad (3)$$

Note: in equation (3) there is always a redundant pairwise comparison. Note that $\Delta \left[\ln \frac{q_{eJt}}{q_{e'Jt}} \right] - \Delta \left[\ln \frac{q_{e'Jt}}{q_{e''Jt}} \right] = \Delta \left[\ln \frac{q_{eJt}}{q_{e''Jt}} \right]$

1.2 Building GMM

In our data we have 3 education levels, 4 skills, J jobs and T periods.

- **Equation (1):** Let x_{ejt} be the row vector $(S_{1ejt} \ S_{2ejt} \ S_{3ejt} \ S_{4ejt})$. Define the matrix X_{ejt} as the 4×4 matrix containing the skill indexes by education. For simplicity, I have ignored the bars in the notation.

$$X_{ejt} = \begin{pmatrix} S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \\ S_{1ejt} & S_{2ejt} & S_{3ejt} & S_{4ejt} \end{pmatrix} = \begin{pmatrix} x_{ejt} \\ x_{ejt} \\ x_{ejt} \\ x_{ejt} \end{pmatrix}$$

Let X be the matrix stacking all the X_{ejt} in some order that for now is irrelevant. The only thing that matters is that I use the same order across all definitions. This matrix

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is of dimension $N \times 4$.

$$X = \begin{pmatrix} X_{111} \\ \vdots \\ X_{3JT} \end{pmatrix}$$

Let I_e be the $N \times 4$ matrix with columns given by education level e dummies:

$$I_e = (\iota_e \quad \iota_e \quad \iota_e \quad \iota_e)$$

The set of instruments for equation 1 is then:

$$Z_1 = (I_1 \odot X \quad I_2 \odot X \quad I_3 \odot X) \quad (4)$$

- **Equation (1):** set of instruments are a set of occupation by year dummies Z_2 .
- **Equation (2):** let \tilde{X}_e be the $N_e \times 4$ matrix stacking all the x_{ejt} for education level e . The instruments for equation 2 are the block diagonal matrix with elements \tilde{X}_e :

$$Z_3 = \begin{pmatrix} \tilde{X}_1 & 0 & 0 \\ 0 & \tilde{X}_2 & 0 \\ 0 & 0 & \tilde{X}_3 \end{pmatrix}$$

- **Equation (3)** let \bar{X} be the matrix stacking the row vectors $(x_{1jt} \quad x_{2jt} \quad x_{3jt})$. Let C be the matrix containing education-pair \times T dummies. Instruments for equation 3 are:

$$Z_4 = (\bar{X} \quad C) \quad (5)$$

- **Putting it all together:** let u_l be the stacked (in the appropriate order) vector of equation l errors. Define:

$$Z' = \begin{pmatrix} Z_1' & 0 & 0 & 0 \\ 0 & Z_2' & 0 & 0 \\ 0 & 0 & Z_3' & 0 \\ 0 & 0 & 0 & Z_4' \end{pmatrix}$$

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}$$

the moment equations from the model are given by $\mathbb{E}(Z'u) = 0$.

2 How did I write the code?

2.1 Basic definitions

$$\Lambda = \begin{pmatrix} \Delta \ln A_{111} \\ \Delta \ln A_{121} \\ \Delta \ln A_{131} \\ \vdots \\ \Delta \ln A_{3JT-2} \\ \Delta \ln A_{3JT-1} \\ \Delta \ln A_{3JT} \end{pmatrix} \quad \Theta = \begin{pmatrix} \theta_1^1 \\ \theta_2^1 \\ \theta_3^1 \\ \theta_4^1 \\ \vdots \\ \theta_1^3 \\ \theta_2^3 \\ \theta_3^3 \\ \theta_4^3 \end{pmatrix} \quad \Sigma = \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_J \end{pmatrix} \quad \Xi = \begin{pmatrix} \xi_{11} \\ \vdots \\ \xi_{3T} \end{pmatrix}$$

I defined the model parameter vector as

$$\Pi = \begin{pmatrix} \Lambda \\ \Theta \\ \Sigma \\ \Xi \end{pmatrix}$$

2.2 Equation (1)

1. Order observations by skill, education, occupation, and year.
2. Create vector $\tilde{\Sigma}$

$$\tilde{\Sigma} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_J \end{pmatrix}$$

this vector of is has the same dimensions as Λ . It places all elements of Σ in the “right” order **I should say what the right order is here**.

3. Define D as a matrix of dimensions $(E \times J \times T) \times (J \times T)$. Define Z_1 as:

$$X_1 = \begin{pmatrix} -D & 0 & 0 \\ 0 & -D & 0 \\ 0 & 0 & -D \end{pmatrix}$$

This matrix contains occupation by year dummies. Equation (1) can be written as:

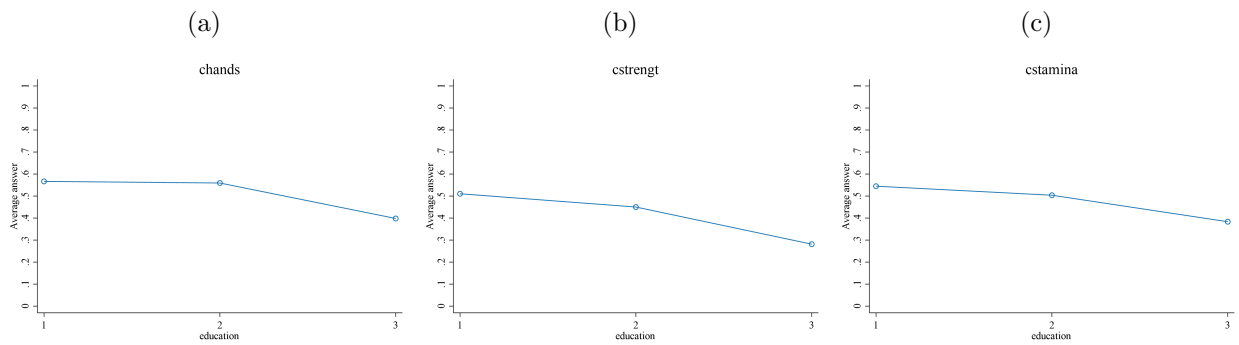
$$y_1 = X_1 \times (\Lambda \odot \tilde{\Sigma})$$

where y_1 was defined in the appropriate order.

3 Summary stats

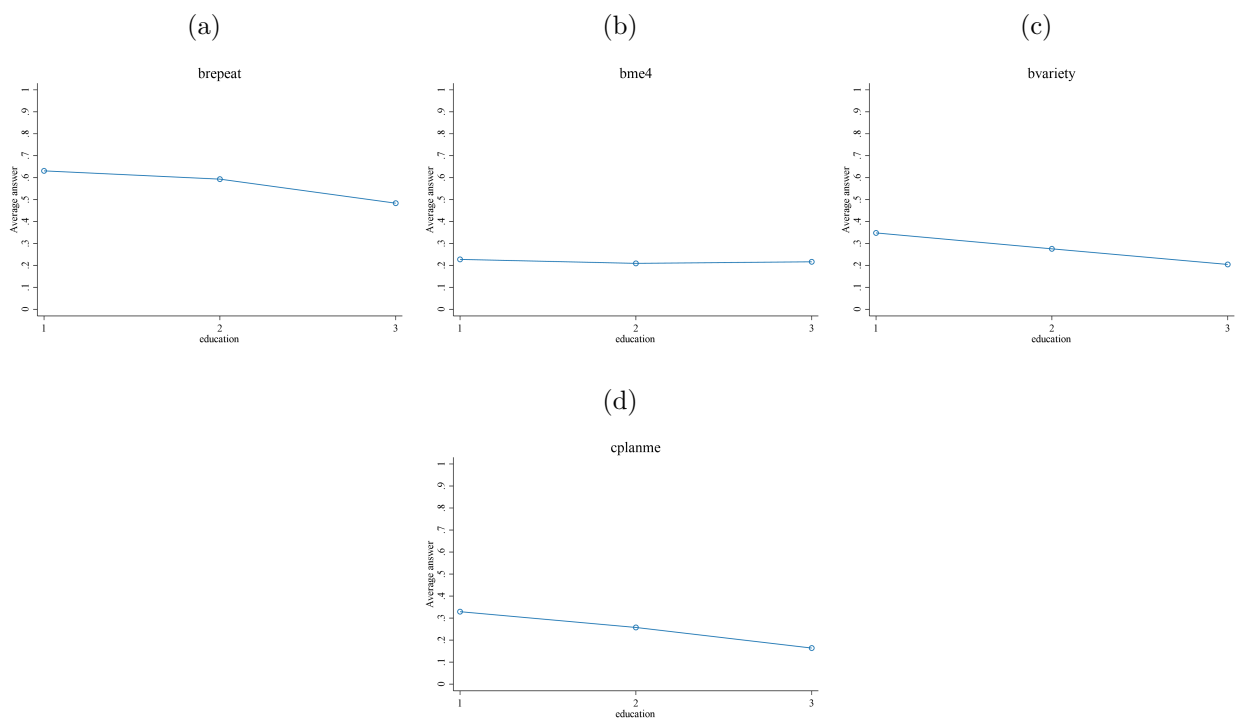
3.1 Averages

Figure 1: Manual index variables



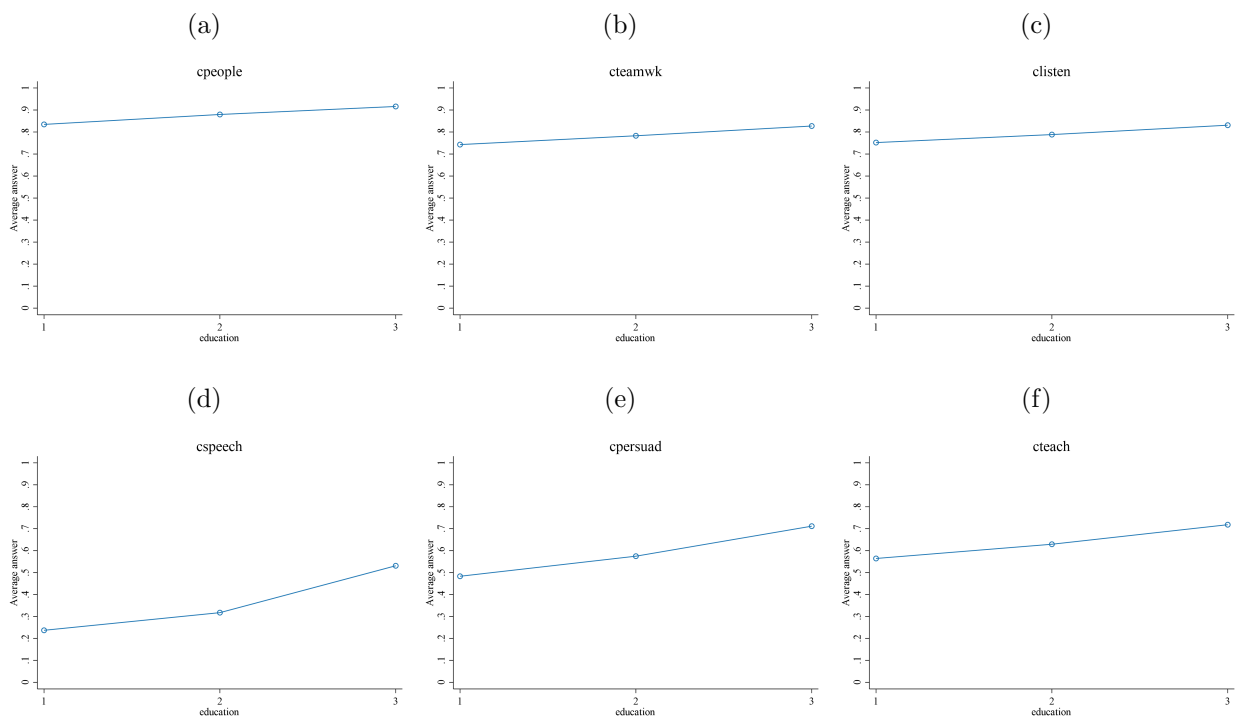
Note: figure note. Figure generated on 28 Jul 2022 at 10:26:01.

Figure 2: Routine index variables



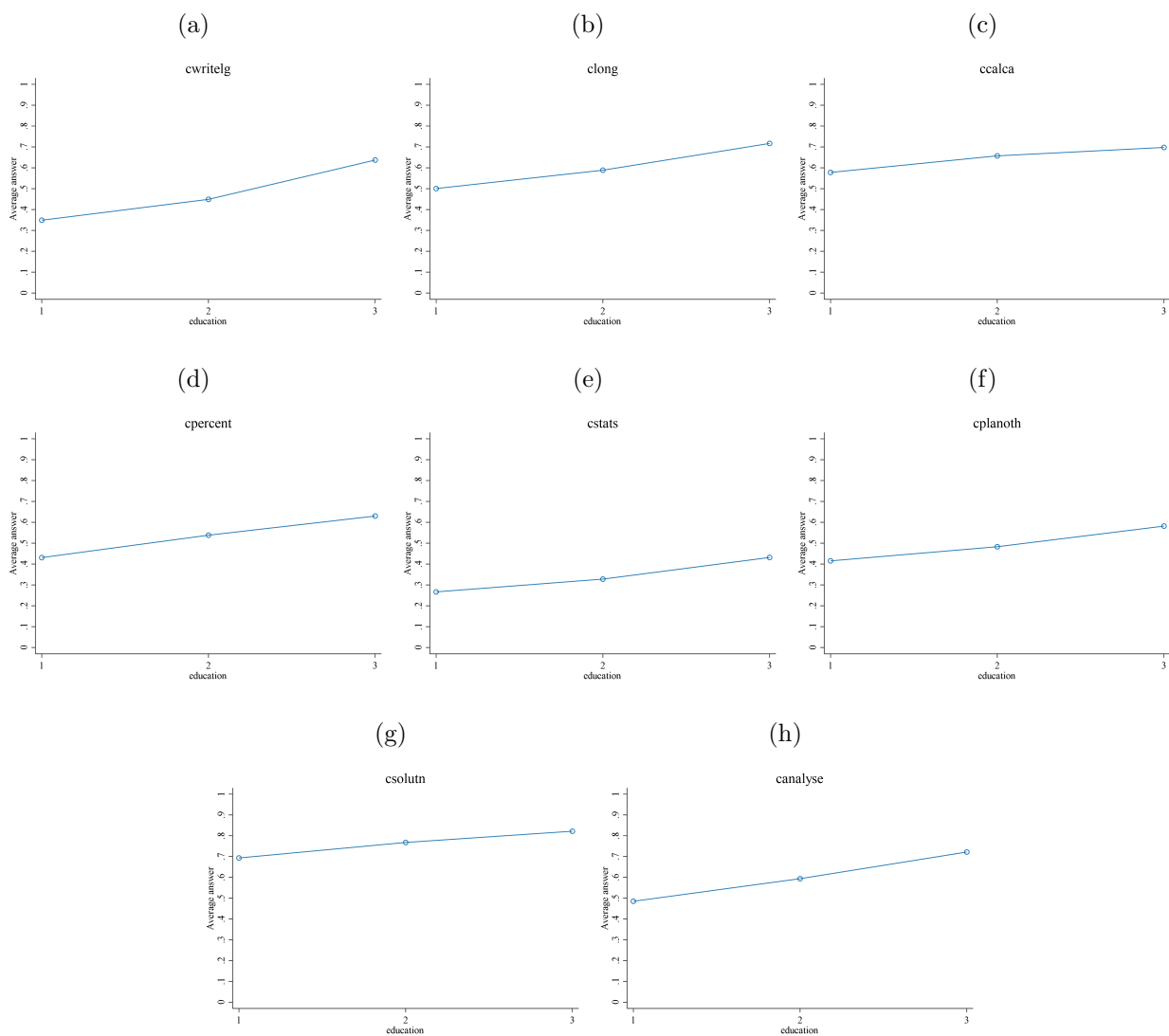
Note: figure note. Figure generated on 28 Jul 2022 at 10:26:01.

Figure 3: Social index variables



Note: figure note. Figure generated on 28 Jul 2022 at 10:29:48.

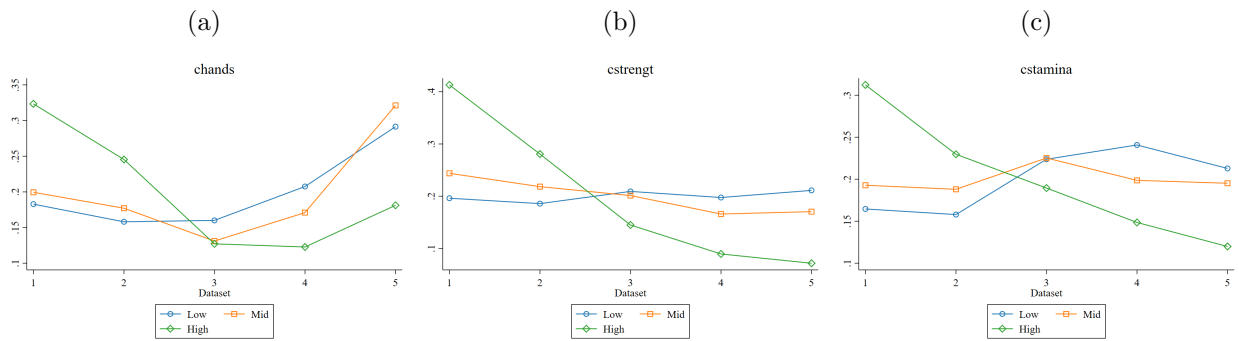
Figure 4: Abstract index variables



Note: figure note. Figure generated on 28 Jul 2022 at 10:29:48.

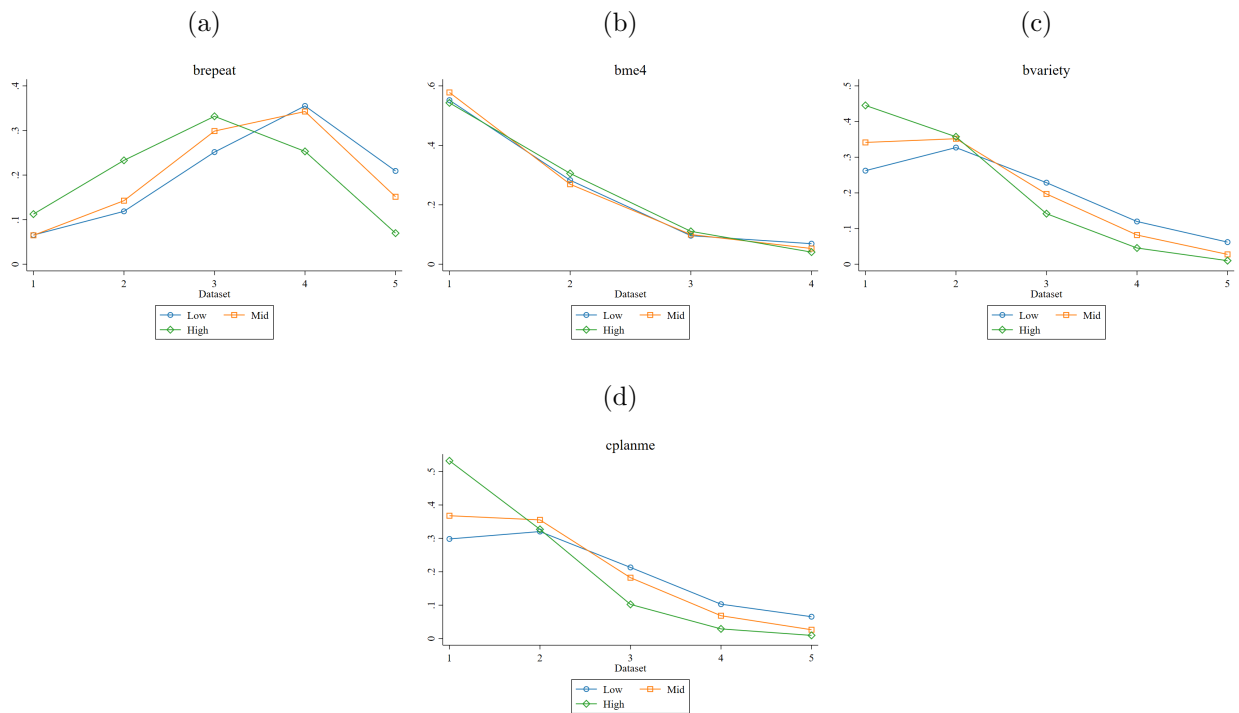
3.2 Distributions

Figure 5: Manual index variables



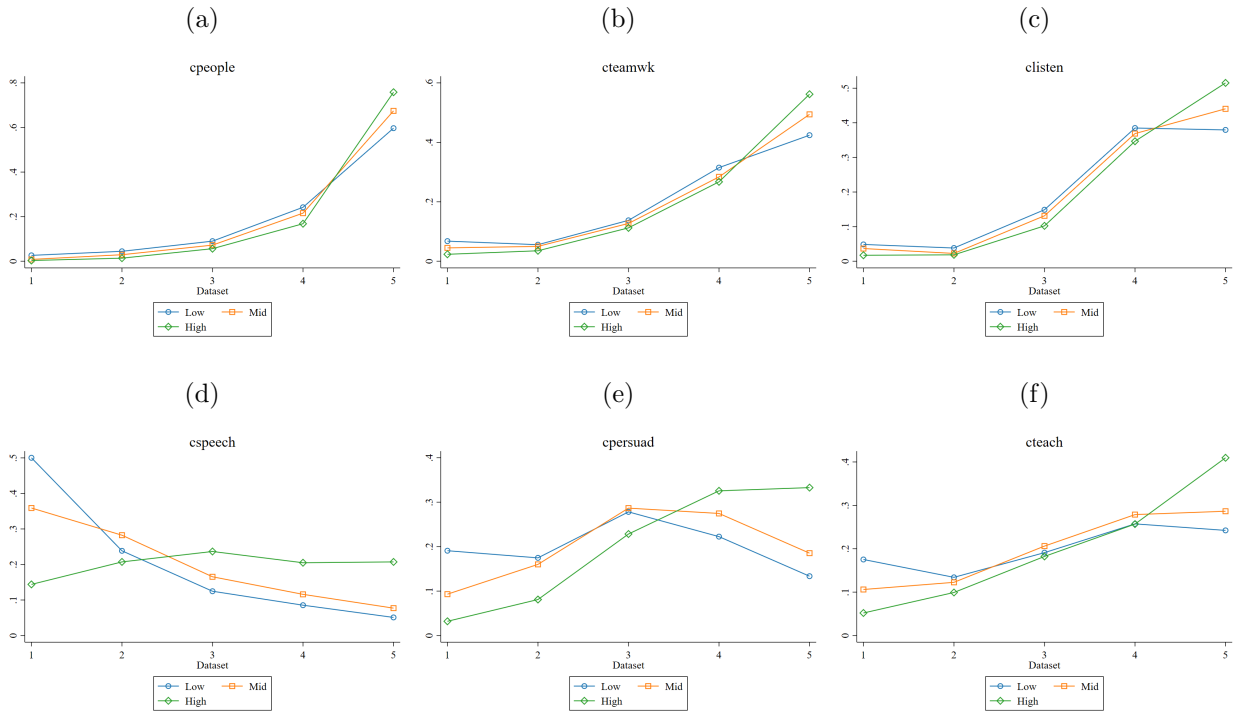
Note: figure note. Figure generated on 28 Jul 2022 at 10:41:45.

Figure 6: Routine index variables



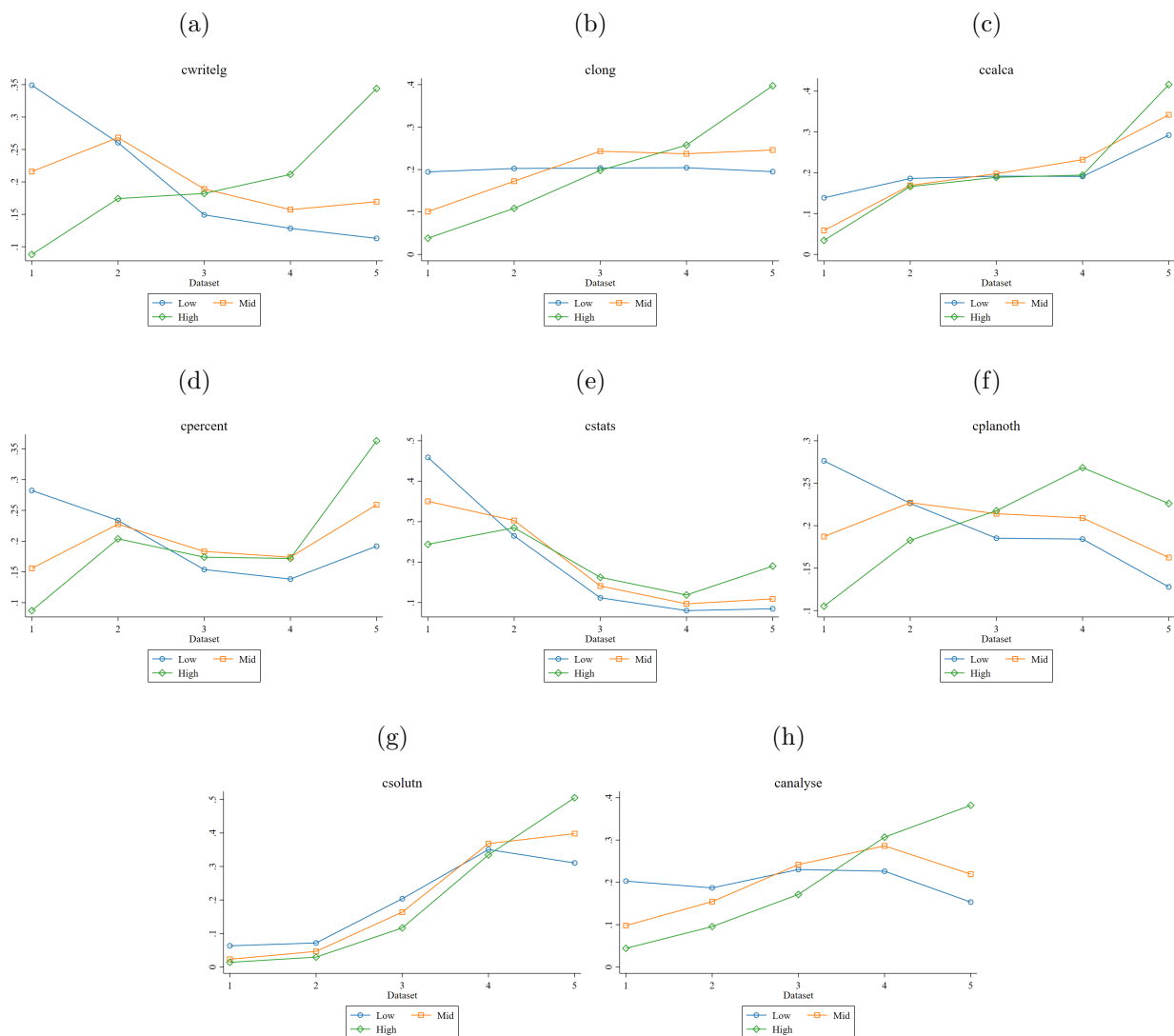
Note: figure note. Figure generated on 28 Jul 2022 at 10:41:45.

Figure 7: Social index variables



Note: figure note. Figure generated on 28 Jul 2022 at 10:41:45.

Figure 8: Abstract index variables



Note: figure note. Figure generated on 28 Jul 2022 at 10:41:45.