Code documentation

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1 Estimation

We estimate the three-equation model:

$$\begin{aligned}
\Delta\overline{\ln S^{e}_{ijet}} &= \sum_k \theta_{ke}\overline{S_{kejt}}\pi_{kjt} - \pi_{ijt} && (1)\\
1 &= \sum_k \theta^{e}_{k}\overline{S_{kejt}} && (2)\\
\Delta\left[\ln\frac{q_{eJt}}{q_{e'Jt}}\right] &= \beta_j\left[\sum_k\left(\theta^{e}_{k}\overline{S_{kJet}} - \theta^{e'}_{k}\overline{S_{kJe't}}\right)\pi_{kjt}\right] + \gamma_{e,e't} && (3)
\end{aligned}$$

where $\pi_{kjt} = \frac{\sigma_j}{\sigma_j - 1}\Delta\ln A_{kjt}$ and $\beta_j = \frac{\sigma_j - 1}{\sigma_j}$.

Let ε_l be vector of stacked errors from equation j and let ε the vector of stacked errors of the full model ($\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3)'$). We estimate the model parameters μ by GMM, minimizing the quadratic form:

$$Q = \varepsilon(\mu)'Z'Z\varepsilon(\mu) \qquad (4)$$

where Z is the vector of instruments.

1.1 Instruments

The matrix of instruments Z is a block-diagonal matrix with diagonal elements given by the matrix of instruments for each equation (Z_l).

- **Equation 1 instruments:** we instrument the skill levels $\overline{S_{kejt}}$ with the size-weighted skill level of the other education groups in that job:

  $$\overline{S_{k-ejt}} = w_{e'jt}\overline{S_{ke'jt}} + w_{e''jt}\overline{S_{ke''jt}}$$

  where $w_{e'jt}$ is the employment share of education e' in that job and year. We also include as instruments the appropriate set of –negative– job by skill by year dummies. Therefore, Z_2 is a $12JT \times (12 + 4JT)$ matrix.

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- **Equation 2:** instruments for this equation are the skill levels by education level. Therefore, Z_2 is a $3JT \times 12$ matrix.
- **Equation 3 instruments:** there are three types of instruments for this equation: education pair dummies, job by year dummies, and job by education dummies. Thus, Z_3 is a $2JT \times (6 + JT + JE)$ matrix.

1.2 Identification

The GMM model is overidentified. In total we have $30+5JT+JE$ instruments to identify $18 + 4JT + J$ parameters.

1.3 Standard errors

The GMM estimates are distributed as follows:

$$\sqrt{N}(\hat{\mu} - \mu) \rightarrow N(0, V)$$

where $(D'D)^{-1}D'VD(D'D)^{-1}$, with D the model gradient, and

$$\hat{V} = \frac{1}{N}\sum_{n=1}^{N} \varepsilon_n(\hat{\mu})\varepsilon_n(\hat{\mu})'$$

1.4 Calculating the gradient

Reminder:

$$\frac{\partial Q}{\partial x} = x'(A + A')$$

I can write the quadratic from as:

$$Q(\gamma) = \frac{1}{N}\left(u(\gamma)'Z(Z'Z)^{-1}Z'u(\gamma)\right)$$

then,

$$\frac{\partial Q(\gamma)}{\partial \gamma} = \frac{2}{N}\left(\frac{\partial u(\gamma)}{\partial \gamma}\right)' Z(Z'Z)^{-1}Zu(\gamma)$$

now, I start element by element:

$$\begin{aligned}
\frac{\partial \Delta \ln S_{iejt}}{\partial \ln A_{kjt}} &= \theta_{ke}S_{kejt} \\
\frac{\partial \Delta \ln S_{iejt}}{\partial \ln A_{ijt}} &= \theta_{ie}S_{iejt} - 1 \\
\frac{\partial \Delta \ln S_{iejt}}{\partial \ln \theta_{ke}} &= S_{kjt}\Delta \ln A_{kjt}
\end{aligned}$$

for the sum to 1 restrictions we have:

$$\frac{\partial g_{ejt}(\gamma)}{\partial \theta_{ke}} = -S_{kejt}$$

next thing to do: write the gradient