UC Berkeley
Electrical Engineering
& Computer Sciences

New Silicon Initiative

You’re invited to a special event

Considering a career in VLSI, chip design, circuit design, or computer architecture?

Join Apple and UC Berkeley EECS faculty for a special presentation about career paths in these exciting fields!

Afterwards, network with Apple engineers and UC Berkeley over burritos and boba.

New Silicon Initiative Fall Kickoff
Friday, September 5, 2025
11:00 a.m. – 1:00 p.m. PT
Banatao Auditorium

Check-in begins at 10:45 a.m. PT

Register by scanning or clicking the QR code.

Announcements

- New Zoom link: cs61c.org/fa25/lecture-zoom
- Waiting on dept word about class expansion (+CEs); I'm hopeful
- Planning to enroll but need Ed access? forms.gle/Qv2riacbtKCp2cZy6
- Discussion section format
  - On the fence about Regular vs. Bridge vs. Video discussion? Choose Regular/Bridge to be assigned a time that works for you.
  - Fill out welcome survey by **Friday (today) 11:59pm** to get assigned a regular time (if applicable). **All other students default to video.**
  - Can switch section times/formats until Add/Drop deadline (Week 4)
- 61C Scholars Pilot Program (self-identify on welcome survey)
  - Scholars-specific regular discussion + other activities/socials

CS61C

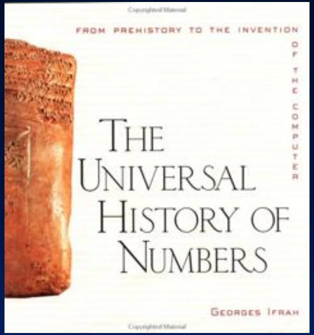
Great Ideas
in
Computer Architecture

(a.k.a. Machine Structures)

Teaching Professor
Dan Garcia

Number Representation

Great book ⇒
The Universal History of Numbers
by Georges Ifrah



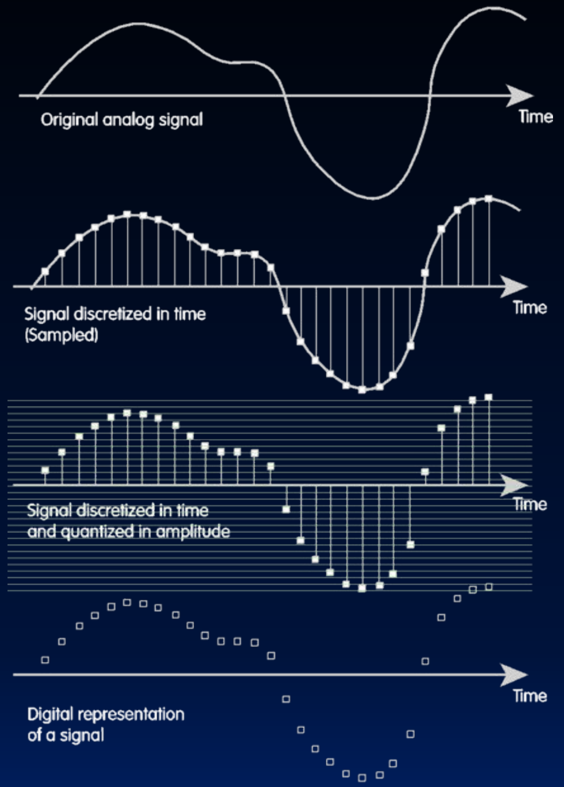
Digital data not necessarily born Analog…

hof.povray.org

Data input: Analog → Digital

Real world is analog!
To import analog information, we must do two things

- Sample
  - E.g., for a CD, every 44,100ths of a second, we ask a music signal how loud it is.
- Quantize
  - For every one of these samples, we figure out where, on a 16-bit (65,536 tic-mark) “yardstick”, it lies.



Agenda

Binary, Decimal, Hex

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

Number vs Numeral

Numeral

A symbol or name that stands for a number

e.g., *4* , *four* , *quatro* , *IV* , *IIII* , ...

...and **Digits** are symbols that make numerals

Above the abstraction line

Abstraction Line

Below the abstraction line

Number

The "idea" in our minds...there is only ONE of these

e.g., *the concept of "4"*

Decimal: Base 10 (Ten) #s

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Example:

$\mathbf{3271} = \mathbf{3271}_{10} = (\mathbf{3} \text{x} 10^3) + (\mathbf{2} \text{x} 10^2) + (\mathbf{7} \text{x} 10^1) + (\mathbf{1} \text{x} 10^0)$

Base 2 (Two) #s, Binary

Digits: 0, 1 (**bi**nary digi**ts** → bits)

Example: Binary number "**1101**"

Convert to decimal:

0b1101 = 1101_2 = $(1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

= 8 + 4 + 0 + 1

= 13

Common binary shorthand:
`0b1101`

Base 16 (Sixteen) #s, Hexadecimal

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

10, 11, 12, 13, 14, 15

Example: Hexadecimal number "**A5**"

Convert to decimal:

$$\mathbf{0xA5} = \mathbf{A5}_{16} = (\mathbf{10} \times 16^1) + (\mathbf{5} \times 16^0)$$
$$= 160 + 5$$
$$= 165$$

"Hex" for short.
Common hex shorthand: `0xA5`

Convert from Decimal to Binary

E.g., 13 to binary?
Start with the columns

~~13~~
~~5~~
~~1~~
0

$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
1	1	0	1

`0b1101`

Left to right, is (column) ≤ number **n**?

- If yes, put how many of that column fit in **n**, subtract col * that many from **n**, keep going.
- If not, put 0 and keep going. (and Stop at 0)

Convert from Decimal to Hexadecimal

E.g., 165 to hexadecimal?
Start with the columns

~~165~~
~~5~~
0

$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
0	0	10(A)	5

0x00A5
0xA5

Left to right, is (column) ≤ number **n**?

- If yes, put how many of that column fit in **n**, subtract col * that many from **n**, keep going.
- If not, put 0 and keep going. (and Stop at 0)

Nibbles and Bytes

Memorize this table.

- 4 Bits
  - 1 "Nibble"
  - 1 Hex Digit = 16 things
- 8 Bits
  - 1 "**Byte**"
  - 2 Hex Digits = 256 things

Dec	Hex	Bin
00	0	0000
01	1	0001
02	2	0010
03	3	0011
04	4	0100
05	5	0101
06	6	0110
07	7	0111
08	8	1000
09	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Convert Binary → Hexadecimal

Memorize this table.

- Binary → Hex? Easy!
  - **Left-pad** with 0s
  - (Group into full 4-bit values)
  - Look it up
  - `0b11110`
    `→ 0b00011110`
    `(→ 0b0001 1110)`
    `→ 0x1E`
- Hex → Binary? Easy!
  - Just look it up
  - `0x1E`
    `→ 0b00011110`
    `→ 0b11110` (drop leading 0s)

Dec	Hex	Bin
00	0	0000
01	1	0001
02	2	0010
03	3	0011
04	4	0100
05	5	0101
06	6	0110
07	7	0111
08	8	1000
09	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Binary odometer

Car odometer

Which base do we use?

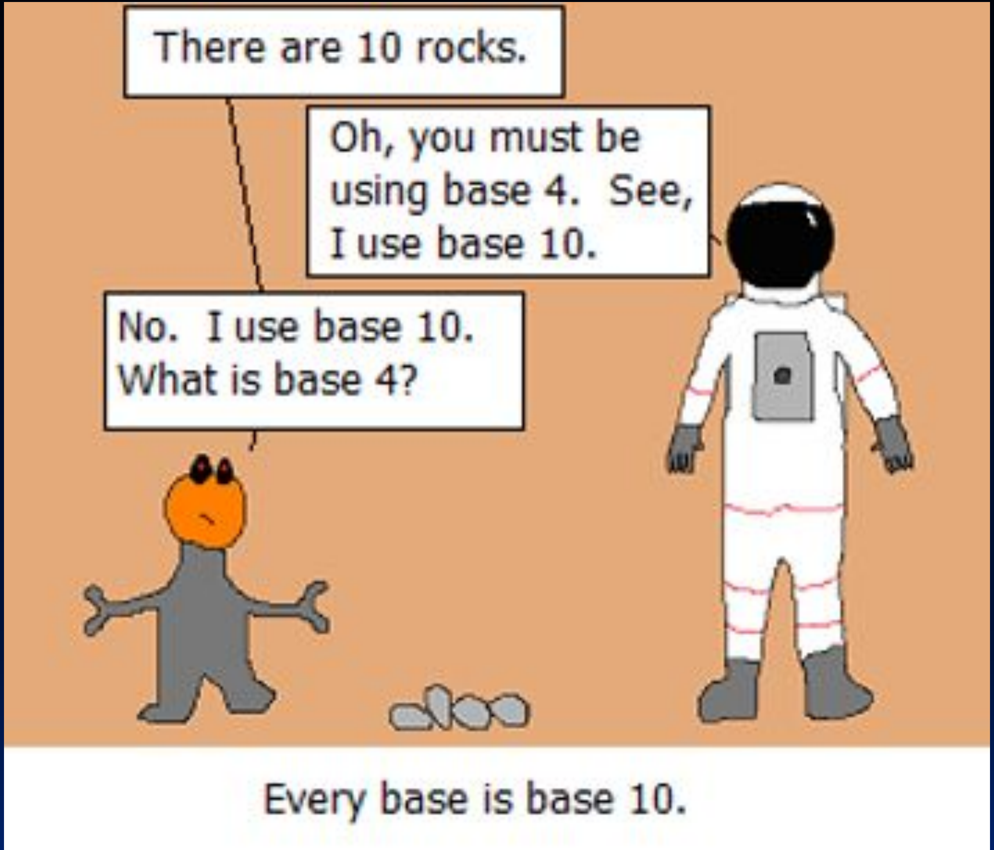
- **Decimal:** great for humans, especially when doing arithmetic
- **Hex:** if human looking at long strings of binary numbers, its much easier to convert to hex and see 4 bits/symbol
  - Terrible for arithmetic on paper
- **Binary:** what computers use
  - To a computer, numbers are always binary
  - Regardless of how number is written:
  - 32_{ten} == 32_{10} == 0x20 == 100000_2 == 0b100000

- To avoid confusion:

Decimal:	subscript "ten"	or	prefix (none)
Hex:	subscript "hex"	or	prefix `0x`
Binary:	subscript "two"	or	prefix `0b`

Every Base is Base 10...?

How many rocks?



The computer knows it, too…

(more next time)

```c
#include <stdio.h>
int main() {
     const int N = 1234;
     printf("Decimal: %d\n",N);
     printf("Hex:     %x\n",N);
     printf("Octal:   %o\n",N);

     printf("Literals (not supported by all compilers):\n");
     printf("0x4d2         = %d (hex)\n", 0x4d2);
     printf("0b10011010010 = %d (binary)\n", 0b10011010010);
     printf("02322         = %d (octal, prefix 0 - zero)\n", 0x4d2);
     return 0;
}
```

Output

```
Decimal: 1234
Hex:     4d2
Octal:   2322
Literals (not supported by all compilers):
0x4d2         = 1234 (hex)
0b10011010010 = 1234 (binary)
02322         = 1234 (octal, prefix 0 - zero)
```

BIG IDEA: Bits can represent anything!!

- Logical values? 1 bit
  - One possible convention: 0 → False, 1 → True
- Characters? Several options:
  - A, ..., Z: 26 letters → 5 bits (26 ≤ 32)
  - ASCII: upper/lower case + punctuation → 7 bits → round to 1 byte
  - Unicode (www.unicode.com): standard code to cover all the world's languages ⇒ 8, 16, 32 bits
- Colors?
  - HTML color codes: 24 bits (3 bytes)
- Locations / addresses? Commands?
  - IPv4 (32 bit), IPv6 (64 bit), etc.

California Gold
0xFDB515

Red (FD)	Green (B5)	Blue (15)

With N bits, you can represent at most 2^N things.

How many bits do you need to represent π (pi)?

A. 1
B. 9 (π=3.14, so 0.011"." 001100)
C. 64 (Macs are 64-bit machines)
D. Every bit the machine has
E. ∞

Number Representation: How many bits to represent π (Pi)?

0

1

0%

9 (π = 3.14, so that's 011 "." 001 100)

0%

64 (Since Macs are 64-bit machines)

0%

Every bit the machine has!

0%

∞

0%

Pop quiz??

1. How many "things" can be represented by 4 bits?

   A. 4
   B. 8
   C. 16
   D. 64
   E. Something else

2. [no pollEV, just discuss] How many bits do you need to represent π (pi)?

   A. 1
   B. 9 (π=3.14, so 0.011"." 001100)
   C. 64 (Macs are 64-bit machines)
   D. Every bit the machine has
   E. ∞

3. [no pollEV, just discuss] What does this particular 4-bit pattern represent?

   1011

How many things can be represented using 4 bits?

0

(A) 4 — 0

(B) 8 — 0

(C) 16 — 0

(D) 64 — 0

(E) Something else — 0

1. How many "things" can be represented by 4 bits?
   - A. 4
   - B. 8
   - C. 16
   - D. 64
   - E. Something else
2. [no pollEV, just discuss] How many bits do you need to represent π (pi)?
   - A. 1
   - B. 9 (π=3.14, so 0.011"." 001100)
   - C. 64 (Macs are 64-bit machines)
   - D. Every bit the machine has
   - E. ∞
3. [no pollEV, just discuss] What does this particular 4-bit pattern represent?

   1011

Start the presentation to see live content. For screen share software, share the entire screen. Get help at **pollev.com/app**

Pop quiz??

1. How many "things" can be represented by 4 bits?

A. 4
B. 8
C. 16 =2^4
D. 64
E. Something else

2. [no pollEV, just discuss] How many bits do you need to represent π (pi)?

A. 1
B. 9 (π=3.14, so 0.011"." 001100)
C. 64 (Macs are 64-bit machines)
D. Every bit the machine has
E. ∞

3. [no pollEV, just discuss] What does this particular 4-bit pattern represent?

1011

Agenda

Integer Representations

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

How do we pick a representation for integers?

- Want a representation that supports common integer operations:
  - Add them
  - Subtract them
  - Multiply them
  - Divide them
  - Compare them (<, =, ≠, ≤, etc.)

- Example: 10 + 7 = 17
  - 10, 7 can be represented with 4 bits:
  - Addition, subtraction just as you would in decimal!!
  - So simple to **add** in binary that we can build circuits to do it!
    - **This design decision would make hardware simple!**
  - ...wait...

```
  11     carry bits
  1010
+ 0111
 10001
```

Garcia, FA25

What if “too big”? Overflow

- Strictly speaking, base 2 numerals have an ∞ number of digits.
  - With almost all being same (00…0 or 11…1) except rightmost digits
  - Just don’t normally show leading digits

…0000001010

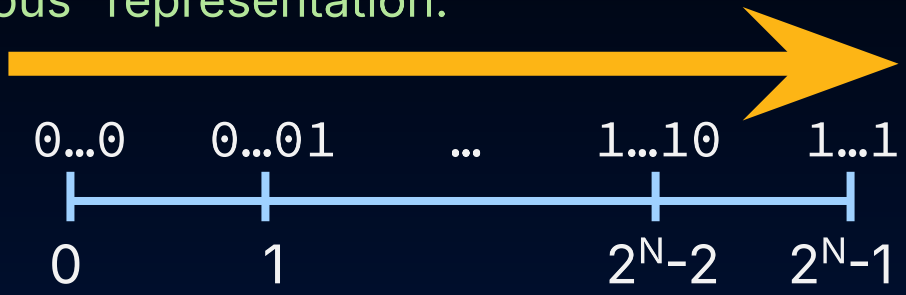
- However, **hardware has physical limits**. No infinite bits!
  - Common representations: 8 bits, 16 bits, 32 bits, 64 bits, …
  - Again: With N bits, you can represent at most 2^N things.
- If integer result of operation (+, -, *, /, >, <, =, etc.) cannot be represented by HW bits, we say **integer overflow** occurred

Integer overflow: The arithmetic result is outside the representable range.

Many Possible Number Representations

- So far, we have only discussed **unsigned numbers** (non-negative).
  - C's `uint8_t`, `uint16_t`, etc.: [0, 2^N-1]
  - Most computers use the "obvious" representation:

Binary odometer



- What about **signed numbers**? Need a way to represent **negative numbers**. Let's discuss a few:
  - Sign-Magnitude
  - Ones' Complement
  - Two's Complement (C23: the only signed integer rep permitted)
  - Bias Encoding (if time, otherwise review on your own)

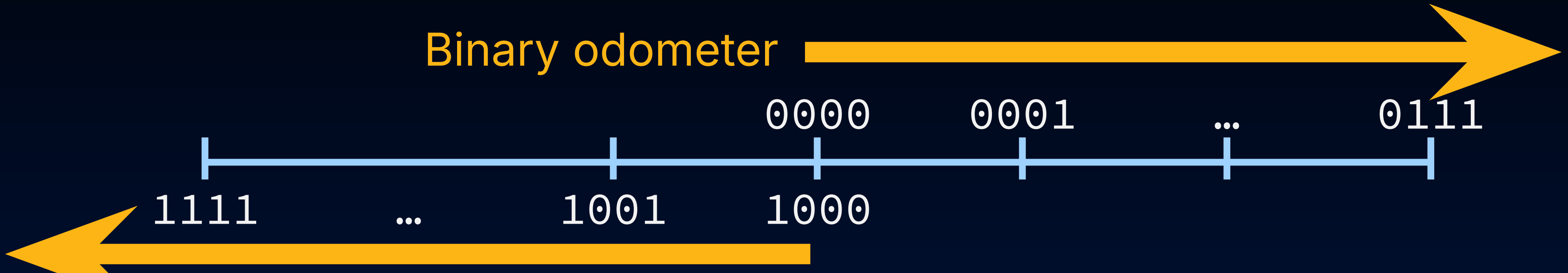
Agenda

Sign-Magnitude, Ones' Complement

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

Sign-Magnitude: Ain't No Free Lunch (tell story)

- Strawman ("obvious") solution:
  - Leftmost **sign bit**: 0 → +, 1 → –
  - Rest of bits: numerical value



- Sign-magnitude is **rarely used**, due to many shortcomings:
  - Incrementing "binary odometer" increases then decreases values
  - Arithmetic circuit complicated: depends on signs same/different
  - **Two zeros** (how to compare??)
- Reasonable for signal processing, not for general purpose computers

`0x00000000` = $+0_{ten}$

`0x80000000` = -0_{ten}

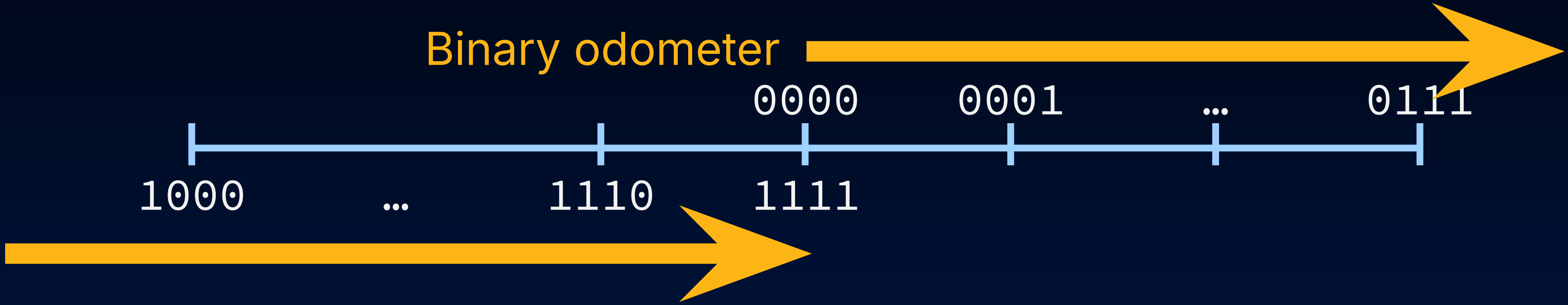
`0b 1000 0000 … 0000`

Ones' Complement: Another try

- To represent a negative number, complement ("**flip**") the bits of its positive representation:

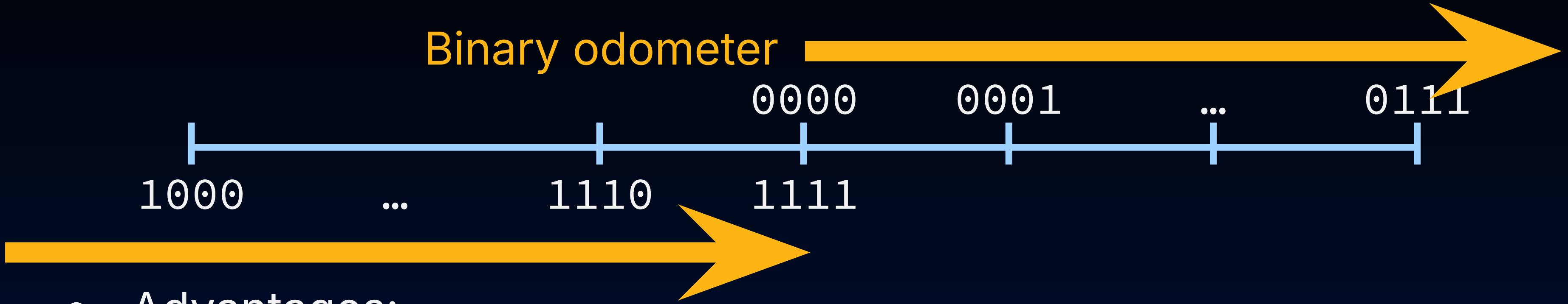
$+7_{ten}$ = 0b 0000 0111

-7_{ten} = 0b 1111 1000



- Observations:
  - Positive numbers: leading 0s
  - Negative numbers: leading 1s
- #s represented in N bits:
  - Zero: 2
  - Positive: $2^{N-1} - 1$
  - Negative: (same as positive)

Shortcomings of Ones' Complement?



- Advantages:
  - Leftmost bit ("**most significant bit**") is still effectively sign bit
  - Incrementing binary odometer consistent on the # line
- Some disadvantages still persist:
  - Still two zeros
  - Arithmetic still somewhat complicated (more later)
- While used for a while on some computer products
  - It's not currently used in current hardware

Agenda

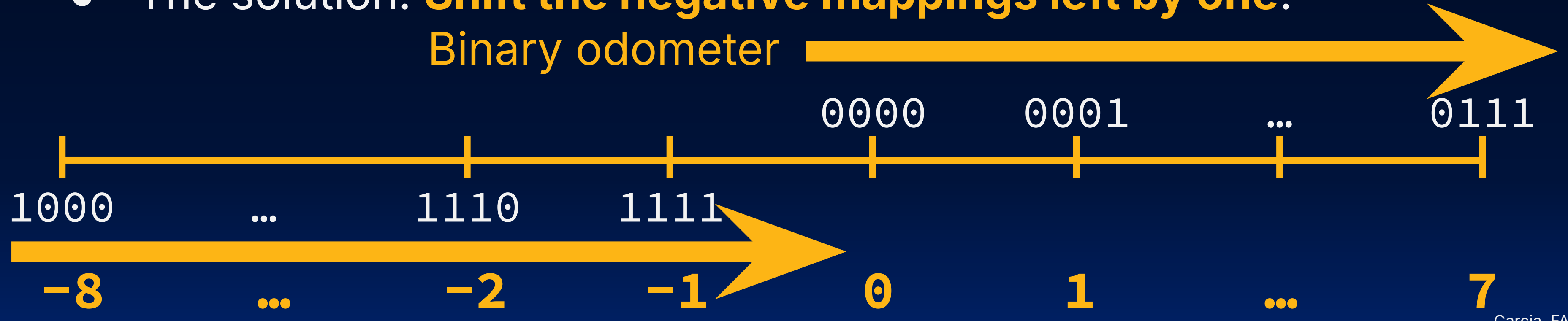
Two's Complement

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

Two's Complement: C23 standard number rep.

- Ones' complement:

- The problem: Negative mappings "overlap" with the positive ones, creating the two 0s.
- The solution: **Shift the negative mappings left by one**.



Arithmetic in Two's Complement is simple

1000	…	1110	1111	0000	0001	…	0111
-8	**…**	**-2**	**-1**	**0**	**1**	**…**	**7**

- Advantages:
  - Leftmost bit ("**most significant bit**") is still effectively sign bit
  - Incrementing binary odometer consistent on the # line
  - One zero, and one extra negative number (here, -8 vs 7)
  - **Simple hardware for addition**

Decimal		Two's complement	
		carry bits	111
	5		0101
	+ -5		+ 1011
	0	dropped	10000

Garcia, FA25

Two's Complement: Formula

- Positive and negative numbers can be computed using the same formula:
  - Highest bit multiplied by neg power of 2!

0b1011

$= (1 \times -2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= -8 + 0 + 2 + 1$

$= -5$

0b0101

$= (0 \times -2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

$= 0 + 4 + 0 + 1$

$= 5$

Two's Complement: Algorithm

At home: Prove algorithm is equivalent to formula!

- Positive <u>and</u> negative numbers can be computed using the same formula:
  - Highest bit multiplied by neg power of 2!

0b1011

$= (\mathbf{1} \times -2^3) + (\mathbf{0} \times 2^2) + (\mathbf{1} \times 2^1) + (\mathbf{1} \times 2^0)$

$= -8 + 0 + 2 + 1$

$= -5$

0b0101

$= (\mathbf{0} \times -2^3) + (\mathbf{1} \times 2^2) + (\mathbf{0} \times 2^1) + (\mathbf{1} \times 2^0)$

$= 0 + 4 + 0 + 1$

$= 5$

- Hardware to convert positive to negative (& vice versa) is **simple**.
  1. Complement all bits
  2. Then add 1

5_{ten} `0101` → `1010` → `1011` -5_{ten}

-5_{ten} `1011` → `0100` → `0101` 5_{ten}

Two's Complement: C standard (as of 2025)

- Two's complement is the C23 standard number representation for signed integers.

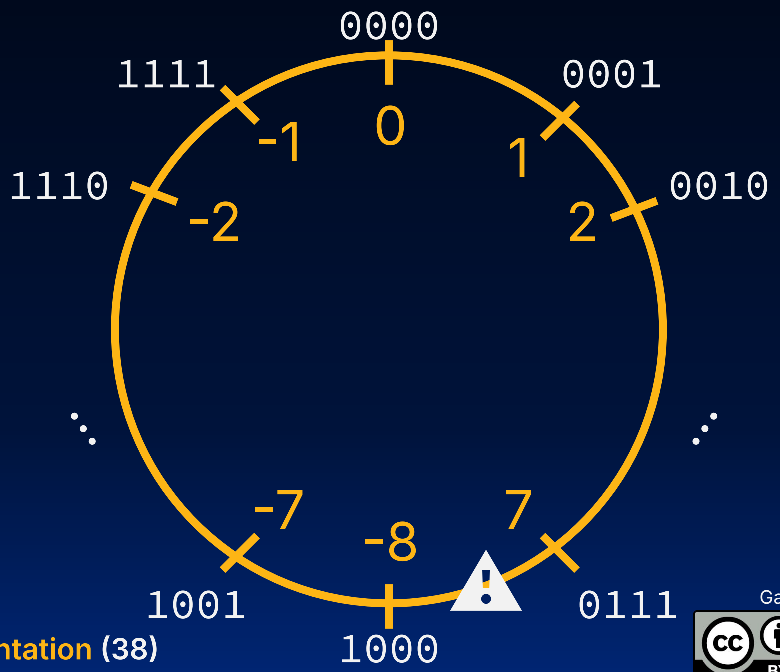
  - 2^{N-1} negatives
  - 2^{N-1} non-negatives
    - 1 zero
    - How many positives?

Two's Complement: Integer Overflow

- Two's complement is the C23 standard number representation for signed integers.

  - 2^{N-1} negatives
  - 2^{N-1} non-negatives
    - 1 zero
    - How many positives?

- **Integer overflow** in two's complement can be conceptualized via a "number wheel":



Agenda

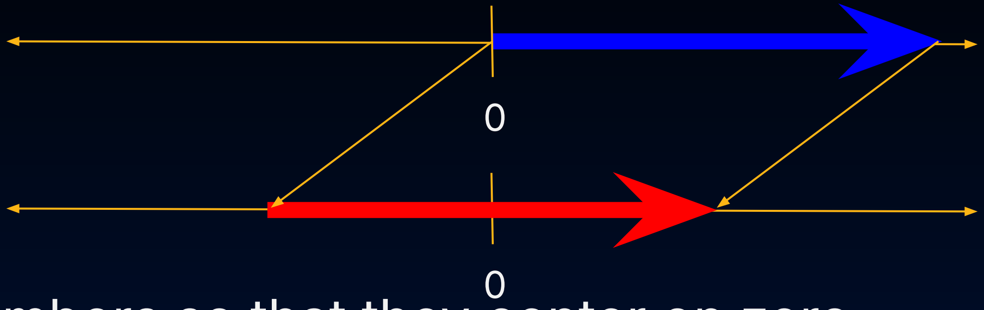
Bias Encoding

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

Bias Encoding

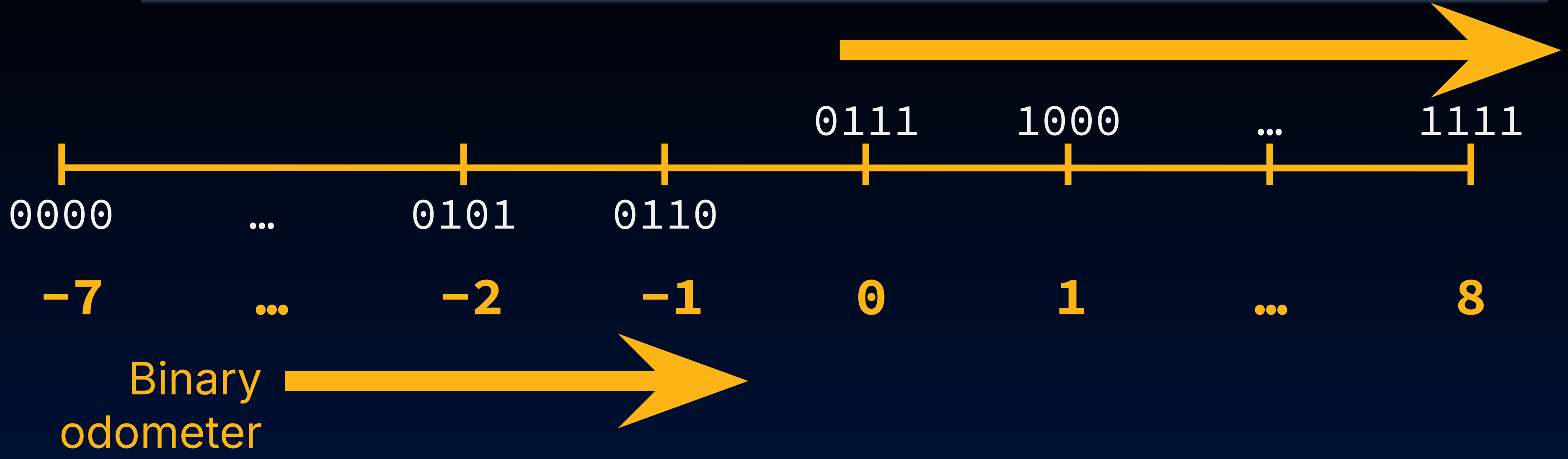
Think of an electrical signal from 0v to 15v. How to center on 0?

- We have a system
  that can represent this:
- We want to represent this:



- **Bias encoding**: "Shift" the numbers so that they center on zero
- Formally:
  - Define a "bias"
  - To interpret stored binary: Read the data as an unsigned number, then **add the bias**
  - To store a data value: Subtract the bias, then store the resulting number as an unsigned number

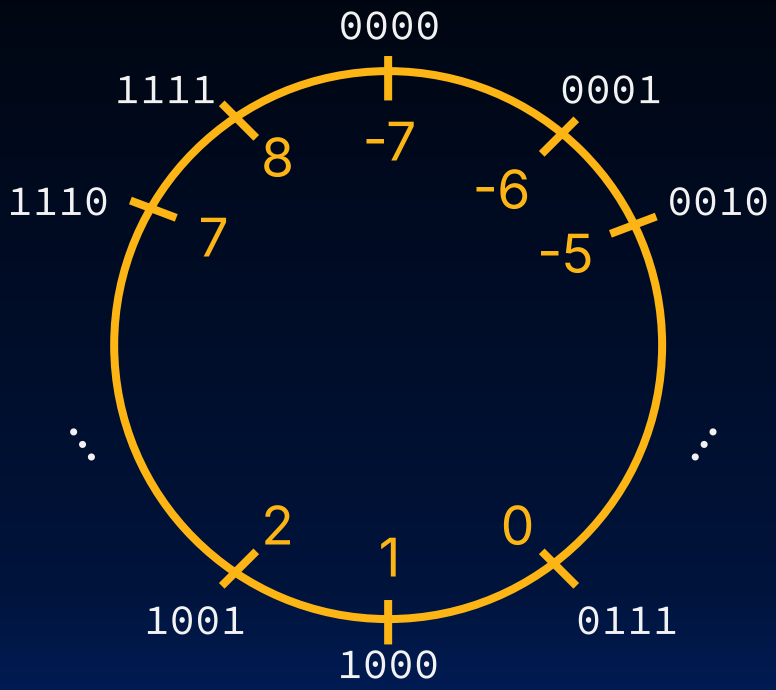
Bias Encoding



- Number = (unsigned rep) + (bias)
- With N bits, default bias is $-(2^{N-1} - 1)$
  - E.g., 4 bits, bias = $-(2^3-1) = -(8-1) = -7$
- Bias could be anything we want! (i.e., 4 bits could be #s 800-815)

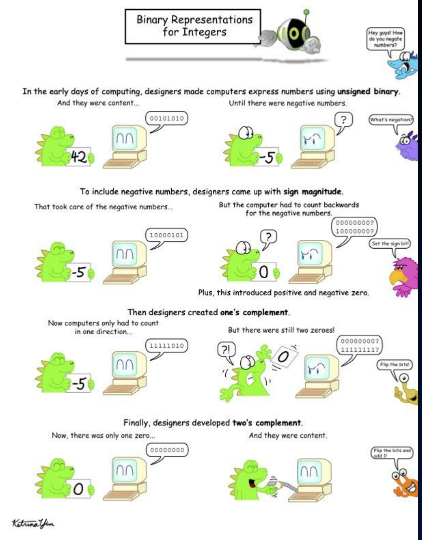
Bias Encoding: N = 4, bias = -7

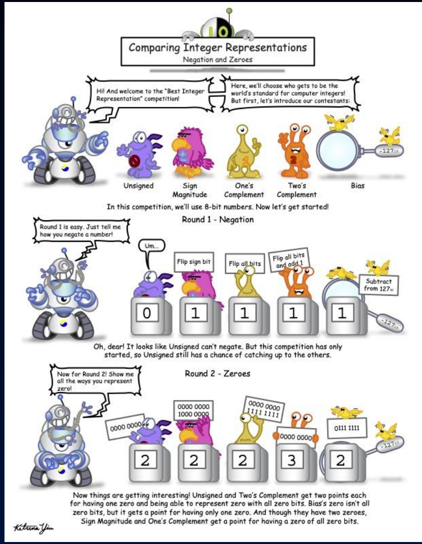
Example: N = 4, bias = -7

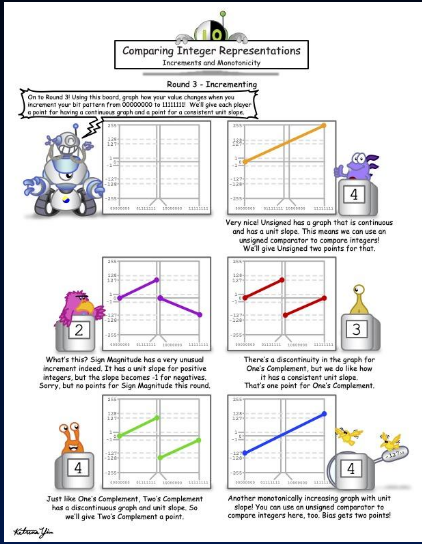


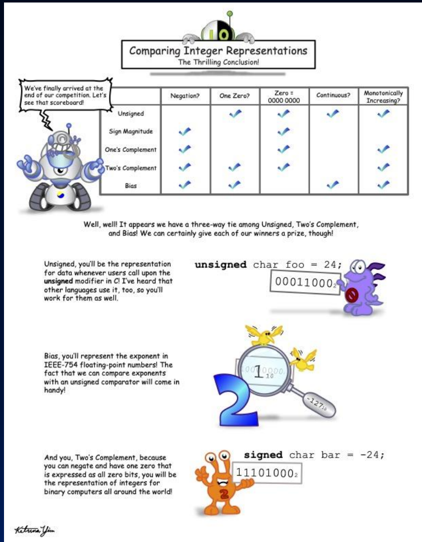
- Consider:
  - One zero
  - How many positives?
  - How many negatives?

Amazing Illustrations by Ketrina (Yim) Thompson









And in summary...

- We represent "things" in computers as particular bit patterns:
  - With N bits, you can represent at most 2^N things.
- Today, we discussed five different encodings for integers:
  - Unsigned integers
  - Signed integers:
    - Sign-Magnitude
    - Ones' Complement
    - Two's Complement
    - Bias Encoding
- Computer architects make design decisions to make HW simple
  - Unsigned and Two's complement are C standard. Learn them!!
- Integer overflow: The result of an arithmetic operation is outside the representable range of integers.
  - Numbers have infinite digits, but computers have finite precision. This can lead to arithmetic errors. More later!

For you to consider:
How could we represent -12.75?

Respond at **pollev.com/ddg**

Text **DDG** to **22333** once to join, then **A, B, C, D, or E**

L02b How best to represent -12.75? (explain shifting binary point)

- 2s Complement (but shift binary point)
- Bias (but shift binary point)
- Combination of 2 encodings
- Combination of 3 encodings
- We can't

Powered by Poll Everywhere

Start the presentation to see live content. For screen share software, share the entire screen. Get help at **pollev.com/app**

Respond at **pollev.com/ddg**

Text **DDG** to **22333** once to join, then **A, B, C, D, or E**

L02b How best to represent -12.75? (explain shifting binary point)

- 2s Complement (but shift binary point)
- Bias (but shift binary point)
- Combination of 2 encodings
- Combination of 3 encodings
- We can't

Poll locked. Responses not accepted.

L02b How best to represent -12.75? (explain shifting binary point)

- 2s Complement (but shift binary point)
- Bias (but shift binary point)
- Combination of 2 encodings
- Combination of 3 encodings
- We can't