Number Representation Notes

CS Lecture Notes

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1 Representation Scheme

Representation Scheme: A method that tells us how to store information (e.g., numbers, addresses).

Example: Number representation

2 Number Representation

Number Representation: A set of rules for how numbers are interpreted and encoded.

Example: Base systems

3 Base / Radix

Base: A number system that defines the number of unique digits used to represent an integer.

Examples of Common Base Systems

1. Binary (Base-2) = 0, 1
2. Octal (Base-8) = 0, 1, 2, 3, 4, 5, 6, 7
3. Decimal (Base-10) = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
4. Hexadecimal (Base-16) = 0–9 and A–F, where A = 10, B = 11, ..., F = 15

All of these are number systems.

We typically index digits starting from 0, with the rightmost digit (the one's place) being position 0.

Number Value:	1	3	2	9
Place Value:	thousand's	hundred's	ten's	one's
Digit Index:	10^3	10^2	10^1	10^0

Number Value:	1	0 .	2	9
Place Value:	ten's	one's	tenth's	hundredth's
Digit Index:	10^1	10^0	10^{-1}	10^{-2}

4 Decimal (Base-10)

Decimal: A positional number system that represents any integer.

Digits for base-10 system: 0–9

Example:

$$12 = (1 \times 10^1) + (2 \times 10^0) = 12$$

We use powers of 10 because there are 10 distinct digits. After 9, we roll over to the next place value:

$$9 \to 10, \quad 10 \times 10 = 100$$

5 Binary (Base-2)

Binary: A positional number system that represents two digits.

Digits: 0 and 1.

Binary prefixes: `0b`, subscript 2, or just binary digits.

Example:

$$0b0100 = (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) = 4$$

Binary rolls over after 1, just like decimal rolls over after 9.

$$\text{Decimal } 2 = \text{0b0010}, \text{ Decimal } 4 = \text{0b0100}$$

A **bit** is a binary digit (0 or 1). A single bit can represent anything, e.g., 0 = False, 1 = True.

Example: The binary `0b1010` has 4 bits, it is just the total number of 0's and 1's.

6 More on Bits

Common groupings:

- Nibble = 4 bits
- Byte = 8 bits
- 2 Bytes = 16 bits
- 4 Bytes = 32 bits

Formulas

$$2^n = \text{Total number of values representable with } n \text{ bits}$$
$$2^n - 1 = \text{Maximum decimal value with } n \text{ bits}$$
$$2^{(n-1)} = \text{Value of the leftmost (most significant) bit}$$

Example: With 4 bits:

$$2^4 = 16 \quad \Rightarrow \quad \text{16 values total}$$
$$2^4 - 1 = 15 \quad \Rightarrow \quad \text{Max decimal value} = 15$$
$$2^{(4-1)} = 2^3 = 8 \quad \Rightarrow \quad \text{Significant bit value} = 8$$

7 Examples of Binary and Decimal

Binary to Decimal

Example 1:
`0b11001011`

Binary Value:	1	1	0	0	1	0	1	1
Digit Index:	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Thus, `0b11001011`

$$(1 \times 128) + (1 \times 64) + (0 \times 32) + (0 \times 16) + (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1)$$
$$= 128 + 64 + 8 + 2 + 1 = 203$$

Example 2:
`0b00111111`

Binary Value:	0	0	1	1	1	1	1	1
Digit Index:	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Thus, `0b00111111`:

$$(0 \times 128) + (0 \times 64) + (1 \times 32) + (1 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + (1 \times 1)$$
$$= 32 + 16 + 8 + 4 + 2 + 1 = 63$$

Decimal to Binary

Example 1:

`143, using 8 bits, our leftmost digit is 128 which is` 2^{8-1}

Digit Index:	128	64	32	16	8	4	2	1
Binary Value:	1	0	0	0	1	1	1	1

Thus, `143` = 0b10001111

Example 2:

`9, using 4 bits, our leftmost digit is 8 which is` 2^{4-1}

Digit Index:	8	4	2	1
Binary Value:	1	0	0	1

Thus, 9 = 0b1001

Example 3:

`3.14, using 4 integer bits for the decimal 3, and 4 fractional bits for .14 decimal`

`3`

Digit Index:	8	4	2	1
Binary Value:	0	0	1	1

Thus, 3 = 0b0011

`.14, for fractions, multiply by 2`

fraction:	.14	.28	.56	.12
$fraction * 2$:	.28	.56	1.12	.24
Binary:	0	0	1	0

So, the binary of .14 using 4 bits fractional is

$$.14 \approx 0b0.0010.$$

We put a 1 at the third step because $0.56 \times 2 = 1.12 \geq 1$. That means the 2^{-3} place (the 1/8 place) is filled with a 1. The earlier steps gave 0s in the 2^{-1} and 2^{-2} places.

Thus, the binary of 3.14 using 4 bits and 4 fractional bits is

$$3.14 \approx 0b0011.0010.$$

Example 4:

`.36`

fraction:	.36	.72	.44	.88
$fraction * 2$:	.72	1.44	.88	1.76
Binary:	0	1	0	1

So,

$$.36 \approx 0b0.0101.$$

8 Hexadecimal (Base-16)

Hexadecimal: A positional number system that includes 16 digits.

Digits: 0-9, and A-F, where A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.

Prefix: 0x

Example: "0x14"

$$0x14 = (1 * 16^1) + (4 * 16^0) = 20$$

What we did is convert hexadecimal to decimal using the power of 16 because we have 16 different digits. Additionally, we know that $16^1 = 2^4 = 16$, and $16^0 = 2^0 = 1$ thus we can write,

$$0x14 = (1 * 16^1) + (4 * 16^0)$$

$$0x14 = (0b0001) * 2^4 + (0b0100) * 2^0 (\textit{Converted the values 1 and 4 to binary})$$

$$0x14 = (0b0001)0000 + 0b0100(\textit{concatenate})$$

$$0x14 = 0b00010100(\textit{Binary})$$

Now we converted hexadecimal to binary. We converted 1 and 4 to binary because 2^4 is 16^1, and each hex digit maps to a nibble, then we concatenate.

Steps from hexadecimal to decimal.

1. Multiply the individual values by 16 with its respective position.
2. Then add those values to obtain the decimal.

Steps from decimal to hexadecimal.

1. Convert the whole decimal to binary.
2. Then split the binary into nibbles, resulting to hex form.

Steps from hexadecimal to binary.

1. Convert the individual values to binary.

2. Then concatenate both binary values to obtain the final binary.

Steps from binary to hexadecimal.
1. Split the binary into two nibbles.
2. Then match it to the right digit.

9 Examples of Hexadecimal

Hexadecimal to Decimal

Example 1: $0x5A$

$$0x5A = (5 * 16^1) + (10 * 16^0) = (5 * 16) + 10 = 90$$

Example 2: $0xF1$

$$0xF1 = (15 * 16^1) + (1 * 16^0) = (15 * 16) + 1 = 241$$

Decimal to Hexadecimal

Example 1: 21 using 8 bits because we want two nibbles to represent a hexadecimal, so our leftmost digit is $2^{8-1} = 2^7 = 128$.

Digit Index:	128	64	32	16	8	4	2	1
Binary Value:	0	0	0	1	0	1	0	1

So,

$$23 = 0b00010101 = 0x15$$

Example 2: 9

Digit Index:	128	64	32	16	8	4	2	1
Binary Value:	0	0	0	0	1	0	0	1

So,

$$9 = 0b00001001 = 0x09$$

Example 3: 233

Digit Index:	128	64	32	16	8	4	2	1
Binary Value:	1	1	1	0	1	0	0	1

So,

$$233 = 0b11101001 = 0xE9$$

Hexadecimal to Binary

Example 1: $0x45$

$$0x45 = (4 * 16^1) + (5 * 16^0)$$

$$0x45 = (0b0100 * 2^4) + 0b0101$$

$$0x45 = (0b0100 * 0000) + 0b0101 = 0b01000101$$

Example 2: $0x6B$

$$0x6B = (6 * 16^1) + (11 * 16^0)$$

$$0x6B = (0b0110 * 2^4) + 0b1011$$

$$0x6B = (0b0110 * 0000) + 0b1011 = 0b01101011$$

Binary to Hexadecimal

Example 1: $0b11001100$

Binary in two nibbles:	1100	1100
Hexadecimal value:	C	C

So,

$$0b11001100 = 0xCC$$

Example 2: $0b10100111$

Binary in two nibbles:	1010	0111
Hexadecimal value:	A	7

So,

$$0b10100111 = 0xA7$$

10 Octal (Base-8)

Octal: A positional number system that includes 8 digits.

Digits: 0, 1, 2, 3, 4, 5, 6 ,7.

Prefix: 0o.

Example 1: "0o12"

$$0o12 = (1 * 8^1) + (2 * 8^0) = 8 + 2 = 10$$

We converted octal to decimal. Similar to hexadecimal, if you want to go from decimal to octal, you follow the same steps.

Example 2: 83

Digit Index:	256	128	64	32	16	8	4	2	1
Binary Value:	0	0	1	0	1	0	0	1	1

So, 1 octal = 3 bits, we can group the binary to 3 bits, because $8^1 = 2^3$, where n = 3, number of bits.

$$83 = 0b001010011 = 0o123$$

Example 3: $0b101101$ to octal form

$$0b101101 = 0b101 \quad 0b101 = 5 \quad 5 = 0o55.$$