ECE3230 - Practicum II

Sampling, Aliasing, Decimation, Interpolation

Reporting Requirements: Follow report instructions for Practicum I.

Objective: To gain experience with and an understanding of sampling of a continuous time signal, x(t), at different sampling rates (sampling period T_s or sampling frequency $f_s = 1/T_s$) to reach the discrete time version of it, x[n], and what kind of effects these operations can have on the signal's frequency content. Furthermore, decimation (reduction of the sampling rate, down-sampling) and interpolation (increasing the sampling rate, up-sampling) concepts will be studied.

Background:
As mentioned in Practicum I, sampling is discussed on an elementary level (adequate for this Practicum) in Section 1 of Dr. Kevin Buckley's notes. Please review this section again.

Basically, the ideal A/D converter assumed here forms a discrete time signal x[n] from an underlying continuous time signal x(t) as

$$x[n] = x(nT_s) \tag{1}$$

where T_s is the sampling interval in seconds (so $f_s = 1/T_s$ is the sampling frequency in samples/second).

A D/A converter operates to reconstruct x(t) by effectively interpolating between the samples (in x[n]). As you will see later in the lecture in ECE 3225, x(t) can be exactly reconstructed only if $f_s > 2f_{max}$ where f_{max} is the maximum frequency component of x(t).

Review these topics again before the practicum session.

Procedures: This practicum partially follows the Lab on Sampling and Aliasing concepts in the reference text "DSP First: A Multi-media Approach", by McClellan, Schafer and Yoder, Prentice Hall, 1998. This Lab consists of 2 procedures, both are to be performed within 1 week.

1. ***Generating and plotting of cosines sampled at different rates*:**

(a) For a sinusoidal signal *x(t)*:

$$x(t) = \mathrm{A}\cos(\omega_o \mathrm{t} + \varphi)$$

Generate a vector *x1* as samples of *x(t)* with A = 1, $\omega_0 = 2\pi(5)$ and $\varphi = 0$. Use a sampling rate of fs=100 samples/second, for a duration of 10 seconds. Plot (use the "plot" command) *x1* for $0 \leq t \leq 0.5$ seconds (You can use the command axis([0 0.5 -1 1]) to zoom in a certain portion of a plot, type help to learn more on the "axis" command). Also, plot the spectrogram of *x1* using the "spectrogram" command. (Make sure your axes are properly labelled with time and frequency, *e.g. use spectrogram(x1,256,250,512,fs, 'yaxis').) Does the time domain signal, x1 look like a sinusoid? Calculate its frequency from the time domain plot and compare it with the frequency found in spectrogram plot, is it the same as the sinusoid you have generated using the cos( ) function in Matlab?*

(b) Using the sinusoidal signal given in part 1(a), compute a vector *x2* as its samples, over a 10 second duration, of a sinusoid with A = 2, $\omega_o = 2\pi(90)$ and $\varphi = \pi/3$ for fs=100 samples/second. Plot *x2* for $0 \leq t \leq 0.5$ seconds. Also, plot the spectrogram of *x2* using the "spectrogram" command as in 1(a). *Does the time domain signal look like a sinusoid? What* *is the frequency of the signal x2 (calculate it from the time domain plot and obtain it from the spectrogram plot), is it the same as the sinusoid you have generated? Comment on what could be the reason if there is a discrepancy.*

(c) Repeat part 1(b) and generate *x3* for the same values except for fs=1000. *Comment on your observations, what kind of an effect increasing the sampling frequency had on the discrete time signal and its frequency content.*

2. *Down-sampling and up-sampling for image resizing*:

Images that are stored in digital form on a computer have to be sampled images because they are stored in an M ×N array (a matrix). The sampling rate in the two spatial dimensions was chosen at the time the image was digitized. If we want a different sampling rate, we can simulate a lower sampling rate by simply throwing away samples in a periodic way. For example, if every other sample is removed (x[2n]), the sampling rate will be halved (fs/2). Usually this is called sub-sampling or down-sampling. Down-sampling throws away samples, so it will shrink the size of the image. If xx is the original image and xxds is the down-sampled image, for the downsampling by a factor of p, it can be done by the following scheme:

```
xxp = xx(1:p:end,1:p:end);
```

(a) Down-sampling – Image compression: One potential problem with down-sampling is that aliasing (overlap in frequency domain) might occur. This can be illustrated in a dramatic fashion with the lighthouse image. Load the lighthouse.mat file which has the image stored in a variable called *xx*. When you check the size of the image, you'll find that it is not square. *Plot the original lighthouse image using the show_img.m file given in the course website.* Down-sample the lighthouse image by a factor of p=3. *What is the size of the down-sampled image?* Notice the aliasing in the down-sampled image. *Describe how the aliasing appears visually. Which parts of the image show the aliasing effects most dramatically and why?* (Notice that the fence provides a sort of increasing frequency from left to right of the image. You can explain aliasing in the lighthouse image by using a "frequency domain" explanation similar to the ones in part 1.)

(b) Up-sampling – Images Reconstruction: When an image has been down-sampled, we can fill in the missing samples by up-sampling in different ways (stretching - filling in the missing values by zeros, holding - filling in the missing values by the same prior values or doing interpolation – linear or other kinds).

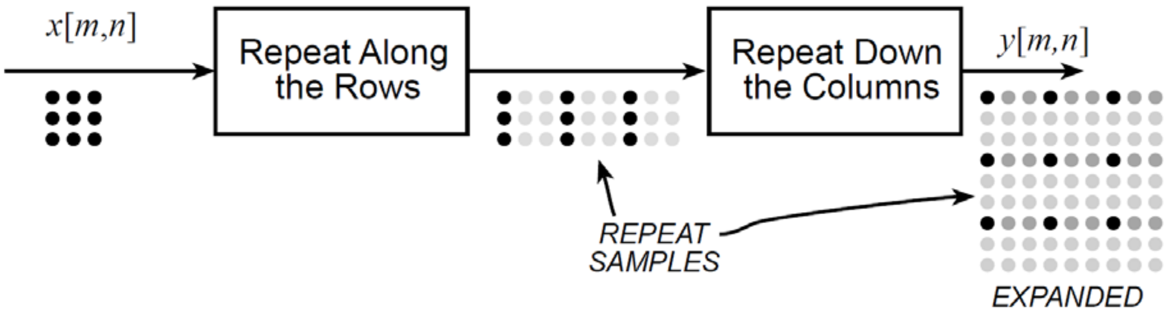

Figure 1: 2-D Interpolation broken down into row and column operations: the gray dots indicate repeated data values created by a zero-order hold; or, in the case of linear interpolation, they are the interpolated values.

For the following three reconstruction experiments, use the lighthouse image, *xx,* down-sampled by a factor of 3 as in 2(a). The down-sampled lighthouse image should be created and stored in the variable *xx3*. The objective will be to reconstruct an approximation to the original lighthouse image from the smaller size down-sampled image.

(i) The simplest up-sampling would be through stretching the image both in its rows and its columns (xxs[n,n]=xx3[n/p,n/p] as would be performed analytically in signal processing applications – not a Matlab script!) simply by inserting zeros in the samples corresponding the samples n in xxs[n,n] where n/p in xx3[n/p,n/p] does not exist (when n/p is not an integer). Generate and plot *xxs* in Matlab for p=3 (insert appropriate number of zeros in between existing samples to do this operation) which can be done by the Matlab commands as given below:

```
xxs=zeros(size(xx));
xxs(1:3:size(xx,1),1:3:size(xx,2))=??? %fill in
show_img(xxs)
```

Compare xxs with the original lighthouse image xx. Comment on the visual appearance of the "reconstructed" image versus the original; point out differences and similarities. Comment on your observations for this type of up-sampling operation.

(ii) Another simple but more effective up-sampling procedure would be reconstruction with a square pulse which produces a "zero-order hold." For the up-sampling factor p, the zero-order hold method fills in the missing values by the same prior values by p times as shown in Figure 1. Using the down-sampled lighthouse image, *xx3*, obtain the up-sampled image *xxh* by a factor p=3 xxh using the zero-order hold idea. An example Matlab script to perform this job is given below:

```
[Nr,Nc] = size(xx3);
ip = ceil((0.9:1:p*Nr)/p);
jp = ??? %fill in
xxh = xx3(ip,jp);
show_img(xxh)
```

Compare the result (xxh) to the original lighthouse image, xx and the stretched image xxs. Point out differences and similarities (regions where they differ such as the regions of "low-frequency" content (background) and "high-frequency" content (fence, edges, etc.)). In general can the reconstruction process remove the aliasing effects from the down-sampled lighthouse image, xx3?

Practicum II

Instructor/TA Sign Off Sheet

Student's Name: ____

1. Pract. II, Procedure 1: plots for (a,b,c) ______________________________________
   Record requested comments for (a,b,c)

2. Pract. II, Procedure 2(a,b): image plots ___________________________________
   Record requested comments for (a,b)