ECE 3230 - Practicum VI

CTFS Spectrum Analysis

Reporting Requirements: Follow report instructions for Practicum I.

Introduction

In this practicum you will investigate Fourier transformation, specifically CTFS in more depth. You will study the frequency content of CT signals where you will numerically derive and plot their spectra. In effect, you will be designing, implementing and testing a digital spectrum analyzer (i.e. a spectrum analyzer which computes spectrum estimates using digital hardware and/or software). These days, the vast majority of spectrum analyzers are digital. Here, you will develop a spectrum analyzer for CT periodic signals. That is, you will design and implement a Matlab function for computing the CTFS coefficients from samples of one period of a CT periodic signal. In addition, you will test this CTFS spectrum analyzer on a signal with known frequency content such as an Amplitude Modulated (AM) signal. You will then use the CTFS spectrum analyzer on an Electrocardiogram (ECG) signal. For each of these signals, you will draw conclusions from its spectrum.

Background

For a real-valued periodic signal $x(t)$ with period T_o and fundamental frequency $\omega_o = 2\pi/T_o$, the signal can be written using the CTFS coefficients a_k (in cosine or trigonometric form) as:

$$x(t) = a_o + 2\sum_{k=1}^{\infty} |a_k| \cos(\mathrm{k}\omega_o \mathrm{t} + \measuredangle a_k) \tag{1}$$

where the exponential CTFS coefficients, a_k are found as:

$$a_k = \frac{1}{T_o}\int_0^{T_o} x(t) e^{-jk\omega_o t} dt \tag{2}$$

We know that we cannot generate or have a CT signal for use in computers, but we can generate or use samples of the underlying CT signal, $x(t_n) = x(nT_s)$ at time instants $t_n = nT_s \; for \; n = 0,1, \dots, (N-1)$ where $f_s = 1/T_s$ is the sampling rate. For a periodic CT signal of period T_o, the number of samples, N in the sampled signal $x(t_n)$ required for one period T_o of it with the sampling interval T_s, can be found as: $NT_s = T_o$. Then, we can approximate the CTFS integral as a summation to find the approximate CTFS coefficients $\hat{a}_k$ for selected k values from K_1 through K_2, as follows:

$$\hat{a}_k = \frac{1}{NT_o}\sum_{n=0}^{N-1} x(t_n) e^{-jk\omega_o t_n}; \qquad k = K_1, K_1+1, \dots, K_2-1, K_2 \tag{3}$$

is computed, which will in general be an approximation.

1. Periodic Amplitude Modulated (AM) Sinusoid: In AM communications, the transmitted information is embedded as the carrier sinusoid's amplitude. Consider the signal

$$x(t) = (1 + 0.25 \cos(\omega_m t)) \cos(\omega_c t)$$

where ω_c (in radians/second) is called the carrier frequency. $\cos(\omega_c t)$ is the carrier signal that controls what band of frequencies the transmitted signal occupies. ω_m is the frequency of the sinusoid used here to simulate the "message" signal. The constant 0.25 is called the modulation index. It controls the amount of variation in the carrier sinusoid's amplitude.
In this practicum, let $\omega_m = 2\pi$ and $\omega_c = 20\,\omega_m$. (In radio applications, ω_c would be much higher, and $\cos(\omega_m t)$ would be replaced with speech or music.)

(a) Before the practicum session, determine ω_o and T_o, the fundamental frequency and period of x(t)?
(b) Before the practicum session, analytically determine and sketch the CTFS coefficients a_k.

(c) Recall that in Part 3 of Practicum V you took a “black box” approach to analysis. That is, you used the Matlab *fft* function to compute CTFS coefficients. Now develop your own generally applicable spectrum estimation algorithm for CT periodic signals. Specifically, starting with the exponential CTFS coefficient integral equation, use numerical integration to compute the a_k ’s for k = 0, ±1, ±2, ..., ±30. Using the Matlab stem function, plot the power spectrum | a_k |^2 vs. f. Referring to this plot, describe the frequency content of x(t). Is it what you expected?

Your code to calculate CTFS coefficients should start with:

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function [ak] = CTFSmine(x, tn, To, K1,K2)
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where the input and output variables are as explained in equation (3), x being a vector containing the periodic CT signal samples and tn is the vector containing the time samples within one period To starting from time 0 obtained with sampling interval Ts, ak are the approximate CTFS coefficients for k values from K1 to K2 as in equation (3).

2. An ECG Signal: Now that you have tested and verified your spectrum analyzer on data with known frequency content, you will proceed to use it to analyze a signal with unknown frequency content. An ECG signal usually consists of repeated heart beat signals, called QRST complexes. Although the resulting ECG signal is not exactly periodic, because no two QRST complexes are exactly alike and because the rate of these beats changes over time, we will treat the ECG signal as periodic here because it can be an accurate approximation.
Consider data file, ECG_p6.mat given in course website, contains samples of one” period” of an ECG signal x(t), covering time $0 \leq t < 2/3$, in seconds
.
(a) What are ω_o and T_o, the fundamental frequency and period of x(t)?
(b) What is the sampling interval Ts and the sampling rate fs?
(c) Plot x(t) for $0 \leq t \leq 2$.
(d) Using the spectrum analyzer you developed in Procedure 1, numerically compute the CTFS coefficients a_k ’s for k = 0,±1,±2, ...,±K. Due to sampling effects (repetitions in frequency domain at integer multiples of fs), assume that x(t) contains frequency components up to fs/2 but not beyond. Select K with this in mind. Plot the power spectrum | a_k |^2 vs. f (in Hz). Referring to this plot, describe the frequency content of x(t).(Relating characteristics of this plot to the heart’s condition is something a Cardiologist does, assuming he or she understands signal frequency content. A signal processing engineer would design the equipment based on specifications which assure that the Cardiologist is presented the spectrum to the level of accuracy she/he needs.)

Practicum VI: Spectrum Analysis
Instructor/TA Sign Off Sheet, & Report Form

Student's Name: ………………………………………….

1. Procedure 1(a,b): write down To and ωo. "Analytically" derive the a_k (e.g. using the CTFS equation or Table 6 of the Course Notes), and sketch them vs. ω.

2. Procedure 1(c): numerically computed $|a_k|^2$ vs. ω plot…………………………………………………

3. Procedure 1(c): Are you convinced that your spectrum analyzer is functioning correctly? Briefly explain.

4. Procedure 2(a): Record ωo and To for this signal ……………………… Also, record the sampling interval Ts and rate fs for this data ………………………

5. Procedure 2(a): Assume that x(t) contains frequency components up to fs/2 (due to sampling) but no higher. With this in mind, determine the highest harmonic, K, that you will compute a CTFS coefficient for……………..

6. Procedure 2(b,c): x(t), and numerically computed $|a_k|^2$ vs. f (Hz) plots…………………………..

7. Procedure 2(c): Comment on the frequency content of the ECG signal.