ECE3230 - Practicum V

Continuous Time Fourier Series (CTFS)

Reporting Requirements: Follow report instructions for Practicum I.

Introduction: In this practicum, you will study the representation of CT periodic signals using the CTFS. Specifically, you will see, via Matlab, how periodic signals can be built as weighted sums of sinusoids. You will examine the effect of representing a periodic signal using only some of the terms of its CTFS expansion. Finally, you will study the effect that modification of the CTFS coefficients has on a periodic signal.

To simplify coding and interpretation, in this practicum we will consider a real-valued signal and its CTFS in *cosine (i.e. trigonometric)* form. For a real-valued periodic signal with period T and fundamental frequency $\omega_o = 2\pi/T$, the CTFS can be written as

$$x(t) = a_o + 2\sum_{k=1}^{\infty} |a_k| \cos(\mathrm{k}\omega_o \mathrm{t} + \measuredangle a_k) \tag{1}$$

where the a_k as the exponential CTFS coefficients

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt \tag{2}$$

Specifically, we will consider a periodic signal $x(t)$ with period $T = 4$ which is defined over one period as

$$x(t) = (4 - t) \qquad 0 \le t \le 4 \tag{3}$$

Its CTFS coefficients are

$$a_k = \begin{cases} 2 & k = 0 \\ \frac{-j2}{k\pi} = \frac{2}{k\pi}e^{-j\pi/2} & otherwise \end{cases} \tag{4}$$

1. CTFS Representation of a periodic signal:

(a) On paper, determine the power of $x(t)$ using both expressions in the following equation:

$$P = \frac{1}{T}\int_0^T x^2(t)dt = \sum_{k=-\infty}^{\infty} |a_k|^2 \tag{5}$$

This result illustrates a property of the CTFS (and CTFT) called Parseval's theorem. Comment on the result, and discuss how Parseval's theorem can be useful.

(b) Consider the approximation of $x(t)$

$$\hat{x}_N(t) = a_o + 2\sum_{k=1}^{N}|a_k|\cos(k\omega_o t + \measuredangle a_k) = 2 + \sum_{k=1}^{N}\frac{4}{k\pi}\cos\left(\frac{k\pi t}{2} - \pi/2\right) \tag{6}$$

Using Matlab, plot $x(t)$ for $-4 \le t \le 8$. Now plot $\hat{x}_N(t)$ for N = 1, 3, 7, 11 and 51, and compare these to $x(t)$. In your own words, explain why the $\hat{x}_N(t)$ are both similar and different from $x(t)$.

(c) Plot the power spectrum of *x(t)*, $|a_k|^2$ *vs.* *k* and *vs.* ω for $0 \leq \omega \leq 6\pi$.

2. Modification of the CTFS Coefficients:

(a) Now consider a new signal *y(t)* formed by modifying the CTFS coefficients of the *x(t)* above. Specifically, let $a_k^x = a_k$, and let the CTFS coefficients of *y(t)* be

$$a_k^y = a_k^x e^{-jk\frac{2\pi T}{T\,4}} = a_k^x e^{-jk\frac{\pi}{2}} \tag{7}$$

Use Matlab to plot the approximation of *y(t)*

$$\hat{y}_{51}(t) = a_o^y + 2\sum_{k=1}^{51} |a_k^y| \cos\left(\mathrm{k}\omega_o \mathrm{t} + \measuredangle a_k^y\right) \tag{8}$$

Compare your estimate of *y(t)* to the original signal *x(t)*. What effect has this particular change of CTFS coefficients had on the original *x(t)*? In your own words, why does this make sense?

(b) Repeat 2(a) for *z(t)* formed using CTFS coefficients

$$a_k^z = a_{-k}^x \tag{9}$$

3. CTFS and Matlab's fft of a Given Periodic Signal

The data file *pract5_data.mat* found in course website contains N_1 = 2048 samples of a CT periodic signal taken at a sample rate of *fs* = 48, 000 samples/sec.

(a) Copy the data file into your working directory or otherwise assure that you have access to it within Matlab. Load the file into Matlab. In Matlab, what is the data array name? Plot it. How many periods of the signal are present? What is the period N of the signal (in samples)? What is the period T of the signal (in seconds)?

(b) What is the fundamental frequency ω_o of this CT periodic signal (in radians/sec.).

(c) By periodically extending the available data, create 1 sec. of the signal. Use the Matlab *soundsc* function to listen to this 1 sec. segment. Considering your experience from prior Practicums, what does this signal sound like?

(d) Process the first N samples of the data array with the Matlab *fft* function. The first 64 values of the result will give the DC and first 63 positive frequency CTFS coefficients. Specifically, letting X[k]; k = 0, 1, · · · , 63 denote these values, then

$$a_k = \frac{X[k]}{N} \qquad k = 0,1,\ldots,63 \tag{10}$$

Plot the magnitude of these values vs. frequency using 'stem' function over the range $0 \leq \omega \leq 63\omega_0$. How many non-zero coefficients are there over this range? What harmonic frequencies (in Hz.) do they correspond to? Where have you seen these harmonics and harmonic levels before? What musical note and note frequency is this sound closest in frequency to?

Practicum V

Instructor/TA Sign Off Sheet & Report Form

Student Name: ……………………………………………………………………….

1. Procedure 1(a): derive the power of $x(t)$

2. Procedure 1(b): plots of $\hat{x}_N(t)$; N = 1, 3, 7, 11, 51…………………………………………………….

3. Procedure 1(c): plots of $|a_k^x|^2$ vs k and $|a_k^x|^2$ vs w …………………………………………………….

4. Procedure 2(a): write down the equation relating y(t) and x(t). Which CTFS property does this procedure highlight?

5. Procedures 2(a,b): plots of $\hat{y}_{51}(t)$ and $\hat{z}_{51}(t)$ ……………………………………….
How do these signals relate to x(t)? Comment of the significance of these results (i.e. in relation to some application).

6. Procedure 3(a): period in …………………………… samples; in ……………………………….. seconds

7. Procedure 3(b): ω_0…………………………………………..

8. Procedure 3(c): sign off on plot of 1 second of the signal…………………………………………

9. Procedure 3(d): sign off on plot of $|a_k|$ vs. ω…………………………………………………………

10. Procedure 3(d): What note is the audio signal (e.g. middle C)…………………………………