Mathematical model for the evolutionary dynamics of innovations in public transport systems in a city

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Content

Section 1

Adaptive dynamics

The origin of diversity - Speciation I

The formation of new species, called speciation, is one of the central themes of evolutionary theory. It occurs through the genetic and phenotypic divergence of populations of the same species that adapt to different environmental niches in the same or different habitats.

- Allopatric speciation: two populations are separated geographically by natural or artificial barriers.

- Parapatric speciation: two populations evolve towards geographical isolation by exploiting different environmental niches of contiguous habitats.

The origin of diversity - Speciation II

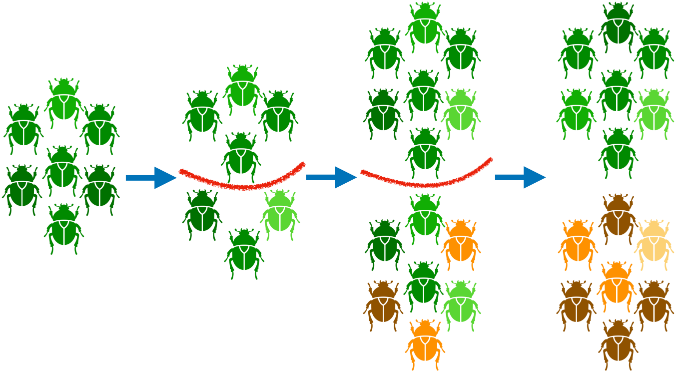

Figura: Different selection pressures and different genetic derivations can act in different environments, isolated populations can eventually become separate species.

Geographic isolation remains an exogenous cause of speciation rather than an evolutionary consequence.

The origin of diversity - Speciation III

- Sympatric speciation: considers populations in the same geographical location.

Key ingredient:

Disruptive selection: selection pressure that favors extreme characteristics over average characteristics.

- It may result, for example, from competition for alternative environmental niches, where being a specialist may be advantageous with respect to being a generalist.

- The population is divided into two initially similar resident groups, which then diverge along separate evolutionary paths (branches), each driven by their own mutations, experiencing what is called evolutionary branch.

The origin of diversity - Speciation IV

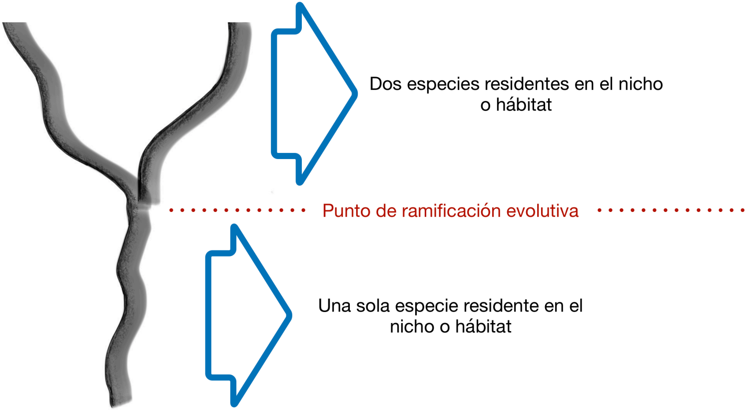

Figura: Under the effect of disruptive selection a monomorphic population may become dimorphic with respect to some relevant attributes

The origin of diversity - Speciation V

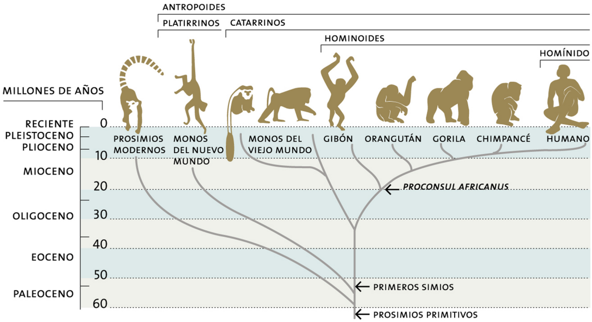

Figura: Phylogenetic tree of the evolution of hominids in the last 7 million years

The causes of the evolutionary ramifications are subject of great discussion

- Origin of articulated language: control of the larynx and mouth, regulated by a particular gene [?].

The origin of diversity - social and technological I

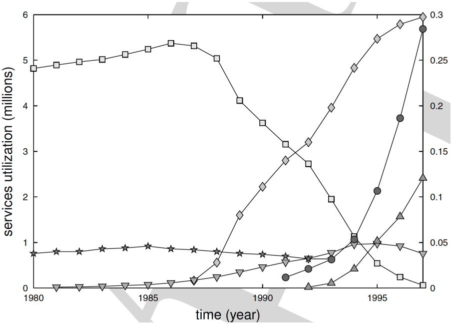

Figura: Use of telecommunication services in Switzerland. **Square:** analog telephones. **Diamonds:**digital phones. **Triangle below:** subscribers to analogue mobile telephony, **Triangle above:** subscribers to digital mobile telephony. **Stars:** public payphones. **Circles:** internet host[**?**].

Attribute substitution: the arrival of digital telephones has been a successful innovation that has led to the replacement of analog telephones.

The origin of diversity - social and technological II

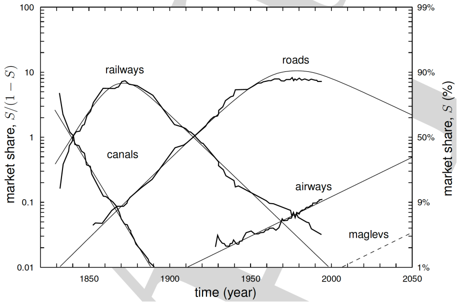

Figura: Evolution of the quota ($S \in [0,1]$) of the transportation infrastructure of USA. By transport system **Thick lines:** historical data. **Thin lines:** estimates based on a substitution logistic model. **Discontinuous line:** a prediction model [**?**].

Under what conditions can the origin of diversity occur in a competitive market between the main public transport systems (TS) of a city?

Section 2

Public transport systems: residents and innovative

Description of assumptions

1. Each transport system will be characterized by a quantitative attribute
   - Average number of passengers transported per unit
2. N transport systems will be assumed residents
3. One (1) transportation system innovator will be assumed
4. Each individual in the city uses a specific TS
5. The budget is increased proportionally to the budget not invested (limited resources)
6. The budget is exhausted to the extent that it is intended to increase the number of users

Generalized Resident Model I

- The population of the city is subdivided into N subpopulations.
- Consider that the N stocks $x_i(t)$ interact through the generalized system:

$$\begin{cases} \dot{x}_i = x_i \left[\alpha(u_i)y_i - \delta(u_i)\right] \left(1 - x_i - \sum_{k=1, k\neq i}^{N} c(u_i, u_k)x_k\right) \\ \dot{y}_i = l(u_i)(1 - y_i) - \epsilon(u_i)\alpha(u_i)x_i y_i \end{cases} \quad (1)$$

- The investor places resources in direct proportion to the budget not yet placed $(1 - y_i)$, with proportionality rate $l(u_i)$.
- $\epsilon(u_i)$ denotes the efficiency with which resources y_i are çonvertedïnto new users x_i.

The model (**??**) will be referred to from now on as the model resident defined in the region of interest:

$$\Omega = \{x_i, y_i \in \mathbb{R} : 0 \leq x_i \leq 1, 0 \leq y_i \leq 1, i = 1, \ldots, N\} \quad (2)$$

Example: a single transport system I

Consider the case $N = 1$, in which there is only one TS available in the city. The equations in (**??**), correspond to the two-dimensional system:

$$\begin{cases} \dot{x}_1 = x_1 \left[\alpha(u_1) y_1 - \delta(u_1)\right](1 - x_1) \\ \dot{y}_1 = l(u_1)(1 - y_1) - \epsilon_1(u_1)\alpha(u_1) x_1 y_1 \end{cases} \tag{3}$$

Instant rate of adoption: $\alpha(u) = a \exp\left(-\frac{1}{2a_1^2} \ln^2\left(\frac{u}{a_2^2}\right)\right)$, as in [**?**].

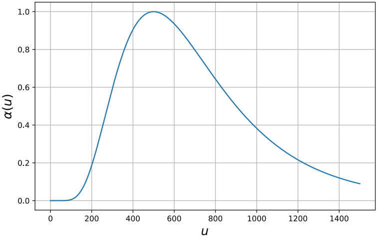

Figura: Picture of $\alpha(u)$ for parameters $a = 1$; $a_1 = 0.5$; $a_2 = 22.36$.

Example: a single transport system II

- It makes perfect sense when x_1 is small and has no competition for other transportation systems.
- A maximum a occurs when $u = a_2^2$, to indicate the value of the easiest absorption attribute (average number of passengers per unit).
- For a TS with low technological content or very sophisticated, $\alpha(u)$ tends to cancel with a controlled sensitivity for a_1. We assume $a > 0$ and $a_1, a_2 \in \mathbb{R}$.

Figura: Representation of a bi-articulated Transmilenio: they have a capacity of 250 people, 62 seated and 188 standing.

It has been considered that the proportion to which fresh resources are invested for the expansion of the TS $l(u) = l$, than the efficiency of the TS i to çonvert"the investment in new users $\epsilon(u) = \epsilon$ the rate at which the TS i is abandoned by the users $\delta(u) = \delta$ are constant.

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Example: analytical and numerical results I

Stationary solutions, for u_1 a fixed value:

- $E_1^a = (0,1)$. Absence of transportation systems in the city.
- $E_1^t = \left(1, \frac{l(u_1)}{l(u_1)+\epsilon(u_1)\alpha(u_1)}\right)$. Total adoption of the transportation system; that is, of maximum possible adoption.
- $E_1^p = \left(\frac{l(u_1)(R_p(u_1)-1)}{\epsilon(u_1)\alpha(u_1)}, \frac{1}{R_p(u_1)}\right)$. Partial adoption of the transport system, where $R_p(u_1) = \frac{\alpha(u_1)}{\delta(u_1)}$. Note that:

$$R_p(u_1) = \underbrace{\alpha(u_1)}_{\text{Adoption rate for TS}} * \underbrace{\frac{1}{\delta(u_1)}}_{\text{Time period while an individual uses the TS}}$$

$R_p(u_1)$ represents a growth threshold for the number of users.

Example: analytical and numerical results II

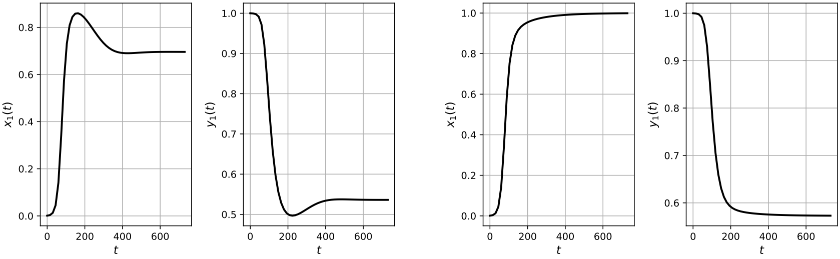

Figura: Two system simulation scenarios are shown **(??)** for $a = 1$; $a_1 = 0.5$; $a_2 = 22.36$; $d = 0.1$ and $u_1 = 200$, thus $l^*(u_1) = 0.0215$ and $R_p(u_1) = 1.8653$. **Left:** solutions for x_1 and y_1 are shown with regards to time t for $l = 0.015$, thus the partial adoption equilibrium point $E_1^p = (0.70, 0.54)$ is LAE. **Right:** we use the same parameter values, but with $l = 0.025$, in this case, the total adoption equilibrium $E_1^t = (1, 0.57)$ is LAE, while $E_1^p \notin \Omega$.

In any case, provided that $\alpha(u_1) > \delta(u_1)$, the system has a non-trivial equilibrium point E_1^p ó E_1^t where it stabilizes

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Example: analytical and numerical results III

Assume now that an innovation appears. The success or failure of the innovation can be studied by extending the system (**??**) into two equations:

$$\begin{cases} \dot{x}_i = x_i\left[\alpha(u_i)y_i - \delta(u_i)\right]\left(1 - x_i - c(u_i,\tilde{u}_j)\tilde{x}_j - \sum_{k=1,k\neq i,j}^{N} c(u_i,u_k)x_k\right) \\ \dot{y}_i = l(u_i)(1-y_i) - \epsilon(u_i)\alpha(u_i)x_i y_i \\ \dot{\tilde{x}}_j = \tilde{x}_j\left[\alpha(\tilde{u}_j)\tilde{y}_j - \delta(\tilde{u}_j)\right]\left(1 - \tilde{x}_j - c(\tilde{u}_j,u_i)x_i - \sum_{k=1,k\neq i,j}^{N} c(\tilde{u}_j,u_k)x_k\right) \\ \dot{\tilde{y}}_j = l(\tilde{u}_j)(1-\tilde{y}_j) - \epsilon(\tilde{u}_j)\alpha(\tilde{u}_j)\tilde{x}_j\tilde{y}_j, \end{cases} \tag{4}$$

Example: innovation for a single TS I

When there is only one transportation system available in the city, the system was obtained

$$\begin{cases} \dot{x}_1 = x_1\left[\alpha(u_1)y_1 - \delta(u_1)\right](1 - x_1) \\ \dot{y}_1 = l(u_1)(1 - y_1) - \epsilon_1(u_1)\alpha(u_1)x_1y_1 \end{cases} \tag{5}$$

An innovation in the attribute u_1, changes its value by $\tilde{u}_1$, which we will denote by comfort u_2, giving rise to a new innovative transport system.

$$\begin{cases} \dot{x}_1 = x_1\left[\alpha(u_1)y_1 - \delta(u_1)\right](1 - x_1 - c(u_1, u_2)x_2) \\ \dot{y}_1 = l(u_1)(1 - y_1) - \epsilon_1(u_1)\alpha(u_1)x_1y_1 \\ \dot{x}_2 = x_2\left[\alpha(u_2)y_2 - \delta(u_2)\right](1 - x_2 - c(u_2, u_1)x_1) \\ \dot{y}_2 = l(u_2)(1 - y_2) - \epsilon_1(u_2)\alpha(u_2)x_2y_2 \end{cases} \tag{6}$$

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Example: innovation for a single TS II

Figura: Representation of a bi-articulated transmilenio: they have a capacity of 250 people, 62 seated and 188 standing.

Figura: Representation of a Metromed train: they have capacity for 1220 passengers (8 passengers per square meter), with 148 seats.

Rate of interaction between transport systems: is taken as in [?]:

$$c(u_1, u_2) = \exp\left(\frac{\ln^2 f_1}{2f_2^2}\right)\exp\left(-\frac{1}{2f_2^2}\ln^2\left(\frac{f_1 u_1}{u_2}\right)\right)$$

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Example: simulations before and after innovation I

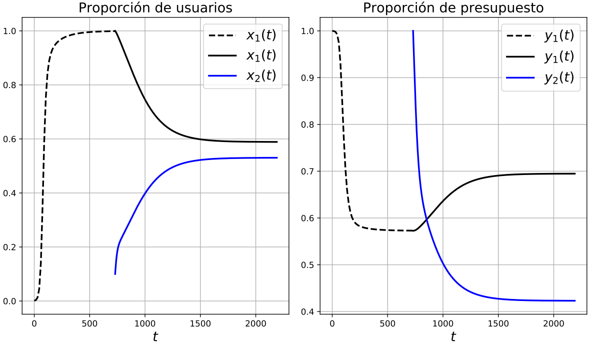

Figura: **Diversification.** System simulation (**??**). **Left:** solutions of x_1 before the innovation (dashed black line) and after the innovation (solid black line) and x_2 (blue line). **Right:** solutions of y_1 before the innovation (dashed black line) and after the innovation (solid black line) and y_2 (blue line). $a = 1$; $a_1 = 0.5$; $a_2 = 22.36$; $\delta = 0.1$, $\epsilon = 0.1$, $l = 0.025$ $f_1 = 0.96$ y $f_2 = 2$. The attributes are $u_1 = 200$ and $u_2 = 800$, thus, $l^*(u_1) = 0.0215$, $l^*(u_2) = 0.01184$, $R_p(u_1) = 1.8653$ and $R_p(u_2) = 6.4287$.

Example: simulations before and after innovation II

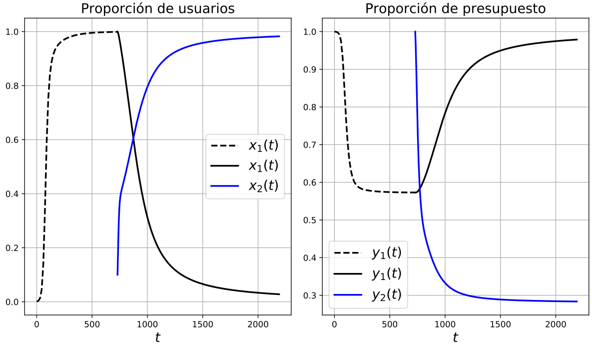

Figura: **Substitution.** System simulation (**??**). **Left:** solutions of x_1 before the innovation (dashed black line) and after the innovation (solid black line) and x_2 (blue line). **Right:** the solutions of y_1 are shown before the innovation (dashed black line) and after the innovation (solid black line) and y_2 (blue line). $a = 1$; $a_1 = 0.5$; $a_2 = 22.36$; $\delta = 0.1$, $\epsilon = 0.1$, $l = 0.025$ $f_1 = 2$ y $f_2 = 2$. Los atributos son $u_1 = 200$ y $u_2 = 800$, por lo tanto, $l^*(u_1) = 0.0215$, $l^*(u_2) = 0.01184$, $R_p(u_1) = 1.8653$ y $R_p(u_2) = 6.4287$.

Section 3

Results and Conclusions

Results I

- The generalized resident model allows to study, under different scenarios, the dynamics of competition between TS of a city.
- The innovative model-generalized resident allows to establish under what conditions in TS innovative can invade and expand in the market
- The approach using adaptive dynamics allows to establish the long-term dynamics of the quantitative attribute (No average passenger per unit)
- The evolutionary dynamics allows to classify the evolutionary equilibriums in branching points (diversification) or terminal points (those where the evolution stops definitively).

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Conclusions I

- In the case of diversification:

  - The resident TS, should be kept in a low number of transported users (< 200 passengers per unit)
  - The innovative TS, must reach a high number of users transported (> 1400 passengers per unit)

  Both TS address different strategies and therefore each will absorb part of the market.

- In the case of substitution:

  - The resident TS, the number of users transported must increase (> 1400 passengers per unit)
  - The innovative TS, must reach a high number of users transported (> 1400 passengers per unit)

  Since both TSs will address the same strategy (> 1400 passengers per unit), finally the innovative TS will absorb all users and replace the resident TS.

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Section 4

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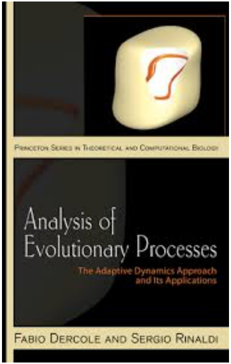

Chapter Four

Evolutionary Branching and the Origin of Diversity

In this chapter we show how continuous marginal innovations subject to severe competition may give rise to increasing diversity in evolving systems. The analysis is performed by pointing out that the AD canonical equations describing the evolution of a family of systems with increasing number of adaptive traits always lead to a branching point. The application that has motivated this study comes from economics, where the emergence of technological variety arising from market interaction and technological innovation has been ascertained. Existing products in the market compete with innovative ones, resulting in a slow and continuous evolution of the underlying technological characteristics of successful products. When technological evolution reaches an equilibrium, it can either be an evolutionary terminal point, where marginal innovations are not able to bring new technological advancement in the market, or a branching point, where new products coexist along with established ones and diversify from them through further innovations. Thus, technological branching can explain product variety without requiring exogenous major breakthroughs. The limitations of the AD approach to economics, as well as

They show how continuous marginal innovations subject to severe competition may give rise to increasing diversity in evolving systems [?].

Main references II

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THE DYNAMICS OF TECHNOLOGICAL CHANGE UNDER CONSTRAINTS: ADOPTERS AND RESOURCES

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"We make use of the adaptive dynamics framework to explain the persistence of closely related technologies as opposed to the usual competitive exclusion of all but one dominant technology [...] The paper illustrates the persistence of closely related technologies and the competitive exclusion in renewable energy technologies and TV sets respectively." [**?**]

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