

# Evolution Models for Urban Metabolism in Bogota

## *Adaptive Dynamics*

Hernán D. Toro & Gerard Olivar

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# What Does the Theory of Adaptive Dynamics Study?

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- ❖ It is a theoretical background originating in evolutionary biology that links demographic (**market**) dynamics to evolutionary changes.
- ❖ It allows for describing long-term evolutionary dynamics when **considering innovations as small and rare events**.
- ❖ It focuses on the long-term evolutionary dynamics of adaptive (quantitative) attributes and **ignores genetic details** through the use of **asexual models**.
- ❖ It considers interactions as the evolutionary driving force and takes into account the feedback between evolutionary change and the forces of selection that agents experience.

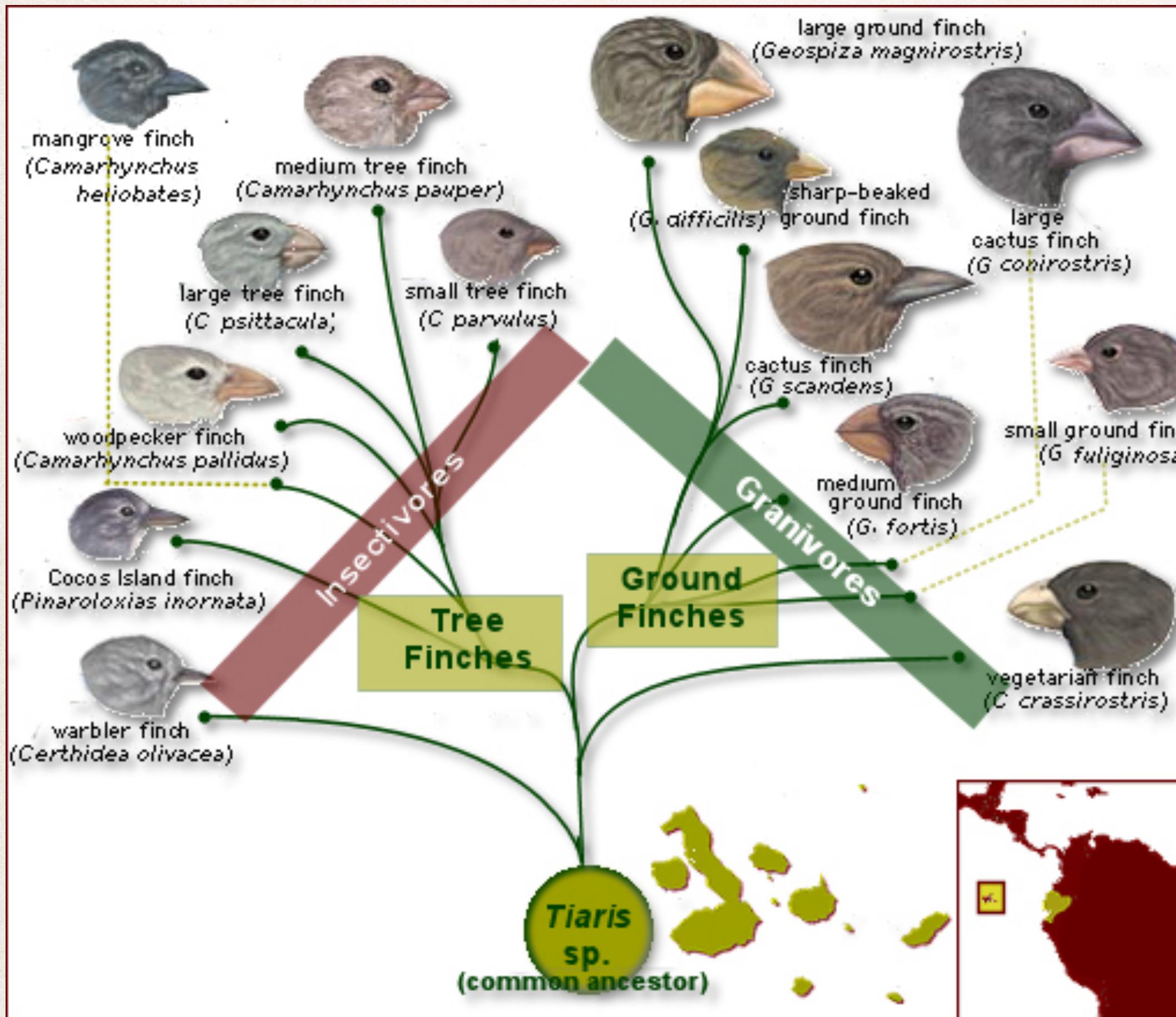
Dercole, F., & Rinaldi, S. (2008). *Analysis of evolutionary processes: the adaptive dynamics approach and its applications*. Princeton University Press.

Dieckmann, U., & Law, R. (1996). The dynamical theory of coevolution: a derivation from stochastic ecological processes. *Journal of mathematical biology*, 34(5-6), 579-612.

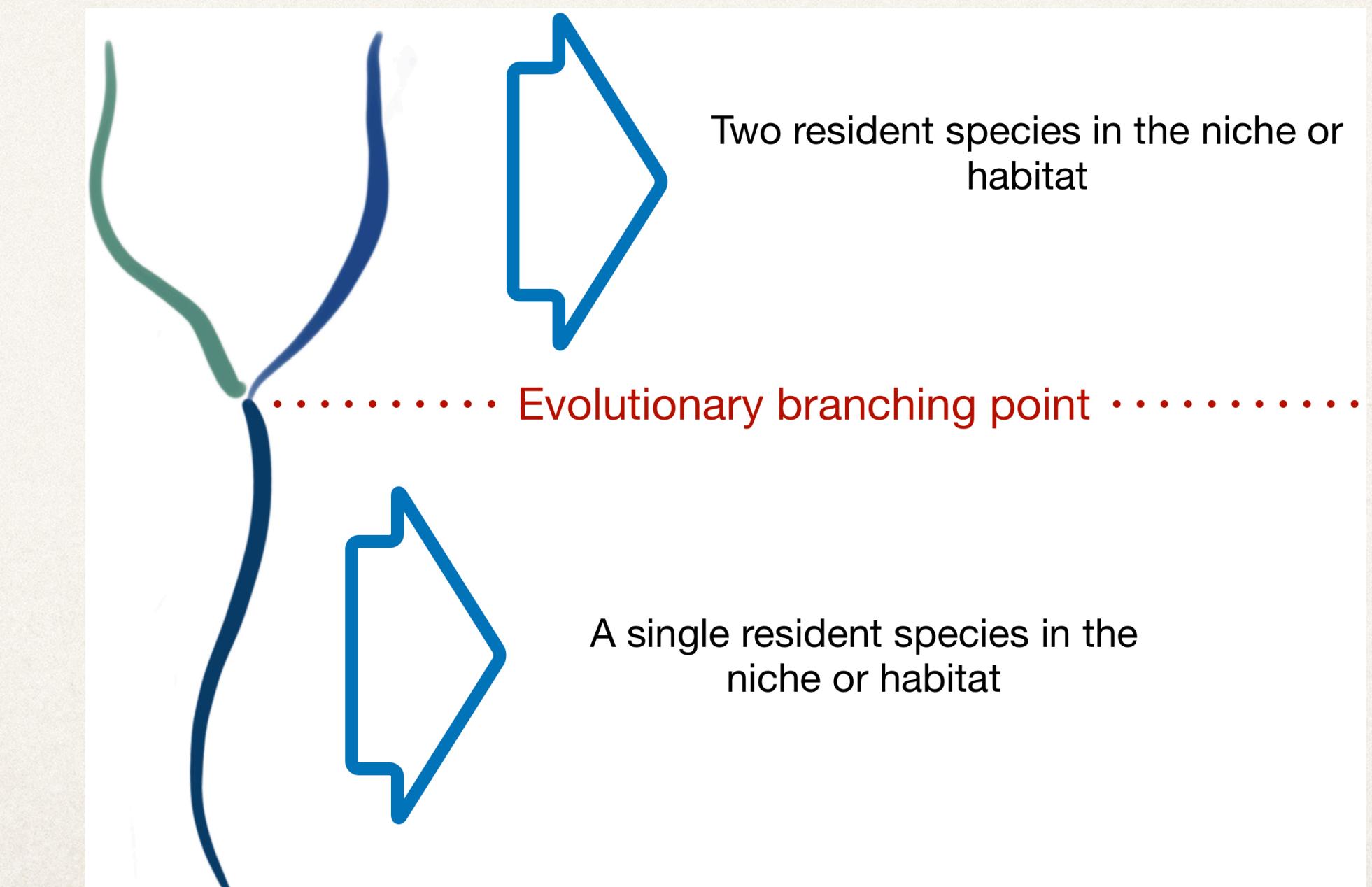
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Geritz, S. A., Metz, J. A., Kisdi, É., & Meszéna, G. (1997). Dynamics of adaptation and evolutionary branching. *Physical Review Letters*, 78(10), 2024.

# The Origin of Diversity: Ecology



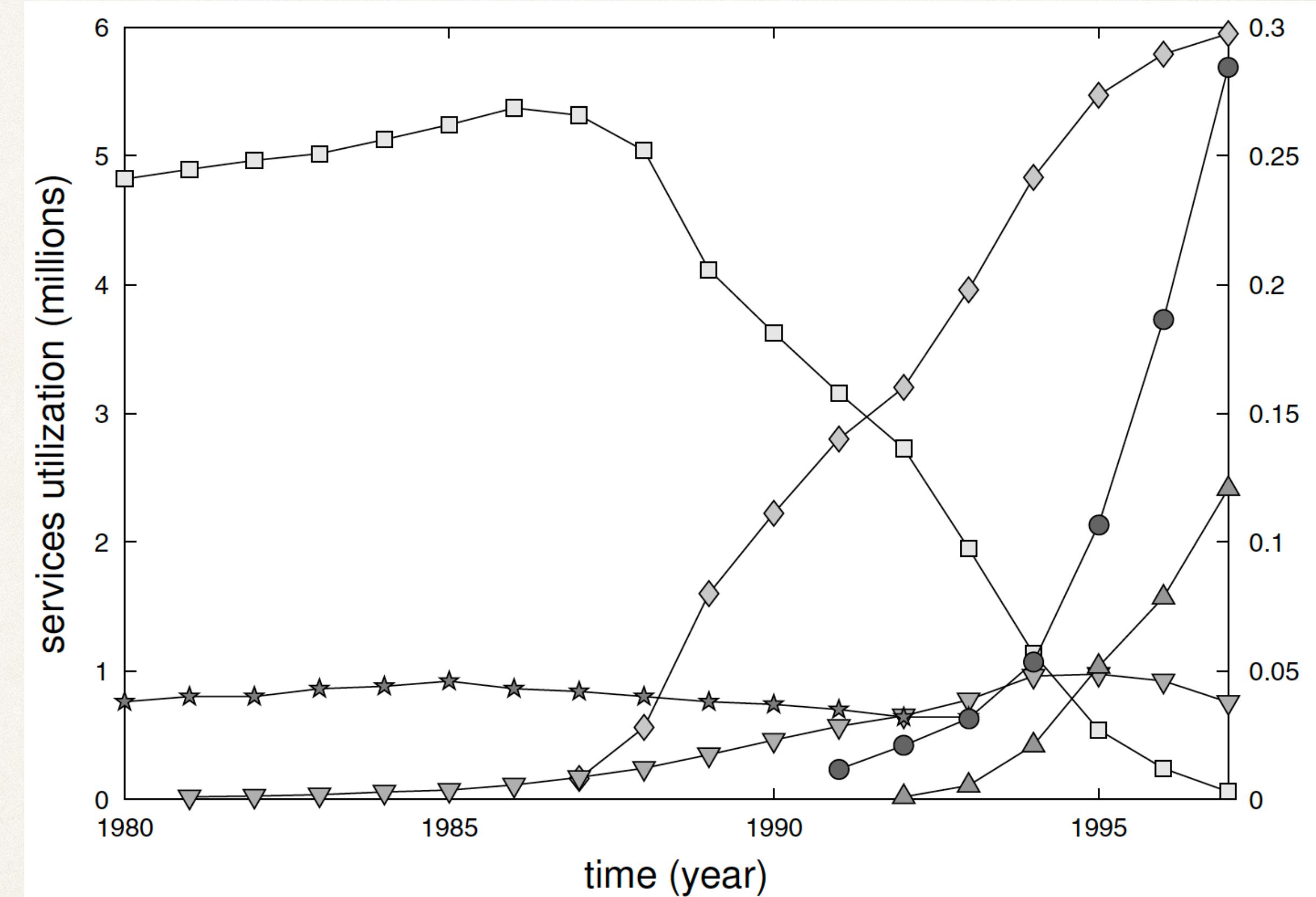
**Sympatric speciation:** The population is divided into two initially similar resident groups, which then diverge following separate evolutionary paths (branches), each driven by their own mutations, experiencing what is called **evolutionary branching**.



# The Origin of Diversity: Technology

The arrival of digital telephones has been a successful innovation that has led to the replacement of analog telephones

Use of telecommunication services in Switzerland. **Square:** analog phones. **Diamonds:** digital phones. **Triangle below:** subscribers to analogue mobile telephony, **Triangle above:** subscribers to digital mobile telephony. **Stars:** public pay phones. **Circles:** internet host.



# Contents of this Talk

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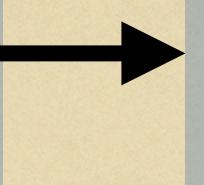
- ♣ Mathematical Models for Evolutionary Competition
- ♣ Application: *Energy Market*  
Conclusions for the *EM*
- ♣ Application: *Public Transport Systems*  
Conclusions for *TS*

# Mathematical Models for Evolutionary Competition: *a Fast Glimpse into the Theory*

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Established-Innovative Model

$$\begin{cases} \dot{n}_1 = n_1 g(n_1, n_2, \mathbf{N}, x_1, x_2, x_1) \\ \dot{n}_2 = n_2 g(n_2, n_1, \mathbf{N}, x_2, x_1, x_2) \\ \dot{\mathbf{N}} = \mathbf{G}(n_1, n_2, \mathbf{N}, x_1, x_2) \end{cases}$$



Invasion equilibrium

$$E_i = (\bar{n}_1(x_1, x_2), 0, \bar{\mathbf{N}}(x_1, x_2))$$

Before innovation

$$n_2 = 0$$

Established Model

$$\begin{cases} \dot{n}_1 = n_1 g(n_1, 0, \mathbf{N}, x_1, \cdot, x_1) \\ \dot{\mathbf{N}} = \mathbf{G}(n_1, 0, \mathbf{N}, x_1, \cdot) \end{cases}$$

Local stability analysis

Market at equilibrium

$$E_s = (\bar{n}_1(x_1), \bar{\mathbf{N}}(x_1))$$

LAS by assumption

Local stability analysis

Invasion eigenvalue

$$\lambda(x_1, x_2) = g(0, \bar{n}_1(x_1), \bar{\mathbf{N}}(x_1), x_2, x_1, x_2)$$

First order Taylor expansion

Invasion conditions - Selection gradient

$$\begin{cases} \frac{\partial \lambda}{\partial x_2}(x_1, x_1) > 0, & x_2 > x_1 \\ \frac{\partial \lambda}{\partial x_2}(x_1, x_1) < 0, & x_2 < x_1 \end{cases}$$

Market timescale

- Branching Point (BP): CC and DC ok
- Terminal Point (TP): if CC or DC fail
- Bifurcation Branching Point (BBP): boundary region between BP and TP

Evolutionary Dynamics

Coexistence Condition - CC

$$\frac{\partial^2 \lambda}{\partial x_1 \partial x_2}(\bar{x}, \bar{x}) < 0$$

Divergence Condition - DC

$$\frac{\partial^2 \lambda}{\partial x_2^2}(\bar{x}, \bar{x}) > 0$$

Evolutionary stability

$$f'(\bar{x}) < 0$$

Canonical Equation of Adaptive Dynamics

$$\dot{x} = \frac{1}{2} \mu(x) \sigma^2(x) \bar{n}_1(x_1) \frac{\partial \lambda}{\partial x_2}(x_1, x_1) = f(x)$$

Evolutionary timescale

Stochastic Process

# Application: *Energy Market*

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Hindawi  
Mathematical Problems in Engineering  
Volume 2018, Article ID 9181636, 15 pages  
<https://doi.org/10.1155/2018/9181636>



*Research Article*

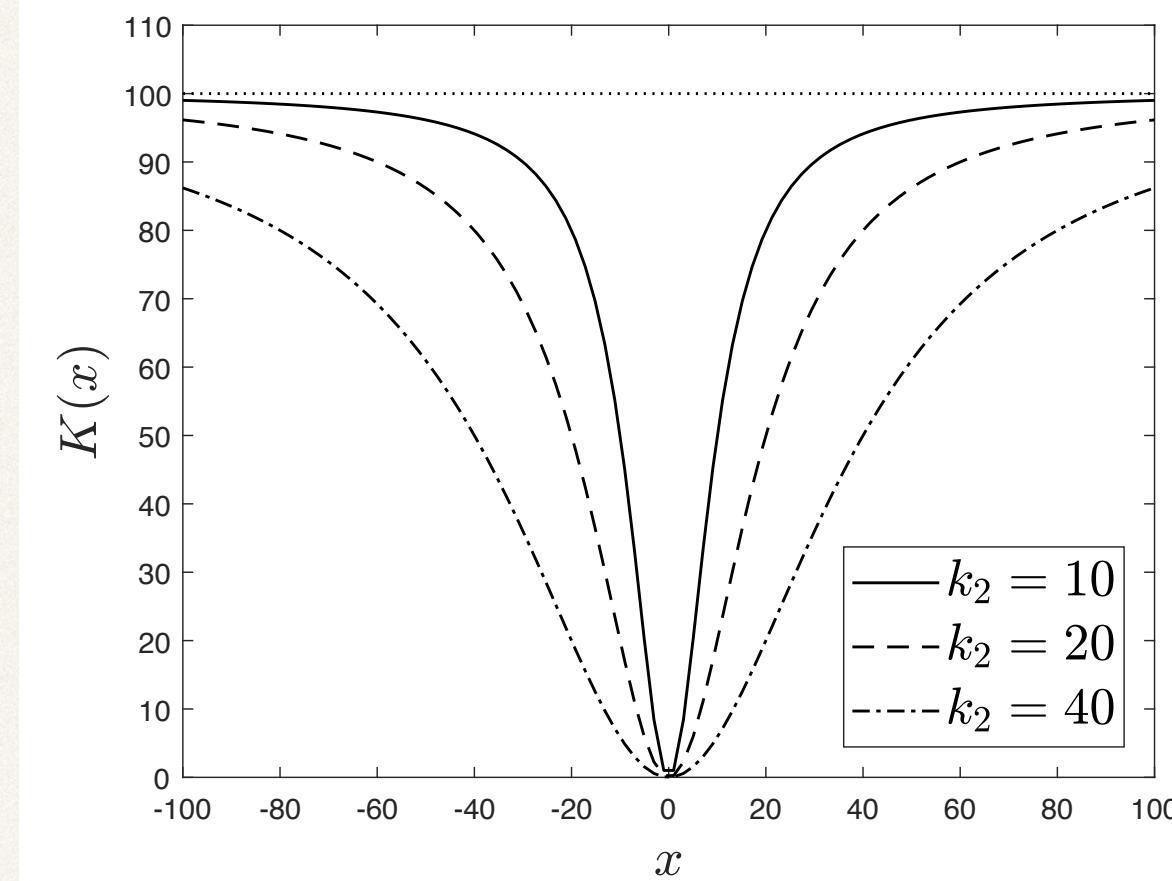
## **Conditions on the Energy Market Diversification from Adaptive Dynamics**

**Hernán Darío Toro-Zapata,<sup>1</sup> Gerard Olivar-Tost<sup>2</sup> and Fabio Dercole<sup>3</sup>**

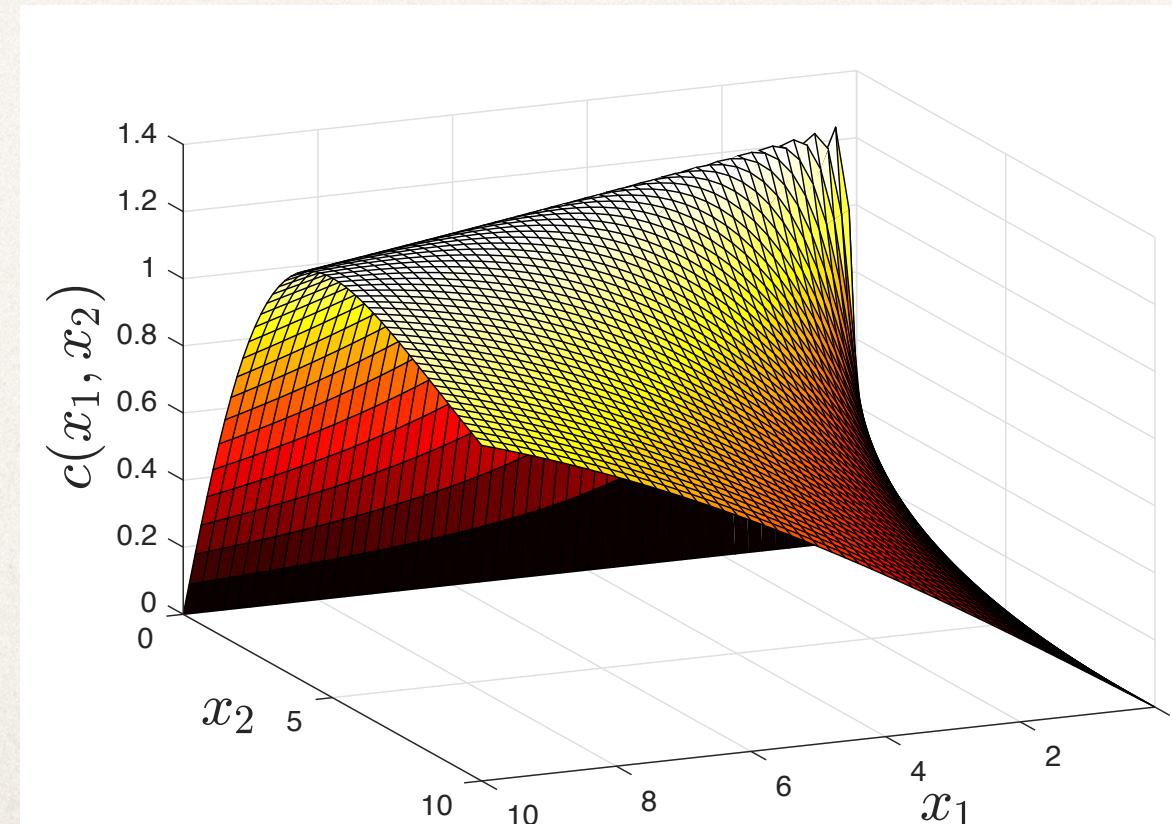
# Conditions on the Energy Market Diversification from Adaptive Dynamics

## Established - Innovative Model

$$\begin{cases} \dot{n}_1 = n_1 r(x_1) \left( 1 - \frac{n_1 + c(x_1, x_2)n_2}{K(x_1)} \right) = n_1 g(n_1, n_2, x_1, x_2, x_1) \\ \dot{n}_2 = n_2 r(x_2) \left( 1 - \frac{n_2 + c(x_2, x_1)n_1}{K(x_2)} \right) = n_2 g(n_1, n_2, x_1, x_2, x_2) \end{cases}$$



$$K(x) = \frac{k_1 x^2}{k_2^2 + x^2}$$



$$c(x_1, x_2) = \frac{(c_1^2 + c_2^2)x_1x_2}{c_1^2x_1^2 + c_2^2x_2^2}$$

Table 1: Description of state variables and coefficients with their corresponding ranges.

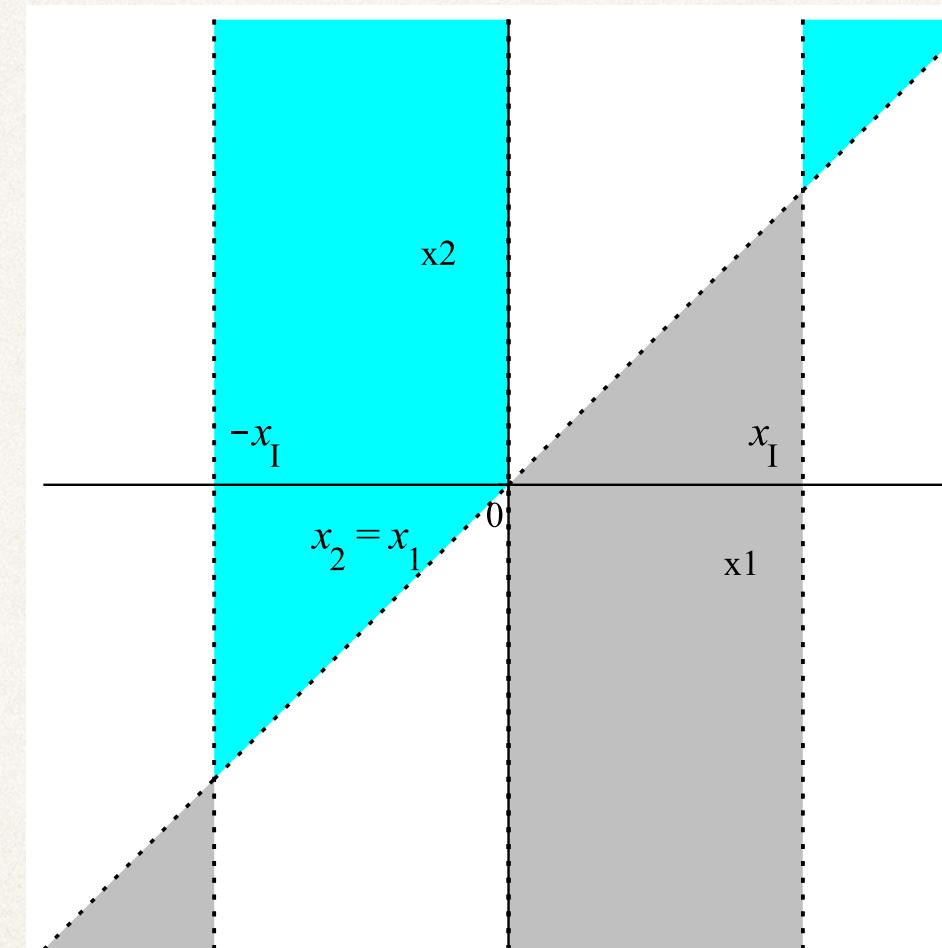
State variables description		Units
$n_1(t)$	CGC* for Standard Energy, characterized by $x_1$	MW
$n_2(t)$	CGC for Innovative Energy, characterized by $x_2$	MW
Parameter description		Ranges
$x_1$	Quantitative continuous characteristic trait defining SE	$x_1 \in \mathbb{R}$
$x_2$	Quantitative continuous characteristic trait defining IE	$x_2 \in \mathbb{R}$
$r(x_i)$	CGC growing rate as a function of $x_i$ , for $i = 1, 2$	$r > 0$
$K(x_i)$	Maximum CGC as function of $x_i$ , for $i = 1, 2$	$K > 0$ MW
$c(x_1, x_2)$	Interaction coefficient between both CGC as a function of $x_1$ and $x_2$	$c \in \mathbb{R}$

\*CGC: cumulative generation capacity. \*\*GT's: generation technologies.

# Conditions on the Energy Market Diversification from Adaptive Dynamics

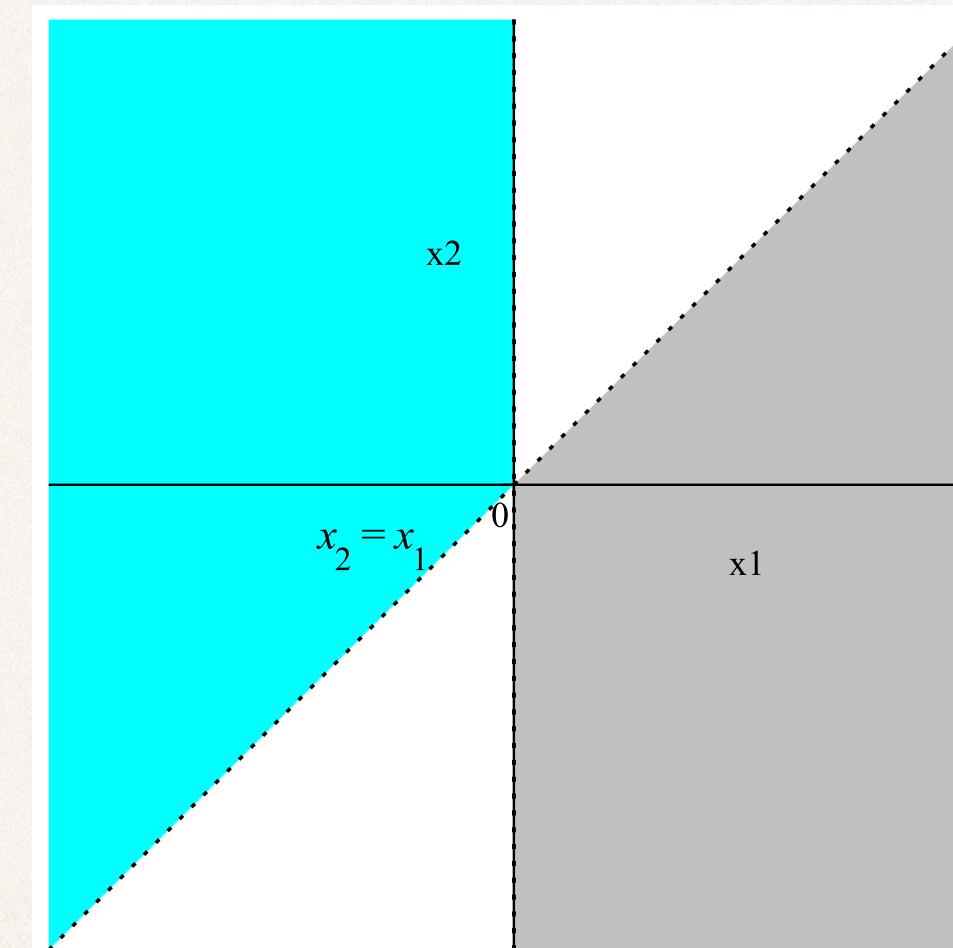
Selection Gradient and invasion conditions:

Different regions in the established-innovative plane where invasion conditions given are satisfied. Blue regions above the diagonal correspond to negative selection gradients, and gray regions below that line correspond to positive selection gradients.



$c_2^2 - c_1^2 = 0.1025 > 0$  ( $c_1 = 1$  and  $c_2 = 1.05$  were used)

$$\frac{\partial \lambda}{\partial x_2}(x_1, x_1) = \frac{[k_2^2(3c_1^2 + c_2^2) - (c_2^2 - c_1^2)x_1^2] r}{x_1(k_2^2 + x_1^2)(c_1^2 + c_2^2)}$$



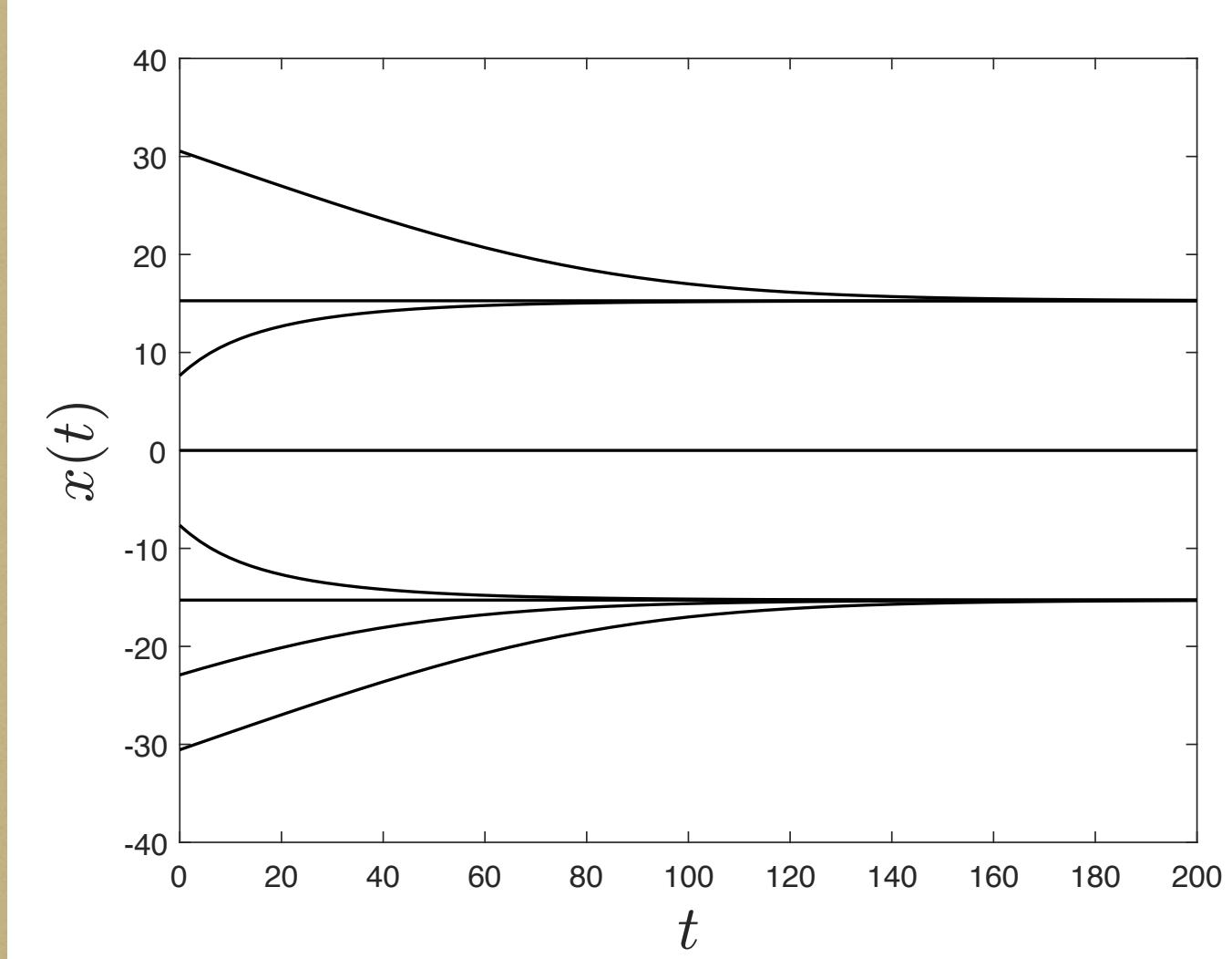
Note that a big innovation is required to have a cooperative market just after an innovation in a market dominated by SE generation technology

$c_2^2 - c_1^2 = -0.1025 < 0$  (with  $c_1 = 1.05$  and  $c_2 = 1$ )

# Conditions on the Energy Market Diversification from Adaptive Dynamics

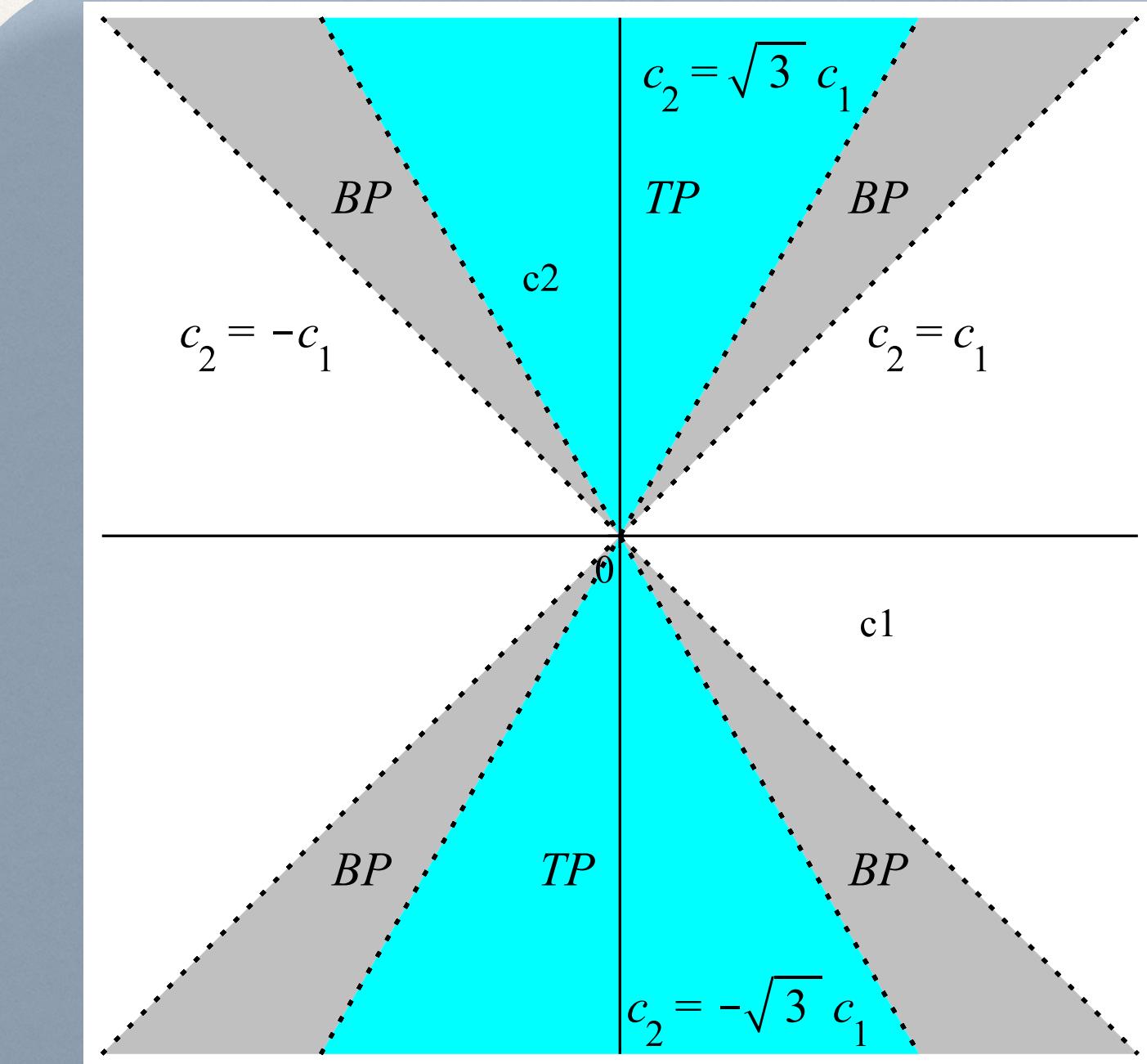
## Canonical Equation of Adaptive Dynamics and classification of evolutionary equilibria

$$\dot{x}_1 = \frac{1}{2}\mu(x_1)\sigma^2(x_1)\bar{n}(x_1)\frac{\partial\lambda}{\partial x_2}(x_1, x_1)$$



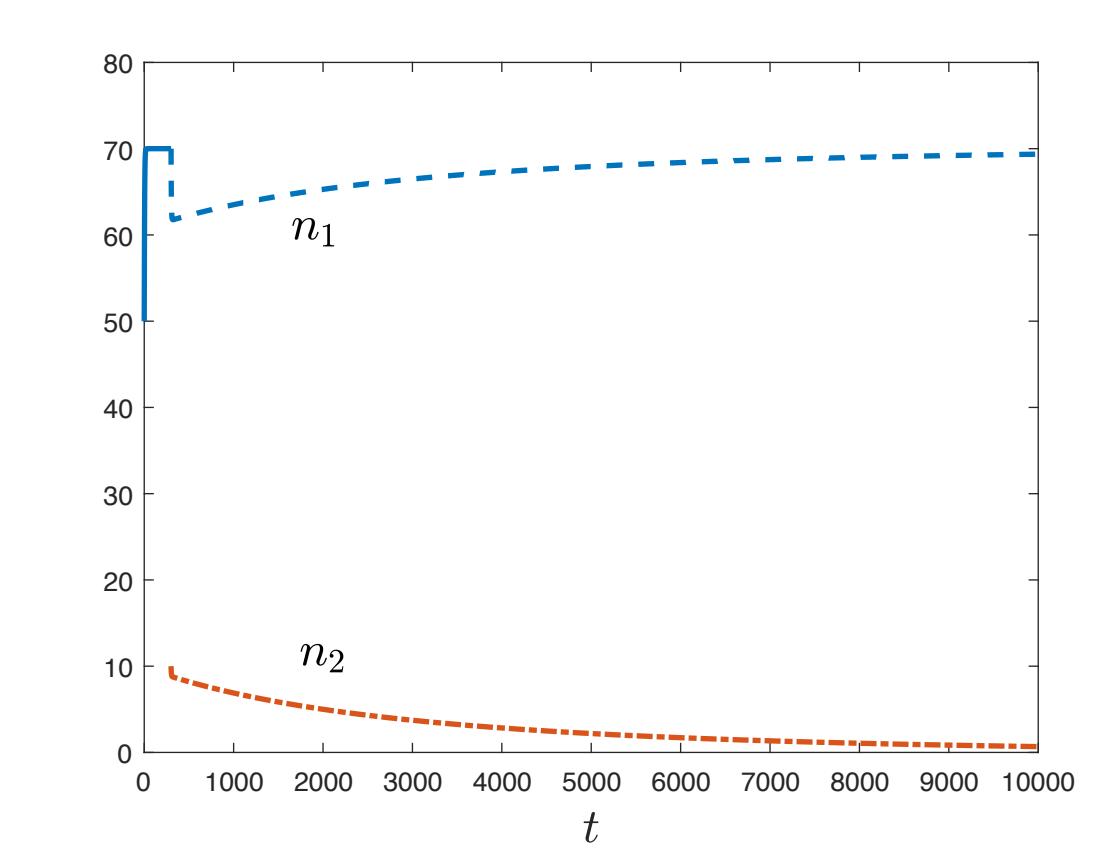
Numeric simulation of evolutionary dynamics of the characteristic trait described by the Canonical Equation

$$r = 0.3, c_1 = 1, c_2 = 2, k_1 = 100, k_2 = 10, \mu = 1, \text{ and } \sigma = 1$$



Classification of stable evolutionary equilibria as BP, TP or BBP.

# Conditions on the Energy Market Diversification from Adaptive Dynamics

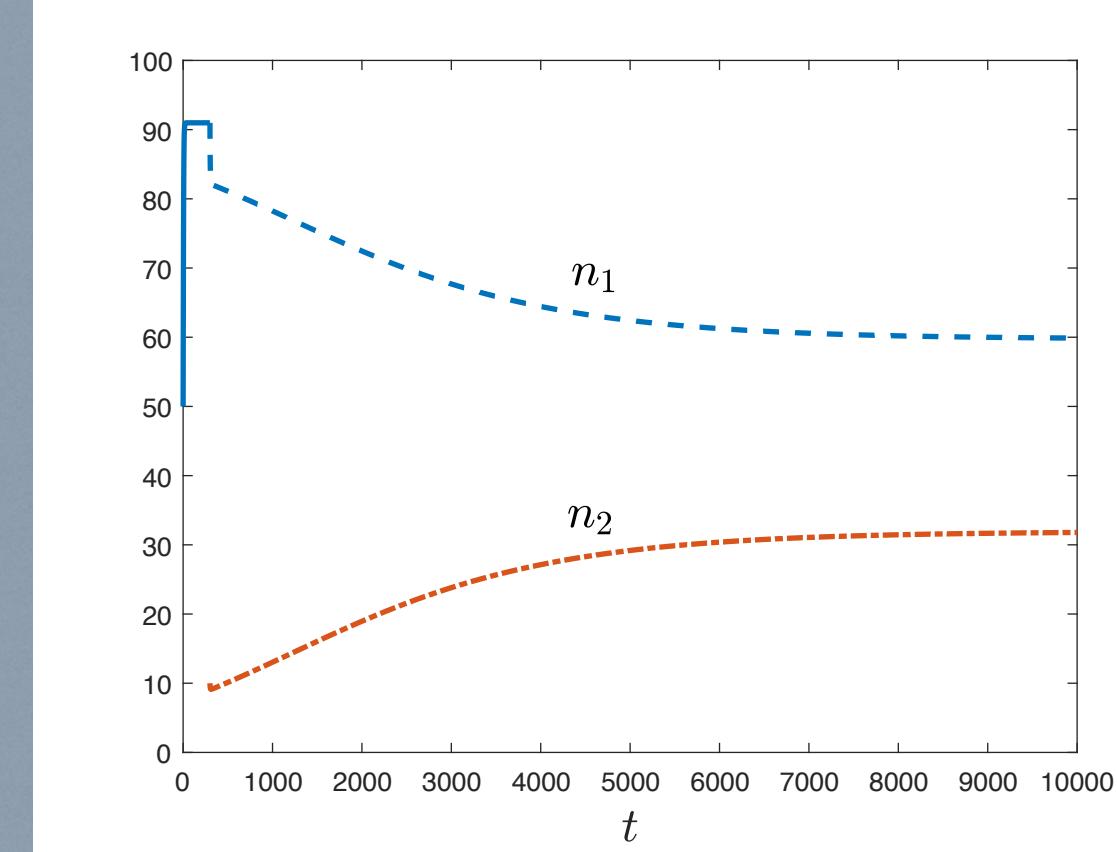


$$c_1 = 1, c_2 = 2$$



Terminal Point

Diversification is not possible

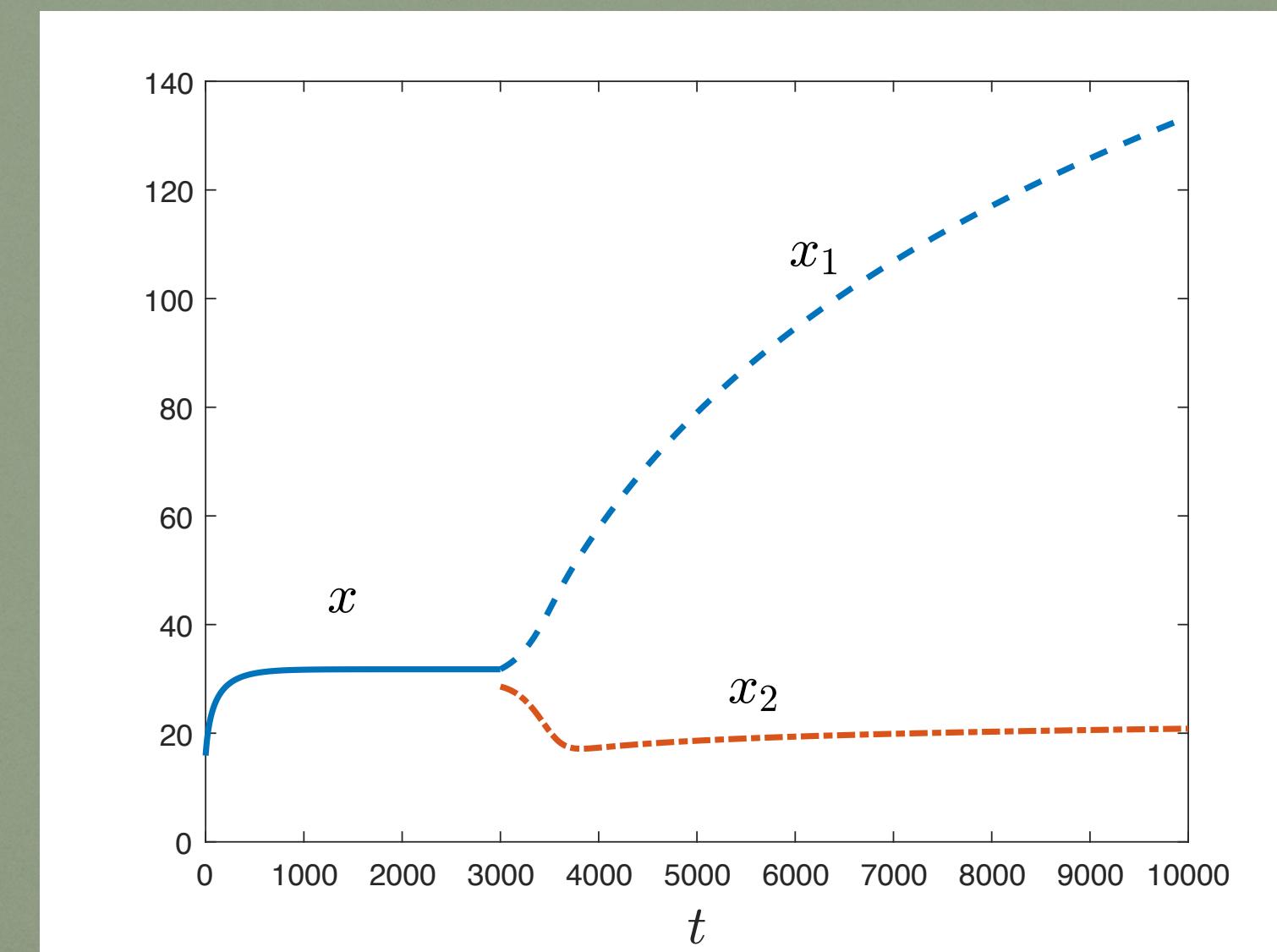


$$c_1 = 1, c_2 = 1.2$$



Branching Point  
Energy market diversifies

**Diversification.** Trait dynamics after branching: two established densities will coexist in the market



# Results and Conclusions for the Energy Market

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- ❖ Local stability analysis of Stablished-innovative model helps to answer the question of **under what conditions can IE spread into the market and interact or even substitute SE.**
- ❖ Energy market will not crash under any circumstances.
- ❖ The instability of invasion equilibria is related to the possibility for an IE to invade the market
- ❖ The existence and stability of equilibria in presence of both, SE and IE, are related to the coexistence, leading to diversification.

- ❖ The interaction dynamics in the market model is well defined for both competition and cooperation market configurations.
- ❖ Invasion conditions determine specific regions of the trait plane under which the invasion of the innovative attribute is possible, and configurations that lead to its disappearance.
- ❖ It was proven that under convenient configurations of subsidies awarded (or taxes imposed), it is possible to determine scenarios in order to evolutionary equilibria to exist and to be locally asymptotically stable.
- ❖ It was shown that evolutionary stability implies coexistence.
- ❖ Evolutionary equilibria can be terminal points, where no marginal innovation can invade into the market. Or can also be branching points, where innovative energy can penetrate, coexist and diversify the market.

- ❖ Both parameters (subsidies/taxes) describe the dynamics in the market timescale, but they finally make a difference in the evolutionary time scale.  
**Indeed, the expressions in the selection gradient controlling the sign, are a measure of the strength of diversification through innovation.**
- ❖ Diversification occurs in markets that are at least slightly asymmetric and in which IE is stimulated over SE, either by the allocation of subsidies or by the imposition of lower taxes.
- ❖ Finally note that repeated process of innovation can give origin to a rich variety of different and complex kinds of technological evolution.
- ❖ Specific situations should be studied in greater depth and detail in order to achieve an informed decision making.

# *Application: Public Transport Systems*

**Copernican Journal of Finance & Accounting**

**2018, volume 7, issue 2**

Toro-Zapata, H.D., & Olivar-Tost, G. (2018). Mathematical model for the evolutionary dynamics of innovation in city public transport systems. *Copernican Journal of Finance & Accounting*, 7(2), 77-98. <http://dx.doi.org/10.12775/CJFA.2018.010>



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**MATHEMATICAL MODEL FOR THE EVOLUTIONARY DYNAMICS  
OF INNOVATION IN CITY PUBLIC TRANSPORT SYSTEMS**

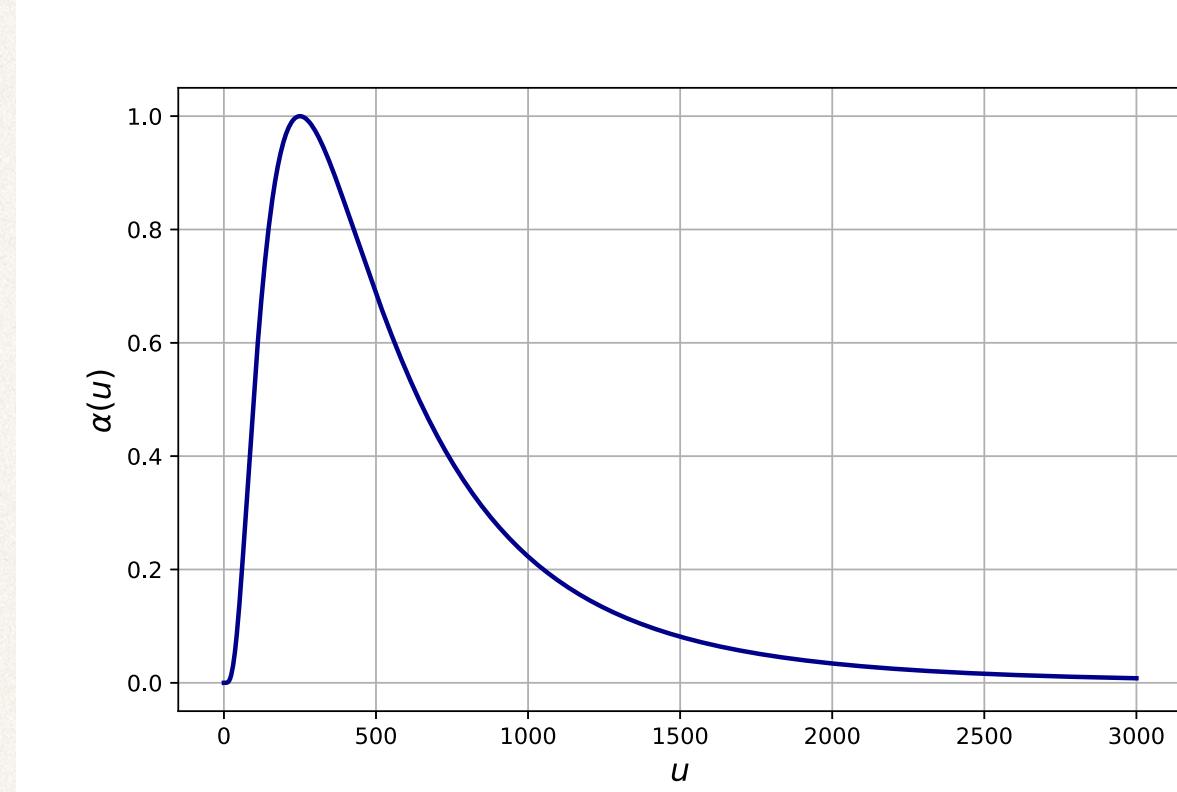
# Mathematical model for the evolutionary dynamics of innovation in public transportation systems

## Established - Innovative Model

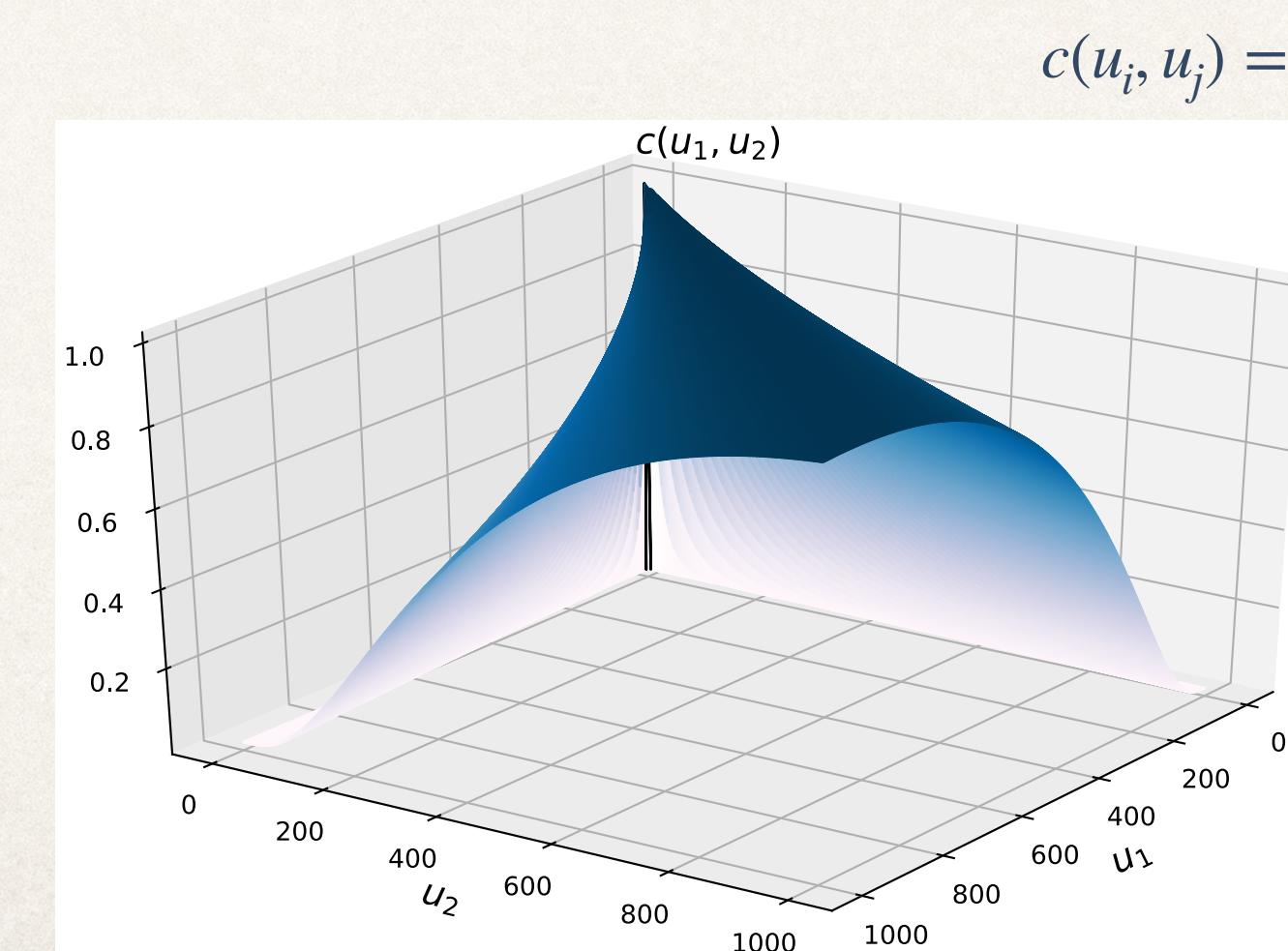
$$\begin{aligned}\dot{x}_i &= (\alpha(u_i)y_i - \delta(u_i)) \left[ 1 - \text{sign}(\alpha(u_i)y_i - \delta(u_i)) \left( x_i + \sum_{k=1, k \neq i}^N c(u_i, u_k)x_k \right) \right] x_i \\ \dot{y}_i &= l(u_i)(1 - y_i) - \epsilon(u_i)\alpha(u_i)x_iy_i,\end{aligned}$$

Table 1: Description of state variables and parameters

Description of the states		Value
$x_i$	Proportion of people using the $i$ -th TS	-
$y_i$	Budget destined to the expansion of the $i$ -th TS	-
Parameter Description		Value
$u_i$	Characteristic attribute describing the $i$ -th TS	-
$\alpha(u_i)$	Instant rate of adoption of the $i$ -th TS	-
$\delta(u_i)$	Rate at which the $i$ -th TS is abandoned by users	-
$l(u_i)$	Rate at which new resources are invested for expansion of the $i$ -th TS	-
$\epsilon(u_i)$	Efficiency of the $i$ -th TS in converting the investment in new users	-
$c(u_i, u_k)$	Interaction rate between $i$ -th and $k$ -th TS's	-



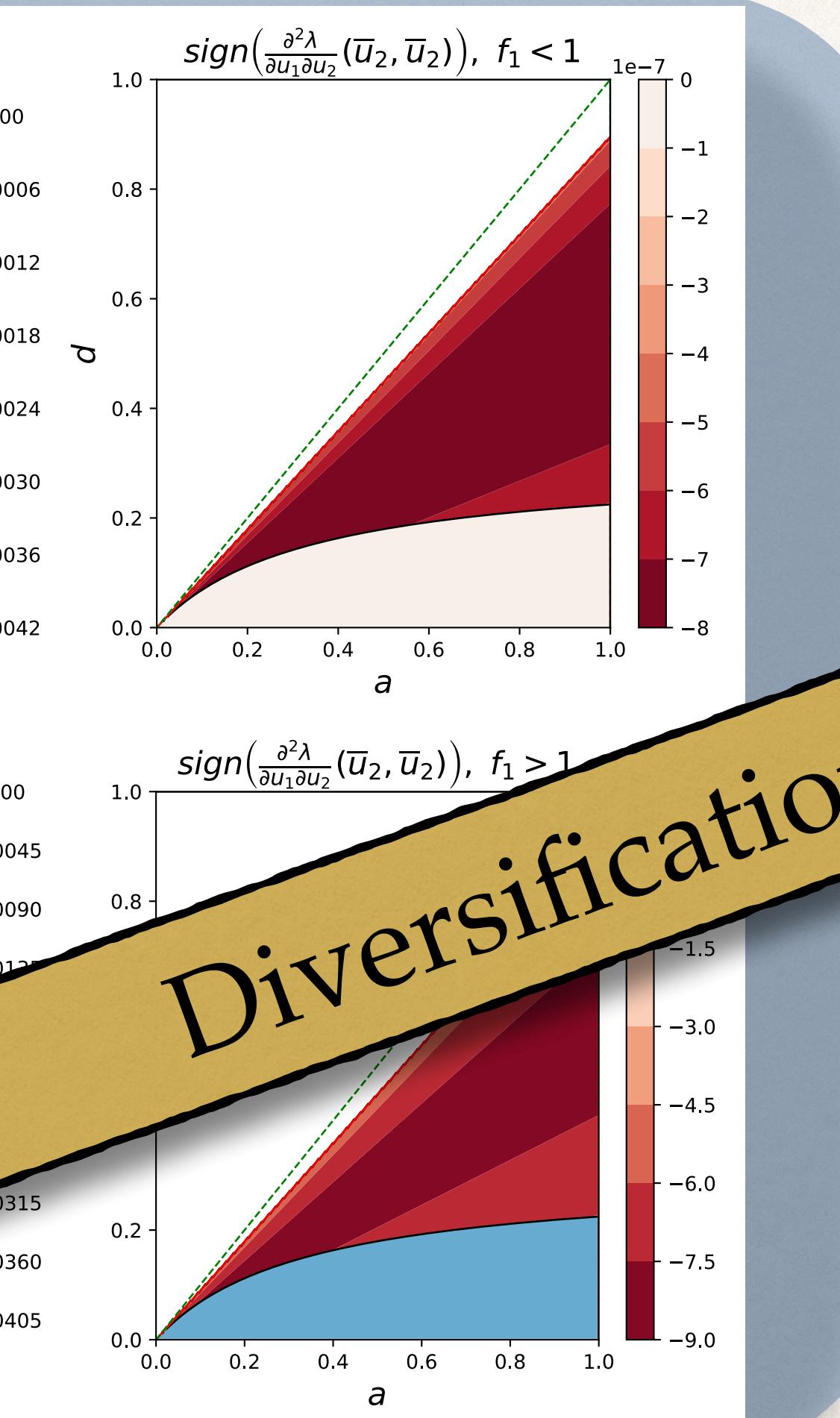
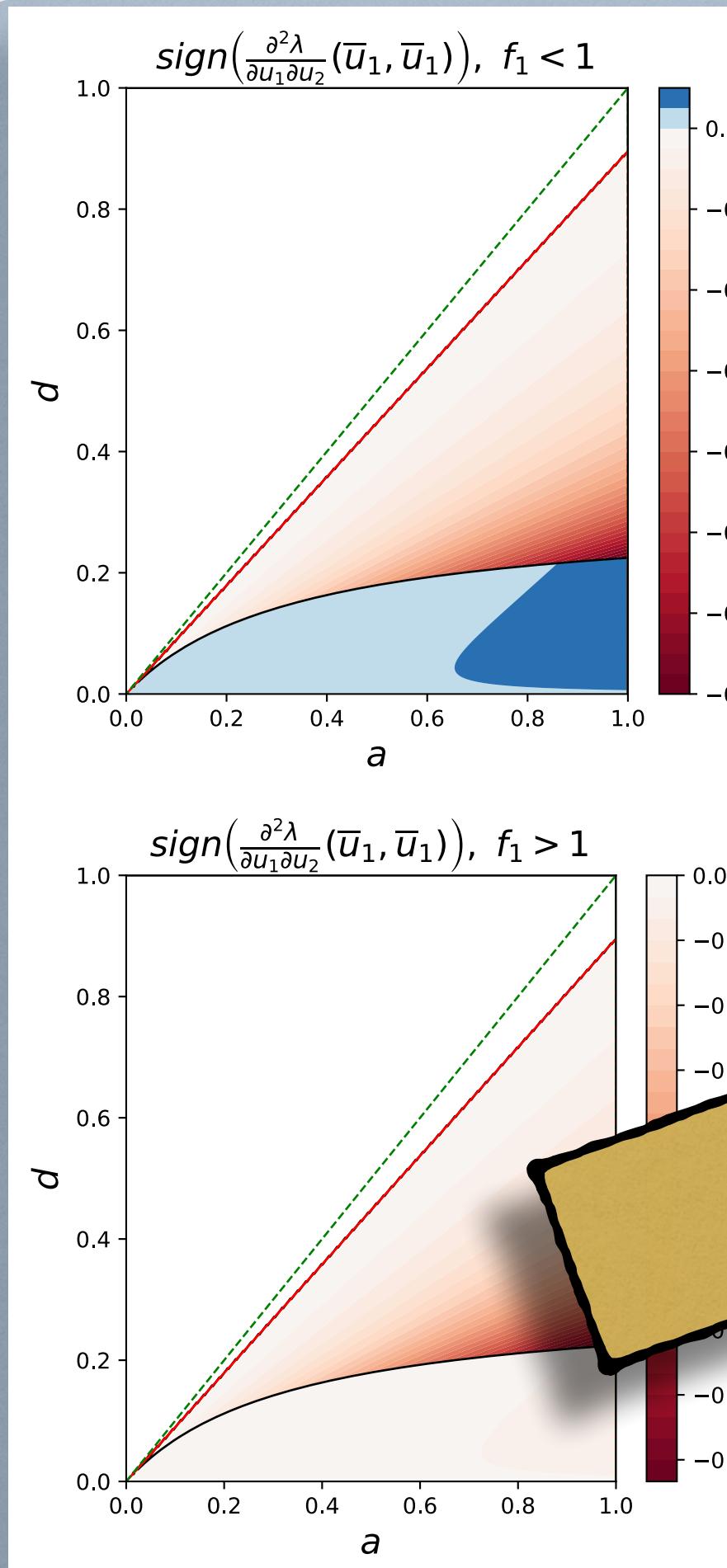
$$\alpha(u) = a \exp\left(-\frac{1}{2a_1^2} \ln^2\left(\frac{u}{a_2^2}\right)\right)$$



$$c(u_i, u_j) = \exp\left(\frac{\ln^2 f_1}{2f_2^2}\right) \exp\left(-\frac{1}{2f_2^2} \ln^2\left(\frac{f_1 u_i}{u_j}\right)\right)$$

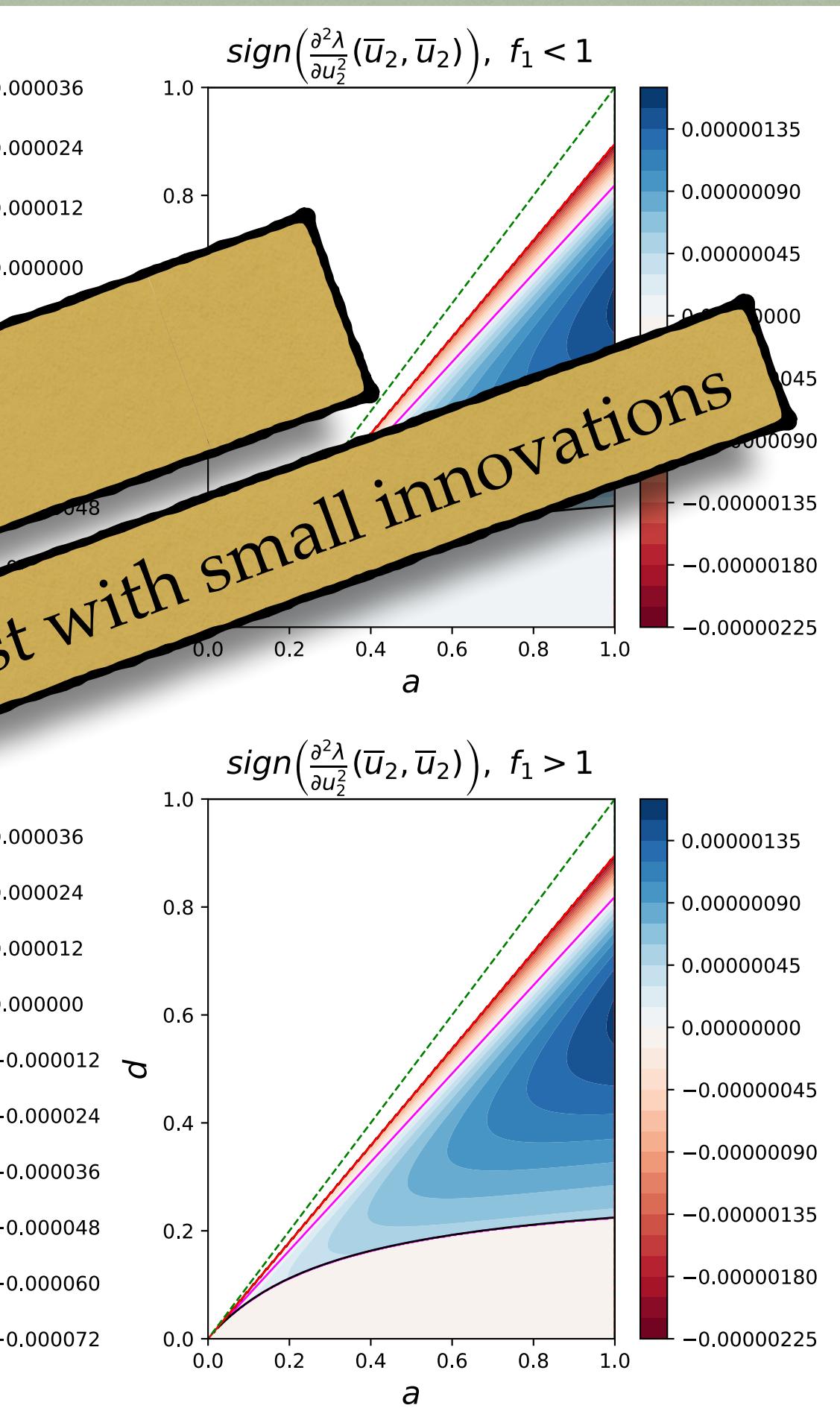
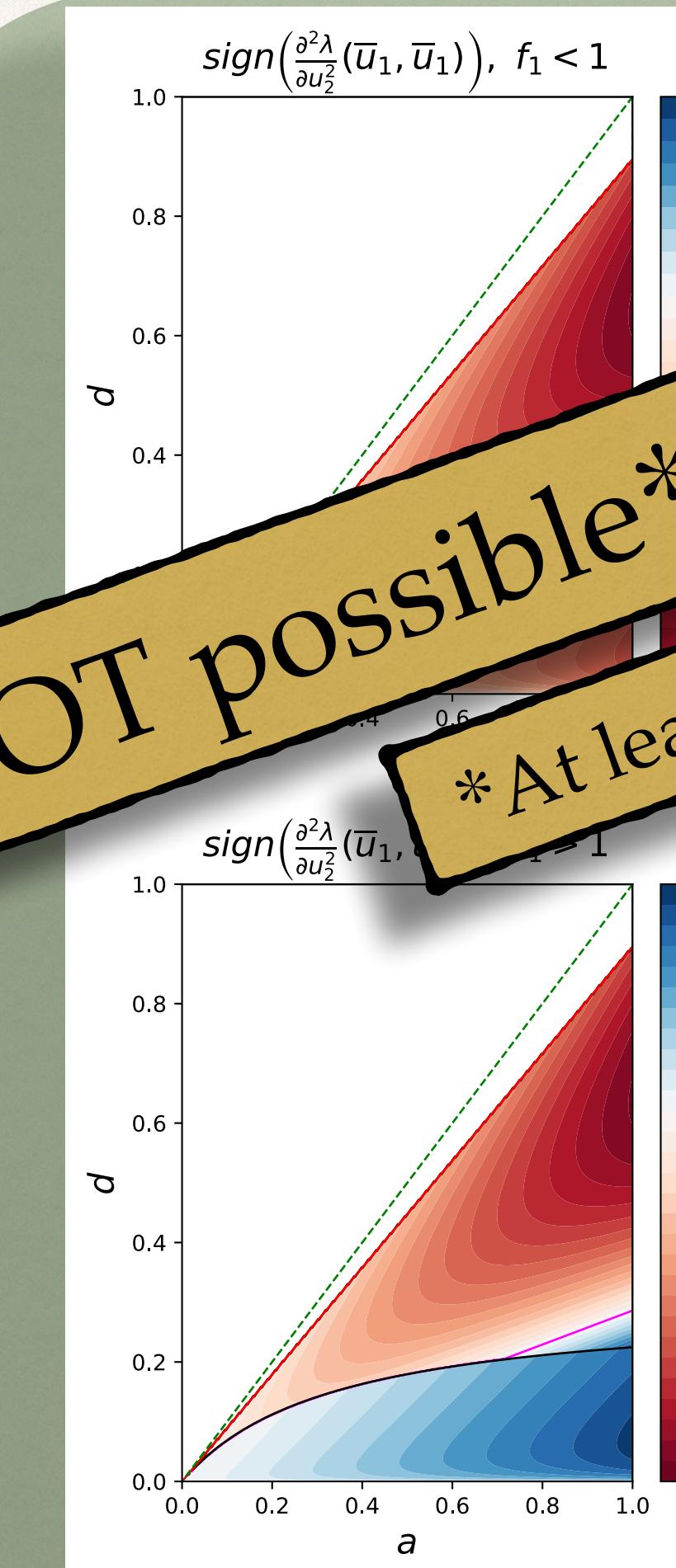
# Mathematical Model for the Evolutionary Dynamics of Innovation in Public Transportation Systems

¿Coexistence < 0?

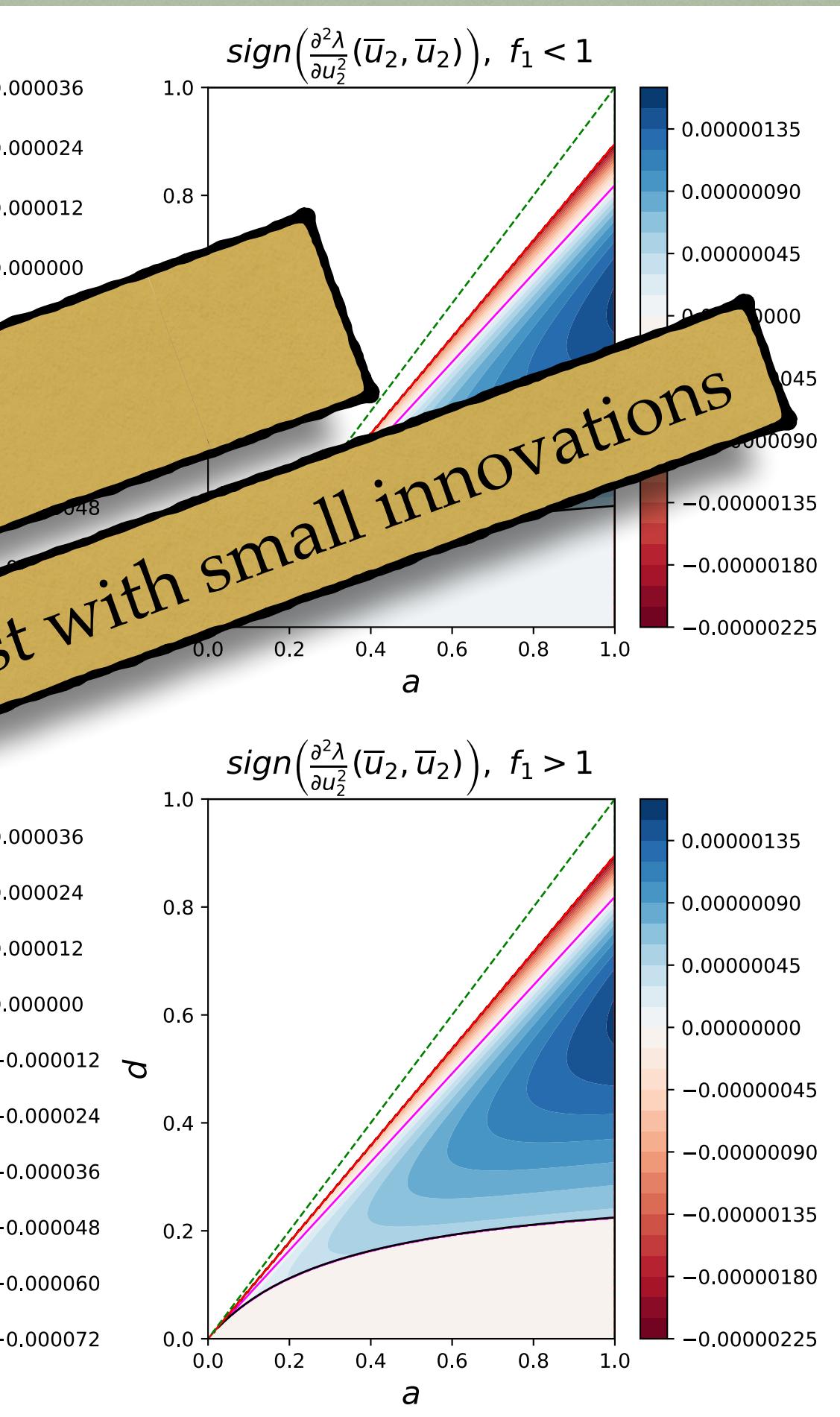
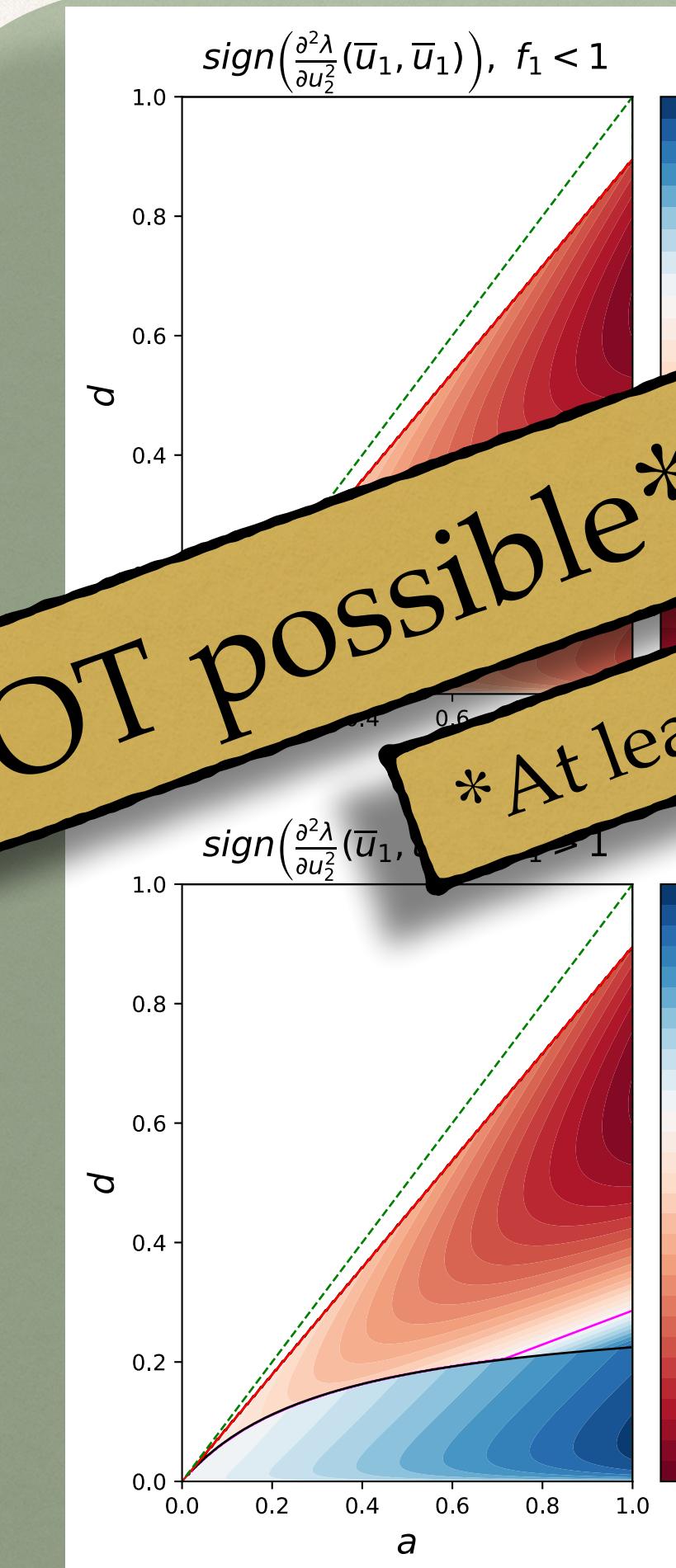
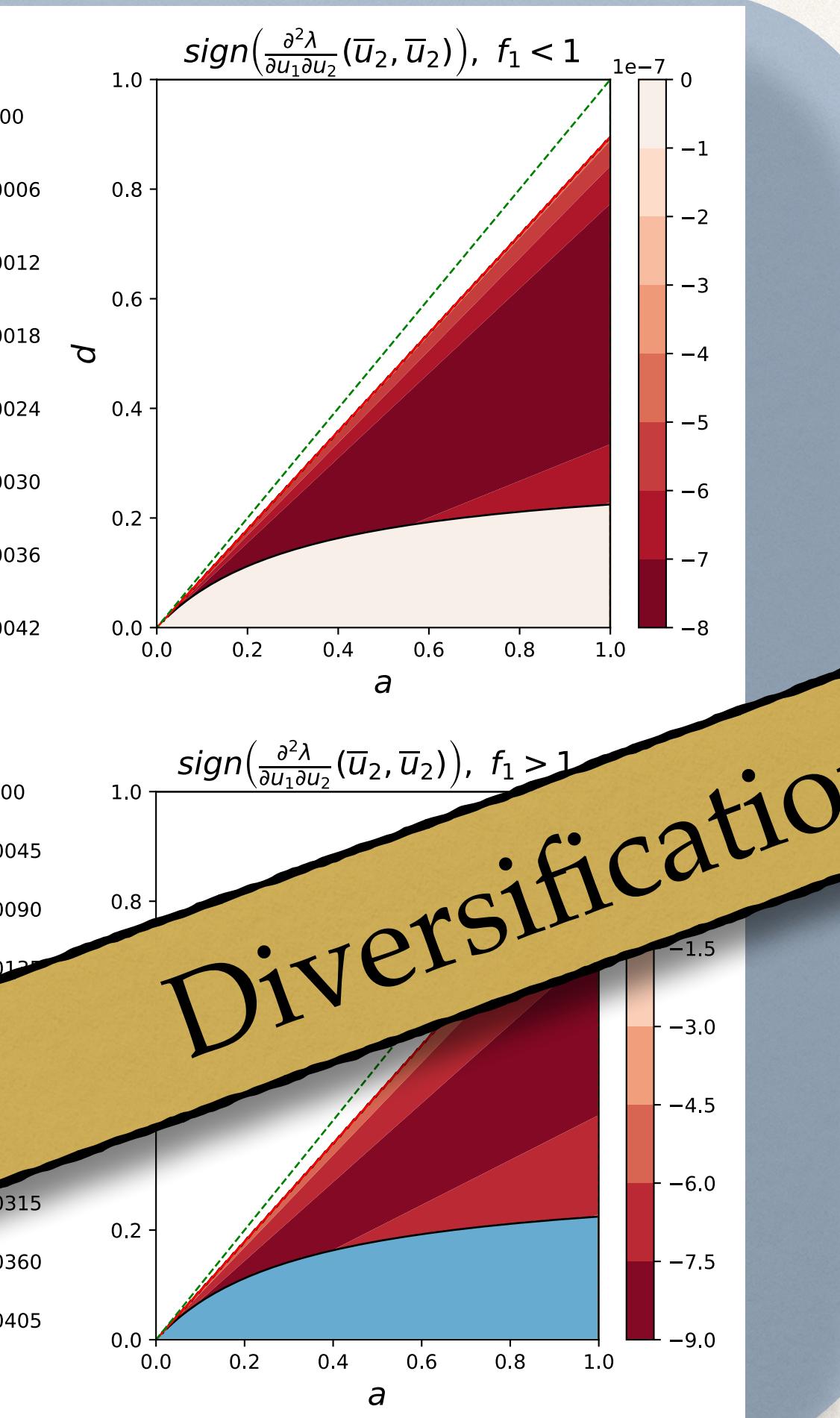
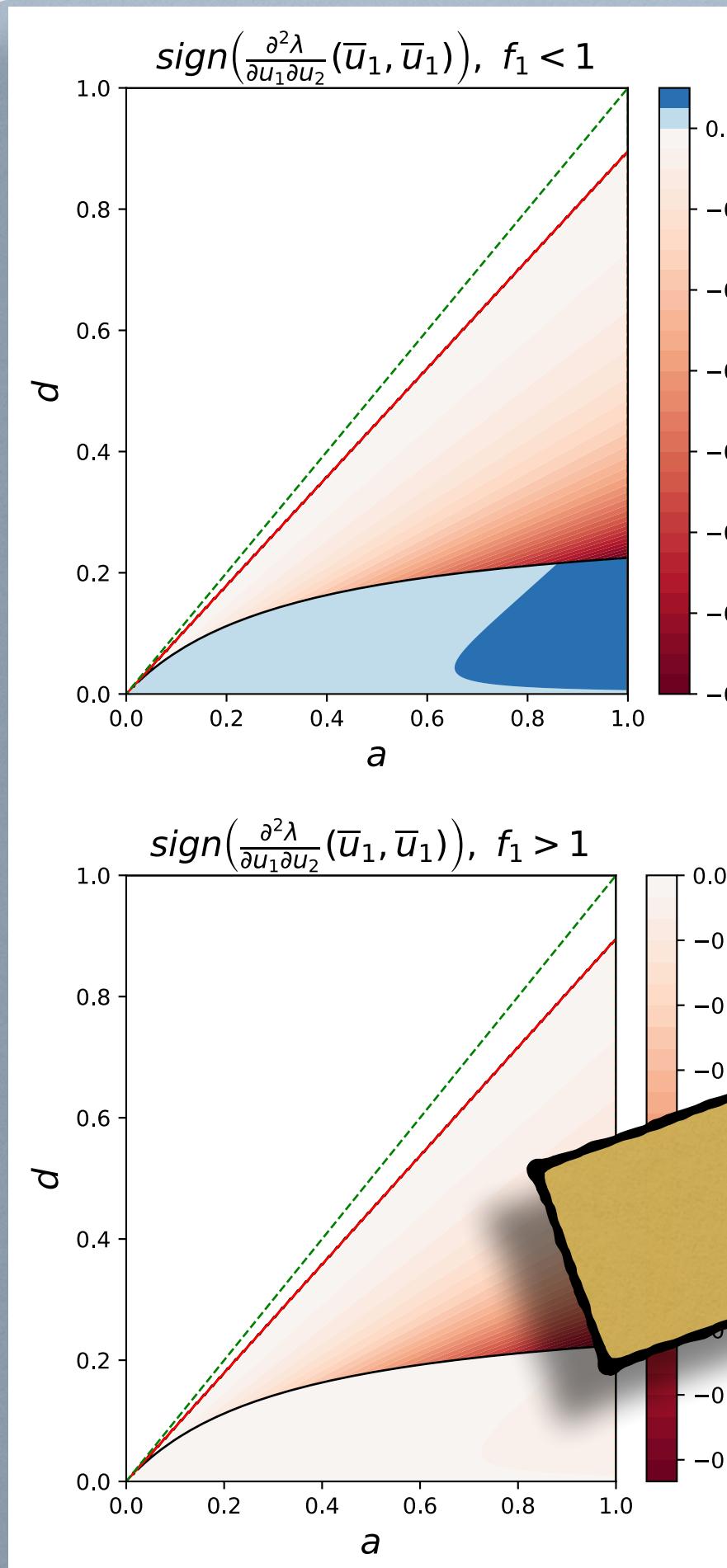


Diversification is NOT possible\*

\*At least with small innovations



¿Divergence > 0?



# Results and Conclusions for Public Transportation

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- A generalized model is formulated, called resident-innovative model, to describe competition in the market when the established TS equilibrium is disturbed by the entry of an innovative TS.
- From study of the fitness function for specific model coefficient expressions. It is possible to determine general conditions that must be met to guarantee or not the success of the innovation, **understanding a successful innovation, one that manages to penetrate and expand in the market.**
- For one TS, it is possible to obtain explicit conditions under which an equilibrium of partial adoption or one of total adoption can be achieved; according to this model, both cases are perfectly achievable without consuming all the available resources.

- ❖ The model makes possible to establish explicit conditions for the level of investment required in each case, information that may be useful in decision making.
- ❖ Coexistence after (large?) innovation is possible, but not the Divergence, this is an example of the situation reported by F. Dercole and S. Rinaldi, where the evolutionary equilibrium corresponds to a TP at which the established and innovative TSs can not coexist **in a neighborhood of the evolutionary equilibrium**; however, both systems evolve towards each other until one of them disappears.
- ❖ Coexistence and Divergence conditions, do not hold simultaneously, then evolutionary branching is not guaranteed by the analysis in this work.

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Announcement!



**Workshop: Problems & Challenges in Applied Mathematics for Aysen region**

- Climate Change and Natural Resources
- Energy and Pollution
- Education and Society

Organizers: Gino Montecinos & Gerard Olivari  
15-17 January 2020, Coyhaique, Aysen, Chile

(after the World Conference on Natural Resource Modeling WCNRM 2020 in Valparaiso)