

R. 2.7

S : Create Vector  $V$ .

Algorithm root (s)

return S.elemAtRank(1)

Algorithm parent ( $p$ )

return S.elemAtRank( $\lfloor p/2 \rfloor$ )

Algorithm leftChild ( $p$ )

return S.elemAtRank( $p * 2$ )

Algorithm rightChild ( $p$ )

return S.elementAtRank( $p * 2 + 1$ )

Algorithm isInternal ( $p$ )

return leftChild( $p$ )  $\neq$  null  $\wedge$  rightChild( $p$ )  $\neq$  null

Algorithm isExternal ( $p$ )

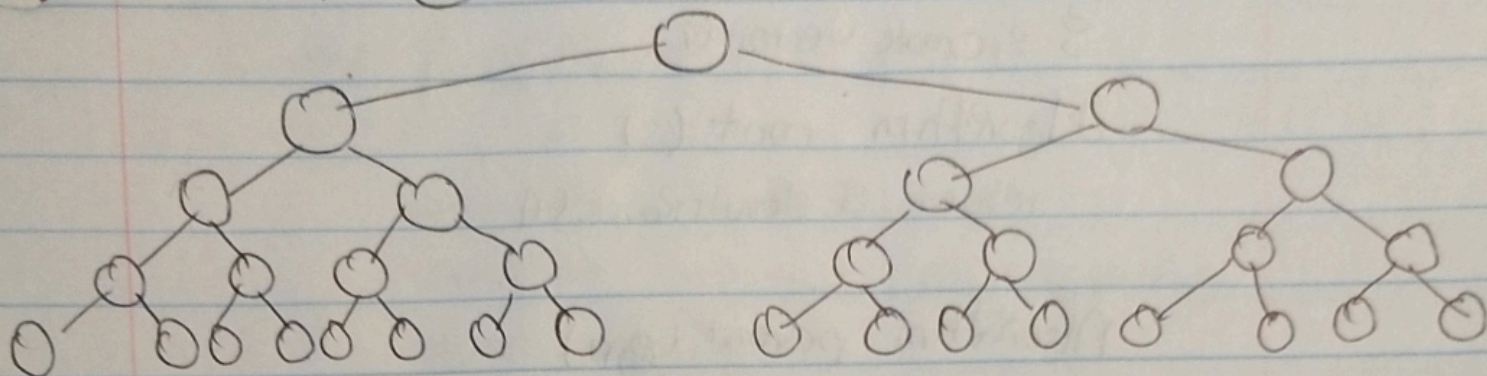
return ! isInternal( $p$ )

Algorithm isRoot ( $p$ )

return  $p == S.elemAtRank(1)$



R.2.8. (a)



(b)  $e = i + 1 \rightarrow i = e - 1$   
 $\downarrow$        $\downarrow$   
 Externos   Internos

$h \leq i \rightarrow h \leq e - 1 \rightarrow h + 1 \leq e$   
 $\downarrow$   
 height

(c)  $e \leq 2^h$

For example we can replace  $h = 2$

min  $h + 1 \leq e \rightarrow 3 \leq e$

max  $e \leq 2^h \rightarrow e \leq 4$

(d)  $n = i + e$   
 $\log_2(n+1) - 1 \leq h \leq \frac{(n-1)}{2}$

$\log_2(n+1) - 1 \leq h \leq i \Rightarrow n = 31$

$\log_2 32 - 1$

$\log_2 5 - 1 \leq h \leq (n-1)/2$

$4 \leq h \leq 15$

The lower bound of  $T$  is greater than 4

The upper bound of  $T$  is less than 15

Internal Node  $\frac{15}{2}$



~~E2.2~~

E.. For which values of  $n$  and  $h$  can the above lower and upper bounds on  $h$  be attained with equality?

When  $n=3$

the upper bound  $\rightarrow (3-1)/2 = 1$

the lower bound  $\rightarrow \log_2(3+1) - 1 = 1$

$$\log_2 2^2 - 1 = 1$$

E2.2 Analyze the queue implementation ADT using stacks from Assignment 2.

Amortized Runtime	action	actual
2	enqueue(a)	1
2	enqueue(b)	1
2	enqueue(b)	1
2	enqueue(d)	1
1	dequeue()	5
1	dequeue()	1
(10)		(10)



2.7

Array Linked List Algorithm shuffle Deck (s)

1 pointcheck = s.size - 1

n while pointcheck > 0 do

n  $n^2$  random := s.rankOf(randomInt(pointcheck + 1))

n  $n^2$  current := s.rankOf(pointcheck)

n n s.swapElements(random, current)

n n pointcheck = pointcheck - 1

(n) ( $n^2$ ) return s