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Theory change as dimensional change: conceptual spaces applied to the dynamics of empirical theories

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Abstract This paper offers a novel way of reconstructing conceptual change in empirical theories. Changes occur in terms of the structure of the dimensions—that is to say, the conceptual spaces—underlying the conceptual framework within which a given theory is formulated. Five types of changes are identified: (1) addition or deletion of special laws, (2) change in scale or metric, (3) change in the importance of dimensions, (4) change in the separability of dimensions, and (5) addition or deletion of dimensions. Given this classification, the conceptual development of empirical theories becomes more gradual and rationalizable. Only the most extreme type—replacement of dimensions—comes close to a revolution. The five types are exemplified and applied in a case study on the development within physics from the original Newtonian mechanics to special relativity theory.

Keywords Conceptual spaces · Dimensional analysis · Incommensurability · Newtonian mechanics · Scientific revolution · Special relativity theory · Theory change

1 Introduction

Within philosophy of science, several accounts of what a theoretical framework is and how theory change should be described have been proposed. During the heyday of logical empiricism, an empirical theory was considered to be a set of *sentences*: laws and others. Describing theory change was mainly a question of how new sentences could be added. Science was generally seen to be cumulative.

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Reacting to this conception, Thomas Kuhn (1962/1970) presented an account that posited two types: revolutions as dramatic paradigm shifts, with periods of normal scientific change in between.[1] Here, the historical development of scientific knowledge is not cumulative. On the contrary, old and new paradigms are considered to be mutually incommensurable: acceptable solutions, methods (e.g., of measurement), definitions, and scientists' 'world-views' may all change.[2]

A third account starts out from Lakatos (1978) notion of a theory core that he developed in criticism of Kuhn's notion of a paradigm (Blaug 1975, p. 407 *ff*). The *structuralists* (Balzer et al. 1984, 1987, 2000; Gähde 2002; Moulines 2002; Sneed 1971; Stegmüller 1976) later modified and systematized it. To a structuralist, an empirical theory is a set of mathematical (set-theoretical) *structures*—hence the name. These structures happen to satisfy some axioms, which can be expressed as set-theoretical predicates (Suppes 1957, chap. XII; Sneed 1971). Because the structures and not the axioms are central, the structuralist program is characterized as a *non-statement* or *semantic view* of empirical theories. The program seeks to provide exact representations of the logical structure of an empirical theory, and to achieve rigorous reconstructions of changes to a theory and to the conditions of its application to empirical phenomena. For Kuhn and Lakatos, incompatible theory cores are *logically* inconsistent. In contrast, structuralists comprehend incompatible theory cores as *different* complex structures.[3]

The above three positions continue to provide the anchoring points for the contemporary debate on scientific change.[4] We seek to complement this debate by proposing a general analytical apparatus for the reconstruction of changes to a scientific conceptual framework.

The main aim of this paper is to present a new way of analyzing changes of the conceptual content of empirical theories. Toward this end, we draw on the theory of *conceptual spaces* (Gärdenfors 2000). Our focus is not on sentential representation (axioms and laws), but on the conceptual framework, i.e., the *dimensions* (and their connection to measurement procedures).[5] Our use of "dimension" is specified in Sect. 2.

[1] This echoes Ludwik Fleck (1935/1979) who took *scientific crisis* to be a third developmental stage.

[2] Kuhn endorses a late Wittgensteinian meaning holism where successive scientific paradigms can save phenomena under quite different conceptual frameworks. His perhaps most famous example of incommensurability concerns the meaning difference of the term mass between Newtonian mechanics (m) and Einsteinian relativity (m_0): "The variables and parameters that in [the statements which Kuhn refers to as] the Einsteinian E_i's represented spatial position, time, mass, etc. still occur in the [Newtonian] N_i's and they still represent Einsteinian space, time and, mass. *But the physical referents of these Einsteinian concepts are by no means identical with those of the Newtonian concepts that bear the same name*. (Newtonian mass is conserved; Einsteinian is convertible with energy. Only at low relative velocities may the two be measured in the same way, and even then they must not be conceived to be the same)" (Kuhn 1961, p. 101–102; *italics added*). See Larvor (2003) on Kuhn's impact, and footnote 29 regarding the meaning of *mass*.

[3] (We are indebted to C. Ulysses Moulines for clarifying this difference). Kuhn (1976, esp. note 17) expresses his praise and reservations against the structuralist.

[4] The contributions in Soler et al. (2009) are an example of this.

[5] This is in line with Gärdenfors' (2000) arguments for distinguishing between symbolic and conceptual levels of knowledge representation.

Our primary thesis is that many types of scientific change can best be understood as systematic change operations applied to a previous framework. We submit that modeling scientific changes on the conceptual level is appropriate, provided that extant accounts of scientific conceptual change remain challenged with exceptions.[6] As we shall argue, the relations between different historical stages of a scientific conceptual framework may become clearer at the dimensional level. In particular, the conceptual frameworks of scientific theories become comparable via the mathematical properties of the dimensions. Consequently, the ground for speaking about incommensurability of frameworks is restricted, as we argue it should be, to ontological claims.

One may classify scientific changes into a number of analytical types, thus avoiding meaning holism (Sect. 3). We present five types of change and claim that these types are applicable to all cases of scientific conceptual change. The conceptual development of theory frameworks thus becomes gradual. We also show that each of the types of change we propose can be given a rationale. Our classification is a constructive contribution to the theory change debate.

As a case study in Sect. 4, we present some of the conceptual changes that took place within Newtonian mechanics before the introduction of special relativity. As should be largely non-controversial, we take much of the conceptual framework of relativity theory to have been prepared in the development of mechanics and electrodynamics. Special relativity indeed did represent a radical change to a conceptual framework, but this shift turns out rather less revolutionary on our account.

Before concluding, we spell out the relevance of our analysis for the history and philosophy of science in Sect. 5.

2 Modeling theory change in conceptual spaces

2.1 Conceptual spaces

Conceptual spaces provide a meta-framework by means of which theory frameworks can be reconstructed. The basic components of a conceptual space are its dimensions. The notion of a dimension should be understood literally. We assume that each dimension is endowed with certain *geometrical* structures.[7]

Examples of such dimensions include, with respect to sensory impressions: color, pitch, temperature, weight, and the three ordinary spatial dimensions (Gärdenfors 2000). Meanwhile, in scientific theories, the dimensions are determined by the variables presumed by the theory. To illustrate: consider *mass* [M], *length* [L], and *time*

[6] "There have been numerous attempts to establish necessary or sufficient conditions for 'progressive incorporation' or 'progressive overthrow'. … Nearly every proposed condition is open to counter-cases such that either we must admit that the condition is neither necessary nor sufficient, or we must revise significantly our understanding of the history of science. This is true of consilience, undersigned scope, incorporation-with-excess content, asymptotic agreement of calculations, the testimony of 'crucial experiments,' and the resolution of anomalies. One exception is the convergence of diverse experimental methods upon a single value" (Losee 2004, p. 156).

[7] In some cases, these are *topological* features or *orderings*. Points in dimensional spaces represent objects; regions represent properties or relations. Degrees of similarity between objects can be determined from the distances between points or regions.

[T] as used in Newtonian mechanics. The first two dimensions have a zero point and only take positive values; they are thus isomorphic to the half number line of the non-negative numbers, while *time* is isomorphic to the full number line.

The dimensions are independent of any symbolic representations: i.e., we can represent the qualities of objects by (partial) vectors in conceptual spaces without presuming an explicit object language in which these qualities are expressed.

Conceptual spaces present a meta-framework that differs both from the positivist approach, where everything is expressed in sentential form, and from analyses which build on the holistic notion of *paradigm* and combine conceptual and ontological considerations.

In contrast to these traditions, we see the meaning of a scientific term as independent of its ontological status. Thus we separate the meaning of the variables of a theory from questions concerning their ontology. On our account, the meaning of a variable is determined by its role in the conceptual framework.

The conceptual spaces approach also differs from the structuralist program. It neither maintains its set-theoretic overhead, nor will the most radical type of conceptual change amount to the rejection of one theory core followed by the adoption of a new theory core. We return to this comparison at the end of this section.

2.2 Separable and integral dimensions

Dimensions can be sorted into *domains*. In psychology, domains are defined via a distinction between *integral* and *separable* dimensions (Melera 1992; Maddox 1992). For example, an object cannot be given a hue without also giving it a brightness. Likewise the pitch of a sound always goes with its loudness. Dimensions that are not integral are *separable*: e.g., the *size* and *hue* dimensions.

Within the context of scientific theories, the distinction is better made in terms of *invariance transformations* (see Suppes 2002, p. 97–128). For example, the three dimensions of ordinary Euclidean space (x, y, z) are separable from the time coordinate t under a Galilean transformation (as *per* Newtonian mechanics), but not under a Lorentz transformation (as *per* Einsteinian special relativity). Similarly, mass is separable from everything else in Newtonian theory, but not from energy in special relativity. It is part of the meaning of "integral dimensions" that the dimensions share a metric. The role of the invariance transformations in the specification of a theory's conceptual space will become clearer in the case study in Sect. 4.

A theory *domain* can now be defined as the set of integral dimensions that are separable from all other dimensions. More precisely: domain C is separable from D in a theory *iff* the invariance transformations of the dimensions in C do not involve any dimensions from D. And similarly: the dimensions of a domain C are integral *iff* their invariance class does not involve any other dimensions. This criterion for identifying domains is tightly connected to measurement procedures for the domains.

To illustrate, consider how, in classical mechanics, distance and duration (trigonometry and chronometry) are treated as separable.[8] Light signals are tacitly assumed to

[8] Even though Newton's *Scholium* argues that measuring time requires measuring angles.

propagate instantaneously rather than at finite speed. Likewise, the mass of an object is presumed to be separable from (or: independent of) its position or velocity. As a final example, heat and work had been considered separable dimensions until the definition of heat as mean kinetic energy established that one could be measured in terms of the other. We return to this example below.

2.3 Dimensional analysis

We can compare our approach, of reconstructing the conceptual framework of empirical theories via their underlying conceptual dimensions, with *dimensional analysis* (Bridgman 1922; Huntley 1952; Palmer 2008; Rayleigh 1915). Again, our proposal is to model theoretical frameworks as domains with integral dimensions. For example, when adding to three-dimensional Euclidean space [L^3] the dimensions of *time* [T] and *mass* [M], and the three integral dimensions of *force* [F^3] (isomorphic to a three-dimensional Euclidean [vector] space), one obtains the conceptual space of the original Newtonian mechanics: an eight-dimensional conceptual space of four domains: [L^3], [T], [M], and [F^3].[9] What some call *derived magnitudes* (e.g., *velocity* [LT^{-1}], *acceleration* [LT^{-2}], and *momentum* [MLT^{-1}] connect domains without affecting the independence of their respective measurement procedures.[10]

Provided that the meaning of a scientific concept is determined by the dimensions that constitute it and by their respective measurement procedures, a scientific law can now be defined as the expression of a constraint on the distribution of points over the dimensions underlying the associated theory. Thus, in Newtonian mechanics, the second law of motion states that all observed measurement points will lie on the hypersurface spanned by $F = ma$ (Gärdenfors 2000, p. 216). This equation connects all eight dimensions of the theoretical framework. As we shall see in Sect. 4, there are two interpretations of the force domain F: as fundamental or as derived (defined). If it is taken as fundamental, then the equation $F = ma$ constitutes the basic empirical claim of the theory. Laws of mechanics that are specific to an application, such as Hooke's law of the pendulum $F = -\mathrm{k}x$,[11] add further constraints that increase the theory's empirical content.

The conceptual spaces approach treats all dimensions principally on par, so that a theory domain need not consist of fundamental dimensions only (in the sense of fundamental measurement magnitudes). The approach thus leaves room for, but does not require assigning epistemological privileges to *fundamental* over *derived* magnitudes (We return to this in Sect. 3.3). It makes no attempt either at answering ontological

[9] For later changes of these domains, see Sect. 4.

[10] A standard objection against representing scientific concepts in terms of conceptual dimensions is that distinctions between vector and scalar quantities are lost: thus, *torque* and *energy* would both take the form [ML^2T^{-2}] in Newtonian mechanics. Following Coulson et al. (2007, p. 20 *ff.*), we can use subscripting to save the distinction. For example, *torque* is the product of a force in, say, the x-direction [F_x], and an arm length [L_y] at a right angle. It may be expressed dimensionally as [$ML_xL_yT^{-2}$]. By contrast, *mechanical energy*—the product of a force and a length in the same direction—can be rendered as [$ML_x^2T^{-2}$].

[11] Here, x is the displacement of the spring's end from its equilibrium position, F the restoring force (dependent on the material), and k the force (or spring) constant.

questions that arise when assessing which dimensions to take as primitive ("do forces exist?"; "can a particle become a wave?"). Nor does it require drawing Sneed's (1971) distinction between T-theoretical terms (e.g., *mass* and *force* in Newtonian mechanics) and T-non-theoretical ones (e.g., *space* and *time*). However, in relying on measurement procedures to define the separability of domains, we follow Sneed's pragmatic stance: a T-theoretical dimension is one for which the value of an object cannot be determined without applying the theory T itself.[12] For example, force becomes a theoretical dimension in Newtonian mechanics (Sneed 1971).[13]

3 Five change operations

When the conceptual framework of an empirical theory is modeled as a conceptual space, changes of a theory framework divide naturally into five types.[14] Obviously, this provides a finer grain than distinguishing normal from revolutionary change. We present the types in order of increasing severity. What is commonly referred to as *scientific revolution*, we categorize primarily under the last type: the replacement of dimensions. Our analysis cannot predict when a change in the framework of a theory takes place. Nevertheless, each of the five types of changes can be given a *rationale* that is in line with general criteria within the philosophy of science.

3.1 Addition and deletion of special laws

Perhaps the most common type of change in empirical theories is the addition of special laws, such as the aforementioned Hooke's law for springs; the pendulum law $T = 2\pi\sqrt{L/g}$(with T being the period, L being the pendulum's length, and g being the gravitational acceleration) in Newtonian mechanics; or Boyles' gas law $pV = \mathrm{k}$ (with p being the pressure, V being the volume and k indicating a constant temperature) in thermodynamics.

The rationale for adding a law is the standard one in the positivist/Popperian tradition: it increases the testability/falsifiability of the theory and thereby its empirical content. Conversely, when experimental data are recalcitrant, it may be reasonable to delete a law (but see below).

It may seem surprising that we assign new special laws a comparatively unimportant status in our classification; after all, predictions come about only by virtue of applying special laws accompanied by a *ceteris paribus* clause (Zenker 2009). As Kuhn (1961) correctly stresses, the ever more accurate and precise determination of

[12] See Suppes (2002, p. 114): "[W]e may define a scale as a class of measurement procedures having the same transformation properties." In this sense, 'measurement procedure' denotes the pragmatic aspect of mathematical invariance.

[13] In Gärdenfors and Zenker (2011) we compare conceptual spaces with the structuralist program. There, we provide analogue to the structuralist's various kinds of models and constraints. We argue that set theory, paired with the use of theory cores, is insufficient to account for changes of theoretical frameworks. In the following sections, our aim is to show that conceptual spaces theory fares better.

[14] Gärdenfors and Zenker (2011) discuss only four types. In that paper, "change in the importance of dimension" is grouped together with "change of scale or metric."

natural constants (e.g., Boltzmann's [$ML^2T^{-2}K^{-1}$],[15] Hooke's [MT^{-2}], or gravitational acceleration [LT^{-2}])—on which predictions *depend*—accounts for a large part of what he called 'normal science'.

Nevertheless, once the appropriate dimensions are specified, formulating special laws may be straightforward. As Lord Rayleigh observed, "it happens not infrequently that results in the form of 'laws' are put forward as novelties on the basis of elaborate experiments, which might have been predicted *a priori* after a few minutes consideration" (Rayleigh 1915; cited after Roche 1998, p. 211). The conceptual framework established, adding a new special law or determining more precisely the value of a natural constant represents a comparatively undramatic type of change of theory, which may nevertheless be far-reaching since predictions then differ for a wide range of applications.

In principle at least, special laws can also be deleted. In the late nineteenth century, new exponents of r in Newton's law of gravitation $F = \mathrm{G}Mm/r^2$ (F being the net system force, G the gravitational constant, M and m being two masses, and r their distance apart) were proposed regularly. For example it was suggested that the exponent 2 should be replaced by 2.00000016 (see below). Yet "deletion" (meaning replacement of the original law) is the wrong term. Anomalous applications of special laws—e.g. with respect to Mercury's orbit (Roseveare 1982)—are normally tolerated. Occasionally, a special law may be moved to another theory in which it proves more successful. [16]

3.2 Change of scale or metric

The technical notion of *scale* goes back to Stevens (1946), who distinguished four types—*nominal*, *ordinal*, *interval*, and *ratio*[17]—ordered in terms of levels of measurement. The scales later received a set-theoretical embedding by Krantz et al. (1971, 1989, 1990). The conceptual dimensions that constitute a domain are measured by different metrics. With each scale type, information content increases between levels, where the next level shows fewer invariances than the last.[18] Thus the rationale for strengthening a scale type is the same as in the previous case: it increases the testability of the theory.

For instance, *interval* scales allow positive linear (affine) transformations, while *ratio* scales allow only multiplication. Thus, the Kelvin scale entails that there is no

[15] K is the dimensional symbol for temperature.

[16] A problematic special law may also persist as part of a theory without any application being assigned to it. Clairaut's law $F = \mathrm{G}Mm/\mathrm{r}^2 + a/r^4$ (a being a constant), which dates to 1747, is a case in point (Chandler 1975). Out of fashion for a hundred years, it was applied (unsuccessfully) to Mercury's orbit in the mid-nineteenth century (Gähde 2002). In structuralist terms, if T_n is the respective theory element to which the law is assigned, its set of applications $I(\mathrm{T}_n)$ may drop to zero. Here, we follow the pragmatic lead of structuralism in identifying intended applications.

[17] Note that nominal scales ("naming") yield no distance information, while ordinal scales only yield comparative information ("less or equal"). Hence, the dimensional account is not restricted to conceptual frameworks which feature the sort of measurable magnitudes that dominate modern physics.

[18] In line with the distinction between integral and separable dimensions, invariance classes are also important here.

temperature below zero degrees. By contrast, the corresponding point −273.15°C lacks any special status, as the Celsius scale is an interval scale. The change from Newton's absolute space and time to relative space and time (introducing the Galilean invariance class) is another example. In this case, the change is a weakening of the content that was motivated by the lack of method to identify the absolute space.

The following quote from Hausdorff (1903) may assist in appreciating the difference between a change in metric and more radical changes:

> We can guarantee that the dimension of [absolute] space is precisely three…. Further, replacing the square by the exponent 2.00000016 (this has recently been proposed… to explain the advance of the perihelion of Mercury) surely is an unfortunate idea. We cannot guarantee, however, that a measure of the curvature of space exactly equals zero…; we know only that a very small…, positive or negative, number estimates that measure. (Hausdorff 1903, p. 2 *f*.; cited after Czyz 1994, p. 251 *ff*.)

In this way, changing the metric may represent a less severe change than conjecturing space to have four (or even more) dimensions.

3.3 Change in the importance of dimensions

We begin with two simple examples. First, until the eighteenth century, *color*, as perceived by the eye, remained important to analytical chemistry. Starting with Lavoisier's oxygen chemistry and the invention of instruments such as the polarimeter (later the photoelectrical detector), the importance of perceived color was demoted. Second, pre-Linnaean botany focused on the holistic dimensions of flowers such as *size* and *color*, while Linnaeus' classification raised the importance of the numbers of pistils and stamens as salient classification features.[19]

We next turn to a more serious form of change in the importance of dimensions. A considerable amount of scientific debate concerns the *ontology* of the dimensions involved in a theory. For example, in Newtonian mechanics it was discussed whether forces really exist, or whether they are just defined entities.

However, changing the importance of a dimension does not change the empirical content of a theory, that is, its testability, so this cannot be the rationale for the change. The ontological debates are important for the rationality of a change, as they concern for the most part which dimensions are fundamental and which are derived, and what is deemed fundamental or derived changes as science develops. As we shall see, changes of the importance of dimensions can also pave the way for the unification of two theories.

Perhaps the most important—and ontologically most versatile—dimension of modern physics is *energy*, even though it was of hardly any importance to Newtonian mechanics. Going back to Leibniz's *vis viva* (Coopersmith 2010) as a description of

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19 Andersen et al. (2006) and Barker (2012) provide further examples of taxonomic change in biology and physics reconstructed with the theory of frames. Zenker (forthcoming) compares frames with conceptual spaces.

kinetic energy, it was revived in the nineteenth century by Young, gradually becoming more important in the development of physics. We return to this discussion in the case study in Sect. 4.

3.4 Change in the separability of dimensions

Recall that a dimension of a domain C is integral if its invariance class does not involve any other dimensions. In Newtonian mechanics, *mass* was originally a separable dimension. After the late nineteenth century observation that heavily charged particles appear to gain in mass,[20] the mass dimension gradually lost its separability (also see Sect. 4.3). In twentieth century relativity physics, mass and energy are effectively treated as interdefinable dimensions.[21]

The paradigmatic twentieth century example of change in separability is the transition (ascribed to Einstein but prepared, amongst others, by Poincaré) from Newton's *space* and *time* to Minkowski's *spacetime*. In Newtonian physics, there is no interaction in the measurements of space and time. Under special relativity, spatial and temporal coordinates (x, y, z, t) become integrated (see Sect. 4).

Another example is *heat*, which at one time was considered to be a substance: i.e., *caloric*, a fluid passing from warmer to colder bodies. Starting already with Clausius' work in thermodynamics in the middle of the nineteenth century, heat began to lose its independent status, and effectively lost it entirely when Boltzmann functionalized temperature as *change in mean kinetic energy* (Chang 2004).

Changing the separability of dimensions is a comparatively radical move, since it involves a change in the *measurement methods* that may be applied. For example, if the mass of a particle is dependent on its charge, traditional Newtonian measurement procedures for mass can no longer be used. In all historical examples we have found, such a change is connected with a unification of two theories. The change can therefore be rationalized as a step in the unification process.

3.5 Addition and deletion of dimensions

The most extreme form of conceptual change occurs when a dimension is added to (or deleted from) its associated conceptual framework. Perhaps the clearest example is the introduction of Newtonian *mass* and *gravitational force* to replace Gallilean *weight*. In modern terms, an object's weight is understood as its mass under the influence of a gravitational field. Notably, mass has, at least once, been a candidate for deletion as a separable dimension. Priestley resisted the Newtonian separation of matter from force. He proposed instead "to reduce matter entirely to the forces of attraction and

[20] J. J. Thomson is widely cited to have introduced the idea that mass depends on velocity when, in 1881, he measured the kinetic energy of charged particles. As Okun (1989, p. 32) points out, this ascription is incorrect, and seems to have come about with Henrik Lorentz whose (1899) work relies on this very claim. Also see the next section.

[21] For a reservation concerning this statement, see Sect. 4.

repulsion" (McMullin 2002, p. 33), thus making force, rather than mass, one of Newtonian mechanics' fundamental dimensions.

Another example comes from thermodynamics where, in 1850, Clausius searched for the conserved quantity in heat change processes. Conjecturing *energia* to be some combination of *heat* and *work* led him to introduce *energy* as a new dimension, integral with heat and work. In fact, already in the eighteenth century, Joseph Black had distinguished the quantity of heat (including latent heat) from its intensity, only the latter being measured by temperature.

Finally, consider the *ether* in its various forms: luminferous, electromagnetic, etc. The ether may be reconstructed as a three-dimensional space. It was introduced—by analogy to air and sound—as the medium carrying light. Following Michelson and Morley's null result, and Einstein's critical results rendering it superfluous, the space of ether was effectively deleted.

Being the most radical type of change, it is also the type that is most difficult to rationalize. Again, the aim to *unify* theories seems to be common to many historical dimensional changes, for example Newton's unification of terrestrial and celestial mechanics by the introduction of the new dimension "mass".[22] It seems that the rationality of a dimensional change can only be evaluated *post hoc*.

The five types of dimensional changes we have presented here constitute a toolbox for the analysis of historical changes of scientific conceptual frameworks. After illustrating in a case study how the changes can be applied, we discuss their general value for the history and philosophy of science in Sect. 5.

4 Case study: conceptual changes from Newtonian mechanics to special relativity

In this section, we will illustrate the five types of change in a case study, tracing the transitional steps in the underlying conceptual spaces from Newtonian mechanics to special relativity. Our aim is to show that the transition from Newtonian mechanics to Einsteinian special relativity is not as revolutionary as sometimes claimed. Within Newtonian mechanics, a number of changes occurred that prepared the way for special relativity. We will model these by using the five types of change described in Sect. 3. Our aim is not to write this history, but rather to highlight some of the conceptual changes that occurred in this transition. Furthermore, we will not enter into discussion of the ontological status of the various dimensions.

4.1 Newtonian mechanics

The quality dimensions of the conceptual space underlying the original Newtonian mechanics are ordinary space s (isomorphic to R^3), time t (isomorphic to R), mass m (isomorphic to R+), and force F (isomorphic to R^3). All spaces are Euclidean.

[22] Considerations of *simplicity* may therefore also be a motive.

Newton originally considered space and time absolute: i.e., he assumed the domains to have fixed origins. However, Newton's laws are invariant under linear (Galilean) transformations, so this assumption is of an ontological nature that has no empirical consequences. The theory was later reformulated without assuming absolute space or time. Instead, it was described in terms of a relational space s and a relational time t, both invariant under linear transformations—representing a change of scale (Sect. 3.2). These invariances imply that *space*, *time*, and *mass* should be understood as separable domains, according to the criteria presented in Sect. 2.

Importantly, the total mass of objects in a system was assumed to be constant over all applications of the theory. One of the contributions of Newtonian theory is that it introduced a distinction between *mass* and *weight*.[23] The dimensions of velocity $[LT^{-1}]$ $v = ds/dt$, acceleration $[LT^{-2}]$ $a = ds^2/dt^2$, and momentum $[MLT^{-1}]$ $p = mv$, along with kinetic energy $e_k = 1/2\ mv^2$, potential energy $e_p = mgh$, and work $w = Fs$ (all three of the non-discriminate dimensional form $[ML^2T^{-2}]$), can be introduced as *defined magnitudes* in Newtonian mechanics.

The second law of motion $F = ma$ introduces a constraint that connects all dimensions of the conceptual space. From it, Newton was able to derive a number of fundamental mechanical laws—amongst others Galileo's and Kepler's laws—as well as the principle of the invariance of momentum. (Newton thus fulfilled Galileo's vision by unifying terrestrial and celestial mechanics.)

In the discussions of Newtonian mechanics over the years, one finds different opinions on the status of the domain of *force*. It is sometimes taken as a fundamental three-dimensional space that is separable from the other dimensions, in which case Newton's second law of motion $F = ma$ introduces empirically testable constraints on observations. Other times it has been seen as a defined magnitude, in which case the second law becomes the very definition. On this latter interpretation, Newtonian mechanics is built on five dimensions: [M], $[L^3]$, and [T] (Kyburg 1984, p. 175 *ff.*). The choice about how to understand force can be taken as a decision about the importance of the relevant domains. It involves a change in the importance of dimensions (Sect. 3.3), and has ontological implications concerning force.

The history of mechanics reveals proponents of both positions (Jammer 1957). Many wish to eliminate force as an independent magnitude—in particular Ernst Mach, but also proponents of the electro-dynamic frameworks preceding him, such as Zöllner and Ritz (Zenker 2009, p. 50–54). Newton seems to have viewed the second law as retaining the fundamental status of the force domain. As mentioned earlier, Priestley, one of the last supporters of phlogiston, resisted the Newtonian separation of matter from force. As late as 1804, he denied that phlogiston had negative weight. Following Boscovich[24], he sought to eliminate *mass* (or matter)—which "can be nothing else than the enumeration of its properties" (Schofield 1964, p. 293)—but retain *force*.

[23] In Newton's *Principia*, mass is defined in terms of density but this dimension seems to play no further role in his theory.

[24] Boscovich's kinematics relied on distance and motion only; the force of gravity was taken to be a derived quantity.

4.2 Enter energy

Leibniz proposed the principle of invariance of kinetic energy (which he called living force: *vis viva*). He argued that it is more fundamental than the invariance of momentum. In our classification of types of change, this represents the addition of a new law (Sect. 3.1) that increases the empirical content of the theory. Leibniz' principle has since been incorporated into classical mechanics (Lindsay 1971). The addition of this principle represented perhaps the first step in the process leading to the greater importance of energy, and it proved helpful in subsequent stages, when other types of energy were added: e.g., heat and electromagnetic energy.

Later, mathematical reformulations of classical mechanics by Lagrange and Hamilton further elevated energy's importance. The Lagrangian of a system is its kinetic energy minus its potential energy. The Hamiltonian of a system is its total energy: for a closed system, the sum of the system's kinetic and potential energy. (One can reasonably claim that the fundamental conceptual domains of Hamiltonian classical mechanics are *space*, *time*, *mass*, and *energy*.) In both cases, as the role of energy is elevated, the role of force is demoted. This constitutes a change in the importance of dimensions (Sect. 3.3). That said, the Newtonian and Hamiltonian versions of classical mechanics are formally inter-translatable and empirically equivalent. Once more, the contentious issue is one of ontology: here, the status of *force*.

Another step towards making energy more central to and at the same time independent of Newtonian mechanics is Joule's (1887) determination, in 1849, of the mechanical equivalent of heat via the temperature change in a liquid.[25] Henceforth, a conserved quantity, *energy*, becomes expressible, one that connects amounts of heat and work.

In thermodynamics, *heat* (originally a fundamental dimension) is reduced to mean *kinetic* (motion) energy, plus the *potential* (bonding) energy of molecules—the latter accounting for aggregate-state transitions. In other words, the dimension of *heat* becomes derived. It can no longer count as separable within mechanics: a change in the separability of dimensions (Sect. 3.4). In consequence, the dimension of *temperature* changes from an interval scale to a ratio scale, with zero kinetic energy as the endpoint: an example of a change of scale (Sect. 3.2).

Leibniz' principle of the invariance of kinetic energy becomes the first law of thermodynamics. Regarding the dimension of heat, one can ask whether *heat* = *mean kinetic energy* represents a law or a definition. The situation is very similar to the interpretation of $F = ma$. Without discussing the issue further, we can conclude that thermodynamics also contributed to the increasing importance of the energy dimension (Clarke 2002).

[25] Sibum (1995, p. 74) points out that "[Joule's] exceptional experimental practice was based on the transformation of different, apparently unrelated traditions … and thermo*metrical* skills which were rare in the early Victorian physics community" (*italics added*). Harman (1982) provides an excellent overview of important measuring devices in nineteenth century physics.

4.3 Electromagnetism

Electrodynamics introduces a new dimension, *electric current* [I], and defines *electric charge* [IT].[26] Maxwell's equations express the connections between current, space, and time. Conservation of energy is assumed to hold.

Thomson's (1881, 1897, 1899) hypothesis—that the increase in apparent mass of heavily charged particles is of electromagnetic origin—violates the fundamental assumption of Newtonian mechanics that the mass of a particle is constant. This prepares the way for a break with the separability of mass, and may today be regarded as the "birthplace" of the electron, and perhaps of subatomic physics more generally. Along with developments in thermodynamics, Thomson's discovery contributes once again to the increasing importance of the energy dimension. The upshot is that energy is no longer seen as a defined entity as it was in Newtonian mechanics: a change in separability (Sect. 3.4).

Another consequence of the development of electromagnetic theory is that *fields* become important representational formats (McMullin 2002): e.g., the electric [$\mathrm{MLT}^{-3}\mathrm{I}^{-1}$] and magnetic [$\mathrm{MLT}^{-2}\mathrm{I}^{-1}$] fields. However, fields can be seen as a special type of conceptual spaces. For example, a scalar value from a dimension S (e.g., temperature) assigned to a point in a space C (e.g., a three-dimensional Euclidean space) can be described as a four-dimensional conceptual space $\langle C, S\rangle$, with a function from C to S specifying the values of the scalar at each point of C.

Similarly, a vector field F assigning a three-dimensional vector (e.g., *force*) to a point in a space C (e.g., a three-dimensional Euclidean space) can be seen as a six-dimensional conceptual space $\langle C, F\rangle$, with a function from C to F specifying the vectors. A similar story can be told for tensor fields. In this way, forces become "hidden" within vector fields. In sum, the use of fields simplifies certain representations, but the results are still compatible with the description of theories in terms of conceptual spaces.

Hendrik Lorentz first obtained the Lorentz (or Lorentz-Fitzgerald) transformations when he sought to find transformations that would leave Maxwell's electromagnetic equations unchanged. The transformations were interpreted as describing the contraction of objects in their direction of motion.[27] Space and time remained separable and Newtonian. However, no mechanism was found to account for the contraction. Poincaré's mathematical insight and Einstein's own bold insight allowed the transformations to express an intrinsic property of space and time or, rather, spacetime. Poincaré favored retaining the ether; Einstein deleted it. The mathematical device that Einstein eventually settled on—the energy-stress tensor—no longer required any substance with ontological status but only specified a functional property.

[26] Particle theorists see electrical current as the fundamental dimension; field theorists see electrical charge as the fundamental dimension. This is evidence of a change in the importance of dimensions (Sect. 3.4) between the two traditions.

[27] An equally plausible interpretation would be that of tweaking the scales: "it is only the relation of the magnitude to the instrument that we measure, and if this relation is altered, we have no means of knowing whether it is the magnitude or the instrument that has changed" (Poincaré 1897, p. 97).

Summing up: energy was introduced as a new dimension in later versions of Newtonian mechanics and was assigned a gradually increasing importance. The cumulative change over time represents a change of the fifth type, although it is difficult to pinpoint exactly when the change takes place: there is no revolutionary *moment*. Note, too, that the change occurs *before* Einstein formulates special relativity. The Lagrangian and Hamiltonian formulations make energy absolutely central. They continue to play an important role in quantum theory today.

4.4 Special relativity

Einstein started out from two fundamental postulates: (1) the speed of light is constant in all inertial frames of reference, and (2) all laws of physics (meaning here electrodynamics) are the same in all inertial frames (i.e., they obey the same transformation rules). The first postulate provides the one fundamentally new constraint of his relativity theory. From these two postulates, Einstein is able to derive the invariance with respect to the Lorentz transformations.

Hence, the Lorentz transformations—originally developed in an ether model—could be retained, while accepting that space and time are no longer separable domains but form an integrated four-dimensional *spacetime*. Since the geometry of spacetime is different from that of either space or time in classical mechanics (i.e., a Minkowskian versus a Euclidean metric), this involves a change of scale (Sect. 3.2).

Unlike Newtonian theory, special relativity no longer allows the passing of time to be treated independently of the state of motion of an observer. Thus, Einstein's postulates entail a change in the separability of dimensions (Sect. 3.4). However, energy remains separable from spacetime.

Following Poincaré's lead—and also Thomson's,—Einstein argued that mass and energy should no longer be viewed as separable dimensions, another change in the separability of dimensions (Sect. 3.4).[28] By the principle of the *inertia of energy*, to each energy E there corresponds a (rest)mass $m = E/c^2$, where "E need not be the total energy of a system, as was first hypothesized, but … a mass (or momentum) may be associated with each individual energy (or energy current)" (Hickman 1984, p. 542).

To his first two postulates, Einstein added a third, regarding the conservation of momentum: (3) the total momentum of a system is preserved in all inertial frames of reference. In this context, momentum—classically given by $p = mv$—is expressed by $p = m_0 v/\sqrt{(1 - (v^2/\mathrm{c}^2))}$, in turn giving rise to well-known allegations of radical meaning change.[29] In contrast to Newtonian theory, this principle *entails* the conser-

[28] It is rather difficult to substantiate the claim that Einstein followed these leads intentionally, as he was rather careful in revealing his sources. It is at least highly plausible that Einstein must have known about the work of, amongst others, Poincaré and Thomson. Yet Einstein resisted Thomson's *metaphysical* preference for 'mechanical continuity' (read: ether) over 'action at a distance' (Navarro 2005, p. 276).

[29] In this equation, v is the velocity and c the speed of light in a vacuum. Both Kuhn and Feyerabend have emphasized this example. The special relativity form $p = m_0 v/\sqrt{(1 - (v/\mathrm{c})^2)}$ converges to the Newtonian mechanics form $p = mv$ as v goes to zero. Their critical claim is that m_0 and m amount to different definitions of mass—i.e., m_0 is literally a different thing than m. After all, or so is the argument,

vation of the total energy of a system. Provided sufficient energy, "massless objects" can travel at the speed of light, c; while accelerating a particle of non-zero rest mass to c requires infinite energy. Energy and mass are interdefinable and no longer count as separable dimensions. In consequence, mass loses its status as a fundamental dimension: a change in separability (Sect. 3.4). Meanwhile the dimension of force plays no central role in the theory, although it can still be defined within it.

The relativistic mass of an object is dependent on its inertial frame. What remains constant is the resting mass of an object which does not move in relation to a given frame of reference. That said, when objects in a system break down into parts—e.g., in the fission of radioactive particles—the sum of the objects' resting masses need not be constant. This contrasts with Newtonian mass as an *invariant* quantity. Of course, it would be unfair to blame Newton for later developments in nuclear physics.

The conceptual dimensions of special relativity are thus *spacetime* [L^3T] and *mass-energy* [E]. Relativistic momentum and the electromagnetic field are derived dimensions, in accordance with the development of mechanics after Newton.

4.5 Summary and contrast

A key question, in engaging with claims that two historically successive conceptual frameworks are incommensurable, is how the meaning of "mass" changes from Newtonian to Einsteinian theory. That mass is one-dimensional and measured on a ratio scale remains constant. The interpretation of resting mass seems to remain fairly constant as well. What changes is, first, the separability of mass from energy; and, second, its frame dependence: i.e., the sum of resting masses in a system need not be constant. Meanwhile, energy gains in importance, while the status of force is demoted.

We believe our account to be more informative about the development of physics than Kuhn's suggestion of a radical shift in the meaning of mass as a consequence of the Einsteinian "revolution." This is so because, on our account, the meaning of scientific terms is exhausted by the structure of conceptual frameworks and their associated measurement procedures. The meanings of the scientific variables do not reside in the symbolically expressed laws that are formulated to express connections between the dimensions.

In our opinion, the crudeness of the notion of a scientific revolution may be a consequence of focusing on linguistic forms—in particular, axioms and laws—when formulating a theory's conceptual content. Given this restriction, it is hardly surprising

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although the term $\sqrt{(1-(v/c)^2)}$ approaches 1 as v approaches 0, the Newtonian form never allowed this possibility. Hence, this becomes evidence of *radical* difference in meaning. Such differences have come to be called *ruptures*, the ruptures allegedly indicate "rationality gaps" (Rehg 2009, p. 33–80). Notably, expert disagreement over the status of m_0 and m persists. See footnote 2 for Kuhn's view (according to which m and m_0 have *different referents*), and Rivadulla (2004) or Okun (1989) for the meaning-constancy standpoint, as well as Falkenburg (2007, p. 161 *ff.*) for the meaning-divergence standpoint.

A typical reaction of those upholding the meaning constancy standpoint is to point out, with Einstein, that: "it is better to introduce no other mass concept that the 'rest mass' m. Instead of introducing [the relativistic mass] M, it is better to mention the expression for the momentum and energy of a body in motion" (Einstein, letter to Lincoln Barnett, 19 June 1948; cited after Okun 1989, p. 32).

when one finds that reconstructions of change episodes which amount to more than the mere addition or deletion of a law result in a revolution.[30]

We believe such characterizations of scientific change to be misleading. We hope to have shown that the conceptual framework of a scientific theory can be repeatedly revised without leading to a scientific revolution. Our analysis can likewise account for the long-term guidance which conceptual frameworks provide in seeking empirical "cooperation" from nature.

We submit that the gradual conceptual transition from Newtonian to Einsteinian physics is best illuminated by our analysis of types of change within a conceptual spaces framework. For the most part, those following in the tradition of Kuhn and Feyerabend tend to neglect the conceptual continuities for which we can account, and focus on ontology (Hoyningen-Huene 1993; Oberheim and Hoyningen-Huene 2009). A consequence of our analysis is that the transition appears much less revolutionary.

5 The relevance of the analysis for history and philosophy of science

We make the programmatic claim that no tool is needed beyond our classification of change into the five types for reconstructing *the conceptual part* of scientific change. On our account, even the most radical changes turn out to be less radical than Kuhn and his proponents seem to believe. After all, if specific applications (e.g., planetary orbits) can be shared between conceptual frameworks, it is possible to compare the new and the old framework *vis à vis* these applications. Consequently, the associated conceptual spaces, with their different properties, can be analyzed via the classification scheme of change types.

In the light of employing conceptual spaces, the "rationalization" of historical episodes of conceptual change in the sciences can now draw to lesser extents on the kinds of factors that the sociology of scientific knowledge offers.

We present our classification of changes of theoretical frameworks as a new tool for history and philosophy of science. Historians of science will hopefully find it useful when treating a field's history. Periods that are allegedly revolutionary normally feature little science-internal historical evidence for a revolution being under way. In contrast, there is often evidence that scientists prefer one among several rivaling conceptual frameworks for reasons that are rationalizable, at least in retrospect. We believe that the conceptual spaces approach is useful for bringing out such a contrast between competing frameworks.

[30] Thus, Kuhn writes: "Revolutionary changes are somehow holistic. They cannot, that is, be made piecemeal, one step at a time, and they thus contrast with normal or cumulative changes like, for example, the discovery of Boyle's law. In normal change, one simply revises or adds a single generalization, all others remaining the same. In revolutionary change one must either live with incoherence or else revise a number of interrelated generalizations together. If these same changes were introduced one at a time, there would be no intermediate resting place. Only the initial and final sets of generalizations provide a coherent account of nature" (Kuhn 1987, p. 19).

Especially Kuhn's (1977, 2000) later work on taxonomic change, also known as "the structured lexicon"—particularly his idea that full inter-translatability between taxonomies is impossible—suggests to us that he continued to endorse a *primarily linguistic* reconstruction of scientific concepts.

More generally, historians of science may find our tool box especially helpful insofar as the addition or deletion of dimensions can be used as a criterion for identifying a radical change. However, even then, such transitions should not count as instances of a rationality-defying shift in scientific knowledge. Likewise for mathematical advances (e.g., from vector to tensor calculus in general relativity), insofar as these do not change the underlying conceptual space.[31] As we have sought to explain in our case study, the conceptual change in transitioning from Newtonian mechanics to special relativity consists in integrating three-dimensional space and one-dimensional time into four-dimensional spacetime, and making *energy* and *mass* convertible, at the expense of demoting *forces*.

Our proposal—to reconstruct scientific change within conceptual spaces—is not conservative in the sense of trying to preserve some form of positivism. Clearly, the development of scientific knowledge does not consist only of adding facts. Much of the important work concerns the status of the underlying conceptual dimensions: how they are related, which are deemed fundamental and which are not, and what are their measurement procedures.

Finally, this approach only applies to meaning changes and not to other aspects of a theory. If this distinction can be defended, a counterexample would amount to showing that the five types of change are not sufficient (or not fine-grained enough) to provide a satisfactory account of meaning change within the development of a theory.

6 Conclusion

Our starting point has been that describing scientific conceptual change merely as addition of symbolically expressed laws is not an appropriate strategy. On the other hand, replacing that strategy with a dichotomy between normal and revolutionary scientific change obliterates many of the changes, within what is now commonly called normal science, that prepare the way for a more radical conceptual shift.

Structuralism does not provide the most satisfactory account of theory change either since it has difficulties explaining what happens when one theory core is replaced by another. Conceptual spaces offer a reconstructive framework for empirical theories that allows a more realistic description of how scientists work than what is offered by either the positivist, a Kuhnian, or the structuralist account.

Our basic strategy has been to identify empirical theories not via theory cores (in the technical sense of structuralism), but by using conceptual dimensions in a more direct way. A reconstruction of the framework underlying a theory in conceptual spaces brings into focus those dimensions that belong to a theory at a particular point of time and the measurement procedures associated with them. We adopt this strategy from dimensional analysis and extend it to revisions of conceptual frameworks.

We reject the notion of a paradigm shift as a historical episode that cannot be given a rational reconstruction without citing extra-scientific factors: e.g., power, fashion, interests, etc. Once the analysis is rich enough to describe all conceptual changes, and

[31] Pragmatic factors pertaining to the successful application of a theory do not fall under what we call conceptual knowledge.

provided conceptual change is well-distinguished from ontological change, revolutionary gaps in the development of conceptual knowledge seem to disappear.

By way of a case study, we have shown that conceptual change in science is a rather regular process that can be rationalized and classified according to severity. We classify such changes into five types of which only the addition/deletion of dimensions is of truly radical nature. We invite historians and philosophers of science to test the viability of our analysis and classification scheme by applying them to other case studies.

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References

Andersen, H., Barker, P., & Chen, X. (2006). *The cognitive structure of scientific revolutions*. Cambridge, UK: Cambridge University Press.

Balzer, W., Moulines, C. U., & Sneed, J. D. (1987). *An architectonic for science: The structuralist program*. Dordrecht: Reidel.

Balzer, W., Moulines, C. U., & Sneed, J. D. (Eds.). (2000). *Structuralist knowledge representation: Paradigmatic examples*. Amsterdam: Rodopi.

Balzer, W., Pearce, D. A., & Schmidt, H.-J. (1984). *Reduction in science: Structure, examples, philosophical problems*. Dordrecht: Reidel.

Barker, P. (2012). The cognitive structure of scientific revolutions. *Erkenntnis, 75*(3), 445–465.

Blaug, M. (1975). Kuhn versus Lakatos, or paradigms versus research programmes in the history of economics. *History of Political Economy, 7*(4), 399–433.

Bridgman, P. (1922). *Dimensional analysis*. Yale: Yale University Press.

Chandler, P. (1975). Clairaut's critique of Newtonian attraction: Some insights into his philosophy of science. *Annals of Science, 32*, 369–378.

Chang, H. (2004). *Inventing temperature: Measurement and scientific progress*. New York: Oxford University Press.

Clarke, B. (2002). From thermodynamics to virtuality. In B. Clarke & L. D. Hernderson (Eds.), *From energy to information: Representation in science and technology, art and literature* (pp. 17–33). Stanford, CA: Stanford University Press.

Coopersmith, J. (2010). *Energy, the subtle concept: The discovery of Feynman's blocks from Leibniz to Einstein*. Oxford: Oxford University Press.

Coulson, J. M., Richardson, J. F., Backhurst, J. R., & Harker, J. H. (2007). *Chemical engineering* (Vol. 1). Burlington, MA: Elsevier.

Czyz, J. (1994). *Paradoxes of measures and dimensions originating in Felix Hausdorffs's ideas*. River Edge, NJ: World Scientific.

Falkenburg, B. (2007). *Particle metaphysics: A critical account of subatomic reality*. Berlin: Springer.

Fleck, L. (1935/1979). *Genesis and development of a scientific fact*. T. Trenn & R. Merton (Eds.). Chicago: University of Chicago Press.

Gärdenfors, P. (2000). *Conceptual spaces: The geometry of thought*. Cambridge, MA: The MIT Press.

Gärdenfors, P., & Zenker, F. (2011). Using conceptual spaces to model the dynamics of empirical theories. In E. J. Olsson & S. Enqvist (Eds.), *Belief revision meets philosophy of science* (pp. 137–153). Berlin: Springer.

Gähde, U. (2002). Holism, underdetermination, and the dynamics of empirical theories. *Synthese, 130*, 69–90.

Harman, P. M. (1982). *Energy, force, and matter: The conceptual development of nineteenth-century physics*. Cambridge, UK: Cambridge University Press.

Hausdorff, F. (1903). Das Raumproblem. *Annalen der Naturphilosophie, 3*, 1–23.

Hickman, F. R. (1984). Electrodynamical origins of Einstein's theory of general relativity. *International Journal of Theoretical Physics, 23*, 535–566.

Hoyningen-Huene, P. (1993). *Reconstructing scientific revolutions: The philosophy of science of Thomas S. Kuhn*. Chicago: University of Chicago Press.

Huntley, H. E. (1952). *Dimensional analysis*. London: McDonald.

Jammer, M. (1957). *Concepts of force: A study in the foundations of dynamics*. Cambridge, MA: Harvard University Press.

Joule, J. P. (1849/1887). On the mechanical equivalent of heat. Reprinted in *The Scientific Papers of James Prescott Joule* (pp. 298–328). London: Dawsons of Pall Mall.

Krantz, D. H., Luce, R. D., Suppes, P., & Tversky, A. (1971, 1989, 1990). *Foundations of measurement* (Vol. I–III). New York: Academic Press.

Kuhn, T. S. (1961). The function of measurement in modern physical science. *Isis, 52*, 161–193.

Kuhn, T. S. (1962/1970). *The structure of scientific revolutions*. Chicago: University of Chicago Press.

Kuhn, T. S. (1976). Theory-change as structure-change: Comments on the Sneed formalism. *Erkenntnis, 10*, 179–199.

Kuhn, T. S. (1977). *The essential tension: Selected studies in scientific tradition and change*. Chicago: University of Chicago Press.

Kuhn, T. S. (1987). What are scientific revolutions? In L. Krüger, L. Daston, & M. Heidelberger (Eds.), *The probabilistic revolution* (pp. 7–22). Cambridge, UK: Cambridge University Press.

Kuhn, T. S. (2000). The Road since Structure: Philosophical Essays, 1970–1993, with an Autobiographical Interview. In J. Conant & J. Haugeland (Eds.). Chicago: University of Chicago Press.

Kyburg, H. E. (1984). *Theory and measurement*. Cambridge, UK: Cambridge University Press.

Lakatos, I. (1978). *The methodology of scientific research programs*. Cambridge, UK: Cambridge University Press.

Larvor, B. (2003).Why did Kuhn's *Structure of Scientific Revolutions* cause a fuss? *Studies in the History and Philosophy of Science, 34*, 369–390.

Lindsay, R. B. (1971). The concept of energy and its early historical development. *Foundations of Physics, 1*, 383–393.

Lorentz, H. (1899). Simplified theory of electrical and optical phenomena in moving systems. *Proceedings Academy of Science, Amsterdam, I*, 427–443.

Losee, J. (2004). *Theories of scientific progress: An Introduction*. New York, NY: Routledge.

Maddox, W. T. (1992). Perceptual and decisional separability. In G. F. Ashby (Ed.), *Multidimensional models of perception and cognition* (pp. 147–180). Hillsdale, NJ: Lawrence Erlbaum.

McMullin, E. (2002). The origins of the field concept in physics. *Physics in Perspective, 4*, 13–39.

Melera, R. D. (1992). The concept of perceptual similarity: From psychophysics to cognitive psychology. In D. Algom (Ed.), *Psychophysical approaches to cognition* (pp. 303–388). Amsterdam: Elsevier.

Moulines, C. U. (2002). Introduction: Structuralism as a program for modeling theoretical science. *Synthese, 130*, 1–11.

Navarro, J. (2005). J. J. Thomson on the nature of matter: Corpuscles and the continuum. *Centaurus, 47*, 259–282.

Oberheim, E., & Hoyningen-Huene, P. (2009). The incommensurability of scientific theories. In E.N. Zalta (Ed.), *The Stanford encyclopedia of philosophy*. http://plato.stanford.edu/archives/spr2009/entries/incommensurability Accessed 24 December 2011.

Okun, L. B. (1989). The concept of mass. *Physics Today, 42*(6), 31–36.

Palmer, A. C. (2008). *Dimensional analysis and intelligent experimentation*. Hackensack, NJ: World Scientific Publishing.

Poincaré, H. (1897). *Science and method* (F. Maitland, Trans.). London: Thomas Nelson and Sons.

Rayleigh, L. (1915). The principle of similitude. *Nature, 95*, 66–68.

Rehg, W. (2009). *Cogent science in context: The science wars, argumentation theory and Habermas*. Cambridge, MA: The MIT Press.

Rivadulla, A. (2004). The Newtonian limit of relativity theory and the rationality of theory change. *Synthese, 141*, 417–429.

Roche, J. (1998). *The mathematics of measurement: A critical history*. London: The Athlone Press.

Roseveare, N. T. (1982). *Mercury's perihelion from LeVerrier to Einstein*. Oxford: Clarendon Press.

Schofield, R. E. (1964). Joseph Priestley, the theory of oxidation and the nature of matter. *Journal of the History of Ideas, 25*, 285–294.

Sneed, J. D. (1971). *The logical structure of mathematical physics*. Dordrecht: Reidel.

Soler, L., Sankey, H., & Hoyningen-Huene, P. (2009). *Rethinking scientific change and theory comparison: Stabilities, ruptures, incommensurabilities? (Boston studies in the philosophy of science,* vol. 255*)*. Berlin: Springer.

Stegmüller, W. (1976). *The structuralist view of theories*. Berlin: Springer.

Stevens, S. S. (1946). On the theory of scales of measurement. *Science, 103*, 677–680.

Sibum, H. O. (1995). Reworking the mechanical value of heat: Instruments of precision and gestures of accuracy in early Victorian England. *Studies in History and Philosophy of Science, 26*, 73–106.

Suppes, P. (2002). *Representation and invariance of scientific structures*. Stanford, CA: CSLI Publications.

Thomson, J. J. (1881). On the electric and magnetic effects produced by the motion of electrified bodies. *Philosophical Magazine, 11*, 229–249.

Thomson, J. J. (1897). Cathode rays. *Philosophical Magazine, 44*, 293–316.

Thomson, J. J. (1899). On the masses of the ions in gases at low pressures. *Philosophical Magazine, 48*, 547–567.

Zenker, F. (2009). *Ceteris paribus in conservative belief revision. On the role of minimal change in rational theory development*. Berlin: Peter Lang.

Zenker, F. (forthcoming). From features via frames to spaces: Modeling scientific conceptual change without incommensurability or aprioricity. In T. Gamerschlag, D. Gerland, R. Osswald, & W. Petersen (Eds.), *Proceedings of CTF09*, Duesseldorf, June 2009.