INTEGRATING VISIBLE LIGHT FOR ENHANCED THERMAL IMAGING: PARAMETERIZED SCENE-BASED NON-UNIFORMITY CORRECTION
SUPPLEMENTARY MATERIAL

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1. SOLVABILITY OF LINEAR NON-UNIFORMITY MODEL WITH MULTIPLE IMAGES

Consider the vectors $\mathbf{x}^{(k)}, \mathbf{z}^{(k)} \in \mathbb{R}^{D\times 1}$, which denote the kth clean and noisy infrared images, respectively, with x_i, z_i representing the pixel values at the ith index. We form arrays of pixel values $\mathbf{x}_i = \left[x_i^0, x_i^1, \ldots, x_i^{k-1}\right]$ and $\mathbf{z}_i = \left[z_i^0, z_i^1, \ldots, z_i^{k-1}\right]$, capturing various scene pixel values from both clean and noisy datasets. By applying the linear noise model parameters (α_i, β_i), the relationship between these two pixel stacks can be expressed as detailed in Eq. 1.

$$\mathbf{x}_i = \alpha_i \mathbf{z}_i + \beta_i + \eta \tag{1}$$

Here the η denotes the zero mean un-modeled noise component which is not captured by our linear non uniformity model. Given that the $\mathbf{x}^{(k)}$ is known, the Eq. 1 is a basic form of the linear regression problem and by defining the augmented vector $\bar{\mathbf{z}}_i = [\mathbf{z}_i | \mathbf{1}]$ the solution can be determined using the ordinary least squares solution as given in the Eq 2.

$$\begin{bmatrix}\alpha_i \\ \beta_i\end{bmatrix} = \left(\bar{\mathbf{z}}_i^{\mathsf{T}} \bar{\mathbf{z}}_i\right)^{-1} \bar{\mathbf{z}}_i^{\mathsf{T}} \mathbf{x}_i \tag{2}$$

For this naive approach to work, $\bar{\mathbf{z}}_i^{\mathsf{T}} \bar{\mathbf{z}}_i$ have to be full-rank, meaning that the $\mathbf{z}_i$ must not be a constant vector. Considering the quantization level (ϵ) of the image, the vector is deemed constant if $|z_i^j - z_i^0| \leq \epsilon/2 \quad \text{for all } j \in \{1, \ldots, k-1\}$. Assuming that the pixel values of the noisy image are independent normally distributed random variables with a distribution of $z_i^j \sim \mathbb{N}(\mu, \sigma)$, the difference between two such variables, defined as $z_i^{j,0} := z_i^j - z_i^0$, also follows a normal distribution with parameters $z_i^{j,0} \sim \mathbb{N}(0, \sqrt{2}\sigma)$. Leveraging the independence of pixel distributions, we can compute the probability that $\mathbf{z}_i$ is constant as described in Eq. 3.

$$\begin{aligned} P_k &= \mathbf{P}(|z_i^1 - z_i^0| \leq \frac{\epsilon}{2}) \mathbf{P}(|z_i^2 - z_i^0| \leq \frac{\epsilon}{2}) \ldots \mathbf{P}(|z_i^{k-1} - z_i^0| \leq \frac{\epsilon}{2}) \\ &= \mathbf{P}(|z_i^1 - z_i^0| \leq \frac{\epsilon}{2})^{k-1} \\ &= \mathbf{P}(-\frac{\epsilon}{2} \leq z_i^{1,0} \leq \frac{\epsilon}{2})^{k-1} \end{aligned} \tag{3}$$

Note that, $z_i^{1,0}$ is a normally distributed random variable and the $\mathbf{P}(-\epsilon/2 \leq z_i^{1,0} \leq \epsilon/2)$ can be computed using the standard normal distribution function Φ as follows.

$$P_k = \left(\Phi(\frac{\epsilon}{2\sqrt{2}\sigma}) - \Phi(-\frac{\epsilon}{2\sqrt{2}\sigma})\right)^{k-1} \tag{4}$$

2. ANALYSIS ON CATS DATASET

Since we are using **8-bit infrared** data, the pixel level difference is denoted by $\epsilon = 1/255$, and based on the analysis on the **CATS** dataset we model the distribution of the normalized image data as $z_i^j \sim \mathbb{N}(\mu = 0.3706, \sigma = 0.1858)$. Substituting these parameters into Eq. 4, it is determined that the probability P_k follows a geometric decay with a factor of $P_k = (0.0168)^{k-1}$, indicating that $\bar{\mathbf{z}}_i^{\mathsf{T}} \bar{\mathbf{z}}_i$ is invertible with a $1 - P_k$ probability. Since each image used in this study has $D = 288 \times 288$ pixels in total, the total probability that the all pixels are invertible will be $Q(k) = (1 - P_k)^D$. Using the Taylor series approximation for $e^x \approx 1 + x$ for $x \ll 1$, we can compute the invertibility of all image pixels as $Q(k) \approx e^{-P_k D}$ and for some values of k, the probabilites are given in Table 1.

	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
$Q(k)$	0.000	0.000	0.672	0.993	0.999	0.999

Table 1. Probability that all of the linear equations are solvable for a 288×288 image where each pixel is independently normally distributed according to the $z_i^j \sim \mathbb{N}(\mu = 0.3706, \sigma = 0.1858)$ distribution.