1. 3 days weather conditions needed to predict, $2^3=8$ states for markov chain.

· Rain all last 3 days, · Rain on previous 2 days but not on 3rd.

· Rain only on 2nd day · Rain only on 1st day

· Rain only on 3rd day · Only 2nd day not rain

· Only 1st day not rain · No Rain all 3days

2. $Pr\{\text{Rain Today} \mid \text{Rained Last 3 days}\} = 0.8$

$Pr\{\text{Rain Today} \mid \text{No rain Last 3days}\} = 0.2$

$Pr\{\text{Rain Today} \mid \text{Rained only on 3rd day}\} = 0.6$

Overall matrix.

$$\begin{vmatrix} 0.8 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \\ 0 & 0 & 0.4 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.8 \end{vmatrix}$$

3. A non-autonomous process is not Markov, as the distribution of X_{n+1} cannot be specified solely by specifying the distribution of X_n. On the other hand, Markov can be made by considering the process $Y_n = (n, X_n)$.

4. P_1: It's a finite chain, simple to check all states $0, 1, 2$ communicate, all states recurrent.

P_2: All states 0,1,2,3 communicate, as well as it's a finite chain,

all states recurrent.

P_3: Contains $\{1\}$, $\{0,2\}$, $\{3,4\}$ 3 classes.

Last 2 are recurrent, $\{1\}$ is a transient one.

P_4: Contain 4 classes: $\{0,1\}$, $\{2\}$ $\{3\}$ $\{4\}$. $\{0,1\}$ and $\{2\}$ recurrent,

$\{3\}$, $\{4\}$ transient.

5. π_j is the unique solution to $\pi_j = \sum_{i=0} \pi_i P_{ij}$ and $\sum_{i \geq 0} \pi_i = 1$

Take $\pi_i = 1$, based on double stochastic nature of the matrix,

$$\pi_j = \sum_{i=0}^{M} \pi_i P_{ij} = \sum_{i=0}^{M} P_{ij} = 1.$$

To make sure $\pi_i = 1$ is also a solution to the 2nd, normalize it by dividing $M+1$.

$$\pi_j = 1 / (M+1)$$

7. Solving $\pi = \pi p$

$$\pi_0 = 0.7\pi_0 + 0.2\pi_1 + 0.1\pi_2$$

$$\pi_1 = 0.2\pi_0 + 0.6\pi_1 + 0.2\pi_2$$

$$\pi_2 = 0.1\pi_0 + 0.4\pi_1 + 0.5\pi_2$$

$$\pi_1 + \pi_2 + \pi_3 = 1$$

$\therefore$ $\pi_1 = 0.35$, $\pi_2 = 0.41$, $\pi_3 = 0.24$

$\pi = [0.35, 0.41, 0.24]$, Percentage of employees:

35 %, 41 %, 24 %.