

**MA 321**

**Project #1**

Project 1

## Problem 1

User Guide:

**Bis.M** : This function is used as the bisection method. The bisection method is used to find the roots of a polynomial equation.

The calling Syntax is `bis(a,b,tolb,func)` where `tolb` is the error for the bisection method.

This method returns the found root, `a`, and, `b`.

**Nwt.M**: This function resembles Newton's Method, with the initial guess `p0`. Newton's method is another numerical method used to find the roots of a polynomial equation.

The calling syntax is `nwt(p0,toln,nmax,func,dfunc)` where `toln` is the error for Newton's Method, and `dfunc` is the derivative of `func`.

This method returns the root, `found`, a logical variable indicating whether the root has been found, and the iteration count.

**Bisnwt.m**: This function resembles a mix of Newton's method and the bisection method. We are using both of these methods combined to find the roots of the polynomial equation. This equation is solely based upon algorithm one in the project description.

The calling syntax is `bisnwt(a,b,tolb,toln,nmax,func,dfunc)`. This includes the calling syntax of both the bisection method and Newton's method combined.

This method returns the root, `iter`, and `itern`, where `iter` is the number of iterations used in the while loop. If the while loop fails to achieve an error less than or equal to the `toln`, it outputs "Method failed to achieve error `toln`".

**Tanom.M**: this function solves the Kepler equation and gives the position of the satellite in its orbital plane.

The calling syntax is `tanom(T,e,n,varargin)`. `T` is the period of the orbit in hours, `e` is the eccentricity of the orbit, and `n` is the number of points where we want to localize the satellite on its orbit.

This method returns `orbit`, which is a matrix consisting of `[t, E, v, r, x, y]`.  $t_i = iT/n$ ,  $E_i$  is the solution to Kepler Equation,  $i$  is the true anomaly corresponding to  $t_i$ ,  $r_i$  is the distance between the satellite and the earth at  $t_i$  and  $(x_i, y_i)$  are the cartesian coordinates.

**demo.M**: This method calls `tanom` and displays the orbit in a graph.

The calling syntax

## Problem 2

The argument `toln` in the Matlab function `tanom.m` represents the error that we allow in the approximation of the Kepler equation. If we want the error in the coordinates of the orbit to be smaller than  $10^{-6}$ , which value should we use for `toln`? Justify your answer.

`Toln` would stay the same because Matlab has precision of up to 16 decimal points. `TolN` would be equal to its original value of  $10^{-12}$ .

### Problem 3

– Intuitively, root-finding solvers are meant for  $x \in \mathbb{R}^n$  and thus, the points in the orbit can be found one by one, applying Algorithm 1 and the transformations described in Section 2.1 for each time of the period. A slight change, though, would allow us to work with a vector  $x \in \mathbb{R}^n$  and find all the points of the orbit at once. Discuss about the differences between the two approaches.

It would be a lot easier to debug the first algorithm over the second algorithm. Although algorithm 2 may be faster, it may not be easier to navigate. With algorithm 2 taking multiple errors at once it appears harder to debug than Algorithm 1.

## Problem 4

Describe the tests that you have done to validate your code, what you have discovered, the problems encountered, and their respective solutions. You must include at least the following test  $T = 4$ ,  $e = 0.25$ ,  $n = 100$ , and tolerance  $1e-12$  in "bisnwt.m".

Test 1: During the bisection method I checked if the signs were the same in the  $\text{func}(a)$  and the  $\text{func}(b)$ . If they were the same, it would not meet the requirement for the bisection method. I would have to return.

Test 2: During newtons method, my first if statement was checking if the function value at  $p_0$  was equal to zero. If it was that meant I did not need to go through the method as the root would be found to be zero.

Test 3: In the Bisection/Newton method, I had the function display after each iteration of the while loop if the method failed to achieve error  $j = \text{toln}$ . This enabled me to see the iterations in my command window so I would be able to visualize the loop and predict any potential errors in code.

Test 4: When using the test with  $T=4$ ,  $e=0.25$ ,  $n = 100$  and tolerance  $1e$  to the  $-12$ , One of the main problems I was having was creating the full aspect of the orbit. At first, only about half of the entire orbit was being shown. I knew this was happening in the bottom half of the orbit, around  $\pi$  to  $2\pi$ . In order to fix this issue I had to add an if statement in  $\text{tanom.m}$  to add  $\pi$  to my  $v$  value when  $E$  was between  $\pi$  and  $2\pi$ .