in common. Further, we don't want to say that there is always any old unrelated third event going on exactly in the period of their overlap. Moreover, we are not going to recognize 'the state of e and e' overlapping' as a serious event.

Yet it does not seem to be far fetched to assume that if e and e' overlap, then there is a *part* of e and a *part* of e' that are completely cotemporal. It is, I think, not even far fetched to assume that there is a part of e and a part of e' that are maximally so. Once we realize that we have to put the parts of events in our model anyway, the idea that, for two overlapping events, there will be events that go on exactly at the period of overlap is, I think, no longer objectionable.

Let us put this in a definition:

An *event structure* is a tuple $\langle E, \preccurlyeq, <, \sqsubseteq, \bigcirc\rangle$ (of which only $\preccurlyeq$ and $<$ are primitives) where:

1. E is a non-empty set of events.
2. $\preccurlyeq$ is a partial order, the relation of part-of.
3. $<$ is a strict partial order.
4. $\sqsubseteq := \lambda e_1 \lambda e_2.\ \forall e[e_2 < e \rightarrow e_1 < e] \wedge \forall e[e < e_2 \rightarrow e < e_1]$
5. $e \preccurlyeq e' \rightarrow e \sqsubseteq e'$
6. $\bigcirc := \lambda e \lambda e'.\ \exists e_1[e_1 \sqsubseteq e \wedge e_1 \sqsubseteq e']$
7. Conjunction: if $e \bigcirc e'$ then
   $\exists e_1 \preccurlyeq e\ \exists e_2 \preccurlyeq e' \colon e_1 \sqsubseteq e_2 \wedge e_2 \sqsubseteq e_1 \wedge$
   $\forall e_3 \colon e_3 \sqsubseteq e \wedge e_3 \sqsubseteq e' \rightarrow e_3 \sqsubseteq e_1 \wedge e_3 \sqsubseteq e_2$

Note that we do not have an operation of $\sqcap$ on the event structure. In fact, in this perspective, there isn't a unique event that is the maximal temporal part of e and e' (and there shouldn't be!), because both e_1 and e_2 have that property. We now define the generated period structure as usual:

$$\langle [e]_{\approx}, <, \sqsubseteq, \bigcirc\rangle \text{ under } \approx\ = \lambda e \lambda e'. e \sqsubseteq e' \wedge e' \sqsubseteq e$$

It is easy to see that this is indeed a period structure satisfying monotonicity, convexity, *and conjunction*. Although there isn't a unique event, temporally including every event that is temporally included in both e and e', there is a unique period, $[e]_{\approx} \sqcap [e']_{\approx}$, namely $[e_1]_{\approx}$ (= $[e_2]_{\approx}$) that is temporally included in $[e]_{\approx}$ and in $[e']_{\approx}$ that temporally includes every period that is included in both e and e'.

From here, then, we can use van Benthem's construction.

INTERVALS, EVENTS AND CHANGE

5.1. INTERVAL SEMANTICS

Whereas the logic and model theory of instant tense logic is relatively well developed, well understood and elegant, such cannot be said of period tense logic and event logic.

There seems to be little agreement on what notions to take as basis for the semantics and what form the semantic clauses should take; the intuitions to base the whole on are more subtle and shaky; and deep logical results that might justify one approach over others are largely absent. In sum, interval semantics is a mine field, a field that is by far more complex than instant tense logic, definitely more fuzzy as well, but also, semantically by far more fascinating.

In order to indicate some reasons for why the subject is so much harder, I will briefly go over some of the different directions one could take.

In the first place, as I have mentioned before, one may not believe that a primitive notion 'truth relative to an interval' makes sense at all, whereas 'truth relative to an instant' is a clean and simple notion. This leads to what I have called reductionistic tense logic: 'truth relative to an interval' is a derived notion which can be reduced to 'truth relative to certain instants'.

I have given some reason earlier for scepticism about the viability of this approach, and I won't dwell on it here. Yet it should be mentioned that when we accept a notion of 'truth relative to an interval' as basic in the semantic recursion, this notion is inherently less clear than a notion of 'truth relative to an instant'. Though instants may be abstract entities, once we accept them the number of choices for truth conditions relative to them is relatively small (that is, not bigger than for truth conditions relative to possible worlds, or for that matter, relative to models *per se*).

As Vlach (1981) points out, with intervals there is from the start a fundamental unclarity concerning how to interpret 'truth relative to an interval'. Suppose *Brutus stabbed Caesar* is true relative to an interval.

Does this mean that it is true *at* that interval? *In* that interval? *Throughout* that interval? *For* that interval?

We could force ourselves to make a decision right from the start; however, a more fruitful approach is to leave it open until the semantics for particular expressions force you one way or an other (which they no doubt will). However, you have to be on your guard: sometimes different choices for semantic clauses may be open, going with different interpretations, and not much ground to choose either one. We will not be concerned with this issue here.

A second question concerns the semantics of the logical connectives. When is $\neg\varphi$ true relative to an interval? and $\varphi \vee \psi$? We could keep their semantics classical, but – as we will see – at a price, and there is a strong temptation to use the ordering of subintervals in the semantics of the connectives (for example, we may want to define $\neg\varphi$ to be true at i iff φ is false at every subinterval, so that $\neg\varphi$ means total absence of φ in the interval). In this way, interval semantics becomes very close to the information semantic systems that we have discussed earlier (in fact, the two are closely related, see van Benthem, 1983), and the different options for such semantic clauses abound. Since such subinterval structure is lacking, no such temptation can be found in the case of instant tense logic.

Yet another question is the following.

It is sometimes said that the difficulties with interval semantics come in primarily because the notion 'truth relative to an interval' is taken to play a central role in the semantic recursion. The notion of a proposition as a set of intervals, or – for that matter – a set (or type) of events, brings certain inherent problems with it. There is a different way of setting up interval (or event) semantics (it is argued), that is truly interval (event) semantics, in that it is not reducible to instant semantics, yet is conceptually less problematic.

Here is the argument: clearly (well, let's assume clearly) in natural language we refer to instants and events, and we quantify over them. So we need intervals and events in our ontological setup. However, it is not clear at all that we should take sentences to denote sets of intervals or events. The only evidence that we have is that intervals and events play at least a role similar to individuals (and properties). But that means that it may be more fruitful to simply add sorts for intervals and events to our semantic domains and quantify over them as we quantify over individuals. That just gives us a many sorted logic that is not harder to handle than standard logic.

Such theories have been quite popular ever since Davidson analyzed *John buttered the toast with a knife at midnight* as: $\exists e$[Butter(e, John, The toast) $\wedge$ With a knife(e) $\wedge$ At midnight(e)]. However, this simplicity is an illusion.

In Chapter Three we discussed the relation between Priorian tense logic, with tense operations, with the semantic recursion based on the notion 'truth relative to an instant', and two sorted first order logic with a sort of individuals and a sort of moments of time.

We defined a translation procedure of the first system into the second. Though we discussed advantages that the one system might have over the other, surely the fact that this translation exists means that the complexities of the Priorian system are carried over at their translations in the first order system.

Such a translation procedure obviously works for any parameter that the notion of truth is indexed for, be it instant, interval or event: we have a similar mapping from a logic with 'truth relative to an interval' to a theory with a sort for intervals and variables over intervals, the same for events. And the question whether $S(b, c)$ is true relative to i is not more complex than the question whether $S(i, b, c)$ is true.

Obviously, if we give a sorted first order theory with a domain of intervals or events, and we do not do anything more with those than we do with individuals, we won't come across any serious complexities.

However, the main reason why interval and event semantics (in whatever form) is so hard, has precisely to do with the fact that we *do* want to do more with them than with individuals. Intervals and events are ordered by relations of precedence, part-of and overlap (and maybe more). If part-of and overlap did not play any semantic role, we wouldn't have such problems. But they do.

In general, the relation between the properties of individuals and the properties of their (bodily) parts is quite arbitrary and non-systematic: given the success of the medical sciences, there is not even a *semantic* relation between the truth of: *John is old* and that of *John's heart is old*, but there is a *semantic* relation between *John is old at p* and *John is old at q*, where q is part of p.

If we could reduce truth at p to truth at moments in p, interval semantics would not be more complex than instant tense logic; if we could stick just with arbitrary assignments of truth values to sentences relative to intervals (as we do for instants), it wouldn't be more complex either. The complexities come in because we can't do either. Take atomic sentences. We cannot arbitrarily assign truth values to them relative to intervals, because also for them the values that they take at a certain interval are semantically related to the values that they take at the parts of that interval. The question that makes interval semantics so hard is: how are they related?

For states like *John is old* the answer is still *relatively* unproblematic: for them we can assume downwards closure: if φ is true at p then φ is true at every subinterval of p. For activities like *John is running*, the question already becomes incredibly complex. We would like some form of downward closure, but we immediately hit on the problem of small, irrelevant interruptions: John is running at p seems to be quite compatible with him standing still for a short period in p to catch his breath.

Secondly, problems arise because interval semantics is crucially tied up with the notion of *change*: to say that a sentence is true at an interval typically involves the claim that a certain change takes place over that interval: that something is the case in the beginning, and something else at the end. Where is the first thing the case, where is the second thing the case and what happens in the middle?

Whereas in instant tense logic we can push the problems of vagueness ahead of us and work fruitfully with the idealization of sharp predicates, in interval semantics these problems are too persuasive and too much at the heart of the whole semantics to be fruitfully ignored (they can be ignored, but they strike back directly).

In the case of event logics we can add to this the problems of event identity and the vagueness and intensionality involved there, to make the situation even more complex.

All these factors contribute to make interval semantics and event semantics so much more complicated (and more interesting) than instant tense logic.

Since the literature on tense and event logic is quite wide-ranging and hardly anything is uncontroversial, I will not here try to give an overview of the different approaches, phenomena studied, etc. See for instance: Dowty (1979), Cresswell (1989), Parsons (1989), Hinrichs (1985), van Benthem (1983), Link (1987).

Rather, in this chapter I will limit myself to discussing some aspects of the problem of becoming.

5.2. THE LOGIC OF CHANGE IN INTERVAL SEMANTICS

We will restrict our attention here to states. That is, we are going to assume that all atomic formulas express states and that changes are changes from one state to another. We will do this by imposing the mentioned condition of downward monotonicity on atomic formulas. This is an idealization made for simplicity's sake. See for instance, Dowty (1979) and Taylor (1977) for conditions that predicates in the different verb classes, states, activities, accomplishments and achievements should satisfy.

We will start out with a classical setup, Humberstone's (1979) interval semantics.

Our language L is just propositional logic with an additional sentence operator F, not to be mistaken for the Priorian tense operator F; $F\varphi$ will stand here for: φ fails to be the case.

A model for L is a pair $\langle P, i\rangle$, where P is a partial order (of periods) and i: $ATFORM \times P \to \{0, 1\}$ with the following condition:

downward monotonicity: $i(\varphi, p) = 1 \wedge q \sqsubseteq p \to i(\varphi, q) = 1$ (for $\varphi \in ATFORM$)

The truth definition for the logical connectives is straightforwardly classical:

$[\![\varphi]\!]_{M,p} = 1$ iff $i(\varphi, p) = 1$; ($\varphi \in ATFORM$); 0 otherwise (the same in the other clauses).

$$[\![\neg\varphi]\!]_{M,p} = 1 \text{ iff } [\![\varphi]\!]_{M,p} = 0$$

$$[\![\varphi \wedge \psi]\!]_{M,p} = 1 \text{ iff } [\![\varphi]\!]_{M,p} = 1 \text{ and } [\![\psi]\!]_{M,p} = 1$$

$$[\![\varphi \vee \psi]\!]_{M,p} = 1 \text{ iff } [\![\varphi]\!]_{M,p} = 1 \text{ or } [\![\psi]\!]_{M,p} = 1$$

We'll come to F in a moment.

Since this logic is classical, it does not allow for truth value gaps, i.e. it does not allow for sentences to be underdefined at certain intervals. We will look at that later.

Let us here ask the question: what happens at intervals where sentences are, so to say, overdefined? Take the following case: at interval p John is not married. Between p and q a change takes place and at interval q John is married. Now what is John at interval $p \sqcup q$?

$p \sqcup q$	?
p John is not married	q John is married

In the semantics I have given above, the answer is clear. Since *John is married* is false at p and since *John is married* is atomic, it is false at $p \sqcup q$ (else the downward monotonicity requirement would force it to be true at p).

How natural is this? We certainly wouldn't want to say that John is married at this big interval, but do we want to say that he is not married? Maybe we'd rather want to say: I don't know, it's not appropriate to say either one. This may lead us to adopt a four valued logic, with a value underdefined, to express what is going on between p and q, and a value overdefined, to express what is going on at $p \sqcup q$.

However, we can make the above choice more natural by introducing F:

$$[\![F\varphi]\!]_{M,p} = 1 \text{ iff } \forall q \sqsubseteq p : [\![\varphi]\!]_{M,p} = 0$$

$F\varphi$, φ fails to be the case, says that φ is false at all subintervals of φ. $F\varphi$, unlike $\neg\varphi$, is a monotonic negation operator: if $F\varphi$ holds for p, it holds for all subintervals of p.

We can think of the difference between $\neg\varphi$ and $F\varphi$ as that between external and internal negation. When we talk about the change from John being not married to John being married, we talk about a change from one state to another. Since downward closure characterizes stativity, $\neg$(John is married) is inappropriate as the representation of *John isn't married*, as understood here, because it is not stative. The stronger statement, F(John is married) is appropriate, however. The above situation, then, can be represented as:

$p \sqcup q$	$\neg$(John is married)
p F(John is married)	q John is married

Hence, we interpret $\neg\varphi$ and the assignment of truth value 0 to φ as meaning: the truth value of φ isn't 1 at $p \sqcup q$, in other words: *John isn't married* in this sense means: we can't maintain that *John is married* holds here. $F\varphi$ makes the stronger claim that φ is actually totally absent from this interval: John is unmarried at p.

$\neg$ is our standard classical negation operation. F is in fact intuitionistic negation.

Let us look at Humberstone's deduction system. This time we give the system in the form of a sequent system. A sequent is a statement of the form: $\Gamma \vdash \varphi$ (φ is provable from Γ), where Γ is a finite set of formulas and φ a formula. Sequent $\Gamma \vdash \varphi$ is derivable iff it can be reached from basic sequents, that is, sequents of the form $\psi \vdash \psi$, by a finite sequence of application of the rules. The rules have the form:

$$\frac{s_1 \cdots s_n}{s_m}$$

meaning: from sequents $s_1 \ldots s_n$ you can reach sequent s_m. As can be seen, Humberstone's rules are basically the standard deduction rules for $\neg$ combined with the intuitionistic deduction rules for F:

$$I\wedge \quad \frac{\Gamma \vdash \varphi \quad \Delta \vdash \psi}{\Gamma, \Delta \vdash \varphi \wedge \psi} \qquad E\wedge \quad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \varphi} \quad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \psi}$$

$$I\vee \quad \frac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \vee \psi} \quad \frac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \vee \psi} \qquad E\vee \quad \frac{\Gamma \vdash \varphi \vee \psi \quad \Delta, \varphi \vdash \chi \quad \Sigma, \psi \vdash \chi}{\Gamma, \Delta, \Sigma \vdash \chi}$$

$$I\neg \quad \frac{\Gamma, \varphi \vdash \psi \wedge \neg\psi}{\Gamma \vdash \neg\varphi} \qquad E\neg \quad \frac{\Gamma \vdash \neg\neg\varphi}{\Gamma \vdash \varphi}$$

$$IF \quad \frac{\Gamma, \varphi \vdash \psi \wedge F\psi}{\Gamma \vdash F\psi} \qquad EF \quad \frac{\Gamma \vdash F\varphi}{\Gamma \vdash \neg\varphi}$$

provided that for every $\alpha \in \Gamma$:
every occurrence of $\neg$ in α is in the scope of F in α.

Remark: rule EF can be replaced with the intuitionistic *ex falso*:

$$\frac{\Gamma, \varphi \vdash \varphi \wedge F\varphi}{\Gamma \vdash \psi}$$

Moreover, we can define material and intuitionistic implication as:

$$\varphi \to \psi := \neg(\varphi \wedge \neg\psi)$$
$$\varphi \Rightarrow \psi := F(\varphi \wedge \neg\psi)$$

Humberstone shows that this deduction system is complete with respect to the above interpretation.

In this system, although $\varphi \wedge \neg\varphi$ and $\varphi \vee \neg\varphi$ are a contradiction and a tautology respectively, $\varphi \wedge F\varphi$ is a contradiction ($F\varphi$ entails $\neg\varphi$), but $\varphi \vee F\varphi$ is not a tautology. Similarly, although $\neg\neg\varphi$ entails φ, $FF\varphi$ does not.

We can then extend the models to period structures and add the Priorian tense operators to the language and add tense axioms to the derivation system. In this way we can study minimal tense logic for period structures and its extensions to special kinds of structures. I will not go into that here. For details, see Humberstone and especially van Benthem (1983).

Let us now turn to the topic of this chapter, the analysis of change.

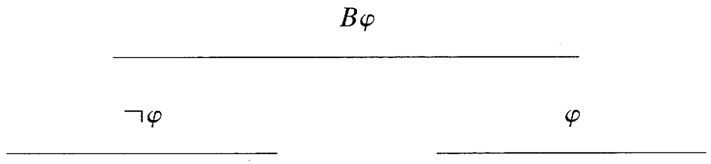
Dowty (1979) gives the above semantics for the connectives (though he doesn't have F). His interval models are point based: they are period sets based on a dense linear order. Furthermore Dowty introduces a sentence operator B : $B\varphi$ means 'φ becomes to be the case'. $B\varphi$ indicates that there is a change from $\neg\varphi$ to φ.

The first semantics that Dowty presents for $B\varphi$ is the following:

> $[\![B\varphi]\!]_{M,p} = 1$ iff there is an interval q overlapping the beginning of p such that $[\![\varphi]\!]_{M,q} = 0$ and there is an interval q' overlapping the end of p such that $[\![\varphi]\!]_{M,q'} = 1$

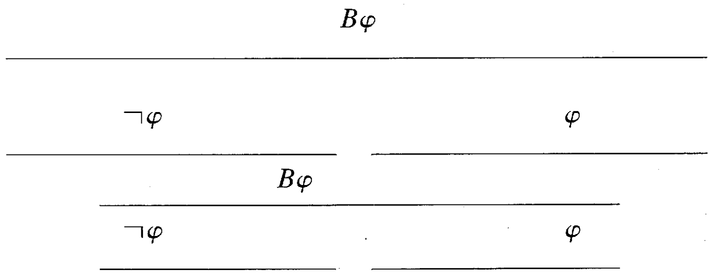
Exercise 1. Dowty formalizes this by using the underlying point structure. Give a formalization purely in terms of period structures.

The intuition is clear: φ becomes to be true at p if at the beginning of p φ is false and at the end of p φ is true:



Dowty points out a problem with this analysis. Let us assume that *The*

door closes is represented as B(*the door is closed*). Suppose that the door is open for six months, then it is closed and it stays closed for six months. Let p be this interval of a year. Then it is true that the door closed at p. Moreover, this will be true at any subinterval, as long as it includes the transition point:



The problem is that it seems inappropriate to say of this one year long interval that it is an interval where the door closed. Of course, it is not inappropriate if we take this to mean that somewhere in this interval the door closed. But that is not the sense that we have in mind here. If the door is opened again directly and stays open for six months, then on the non-intended interpretation we can still call this an interval where the door closed, even though B(*the door is closed*) is false. The sense that Dowty has in mind is the sense where to say that the door closed at an interval means that it took that interval for the door to close, and in this sense it seems indeed inappropriate to say that the door closed at the one year interval.

Or take another example. Let's assume that *John dies* can be represented as B(*John is dead*). Now consider the interval that starts with John's birth and that ends, seventy years later, seven years after John's death. According to the above definition, it is true that John died at that interval, and again this seems weird.

There is a second problem with the above definition. In the above example we dealt with one small change in a large interval. The above definition is also problematic for situations with many changes in a large interval.

Let us look at a traffic light over a five year period. At the beginning of the period it is green. It changes regularly from green to red and back for five years, and at the end of the period it is red. Let us compare this period with the same period plus one minute, in the latter period the light is green again:

$$B\varphi$$

$\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ

$$\neg B\varphi$$

$\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$ φ $\neg\varphi$

The weird conclusion is that in the first period the light becomes red, but in the second it doesn't. The above semantics for B fails the distinguish a macro change, a sequence of changes from a micro change, a single change.

Let me point out that these two problems are independent. It is quite well possible to believe, with Dowty, that the first problem need not be solved by the semantics, but can be related to the pragmatics (i.e. semantically speaking, there is a change from $\neg\varphi$ to φ at this big interval, but pragmatically it is only appropriate to attribute this change to the smallest subinterval of change), while one could maintain that the second problem should be solved in the semantics.

The point is this: one can agree with Dowty that in both cases the statement is only appropriate for intervals that minimally surround a single change from $\neg\varphi$ to φ. Yet one can feel (as I do) that there is a distinction between the two inappropriate situations sketched above: in the first situation one uses a statement that is literally true of the big interval, where one could have narrowed it down more; in the second case however, the statement does not feel literally true at all. More in particular, there doesn't seem to be a difference between the two intervals compared in the second situation (i.e. the green–red vs. the green–green interval). In both cases, if I have to make a decision, I'd call both $B(red)$ in the first case and $B(green)$ in the second case literally false (and moreover inappropriate).

Dowty continues to propose a semantic alternative for the above B-clause, that will make the become-statements in all the above cases semantically false:

$[\![B\varphi]\!]_{M,p} = 1$ iff 1. there is an interval q overlapping the beginning of p such that $[\![\varphi]\!]_{M,q} = 0$ and there is an interval q' overlapping the end of p such that $[\![\varphi]\!]_{M,q'} = 1$

2. there is no proper $p' \sqsubseteq p$ satisfying (1).

Exercise 2. Show that if the period structure is dense, $B\varphi$ is false at every period. In an arbitrary period structure, what are the periods at which $B\varphi$ is true (proof your answer)?

With this condition, in the above situations, $B\varphi$ is indeed false. Take the door: although at the big interval the first clause of $B\varphi$ is satisfied, there are many proper subintervals at which it is satisfied as well, so $B\varphi$ is false.

In the traffic light situation, the same holds: the first clause is satisfied in the green-red case, but any subinterval starting in a green area and ending in a red area satisfies it as well, hence $B\varphi$ is false.

This second condition is rather strong and inelegant, though. In the first place, if it is to be of any use for semantics, it forces you into atomic period structures, a strong claim. Now this pill may be sweetened by the contextual considerations I discussed in Chapter Three: if we don't take an absolute view on atoms, but a contextual (atoms are atoms with respect to a contextually given degree of precision), the above clause may be less unintuitive: it says that changes occur over intervals that consist of two moments (the moment 'before' the change and the moment 'after' the change), where moments are conveniently chosen smallest intervals for the purpose of conversation.

Besides the pragmatic considerations against this revision, Dowty presents yet a different, semantic problem with this revision. This problem concerns simple sentences that express changes that are not simple changes from $\neg\varphi$ to φ, but are changes where two factors change over the same interval, but at different points.

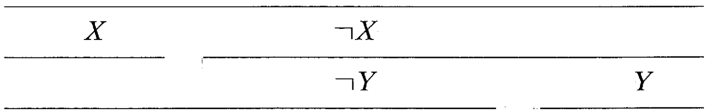
Movement is a case in point. Take the sentence *John walks from X to Y*. This sentence expresses a change in position: at the beginning, John is at X, at the end, John is at Y. But if X and Y are sufficiently apart, then this statement involves crucially *two* changes: a change from X to $\neg X$ and a change from $\neg Y$ to Y.

There are two problems. First, can we express this with the help of B as one complex change? Secondly, as soon as John moves away from

X, the change from X to $\neg X$ has taken place. But if X and Y are apart, the change from $\neg Y$ to Y will only take place *after* that, when John reaches Y. Can we express this with the help of B, either as one complex change, or as a conjunction of two simple changes?

The answer to both questions on the revised definition is no.

The situation is the following:



Concerning the first problem: what would the representation of this complex change be? At the beginning of the interval $X \wedge \neg Y$ holds, at the end of the interval $\neg X \wedge Y$ holds. We need a change from the first situation to the second. Since $\neg X \wedge Y$ is the state to be achieved, the obvious way to express this would be: $B(\neg X \wedge Y)$. However, this is a change from $X \vee \neg Y$ to $\neg X \wedge Y$, and not from $X \wedge \neg Y$ to the latter.

If we cannot follow this lead, i.e. see the sentence as expressing a change of a conjunction, we may want to take the most plausible road and regard it as a conjunction of two changes: $B\neg X \wedge BY$. The problem is that on the revised definition this conjunction can only be true at an interval consisting of two moments, at the first of which both X and $\neg Y$ are true, and at the second both $\neg X$ and Y. Hence, this forces the absurd conclusion that the changes are in fact simultaneous also in the sense that they change at the same point.

Dowty discusses a way out: we can introduce, following Cresswell (1977), a new temporal conjunction, AND, where roughly φ AND ψ is true at an interval iff φ is true at some subinterval and ψ is true at some subinterval (for subtleties of this, see Cresswell, 1977 and van Benthem, 1983). With this new conjunction we can represent the sentence as $B\neg X$ AND BY. I will not go into this way out here.

This problem adds fuel to the pragmatic solution. If we regard the minimality clause as a pragmatic constraint and keep the semantics as in the first clause, there is no problem, we can represent *John walks from X to Y* simply as $B\neg X \wedge BY$.

Semantically this means that over interval i at the beginning X is true, at the end it is false, and at the beginning Y is false, at the end

it is true, but it is unspecified where the X and Y transitions exactly take place (and facts about places will determine that the X transition takes place before the Y transition).

There are some reasons to be unsatisfied with this, though.

In the first place, this keeps us in the situation that I described as unsatisfactory before: in order to get the two changes to hold of the interval, we let the semantics be largely unconstrained: in particular, we do not try to capture the distinction between a sequence of changes and a single change.

Secondly, it doesn't really solve the problem, rather it pushes it from the semantics to the pragmatics. Even on the contextual interpretation of instants, it seems totally unnatural to assume that the second clause of the revised definition can be a pragmatic felicity condition at all.

Suppose I utter the sentence *John walks from X to Y* felicitously. That would mean that, in that context, I am committing myself to the minimality condition, and hence, according to the theory, I am committing myself pragmatically to the simultaneity of the changing points of the X and the Y change. But this is an absurd conclusion: on uttering this sentence there is no such pragmatic committal whatsoever, on the contrary!

Given this, we may want to look for a different semantic analysis. And in fact such an analysis is readily available if we use the idea that I mentioned before, that we are really only interested in changes that are single changes from one state to another.

The characteristics of a state, I said before, is that it is downward monotonic.

We call φ *T-stable at p* iff φ is true at all subintervals of p.

φ is *F-stable at p* iff φ is false at all subintervals of p.

We have seen that $F\varphi$ expresses that φ is F-stable. How would we express that φ is T-stable?

$F\neg\varphi$ is true at p iff $\neg\varphi$ is false at all subintervals of p, iff φ is true at all subintervals of p. Hence, $F\neg\varphi$ expresses that φ is T-stable. If we are interested in changes from one state to another, we hence are interested in a change where φ changes from $F\varphi$ to $F\neg\varphi$, i.e. from being stably false to being stably true. So, for $B\varphi$ to be true at p, we want $F\varphi$ to be true at the beginning of p and $F\neg\varphi$ to be true at the end of p.

However, this is not sufficient to exclude the traffic light situation. To exclude this, we have to require that only one change (concerning

φ) takes place over the interval. Can we do that, without restricting ourselves to intervals consisting of two moments?

Yes, the stability will help us here: we can simply require that the interval of change can be split into a left wing where φ is stably false and a right wing where φ is stably true:

$$[\![B\varphi]\!]_{M,p} = 1 \text{ iff } \exists p_0 \exists p_1 [p = p_0 + p_1 \text{ and } p_0 < p_1 \text{ and } [\![F\varphi]\!]_{M,p_0} = 1 \text{ and } [\![F\neg\varphi]\!]_{M,p_1} = 1]$$

So $B\varphi$ is true at p, if we can partition p into a first part where φ is stably false and a second part where φ is stably true. The operation $+$ is the splitting operation we have defined in Chapter Four.

Note that the stability is crucial in limiting us to single changes. It has the consequence that on the interval containing the macro traffic light changes, $B\varphi$ is false, because this big interval cannot be split into a left wing and a rightwing where *both* at the first φ is *stably* false and at the second φ is *stably* true. We cannot do without, that is, we cannot replace the above clause with, say:

$$[\![B\varphi]\!]_{M,p} = 1 \text{ iff } \exists p_0 \exists p_1 [p = p_0 + p_1 \text{ and } p_0 < p_1 \text{ and } [\![\varphi]\!]_{M,p_0} = 0 \text{ and } [\![\varphi]\!]_{M,p_1} = 1]$$

because this would make $B\varphi$ true at the big interval, because we can split it up in the following ways: (let's assume that φ is atomic and hence φ is equivalent with $F\neg\varphi$)

$B\varphi$

$F\varphi$ φ $F\varphi$ φ $F\varphi$ φ $F\varphi$ φ $F\varphi$ φ $F\varphi$ φ $F\varphi$ φ $F\varphi$ φ $F\varphi$ φ $F\varphi$ φ

$\neg\varphi$ φ

At the leftwing here φ is false, but not stably false. So this partition does count on the second analysis, but not on the first.

So we predict now that $B\varphi$ is false in the traffic light situation. With respect to the door situation we make the same prediction as Dowty (we'll see soon how a more radical change can do away with that one): clearly the one year interval can be partitioned in a left wing where φ is stably false and a right wing where φ is stably true, and so can every subinterval that contains the transition.

However, we have no commitment to instants (i.e. to atomicity): the

only ontological requirement for $B\varphi$ to be applicable to an interval is that it can be splitted in two parts, and this is a minimal requirement that we would need on any analysis.

The advantage of this analysis is that it becomes unproblematic to represent *John walks from X to Y* as $B\neg X \wedge BY$. $B\neg X \wedge BY$ is true at p iff $B\neg X$ is true at p, and BY is true at p, and this is true in the following situation: (note that $F\neg\neg X$ is FX)

$B\neg X \wedge BY$

$F\neg X$ FX

FY $F\neg Y$

5.3. THE MOMENT OF CHANGE

We have not solved Dowty's first problem, the problem that $B\varphi$ is true at a large interval even if φ changes instantaneously in the middle of that interval. As I said, we may want to claim that in this situation $B\varphi$ is literally true, but infelicitous, because of a pragmatic constraint that $B\varphi$ is only appropriately used for intervals consisting of two moments. However, as I argued above, this constraint is highly problematic, and it is not clear that another, non-problematic pragmatic constraint can be found.

I will not continue the search for such a condition here, because I think that the problem is nothing but an innocent looking disguise of a much more serious problem, in fact the fundamental problem of the analysis of change: the problem of the moment of change. I follow Kamp (1979b) here in the formulation of the problem. It can be stated very simply.

Take a traffic light. One moment it is red, then it is green. Clearly a change took place, and for simplicity let us say it was a change from $\neg\varphi$ to φ. When did the change take place?

Clearly not before $\neg\varphi$ has ended and not after φ has started: the change takes place between these two states.

Obviously this is incompatible with classical tense logic, be it instant tense logic, or the classical interval semantics that we have presented here. Classical logic obeys the principle of bivalence:

Bivalence: $\forall p$: either $[\![\varphi]\!]_p = 1$ or $[\![\varphi]\!]_p = 0$

The above statement, on the other hand, expresses a principle of incompatibility:

> *Incompatibility*: at the moment of change neither φ nor $\neg\varphi$ holds.

From these two principles together it obviously follows that there is no time at with the change occurs, i.e. there is no moment of change.

To diagnose why there isn't a moment of change let us step back for a moment to classical instant tense logic. What model of time is encoded in a classical tense logic? We have a sequence of moments of time, where from moment to moment sentences can have different truth values. The picture of time this model is based upon is what I called earlier 'the filmstrip model of time': time is a fixed sequence of frames (states). The present moment, the now, is the projection point, and our sequence runs past the projection point. Just as in a filmstrip, the *illusion* of change comes from distinct states succeeding each other (rapidly enough) in front of the now.

This is essentially a *static* model of time. The closest you can come to defining a change in such a theory is as a *pair* of moments $\langle t, t'\rangle$ where $[\![\varphi]\!]_t = 0$ and $[\![\varphi]\!]_{t'} = 1$. That is, you can characterize the change by its beginning and end. The change itself, however, is in this way eliminated from the theory.

Change is, of course, a dynamic notion. In a static theory of time, like classical tense logic, we can only give a static reconstruction of change, in terms of the states that hold before it and the states that hold after it.

Now it is well known that in all of mathematics and the sciences progress has been made by replacing vague dynamic-sounding notions by mathematically precise static reconstructions: just think of the set theoretic reconstruction of the function concept. Though inevitably *something* is lost in such a reconstruction, most of the time we can convince ourselves that what is lost is nothing more than a dynamic illusion that we do better without anyway.

However, leaving the other sciences aside, in semantics this reconstruction brings us into problems where in fact we lose more than our illusions.

Here is a direct consequence of the above theory of change: it is impossible to give a present tense report of a change: we cannot say: a change from $\neg\varphi$ to φ occurs at this very moment. We can give a *past* tense report of a change: we can analyze *the light changed from red to green* as: it is true at the present moment t_0 that there is a pair of states $\langle t, t'\rangle$ before t_0 with the required conditions. But to say that a change occurs now, we have to set *now* to either t or t', and then by the principle of incompatibility, our present tense report is false.

Although the classical interval tense logic is less perspicuously built on the 'filmstrip' model, the situation is nevertheless the same. As for the instant tense logic, the change in the traffic light is represented as a succession of two states, the only difference being that the states are intervals. Also here, the dynamic moment, the moment or interval of change, is not part of the model itself, but is reconstructed statically as a pair of successive states (intervals).

The situation for the present tense report of the change is the same. Let's follow Dowty in assuming that *now* is an *instant*. Let's look at the sentence: *it is now becominq dark*. Let p be the interval where first $\neg$*Dark* is stably true and then *Dark* is stably true. *Now* is an instant in this interval. The sentence is true (roughly) because there is an appropriate interval surrounding *now* (namely p) where B(*Dark*) is true.

However, depending on where *now* is in p, it is stably true at *now* that it is dark, or stably false at *now* that it is dark, again contradicting the intuition of incompatibility.

If we assume that *now* is an interval, we could try the following way out. If *now* is an interval surrounding the non-existent point of change, then $\neg$*Dark* is true at *now*, but F(*Dark*) is not.

If we could insure that the progressive of B(*Dark*) is only true at *now* if *now* surrounds the point of change, then we can use the same argumentation as we have used before: the fact that the logic is bivalid, means that *Dark* $\vee$ $\neg$*Dark* is true at *now*. The principle of incompatibility, however, is properly expressed with the strong negation F, and indeed $\neg$(*Dark* $\vee$ F(*Dark*)) is true at *now*.

Though we will keep this solution in mind, because it will come back later, it does not seem very iikely that we can insure that the *now* surrounds the change.

The problem is that we would have to build this into the semantics of the progressive. But we want the semantics of the progressive to be general. In general for sentences like, say, *John ran* the following holds: if *John ran* is true of interval p, then for those subintervals of p where *John was running* is true, the requirement on where they are located in p is just: before John stopped running. There is no evidence that in the clause for the progressive there should be a requirement that the interval should surround a certain point. Building in such a requirement for the progressive of become sentences is hence a highly ad hoc move that should be avoided.

However, without it, we get our problems back. Take an interval p of which *it became dark* holds. Look at the subintervals where *it was becoming dark* holds. If the locational requirement that the progressive imposes is just: those intervals are located before the end of the interval p, then nothing prevents the possibility that both *It is becoming dark* and $F(Dark)$ (or $F\neg(Dark)$) hold at such an interval, and nothing prevents us from identifying *now* with such an interval. This then violates incompatibility also in the revised version.

It seems that we would save ourselves a lot of trouble by making the step that has been in the air now for a while: move to a three valued logic. Let us make that step now.

We will be concerned now with a language L which is propositional logic plus five sentence operations: A, P, F, U and B.

A *model* for L is a pair $\langle P, i\rangle$, where P is a period structure and i, the three valued interpretation function, is a function from $ATFORM \times P$ into $\{0, 1, *\}$ with the following two monotonicity constraints: (where $\varphi \in ATFORM$)

$$i(\varphi, p) = 1 \wedge q \sqsubseteq p \rightarrow i(\varphi, q) = 1$$

$$i(\varphi, p) = 0 \wedge q \sqsubseteq p \rightarrow i(\varphi, q) = 0$$

So we have monotonicity for 0 and 1. Note that we could not have the same for $*$. If we would do that then we cannot assign any value to an atomic formula φ at an interval that has subintervals where φ takes different values. This, however, conflicts with the requirement that i is a total function (into $\{0, 1, *\}$).

The semantic clause for atomic formulas is:

$$[\![\varphi]\!]_{M,p} = i(\varphi, p)$$

The connectives get their values according to the strong Kleene truth tables:

$\neg$	
1	0
0	1
$*$	$*$

$\wedge$	1	0	$*$
1	1	0	$*$
0	0	0	0
$*$	$*$	0	$*$

$\vee$	1	0	$*$
1	1	1	1
0	1	0	$*$
$*$	1	$*$	$*$

Note that $\neg$ does not express the weak negation we have discussed before. The connectives are monotonic truth functions, and weak negation is not monotonic. We can introduce weak negation with help of the new (non-monotonic) operator A (for 'assertable'):

$$[\![A\varphi]\!]_{M,p} = 1 \text{ iff } [\![\varphi]\!]_{M,p} = 1;\ 0 \text{ otherwise.}$$

A, hence, is a bivalid connective (it doesn't take value $*$). With the help of A weak negation can be expressed as: $\neg A\varphi$.

Exercise 3. Some feel for this system can be gotten from filling in the values in the following table:

φ	$\neg\varphi$	$A\varphi$	$\neg A\varphi$	$\varphi \wedge \neg\varphi$	$\varphi \vee \neg\varphi$	$\varphi \wedge \neg A\varphi$	$\varphi \vee \neg A\varphi$	$A\varphi \wedge \neg A\varphi$	$A\varphi \vee \neg A\varphi$
1									
0									
$*$									

The operators P (presently), F (fails) and U (undefined) are stability operators for the values 1, 0 and $*$ respectively: they are bivalid as well.

$$[\![P\varphi]\!]_{M,p} = 1 \text{ iff } \forall q \sqsubseteq p\colon [\![\varphi]\!]_{M,q} = 1;\ 0 \text{ otherwise}$$
$$[\![F\varphi]\!]_{M,p} = 1 \text{ iff } \forall q \sqsubseteq p\colon [\![\varphi]\!]_{M,q} = 0;\ 0 \text{ otherwise}$$
$$[\![U\varphi]\!]_{M,p} = 1 \text{ iff } \forall q \sqsubseteq p\colon [\![\varphi]\!]_{M,q} = *\colon 0 \text{ otherwise}$$

Some notation: let us define:

$$p = p_0 + p_1 + p_2 :=$$
$$p = p_0 + (p_1 \sqcup p_2) \text{ and } (p_1 \sqcup p_2) = p_1 + p_2$$

This means that p can be split into three parts: a left wing, a right wing and a middle: these three parts together make up p.

The direct adaptation of the earlier clause for the truth of $B\varphi$ at p would require p to be split into three parts: a left wing where φ is stably false, a middle, where φ is stably undefined, and a right wing where φ is stably true. However, such a clause would be quite pointless, because it would suffer from exactly the same problems as the previous one. However, we have a different option here:

$$[\![B\varphi]\!]_{M,p} = 1 \text{ iff } \exists q \exists p_0 \exists p_1 [q = p_0 + p + p_1 \ \& \ p_0 < p < p_1 \ \&$$
$$[\![F\varphi]\!]_{M,p_0} = 1 \ \& \ [\![U\varphi]\!]_{M,p} = 1 \ \& \ [\![P\varphi]\!]_{M,p_1} = 1]$$

Let us say that a change from $\neg\varphi$ to φ is an interval p that can be split in a beginning where φ is stably false, a middle where φ is stably undefined and an end where φ is stably true. The above clause then says that $B\varphi$ is true at exactly those intervals that form the middle part of a change from $\neg\varphi$ to φ.

This clause successfully deals with most of the problems mentioned.

The solution to the traffic light problems is exactly as before: the stability requirements exclude the possibility that the interval where $B\varphi$ is true is in fact a sequence of changes of φ. Since the interval of change is the interval in the change where φ is stably undefined, neither one of φ, $\neg\varphi$, $F\varphi$, $P\varphi$ are true at it (only $\neg A\varphi$ is).

Concerning the door problem: this is no longer a problem. We do no longer include the F-stable and the P-stable part in the interval where $B\varphi$ holds: $B\varphi$ is true at an interval that is a U-stable interval and that is flanked by an F-stable interval to the left and a P-stable interval to the right. So, although arbitrarily large intervals to the left and the right make $B\varphi$ true at the gap, it is not true at the unions of those intervals, but only at the gap.

Present tense reports of become statements are no problem either. *It is becoming dark now* is true iff *now* is part of an interval where $B(Dark)$ is true, this means that *now* is part of the U-stable part of a change from $\neg Dark$ to $Dark$. This, in turn means that also at *now* neither φ, $\neg\varphi$, $P\varphi$ and $F\varphi$ are true.

We have regained another problem, however. On the above definition, we can no longer deal with the complex changes involved in movement. This can be seen as follows. Let us again represent *John walks from X to Y* as $B\neg X \wedge BY$. Suppose this is true at p. Then both $B\neg X$ and BY are true at p. This means that p is the gap of a change from X to $\neg X$ and p is the gap of a change from $\neg Y$ to Y. In a picture:

$$\begin{array}{ccc} F\neg X & U\neg X & P\neg X \\ \hline FY & UY & PY \\ \hline \end{array}$$

This means, hence, that the change from X to $\neg X$ does not take place before the change from $\neg Y$ to Y.

Now, again we can try to modify the clause to deal with this problem, but again I think it is more instructive to rather discuss some general problems with the present approach of which, I think, the particular problem discussed here is an instance.

These general problems are the well known problems with three valued logic.

The theory that we have presented analyzes change in terms of *vagueness*: an interval of change is a stage where φ does not have a truth value between opposite stages where it does. That vagueness is involved is of course most clear in the case of gradual change: a solution is slowly changing from red to blue. First it is clearly red, it becomes a borderline case of a red solution, it becomes a borderline case of a blue solution, and it becomes clearly blue.

However, as we know from the discussion of vagueness in chapter three, the three valued approach cannot be the whole story, because just going to three valued models cannot account for the semantic, logical connections between sentences that enter in the determination of the semantic values of expressions that take those sentences as parts. This shows up most clearly in the contradictions and tautologies. In the three valued approach, as can be seen from the tables above, $\varphi \wedge \neg\varphi$ is not a contradiction, neither is $\varphi \vee \neg\varphi$ a tautology. As we can see from the table in the exercise given earlier, going to weak negation is not much of a help. Strangely enough, with weak negation the law of excluded middle becomes valid: $\varphi \vee \neg A\varphi$ is a tautology, but the law of non-contradiction still is not valid: $\varphi \wedge \neg A\varphi$ is not a contradiction.

Of course, $A\varphi \wedge \neg A\varphi$ is a contradiction, but though we may have some motivation to assume that negation can be read as weak negation, and that the law of non-contradiction should involve that, no such intuitive motivation seems to be forthcoming for thinking that φ is ambiguous between φ and $A\varphi$. Trying to push this line seems to be a mere technical way out.

All the problems that we have seen earlier for a simple three valued theory of vagueness can easily be constructed for a simple three valued theory of change as well.

It seems thus that the next step to be made is to impose upon our three valued semantics the supervaluation approach to vagueness.

This is our next move.

5.4. SUPERVALUATIONS

On the supervaluation approach our predicates are vague, but potentially sharp. The semantics for vague predicates is defined in terms of the different possible ways in which the vagueness can be removed: P is vague if there are still different ways in which it can be made sharp. The situation here is a bit more complicated than in the simple case of vague predicates, though.

We are interested here in different ways in which the vague interval in a change can be made precise, in which the moment of change can be determined precisely. We are not just interested in replacing undefined values at intervals with defined values (0 and 1), but they have to be replaced in a particular way.

Suppose we have a change from $\neg\varphi$ to φ: an $F\varphi$ interval, a $U\varphi$ interval and a $P\varphi$ interval. Making this change precise means filling in the $U\varphi$ part with a precise change: a truth-value-gapless jump from an $F\varphi$ interval to a $P\varphi$ interval. The different ways of removing the vagueness are the different ways in which that can be done. But this is not an arbitrary assignment of 0 and 1 values.

Let M be a three valued model for our language L.

Let L_0 be the sublanguage of L built from atomic formulas with just $\neg$, $\wedge$, $\vee$ and F. These will be the formulas in whose changes we are interested.

A *completion* of M is a bivalid model C for L_0 (that is, a model in the classical sense, including the classical monotonicity requirement for 1), with the same frame as M, that assigns the same 0, 1 values to the atomic sentences as M does, and that satisfies the following condition: for every interval p, for every formula $\varphi \in L_0$:

if $[\![B\varphi]\!]_{M,p} = 1$ then:
either 1. $[\![F\varphi]\!]_{C,p} = 1$
or 2. $[\![F\neg\varphi]\!]_{C,p} = 1$
or 3. $\exists p_0 \exists p_1 : p = p_0 + p_1$ and $p_0 < p_1$ and $[\![F\varphi]\!]_{C,p_0} = 1$ and $[\![F\neg\varphi]\!]_{C,p_1} = 1$

So when we have an interval in a change of φ where φ is U-stable, we either make φ F-stable at p (we extend the F-part all over p, or we make φ T-stable at p (we extend the T-part all over p), or we split p into a left wing where we make φ F-stable and a right wing where we make φ T-stable.

So completions are not just classical models, but models in which the changes have been replaced by pairs of stable states: an F-state and a T-state.

Now we can define:

φ is *supertrue in M at p* iff φ is true at p in all completions of M

φ is *superfalse in M at p* iff φ is false at p in all completions of M

φ is *supertrue in M iff* φ is supertrue in M at all intervals. The same for superfalse.

Let us consider again the change from $\neg$*Dark* into *Dark*. The situation in M is:

FDark	*UDark*	*PDark*

In the normal situation there will be different ways of filling in the gap, locating the moment of change at different places. In some models *FDark* will hold throughout, in others $F\neg$ *Dark* will hold throughout, in others *FDark* will hold at some first part and $F\neg$ *Dark* at the rest. Consequently *Dark* will neither be supertrue, nor superfalse at p. Yet *Dark* $\wedge\ \neg$*Dark* will be superfalse and *Dark* $\vee\ \neg$*Dark* supertrue.

Let us now be concerned with changes that take place at the same interval. Suppose that both φ and ψ change at p. The situation in M is:

$F\varphi$	$U\varphi$	$P\varphi$
$F\psi$	$U\psi$	$P\psi$

How can we express that the φ change took place before the ψ change?

In the following way. We can introduce a new sentence connective

$<$, which takes two sentences of the form $B\varphi$ and $B\psi$ to give a sentence. Intuitively $B\varphi < B\psi$ means: φ becomes the case before ψ becomes the case. We then can give this the following semantics:

Assume that $B\varphi$ and $B\psi$ are true at p in M.

$$[\![B\varphi < B\psi]\!]_{M,p} = 1 \text{ iff for every completion } C \text{ of } M\text{:}$$
$$\exists p_0 \sqsubseteq p\, \exists p_1 \sqsubseteq p\, [p_0 \bigcirc p_1 \text{ and } [\![F\neg\varphi]\!]_{C,p_0} = 1 \wedge [\![F\psi]\!]_{C,p_1} = 1]$$

This means that the gap can be filled in any of the following ways:

$F\varphi\ F\psi$	$U\varphi\ U\psi$	$P\varphi\ P\psi$
$F\varphi$	$F\neg\varphi$	$F\neg\varphi$
$F\psi$	$F\psi$	$F\neg\psi$

or

$F\varphi\ F\psi$	$U\varphi\ U\psi$	$P\varphi\ P\psi$
$F\varphi$	$F\neg\varphi$	$F\neg\varphi$
$F\psi$	$F\psi$ $\quad$ $F\neg\psi$	$F\neg\psi$

or

$F\varphi\ F\psi$	$U\varphi\ U\psi$	$P\varphi\ P\psi$
$F\varphi$	$F\varphi$ $\quad$ $F\neg\varphi$	$F\neg\varphi$
$F\psi$	$F\psi$ $\quad$ $F\neg\psi$	$F\neg\psi$

We are now able to express, of two changes that occur simultaneously, that the one takes place before the other: this holds if every way we can fill in the gap the transition of the first is before the transition of the second. This is of course very useful.

However, we should ask whether we have really solved the problem of the two changes that we started out with. And to be fair, we haven't (though it could be argued that we have solved a more interesting problem).

The problem is that in the case of *John walks from X to Y* it is not just obvious that the transition from X to $\neg X$ takes place before the transition from $\neg Y$ to Y. But in fact in this situation it is completely

obvious that, if X and Y are far enough apart, the transition from $\neg Y$ to Y in fact takes place *after* $\neg X$ has become true:

PX	UX	FX	
FY		UY	PY

Should we give up the analysis for *become* for this problem after we have gone through these revisions? By doing so we will necessarily get some of the other problems back.

It seems after all more plausible that the representation $B\neg X \wedge BY$ is too simple a representation for *John walks from X to Y*. One way out is to follow Dowty in assuming a representation with Cresswell's AND: $B\neg X$ AND BY.

Another suggestion may be the following (this is inspired somewhat by Hinrichs, 1985). X and Y are places that are connected by a path. Also in the case of *The solution changes from red to blue* we can think of the colours red and blue as ordered on some scale and connected by a path through this scale. Let $\langle X, Y\rangle$ stand for the path from X to Y. Then we can interpret *John walks from X to Y* as: *John covers* $\langle X, Y\rangle$ (*by walking*), and this in turn we can represent as B(John covers $\langle X, Y\rangle$), which is correctly interpreted as a change from the situation where $\langle X, Y\rangle$ is clearly not covered (where John is in X) to the situation where the path is clearly covered (where John is in Y), say:

PX	UX	FX	
FY		UY	PY
$F\langle X, Y\rangle$	$U\langle X, Y\rangle$	$P\langle X, Y\rangle$	

Now that we have come a reasonably far way in solving our semantic problems it is time to turn a conceptual question.

We presented the problems of change earlier as arising out of the attempt to capture a dynamic phenomenon in a static, reconstructionist theory. How dynamic is the solution we have come up with?

5.5. KAMP'S LOGIC OF CHANGE

The basic problem of the theory of change is to do justice to the principle of incompatibility: at the moment of change from $\neg\varphi$ to φ, neither φ nor $\neg\varphi$ hold, and to combine this with our views on classical principles like the principle of bivalence.

At first sight, it may seem that in the theory we have come up with now, we are succesful in this. The principles are made compatible by drawing a distinction between weak and strong negation, where incompatibility is formulated in terms of strong negation and bivalence in terms of weak negation. This much will be an ingredient of any theory that tries to combine those two principles, because without it there just is no way of escaping the contradiction.

Now as far as incompatibility itself is concerned: we have divided changes into F-stable, U-stable and P-stable areas: the U-stable area we interpret as the area where the change takes place, and indeed at this interval $\neg F\varphi$ and $\neg P\varphi$ are both true. So we seem to have been successful.

However, the case of two changes occurring simultaneously is instructive here. Let's analyze the situation again. What are we now saying about change?

What we are saying is in fact that the theory of change is exactly the same static theory that we had before, except that the *illusion* of change comes in through *vagueness*. This can be seen as follows.

We have two changes c and c' both going on right now, say solution A changing from colorless to blue and solution B changing from red to green. Now we want to express that, though both are going on right now, the first change *precedes* the second, in the sense that they have exactly the same initial and end state, but c goes more rapidly than c' in the sense that the turning point for c precedes the turning point for $c' : c$ is colorless at the same time as c' is red, c is blue at the same time as c' is green, but c is becoming blue before c' is becoming green.

As explained earlier, the way we express this in the supervaluation theory, is: c precedes c' iff for every way of resolving the vagueness (i.e. in every completion) the moment of change of c precedes the moment of change of c'.

Of course, in these completions, there are no moments of change, there are at most pairs $\langle p, q\rangle$ and $\langle p', q'\rangle$ of F-stable and T-stable intervals, and these are our old familiar stative pairs.

The problem, then, is this: we accept the principle of incompatibility, but attribute its source to vagueness. The theory does not really accept that there are any changes and that there is really an area of change where incompatibility holds.

What it accepts is that there is an area where we cannot *decide* whether to call φ there true or false. This area then cannot be regarded as the area where φ in fact changes, and this is brought out by the fact that we compare our vague model with the completions where the areas of change are narrowed down, and in the end eliminated: if in our original model the U-stable area would really formalize the area where the change takes place, then it would be incoherent to look at completions, where this area is filled in with states that are not changes.

So the U-stable area is not the area of change, but the area of which we are not able to determine what holds at it. And, in fact, there is no area of change in the theory: we can narrow down the vagueness area up to the completions, where it has disappeared, and we see that there the incompatibility disappears as well.

In other words, we haven't really done justice to the principle of incompatibility and with it to the dynamic nature of change at all!

The principle that we have formulated does not express anything about change whatsoever, but rather it expresses a widely accepted principle about vagueness, and is, in as far as the supervaluation theory is an epistemic theory about our perception of vagueness, and the different possible ways in which we can think of it being removed, an uncontroversial epistemic principle: in the truth value gap of a vague sentence, we are not able to positively verify φ, nor to positively refute φ, without this meaning that on sharpening only one of φ and $\neg\varphi$ is true.

We see then that the theory does not allow changes at all: the illusion of change only comes in because our conceptual apparatus and the measurements that we can develop are not precise enough to eliminate intervals that appear to us as intervals of change.

At this moment the issue becomes a conceptual issue. Do we really want to eliminate change in this way? Do we really want to claim that the change we observe when a solution changes its color is due solely to our incapacity to look more precisely at what is going on?

Classical model theory has trained us to reconstruct dynamic notions like change in terms of static (=classical) terms. This goes so far that for several of them we have learned to think in terms of these reconstructions, rather than in terms of the original dynamic notions. In such static reconstructions, the essential dynamic moment is lost. This is most clear in instantaneous change: we characterize the change in terms of what holds before and after and the change itself drops out of the theory.

However, we are using the theory to characterize the meaning of statements like *The traffic light jumped from red to green*. In this we are less than physicists dealing directly with what goes on in the world, but rather with how human beings conceptualize what is going on in the world. Clearly, language users treat changes as real things, as more than their static reconstructions: the meaning of the above traffic light sentence so to say slams you in the face with its dynamic moment.

Although it is not the place here to argue for it, it may very well be that, in the end, a static theory of time will not be able to capture enough of the meaning of temporal statements. This should be sufficient motivation to at least be interested in finding out what the prospects are for an alternative theory.

Kamp's aim in Kamp (1979b) is to resist the temptation of classical reconstruction and see what the prospects are for a truly dynamic theory: a theory where changes exist in their own right and are not reconstructed in terms of succeeding stages.

In fact, he goes a step further. As we have seen, in instant tense logic, we take the most abstract notion as primitive (instant structures) and try to define the conceptually prior, finitary notions of period and event in terms of that. Kamp takes the opposite view: it is the finitary notions of event and change that are basic.

We have seen before how periods and instants can be constructed out of event structures. In Kamp's paper the task of constructing these abstract notions out of dynamic primitives is formulated as follows: it is changes that are constitutive of time: our notion of time is an abstraction of temporal ordering out of the changes that our realm of experience consists of.

So we will take changes as primitive. Changes are, of course, events. Consequently, we will start out with an event structure. We will use here the Kamp structures that were discussed in the last chapter, that is, structures $\langle E, <, \bigcirc \rangle$, based on a strict partial order of temporal precedence and a reflexive, symmetric relation of overlap (with inclusion defined) and further satisfying principles (1) and (2) and possibly (3):

(1) $e < e' \rightarrow \neg(e \bigcirc e')$ (separation)

(2) $e_1 < e_2 \wedge e_2 \bigcirc e_3 \wedge e_3 < e_4 \rightarrow e_1 < e_4$ (transfer)

(3) $e < e' \vee e \bigcirc e' \vee e' < e$ (linearity)

As we have seen, we know we can build a period structure out of this, by taking equivalence classes under the rèlation of mutual inclusion, and by taking filters to be maximal sets of pairwise overlapping elements, we can get an instant structure. The instant structure is linear iff the event structure is. We will call $T(E)$ the instant structure of E. We will not assume our basic event model to be linear, but will discuss linearity later.

The first problem that needs to be discussed is the status of events. As both the linguistic and the philosophical literature show clearly, the concept of an event is not absolute. What is an event? When are two events identical? What parts does an event have? etc. All this depends on what singles it out as an event, a conceptual scheme. How a conceptual scheme singles out events is a mysterious matter, but for simplicity we can assume that it does this by assigning descriptive content to events.

For this, we add to the event structures two functions $A1$ and $A2$, that will assign descriptive content to the events. Our new event structures thus are tuples of the form $\langle E, <, \bigcirc, A1, A2 \rangle$.

$A1$ conveys for every event *e what sort of event e* is (this comes close to the telling of what type events are in situation semantics). The descriptive content of an event is given in terms of an *atomic formula* of the language (for simplicity) and one of the four symbols P (presently), F (fails), B (becomes) C (ceases):

> $A1$ assigns to every event $e \in E$ an atomic formula of the language and a value P, F, B or C.

Examples:

> $A1(e) = \langle Walk(a), P \rangle$ means: e is an event of a's walking being presently the case.
>
> $A1(e) = \langle Walk(a), F \rangle$ means: e is an event of a's walking failing to be the case.

The changes are:

> $A1(e) = \langle Walk(a), B \rangle$, meaning: e is an event of a's walking becoming to be the case.
>
> $A1(e) = \langle Walk(a), C \rangle$, meaning: e is an event of a's walking ceasing to be the case.

$A2$ conveys what was the case when a given event occurred (this is close to structural constraints in situation semantics).

For every event $e \in E$:

$A2(e)$ maps atomic formulas onto the values P, F, B, C, where:

$A2(e)(\varphi) = P$ means: φ is true throughout e

$A2(e)(\varphi) = F$ means: φ is false throughout e

$A2(e)(\varphi) = B$ means: e is a change from $\neg\varphi$ to φ

$A2(e)(\varphi) = C$ means: e is a change from φ to $\neg\varphi$

To this we add the obvious consistency requirement between $A1$ and $A2$:

If $A1(e) = \langle\varphi, \alpha\rangle$ (α one of P, F, B, C) then $A2(e)(\varphi) = \alpha$

Kamp also adds the following consistency requirement on A2 values for overlapping events:

If $e \bigcirc e'$ and $A2(e)(\varphi)$ and $A2(e')(\varphi)$ are defined then $A2(e)(\varphi) = A2(e')(\varphi)$

$A2$, thus, must be regarded as a partial function and φ will be undefined at large events overlapping with events where φ takes different values (more on this later).

An *event model* is a tuple $\langle E, <, \bigcirc, A1, A2\rangle$ satisfying the above conditions.

Now let us come back to linearity.

We will now have instants as abstractions out of events. If we are only concerned with actual events and leave modality aside for the moment, there is a strong inclination to think of time, instant time, as linearly ordered. Under these restrictions we really would want to get a linear order of instants out of the construction.

However, as we have seen, such a linear order is only obtained if the event structure itself obeys linearity. This is problematic, precisely for the phenomena that we are concerned with here.

Take again the two simultaneous changes.

The event model represents the way our conceptual apparatus, our conceptual scheme carves up the world in stative events and events of change. Given the nature of this level, we should not want to assign more structure to this level than our conceptual apparatus gives us.

This means that if at a certain moment, our conceptual apparatus is not capable of precisely determining the temporal relations between these two events of change (whether the one is before the other, or whether they are really simultaneous) – a situation that is quite normal – this indeterminacy should be reflected at the level of event models, because that is precisely the level that realizes just the conceptual distinctions we are able to make. So this level, unlike abstract, idealized levels of instants, cannot and should not make more distinctions.

However, this means that the basic event model cannot be linear, because linearity precisely tells us whether those two changes are simultaneous or, if not, which one comes first.

This is, where in the present theory vagueness comes in once again. Our conceptual apparatus leaves room for vagueness in the temporal relations, hence the event model should reflect this vagueness.

Such vagueness of the temporal relations is removed in the linear structures. This means, then, that linear structures come in as completions: linear event models, that respect the structure of our starting event model M, but that add events to satisfy linearity (and satisfy maybe some more constraints, see later), are the completions of our event model, the idealized event model extensions in which the vagueness is removed.

We will have to accept then, that linearity only comes in in the idealized limit case, where vagueness is removed, and hence that the linear instant structures we want to end up with are the instant structures corresponding to the completions of M, not to M itself.

Suppose we have an event model $M = \langle E, <, \bigcirc, A1, A2\rangle$. We remove all vagueness, and get a linear structure $M' = \langle E', <', \bigcirc', A1', A2'\rangle$. Since in the instant stucture $T(M')$, the instant structure corresponding to the completion, all vagueness is resolved, we are assuming that there there are no longer truth value gaps stemming from vagueness. This means that the only indeterminacy of the truth value of φ at an *instant* can come in from the fact that φ is a change at i.

So in setting up an interpretation of truth at an instant in an instant model corresponding to a completion, we have to add a condition that insures this:

If for no $e \in t$: $A2(e)(\varphi) = B$ or $A2(e)(\varphi) = C$ then

$[\![\varphi]\!]_{T(M'),t} = 1$ or $[\![\varphi]\!]_{T(M'),t} = 0$

I.e. if no change in φ goes on at t, φ has a definite truth value.

Now, we might want to go on and use this condition, then, to define truth and falsity in that instant structure:

$$[\![\varphi]\!]_{T(M'),t} = 1 \text{ iff } \exists e \in t\text{: } A2(e)(\varphi) = P$$
$$[\![\varphi]\!]_{T(M'),t} = 0 \text{ iff } \exists e \in t\text{: } A2(e)(\varphi) = F$$

But this would be problematic, because on this definition, either the truth definition is bivalid, and there are no changes in the corresponding event model, or there are changes and the truth definition is not bivalid, which is a problem, because we are talking about the model where the vagueness is removed. So we have to come up with a different truth definition.

Let us, for later use, catalogue what can be the case for φ at a certain instant. Here the consistency requirement on overlapping events is important. Since instants are sets of overlapping events, we know:

for every t for every e, $e' \in t$: if $A2(e')(\varphi)$ and $A2(e')(\varphi)$ are defined then $A2(e)(\varphi) = A2(e')(\varphi)$

This means that we have the following possibilities: either t consists solely of events for which φ is undefined, then by the above condition $[\![\varphi]\!]_t = 1$ or $[\![\varphi]\!]_t = 0$; or t consists of events for which φ is undefined together with events where φ takes one of the values P, F, B, C. Important is that no instant contains events that are defined on φ and assign it a different value (like one event P and the other event B).

Although Kamp doesn't do it, we may want to add a condition on the completions, excluding the first possibility, i.e.:

for every t: $\exists e \in t : A2(e)(\varphi) = B, P, C$ or F

The rationale behind this condition has to do with the semantics to be given later.

Now we have to face the relation between the principle of bivalence and the principle of incompatibility. As I mentioned above, if there are times of change, that is, if there are instants t such that for some $e \in t$: $A2(e)(\varphi) = B$ or $A2(e)(\varphi) = C$, then the above semantic definition is not bivalid. If it is bivalid, again, there are no times of change.

As said before, the proper way to deal with this problem seems to be to introduce a weak and a strong negation.

Let us extend the language with four new operators on atomic formulas: $P\varphi$ means 'φ is presently the case'; $B\varphi$ means 'φ is becoming'; $C\varphi$ means 'φ is ceasing'; $F\varphi$ means: 'φ fails to be the case'.

The semantics that we will give in a moment will make $\neg\varphi$ weak negation, which is bivalid on the instant models. It will furthermore insure that:

$F\varphi$ is equivalent to $\neg\varphi \wedge \neg B\varphi \wedge \neg C\varphi$

with the consequence that if a moment is a moment of change, φ won't hold and neither will $F\varphi$. Kamp is able to give a compatible interpretation to bivalence and incompatibility by interpreting them as:

Bivalence: for every t: φ is true at t of $\neg\varphi$ is true at t.

Incompatibility: at the moment of change neither φ nor $F\varphi$ hold (but $\neg\varphi$ does).

Given starting model $M = \langle E, <, \bigcirc, A1, A2\rangle$. The completions are linear models of the form $M' = \langle E', <', \bigcirc', A1', A2'\rangle$ where the vagueness is resolved. With these completions correspond the linear instant structures: $N' = \langle T(M'), T(<'), i'\rangle$ given by the construction of instant structures.

i' is the interpretation function, which assigns the values P, F, B, C to atomic formulas at instants in the following way:

$$i'(\varphi, t) = P \text{ iff } \exists e \in t : A2(e)(\varphi) = P$$
$$i'(\varphi, t) = F \text{ iff } \exists e \in t : A2(e)(\varphi) = F$$
$$i'(\varphi, t) = B \text{ iff } \exists e \in t : A2(e)(\varphi) = B$$
$$i'(\varphi, t) = C \text{ iff } \exists e \in t : A2(e)(\varphi) = C$$

Here we see why it is advisable to add the condition excluding instants where φ is totally undefined: i' would not assign it any value, which would in fact invalidate the claims about bivalence. Note that this condition is very plausible if we think of the partiality of $A2(e)(\varphi)$ as meaning that φ is *overdefined* at e.

Now we can give the bivalid truth definition:

Let φ be an atomic formula.

$$[\![\varphi]\!]_{N',t} = 1 \text{ iff } [\![P\varphi]\!]_{N',t} = 1 \text{ iff } i'(\varphi, t) = P\text{; 0 otherwise}$$
$$[\![B\varphi]\!]_{N',t} = 1 \text{ iff } i'(\varphi, t) = B\text{; 0 otherwise}$$
$$[\![C\varphi]\!]_{N',t} = 1 \text{ iff } i'(\varphi, t) = C\text{; 0 otherwise}$$

$$[\![F\varphi]\!]_{N',t} = 1 \text{ iff } i'(\varphi, t) = F; \text{ 0 otherwise}$$

and for weak negation:

$$[\![\neg\varphi]\!]_{N',t} = 1 \text{ iff } [\![\varphi]\!]_{N',t} = 0; \text{ 0 otherwise}$$

This semantics indeed makes $\neg\varphi$ classical bivalid negation. Furthermore, it can easily be observed that (assuming the condition just mentioned) indeed $F\varphi$ is equivalent to $\neg\varphi \wedge \neg B\varphi \wedge \neg C\varphi$. The left to right side is obvious. For the other side, if $\neg\varphi \wedge \neg B\varphi \wedge \neg C\varphi$ is true at t, then apparently t does not contain any event where φ is P, nor an event where φ is B, nor an event where φ is C. Then given the above condition that it cannot be totally undefined, it has to contain an e where φ is F, hence $i'(\varphi, t) = F$, hence $F\varphi$ is true at t.

Given all this, we can now define (super)truth relative to the original model:

φ is *true in M* at e iff for every completion M' of M and every t in the instant structure N' corresponding to M':

$$\text{if } e \in t \text{ then } [\![\varphi]\!]_{N',t} = 1$$

φ is *false in M* at e iff for every completion M' of M and every t in the instant structure N' corresponding to M':

$$\text{if } e \in t \text{ then } [\![\varphi]\!]_{N',t} = 0$$

So x *becomes red* is true in M at e iff for every way of removing the temporal vagueness and for every instant there, at which e goes on, there is some event e' that goes on at that instant such that e' is an event of x becoming red.

Suppose that we have an event e that goes on just at the present moment. Then x *becomes red* is true in M at e (i.e. now) if an event of x becoming red is going on now (is an element of the present moment).

Let's reflect for a moment on what we have achieved. In the present theory, we separate the indeterminacy of truth value that comes in through vagueness from the indeterminacy that comes in through change. The first is treated with help of supervaluations, giving us the principle of bivalence. The second is not eliminated, giving us the principle of incompatibility of change, formulated with strong negation.

The essential difference with the previous theories is of course that the present theory relies on changes as primitives. It is precisely this that allows us to separate between the indeterminacy brought in by vagueness and the essential indeterminacy that characterizes change: in the previous theory we characterized change with a U-stable interval. Supervaluation theory removes the vagueness here by looking at the completions, but removes in the same process the indeterminacy of the change, and hence, it removes the change.

By introducing the changes as entities in their own right and resisting the temptation to define them in terms of undefinedness, we can remove the vagueness (i.e. locate the change as precise as possible, and in the same process determine whether it precedes other changes or not), without removing the change itself. This allows us to keep the benefits of the supervaluation theory, while having real changes.

Let us say, for the sake of comparison, that a sentence is true at an interval if it is true at some event that goes on at that interval. Then with respect to the puzzles discussed earlier, the present theory makes the same predictions as the last (supervaluation) theory.

The door is becoming closed is true at p if it is true at some event e that goes on at p, which means that for every way of removing the temporal vagueness an event of the door becoming closed is going on at every instant at which e goes on. Assuming that e goes on at every instant in p, this means (roughly) that an event of the door becoming closed goes on at every instant in p. This in turn means, by the compatibility condition, that no event of the door being open or the door being closed goes on at any instant in p. So here too $B\varphi$ is only true at the interval of change, not at the larger intervals containing the states before and after the change. (Consequently, the theory also makes the same predictions as the supervaluation theory with respect to the complex changes.)

Concerning the traffic light problem, at the macro interval $B\varphi$ is not true, simply because it cannot be going on at all of the instants in this interval (because on some of those φ or $F\varphi$ is going on).

Note that the present theory in a sense is again a reductionistic interval semantics: truth with respect to intervals is not taken as a primitive. We could try to recast the theory in terms of intervals rather than instants. I won't try to do that here, though. Moreover, the theory is reductionistic only in a sense: the theory is crucially an *event* semantics: it takes events as primitives, and lets events play a central role in the semantic recursion: just as we saw that an interval semantics is richer than an instant semantics because it deals with the relation between truth at an interval and truth at its subintervals, the present event semantics is richer than a truly instant semantics (or an interval semantics, for that matter), because it deals with the relation between truth at an event and truth at its subevents.

Let us finally discuss some principles that we may want to impose as constraints on the meanings of B, C and F. We haven't yet imposed, for instance, that a change from $\neg\varphi$ to φ is flanked by an $F\varphi$ state and a $P\varphi$ state.

Here are some principles that Kamp discusses. For a discussion of their influence on the logic, see Kamp's paper.

The principle of Separation tells us that every two incompatible states are separated by a change:

Principles of Separation

PS1: if $e_1 < e_2$, $A2(e_1)(\varphi) = F$, $A2(e_2)(\varphi) = P$, then $\exists e_3$: $e_1 < e_3 < e_2$ and $A2(e_3)(\varphi) = B$

in a picture:

PS1: $\underline{F}\quad\underline{P}\quad\Rightarrow\quad\underline{F}\quad\underline{B}\quad\underline{P}$

PS2: $\underline{P}\quad\underline{F}\quad\Rightarrow\quad\underline{P}\quad\underline{C}\quad\underline{F}$

PS3: $\underline{C}\quad\underline{P}\quad\Rightarrow\quad\underline{C}\quad\underline{B}\quad\underline{P}$

PS4: $\underline{F}\quad\underline{C}\quad\Rightarrow\quad\underline{F}\quad\underline{P}\quad\underline{C}$

PS5: $\underline{B}\quad\underline{F}\quad\Rightarrow\quad\underline{B}\quad\underline{C}\quad\underline{F}$

PS6: $\underline{P}\quad\underline{B}\quad\Rightarrow\quad\underline{P}\quad\underline{C}\quad\underline{B}$

PS7: $\underline{C}\quad\underline{C}$

$$\underline{\qquad C \qquad}$$
$$\underline{C}\qquad\qquad\underline{C}$$

or

$$\underline{C}\quad\underline{B}\quad\underline{C}$$

PS8: $\underline{B}\quad\underline{B}\quad\Rightarrow$

$$\underline{\qquad B \qquad}$$
$$\underline{B}\qquad\qquad\underline{B}$$

or

$$\underline{B}\quad\underline{C}\quad\underline{B}$$

The *second principle of separation* says that between two incompatible changes, there is a state:

$PS_2 1$: $\underline{B}\quad\underline{C}\quad\Rightarrow\quad\underline{B}\quad\underline{P}\quad\underline{C}$

$PS_2 1$: $\underline{C}\quad\underline{B}\quad\Rightarrow\quad\underline{C}\quad\underline{F}\quad\underline{B}$

The above principles tell us what happens between incompatible states and incompatible changes. The principle of *completed change* tells us what happens at the sides:

PCC1: $\underline{B}\quad\Rightarrow\quad\underline{F}\quad\underline{B}\quad\underline{P}$

PCC2: $\underline{C}\quad\Rightarrow\quad\underline{P}\quad\underline{C}\quad\underline{F}$

This principle can be strengthened to a *principle of discreet change*: every B is directly flanked by an F and a P (similarly for C). See Kamp's paper for its formulation in a weak and a strong form and for further conditions.

All these principles together will tell us that we only find sequences of changes in the following pattern:

$$F\ B\ P\ C\ F\ B\ P\ C\ F \ldots$$

When we extend the language and we allow the operators B, C, F, P to iterate, the above principles have to be modified. Also new principles may then be entertained: for instance, we will have to take a stand upon the question whether between a state where $F\varphi$ holds and a change $B\varphi$, there should be another change where neither $F\varphi$ nor $B\varphi$ holds (applying the principle of incompatibility once more), a change that we could express as $BB\varphi$ and, similarly, whether between $B\varphi$ and $P\varphi$ there should be a change $CB\varphi$. And we should then take a stand upon the question whether this should iterate *ad infinitum*, or whether this process should stop somewhere (and where). These topics are discussed in the second part of Kamp (1979b). I will not go into that discussion here and refer the reader to Kamp's paper for details.