

Structures for Semantics

by

FRED LANDMAN

*Department of Modern Languages & Linguistics,  
Cornell University, U.S.A.*

*Structures for Semantics* offers an advanced course in logical and mathematical techniques and structures that are used in semantics, in relation to their semantic applications. The book helps students with a background in semantics to develop their skills of formalization and it makes research in semantics accessible. Workers in other disciplines will use it to discover more about the role of formal modelling in current semantic research, and about semantics itself.

Following a chapter on logic and set theory there are three pairs of chapters: two pairs of chapters on partial order and equivalence relations in relation to semantic analyses of tense, partial information and vagueness; two chapters on methods for creating ordered structures in relation to intervals, events, and the semantics of change; two chapters on lattices and Boolean algebras in relation to types for noun phrases and verbs, and the semantics of plurals and mass nouns.

*Audience*

Upper-level undergraduate students and graduate students in semantics, theoretical linguists, logicians, philosophers of language and computer scientists interested in natural language semantics.

45

F. Landman – Structures for Semantics

Fred Landman

Structures  
for Semantics

Studies in Linguistics and Philosophy

---

Volume 45

---

*Managing Editors:*

GENNARO CHIERCHIA, *Cornell University*  
PAULINE JACOBSON, *Brown University*  
FRANCIS J. PELLETIER, *University of Alberta*

*Editorial Board:*

JOHAN VAN BENTHEM, *University of Amsterdam*  
GREGORY N. CARLSON, *University of Rochester*  
DAVID DOWTY, *Ohio State University, Columbus*  
GERALD GAZDAR, *University of Sussex, Brighton*  
IRENE HEIM, *M.I.T., Cambridge*  
EWAN KLEIN, *University of Edinburgh*  
BILL LADUSAW, *University of California at Santa Cruz*  
TERRENCE PARSONS, *University of California, Irvine*

FRED LANDMAN

*Dept. of Modern Languages and Linguistics, Cornell University*

# STRUCTURES FOR SEMANTICS



KLUWER ACADEMIC PUBLISHERS

DORDRECHT / BOSTON / LONDON

*The titles published in this series are listed at the end of this volume.*

Landman, Fred.

Structures for semantics / Fred Landman.

p. cm. -- (Studies in linguistics and philosophy ; v. 45)

Includes bibliographical references and index.

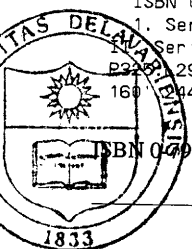
ISBN 0-7923-1239-2

1. Semantics. 2. Semantics (Philosophy) 3. Logic. I. Title. Series.

P22.L294 1991

160 244--dc20

91-13008



ISBN 0-7923-1239-2 (HB)

Published by Kluwer Academic Publishers,  
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Kluwer Academic Publishers incorporates  
the publishing programmes of  
D. Reidel, Martinus Nijhoff, Dr W. Junk and MTP Press.

Sold and distributed in the U.S.A. and Canada  
by Kluwer Academic Publishers,  
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed  
by Kluwer Academic Publishers Group,  
P.O. Box 322, 3300 AH Dordrecht, The Netherlands.

*Printed on acid-free paper*

All Rights Reserved

©1991 Kluwer Academic Publishers

No part of the material protected by this copyright notice may be reproduced or  
utilized in any form or by any means, electronic or mechanical,  
including photocopying, recording or by any information storage and  
retrieval system, without written permission from the copyright owner.

Printed in the Netherlands

## TABLE OF CONTENTS

PREFACE	vii
CHAPTER ONE: LOGIC AND SET THEORY	1
1.1. First Order Logic	2
1.1.1. Basic Concepts	2
1.1.2. Metalogic	13
1.2. Second Order Logic	24
1.2.1. Basic Concepts	24
1.2.2. The Expressive Power of Second Order Logic	28
1.3. First Order Theories	33
1.3.1. Some Examples of First Order Theories	33
1.3.2. Peano Arithmetics (PA)	37
1.4. Zermelo-Fraenkel Set Theory	44
1.4.1. Basic Set Theory	44
1.4.2. The Set Theoretic Universe	52
Appendix	61
CHAPTER TWO: PARTIAL ORDERS	70
2.1. Universal Algebra	70
2.2. Partial Orders and Equivalence Relations	83
2.3. Chains and Linear Orders	105
CHAPTER THREE: SEMANTICS WITH PARTIAL ORDERS	121
3.1. Instant Tense Logic	121
3.2. Algebraic Semantics, Functional Completeness and Expressibility	127
3.3. Some Linguistic Considerations Concerning Instantants	135
3.4. Information Structures	145
3.5. Partial Information and Vagueness	156
CHAPTER FOUR: CONSTRUCTIONS WITH PARTIAL ORDERS	171
4.1. Period Structures	171
4.2. Event Structures	186

CHAPTER FIVE: INTERVALS, EVENTS AND CHANGE	197
5.1. Interval Semantics	197
5.2. The Logic of Change in Interval Semantics	201
5.3. The Moment of Change	211
5.4. Supervaluations	218
5.5. Kamp's Logic of Change	221
CHAPTER SIX: LATTICES	234
6.1. Basic Concepts	234
6.2. Universal Algebra	248
6.3. Filters and Ideals	267
CHAPTER SEVEN: SEMANTICS WITH LATTICES	284
7.1. Boolean Types	284
7.2. Plurals	299
7.3. Mass Nouns	312
ANSWERS TO EXERCISES	325
REFERENCES	355
INDEX	359

## PREFACE

Formalization plays an important role in semantics. Doing semantics and following the literature requires considerable technical sophistication and acquaintance with quite advanced mathematical techniques and structures. But semantics isn't mathematics. These techniques and structures are tools that help us build semantic theories. Our real aim is to understand semantic phenomena and we need the technique to make our understanding of these phenomena precise.

The problems in semantics are most often too hard and slippery, to completely trust our informal understanding of them. This should not be taken as an attack on informal reasoning in semantics. On the contrary, in my view, very often the essential insight in a diagnosis of what is going on in a certain semantic phenomenon takes place at the informal level. It is very easy, however, to be misled into thinking that a certain informal insight provides a satisfying analysis of a certain problem; it will often turn out that there is a fundamental unclarity about what the informal insight actually is. Formalization helps to sharpen those insights and put them to the test.

Many of our students come to the field with no or little mathematical background. This is not a problem in the first stages, because, I think, there are by now enough textbooks around that do a very good job at providing in a 'gentle' way the most essential background and helping to overcome 'math-anxiety'. However, when students reach a more advanced level, where they have to deal directly with the literature, there is a dramatic jump in mathematical sophistication, which often leads to incredible frustration and 'advanced math-anxiety'.

For some this gap is bridged by taking math or logic courses, or reading textbooks in those fields. For others, such courses and textbooks are too much tailored to the audience that they are meant for, and the frustration only increases (with which I do not want to say at all that they shouldn't take such courses). Good students are put off this way, or don't develop a level of sophistication or mastering of the literature that is in principle obtainable for them, or do so only after a long, laborious and painful process.

The present book is aimed at bridging this gap. It presents mathematical and logical techniques and structures that are used in semantics in relation to their semantic applications. Since a major aim is to make the literature more accessible, the book does not compromise (much) on the level: it discusses techniques and structures that, for non-mathematicians, are difficult and go far beyond what a first introduction of math and logic for non-mathematicians would offer. Yet I've done my best to make this book a 'gentle' guide through a difficult subject, and to present things, where possible, in a context that may appeal to semanticists.

This book, then, is meant for people who have an interest in semantics and already some background, basically a one-year course in semantics at the level of Chierchia and McConnell-Ginet (1990) and Dowty, Wall and Peters (1981), or Gamut (1990). Partee, ter Meulen and Wall (1990) may also be helpful as a more elementary discussion of some of the topics discussed here.

The book is not restricted to semanticists, though. Semantics, being as interdisciplinary as it is, is very much part of cognitive science, and cognitive scientists in various disciplines may be interested in the application of the techniques and structures discussed in this book, in their own right, or as a way of finding out more about semantics.

Let me stress that this could be a dangerous book, if used in the wrong way. This is not a 'handbook for the perfect semanticist'. The book focuses on formal techniques and their application in semantics. It is meant to accompany and stimulate linguistic and conceptual argumentation in semantics, not primarily to teach, let alone replace that.

At the same time, this is not just a book of techniques. Besides being a tool for sharpening informal ideas, formalization itself has produced a wealth of semantic insights. Several of these are discussed in the book.

When I originally planned the contents of this book, it was much more ambitious (of course) than it finally turned out to be. There are many topics that I considered including or in fact did include at one time or other, but that in the end didn't make it, enough to fill at least a whole second volume. Though obviously my own preferences and feelings of what is important led me to include the topics that I did, it is mainly reasons of space and coherence that made me exclude those others.

A book like this does not arise out of thin air, of course. This book

was written by using the literature and in particular by using overviews of topics by others. The fact that I have included topics of which thorough overviews exist should not be taken to imply that I felt that I could do a better job, or that my discussion is meant to replace them. On the contrary, the books and papers that I refer to are often high on my list of personal favorites, both in content and presentation. My aim is rather to make them more accessible, because my experience has taught me that there is quite a discrepancy between what I used to consider accessible and my students' opinion on that matter.

Besides the works that I refer to in the book, I should mention here two sets of unpublished class notes that have been very helpful in writing this book. Johan van Benthem's notes on metalogic (written around 1977) were of help in writing the discussion of metalogic in Chapter One. Frank Veltman's notes on tense logic (written around the same time) form the basis for the discussion of properties of linear orders in Chapter Two and Priorian tense logic in Chapter Three.

This book consists of seven chapters: an overview of logic and set theory, and six chapters that come in pairs of two, a chapter discussing certain structures and a chapter discussing semantic applications of those structures.

Chapter One, the chapter on logic and set theory, consists of four sections (and an appendix). Each section consists of a subsection with basic notions and a subsection with more advanced discussion. The later chapters make reference to the material in the basic sections, so the reader who wants to get to these later chapters as soon as possible is advised to read at least through the basic sections of Chapter One.

Chapters Two and Three deal with partial orders. After a general introduction to the notions of structure and homomorphism, Chapter Two discusses partial orders, equivalence relations and partitions; it further deals with properties of partial orders in a temporal setting; and it discusses chains, maximal chains and set theoretic representation of partial orders. In Chapter Three, partial orders are interpreted alternatively as instant time structures, information structures and supervaluation structures for dealing with vagueness.

Chapters Four and Five discuss, again in a temporal setting, techniques for creating new partial orders out of given ones. In Chapter Four, time intervals are created out of instants of time; underlying instant structures are created out of interval structures through filter representations; and interval structures are created out of event struc-

tures through equivalence classes. In Chapter Five, these structures are applied to problems in the semantics of expressions indicating change and becoming.

Chapters Six and Seven discuss lattices. Chapter Six deals with the properties of lattices, join semilattices, and Boolean algebras; it further extends the discussion of homomorphisms and introduces generated and free lattices; and it discusses filters, ideals and set theoretic representation of distributive lattices, join semilattices and Boolean algebras. Chapter Seven discusses the relation between Boolean algebras and the types for noun phrases and verbs; and it discusses the relation between join semilattices and the semantics of plurals and mass nouns.

I am greatly indebted to my students at Cornell who have been exposed to previous versions of this book and helped improve it enormously. Manfred Krifka's very extensive, detailed and helpful comments were of invaluable use in shaping the final version. I thank my friends Nirit Kadmon and Craige Roberts for telling me and reminding me, from the time we first met in 1984 onwards, that I should write this book and what it should be like. Veneeta Srivastav carries the most direct responsibility for my having written this book; she forced me to teach, in the spring of 1988, the course out of which this book grew. I am very grateful for that.

I want to express a great intellectual debt to Hans Kamp. In reading this book (and the topics that didn't make it into this book), it should be obvious how much Hans' thinking has shaped semantics into what it is today.

Finally, I want to thank my friends and colleagues, Gennaro Chierchia, Nirit Kadmon and Sally McConnell-Ginet for their continuous support, moral and otherwise, while writing this book.

## LOGIC AND SET THEORY

Let me first state what the present chapter does not do. It is not intended to give an introduction to logic or set theory. As mentioned in the Preface, such an introduction is presupposed. Secondly, it is not meant to give to any satisfactory degree an overview of the basic results and techniques in logic (see for this Hodges, 1983).

With respect to logic and set theory, semanticists are on the consumers' side. The fruitful application of logical techniques to semantic problems does not require one to be a logician. Yet, something can be said for the view that it is a good thing if semanticists have some idea of what it is that logicians do with these techniques. For one thing, it may be of use if you want to ask your logician-friends for help: you don't have to switch to the automatic pilot at the first mention of, say, completeness or comprehension. Secondly, it may help to get a better grip on what the use of these techniques is committing you to. Thirdly, it is, of course, so inherently rewarding if some such knowledge is part of your intellectual baggage.

More importantly, certain logical concepts are used and mentioned (and, sometimes, mis-used and mis-mentioned) so frequently in the literature, that their proper understanding may make the literature more accessible. The most central of these, and the one that I will focus on, is the notion of 'first order'.

Probably most people who have gone through introductory semantics courses know that 'first order' has to do with quantification over individuals, that many semanticists prefer 'higher order' while many logicians prefer 'first order', and that certain things 'are not first order'. I think that, for the reasons mentioned, some deeper understanding of the notion of 'first order' is useful.

The basic aim of this chapter, then, is to shed some light on notions like first order logic, first order theories and first order definability, in short, on what it means to be first order.