

1 Semantic Competence

The goal of semantics is to properly characterize semantic competence.

What is semantic competence? To see the answer to this question, let us consider what syntactic competence consists of. It consists of the ability to judge which strings of words form grammatical sentences. Similarly, semantic competence consists of the ability to determine the *meaning* of a particular string of words. Since a particular string of words may correspond to more than one syntactic structure, we can take semantic competence to consist of the ability to determine the *meaning* of a particular syntactic structure. This ability also consists of the ability to determine the relationships between the meanings of distinct syntactic structure. These relationships include entailment, equivalence, and contradiction.

It is not clear exactly what kind of objects meanings are. We will treat them as mathematical objects that provide us information about the world. In particular, we will use the truth conditions of a sentence to reason about its meaning.

To make clear the notion of semantic competence, let us construct a thought experiment: suppose a Martian visits earth and develops syntactic competence. What does the Martian need to know in order to be semantically competent? The Martian needs to know what individual words mean. But this is not enough. We know that just knowing the meaning of the words in a sentence is not the same as knowing its meaning.

- (1) a. Jim saw Bill/Bill saw Jim.
- b. Gina saw the girl with a telescope.

The words *Jim*, *saw*, *Bill* can be combined in at least two ways as in (1a). These two orders have distinct meanings. So in addition to knowing the meaning of the words, the Martian must also be aware of the role played by order.

That order is not quite the primitive notion that is relevant is revealed by the ambiguity of (1b), where the same order (but two distinct structures) corresponds to two distinct meanings. The two distinct syntactic structures available for (1b) lead to distinct meanings. So the Martian also needs to be aware of the role played by syntactic structure in the **construction** of meaning.

To sum up, semantic competence consists of:

- (i) knowledge of the meaning of individual lexical items
 - (ii) knowledge of how the syntactic structure guides the construction of sentence (and phrase-level) meaning from the meanings of individual lexical items, and of the operations by which meaning is constructed.
- Both (i) and (ii) are finite i.e. any language has only a finite number of lexical items, and the number of rules that guide the construction of meaning is also finite. However, the nature of the knowledge in (ii) is such that it allows us to compute the meaning of an infinite number of arbitrarily complex syntactic structures. Our goal in this class will be to formally and explicitly characterize what semantic competence consists of. We will find that we will go back and forth between (i) and (ii). Cases where we have clear intuitions about word meaning (i.e. i) as well as sentence meaning will provide us evidence about the nature of the combinatorial rules (i.e. ii). Then once we have some confidence in our combinatorial rules, we can use them and our intuitions about sentence meaning to deduce the meaning of lexical items like *a*, *no*, *every*. We will be adopting the principle articulated by David Lewis that meaning *is* what meaning *does*.

Note that in both directions we make appeal to our intuitions about sentence meaning. These intuitions concern relations of implication, ambiguity, synonymy, contradiction, anomaly, and appropriateness. They are the raw material of semantics.

The goal of semantic theory is to build a theory that can explain why we have these intuitions. Like any theory, it will be based on the basic data of the theory - semantic intuitions. Further, if our theory is any good it must have predictive power i.e. given novel syntactic structures, our theory should be able to predict their meaning i.e. their truth conditions and their semantic relationships with other structures. Quoting from Davidson (1984):

The theory reveals nothing about the conditions under which an individual sentence is true; it does not make those conditions any clear than the sentence itself does. The work of the theory is in relating the known truth conditions of each sentence to those aspects ("words") of the sentence that recur in other sentences, and can be assigned identical roles in other sentences. Empirical power in such a theory depends on success in recovering the structure of a very complicated ability - the ability to speak and understand a language.

2 Pragmatic Competence

There is more to the study of meaning than just understanding the truth-conditions of a sentence. Assume that our Martian has acquired semantic competence. Does it follow that the Martian will be able to respond appropriately to the following utterances?

- (2) a. Could you pass me the salt? (said at a dinner table)
- b. I'd like some bread. (said at a dinner table)
- c. Hi. Is Tom there? (said on the phone)

We have clear intuitions about what the appropriate responses to such utterances are and these responses seem to be related but independent of the semantic meaning of the utterances.

In particular, the additional meaning that we find in (2) has the property that it can be cancelled or strengthened by additional information. This additional component of meaning is called *implicature*.

For the Martian to have a proper understanding of meaning, pragmatic competence is also necessary. We will explicate why cases like (2) have the implicatures they do, and in particular how such implicatures are derived.

3 Background

Our semantic theory will be stated in mathematical terms that use set theory, algebra, and logic. So we will now move on to a brief introduction of set theory and algebra.

3.1 Set Theory Basics

A **set** is a group of objects represented as a unit.

Sets may contain any type of object, including numbers, symbols, and even other sets.

Sets can be described by listing their elements inside braces.

Set1 = {11, 13, 17, 19}

Set2 = {Marge, Bart, Lisa, Homer, Maggie}

The members of a set do not have to have anything to do with each other:

Set3 = {Marge, Bart, 11, 13}

The ordering of elements does not matter. Thus

{11, 13, 17, 19} = {13, 11, 19, 17}

Also repetitions do not count:

{11, 13, 17, 19} = {11, 13, 13, 17, 17, 19}

∈ denotes set-membership.

19 ∈ {11, 13, 17, 19}

∉ denotes nonmembership.

7 ∉ {11, 13, 17, 19}

A is a **subset** of B i.e. $A \subseteq B$ iff everything that is a member of A is a member of B (i.e. for all $x \in A$, $x \in B$).

$A = B$ iff $A \subseteq B$ and $B \subseteq A$

A is a **proper subset** of B iff $A \subseteq B$ and $A \neq B$.

The **empty** set: A set may have no members. This set is called the **empty** set and is represented as $\{ \}$ or ϕ .

The **universal** set: is a set that contains the entire domain. It is sometimes represented by U .

If the set that we are dealing with has many members, then actual listing is not an attractive option.

Further if the set that we are dealing with is infinite, then actual listing is not an option at all.

In such cases, we can use two strategies:

1. Use \dots to indicate **continue the sequence forever**.

$\mathcal{N} = \{0, 1, 2, 3, \dots\}$

$\mathcal{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

2. Give a description of the set:

$\text{SetN} = \{n \mid \text{rule about } n\}$

$\text{Squares} = \{n \mid \exists m \in \mathcal{N}[n = m^2]\}$

$\text{Squares-below-100} = \{n \mid \exists m \in \mathcal{N}[n = m^2 \wedge n < 100]\}$

3.2 Operations on Sets

The following binary operations are defined on sets:

Union : $A \cup B$

Intersection : $A \cap B$

Difference : $A - B$

$A \cup B$ contains everything that is in A **or** B .

$A \cap B$ contains everything that is in A **and** B .

$A - B$ contains everything that is in A but **not in** B .

The following unary operations are also defined on sets:

Cardinality : $\#(X)$, $\text{Card}(X)$

Power Set : 2^X , $\text{Pow}(X)$

Set Complement : X'

The cardinality of a set is the number of elements a set contains.

The power set of a set X is the set of all the subsets of X .

3.3 Set-Theoretic Laws

Idempotency: $X \cup X = X$, $X \cap X = X$

Commutativity: $X \cup Y = Y \cup X$

reversing the order doesn't matter

Associativity: $(X \cup Y) \cup Z = X \cup (Y \cup Z)$, $(X \cap Y) \cap Z = X \cap (Y \cap Z)$

the order in which we apply union/intersection does not matter

Distributivity:

$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$

$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

Union distributes over intersection, and vice versa

Identity Laws:

$X \cup \phi = X$, $X \cap \phi = \phi$

$X \cup U = U$, $X \cap U = X$

Complement Laws:

$X \cup X' = U$, $X \cap X' = \phi$

$(X')' = X$, $X - Y = X \cap Y'$

de Morgan's Law:

$$C - (A \cup B) = (C - A) \cap (C - B)$$

$$C - (A \cap B) = (C - A) \cup (C - B)$$

if we substitute U , the universal set, for C , we get:

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

3.4 Semantic Relations

- Hyponymy as subethood

- the semantics of adjectives

red and round = round and red (Commutativity)

not [round and rough] = not round or not rough (de Morgan)

red and [round or rough] = [red and round] or [red and rough] (Distributivity)

3.5 Relations

The meanings of intransitive verbs and adjectives can be modeled by sets. What about transitive verbs and adjectives?

- (3) a. Bill read Principia.
- b. Jane is fond of Andy.
- c. William is the son of Charles.

Intuitively, *read*, *fond of*, and *son of* denote relationship between two entities.

read represents pairs such that the first thing in the pair read the second thing in the pair. So the meaning of *read* can be thought of as the set of such pairs.

- note that unlike in sets, in a pair **order** matters. If Jane is fond of Olafur, it does not follow that Olafur is fond of Jane. Therefore these pairs are called **ordered pairs**.

There can be any number of elements in an ordered pair. An ordered pair with n elements is called an n -tuple. We need sets of 3-tuples to represent the meanings of ditransitive verbs.

- (4) Paul gave the Interpol CD to Amy.

Two tuples are equal if each of their elements (in order) is equal:

$$\langle x_1, x_2, \dots, x_n \rangle = \langle y_1, y_2, \dots, y_n \rangle \text{ iff } x_1 = y_1 \text{ and } \dots \text{ and } x_n = y_n.$$

Tuples can be created from sets by applying the operation of **Cartesian Product** which is represented by \times , and defined as follows:

$$(5) \quad A \times B = \{ \langle x, y \rangle \mid x \in A, y \in B \}$$

A relation R between a set A and a set B is always a subset of $A \times B$.

read is a relation between *PEOPLE* and *BOOKS* i.e.

$$read \subseteq PEOPLE \times BOOKS$$

3.6 Functions

Functions are a particular kind of relation - they have the following property:

A relation R is a function if for every x, y, z , if $\langle x, y \rangle \in R$ and $\langle x, z \rangle \in R$, then $y = z$.

x is called the argument of the function, and y the value.

R *applies* to x and *yields* y .

Alternatively, R *maps* x to y .

Important concepts:

- domain
- range

- partial function
- into vs. onto
- 1-1 (bijections) vs. many-one (vs. one-many)

Bijections can be used to show that two sets have the same cardinality.

3.7 Sets as Functions

Sets can be described by functions.

Consider the Simpson Nuclear Family =

{Marge, Homer, Lisa, Maggie, Bart }

We could describe this set by the function SNF , which when applied to all the people in the Simpsons World maps the members of the Simpsons Nuclear Family to 1 (or **Yes** or **True**), and everyone else to 0 (or **No** or **False**).

$SNF(Marge) = SNF(Homer) = SNF(Maggie) = SNF(Lisa) = SNF(Bart) = 1$

$SNF(Maude) = SNF(Ned) = \dots = SNF(Itchy) = SNF(Scratchy) = 0$

SNF is the membership function for the set Simpson Nuclear Family.

Such function are also called **characteristic** functions.

4 A simple example

What we want to prove:

- (6) $\llbracket \text{Ann smokes} \rrbracket = 1$ iff Ann smokes.
Ann smokes is true iff Ann smokes.

What we need:

A. Lexicon

$\llbracket \text{Ann} \rrbracket = \text{Ann}$

$\llbracket \text{Jan} \rrbracket = \text{Jan}$

etc. for other proper names.

These are all members of D , the set of actual individuals.

$\llbracket \text{works} \rrbracket = f : D \rightarrow \{0, 1\}$

For all $x \in D$, $f(x) = 1$ iff x works.

$\llbracket \text{smokes} \rrbracket = f : D \rightarrow \{0, 1\}$

For all $x \in D$, $f(x) = 1$ iff x smokes.

B. Rules for interpreting structure.

References

Davidson, D. (1984) *Inquiries into Truth and Interpretation*, Clarendon Press, Oxford.