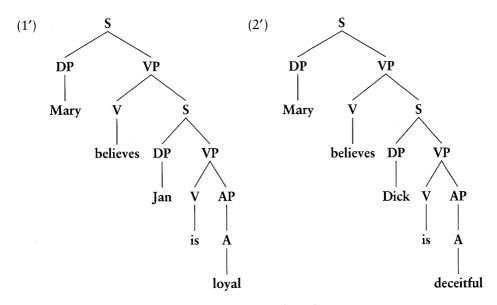
12 First Steps Towards an Intensional Semantics

In chapter 2, we made a move that cost us some effort to sell to our readers. We identified the denotations of sentences with their actual truth-values. We put potential protests to rest by demonstrating that the resulting extensional theory as a whole was still able to pair sentences with their truth-conditions. While sticking to an extensional semantics, we managed to develop a theory of meaning. In this chapter, we will have to face the limitations of the extensional framework we have been working in up to now. We will see that it breaks down in certain contexts, a fact that Frege was well aware of. Fortunately, a minor change in the semantic system will repair the problem (at least to a certain extent), and most of what we have learned will stay intact.

12.1 Where the extensional semantics breaks down

One of the basic assumptions underlying our semantics has been that the extension of a complex expression can be computed from the extensions of its parts in a stepwise fashion, using no more than a handful of composition principles. While this assumption was correct for the examples we have discussed so far, it is plain wrong for examples of the following kind.

- (1) Mary believes Jan is loyal.
- (2) Mary believes Dick is deceitful.



Here is the problem. Our type-driven interpretation system implies that in the two trees above, the denotations of the higher VP-nodes are computed from the denotations of the matrix V-nodes and the denotations of the embedded S-nodes by the same mode of composition. The two V-nodes dominate the same lexical item, hence are assigned the same denotation. The two S-nodes both denote a truth-value. Suppose now that in the actual world, Jan is indeed loyal, and Dick deceitful; that is, [Jan is loyal] = [Dick is deceitful] = 1. Consequently, the two embedded S-nodes have the same denotation, and we predict that $[[V_{PP}believes [V_{PP}believes [V_{PP}beli$

We want to say that sentences embedded under believe are nonextensional contexts. Nonextensional contexts are also called "oblique", "opaque", or "indirect". Other words that create nonextensional contexts include the verbs hope, fear, look (as in look smart), seem, seek, the adjectives alleged or fake, the preposition about, the connective because, and modal words of various categories like must, may, probably, obviously, provable, and permissible.

Exercise

Show that alleged, seem, might, about, and because create nonextensional contexts. Demonstrate this by constructing compositionality arguments of the

sort we have just given for **believe**. Note that nonextensional contexts do not have to be sentential. The nonextensional context created by **alleged** is the modified noun phrase: for example, **son of a sphinx** in **alleged son of a sphinx**.

12.2 What to do: intensions

Frege proposed that in opaque contexts, expressions denote their *Sinn* (sense). But what is a Fregean *Sinn*? Recall the quote from Dummett in chapter 2 that addresses a frequently voiced complaint about Frege being rather vague as to what the senses of expressions are:

It has become a standard complaint that Frege talks a great deal about the senses of expressions, but nowhere gives an account of what constitutes such a sense. This complaint is partly unfair: for Frege the sense of an expression is the manner in which we determine its reference, and he tells us a great deal about the kind of reference possessed by expressions of different types, thereby specifying the form that the senses of such expressions must take. . . . The sense of an expression is the mode of presentation of the referent: in saying what the referent is, we have to choose a particular way of saying this, a particular means of determining something as a referent.³

The Fregean sense of an expression, then, is the mode of presentation of its extension (reference, *Bedeutung*). It's a particular means of determining the extension. But what kind of formal object is a "means of determining an extension"? It could just be a linguistic expression – an expression of set theory, for example. Or it could be an algorithm computing the values of a function for arbitrary arguments. Different expressions might specify the same set, and different algorithms might compute the same function. Expressions of set theory and algorithms, then, are means of determining extensions. But there are other, more abstract possibilities. One was proposed by Rudolf Carnap, a student of Frege's. Here is the idea.

The truth of a sentence depends on the circumstances. It's now true that you are in Amsterdam, but in a little while, that's not going to be the case any more. And if circumstances had been different, you might never have left your native Buffalo at all.

The extension of a predicate depends on the circumstances as well. You are a member of the garden club, but you haven't always been, and you might never have joined. Quite generally, then, the extension of an expression depends on possible circumstances. An *intension* in Carnap's sense is something that determines *how* extensions depend on possible circumstances. David Lewis tells us where to go from there:

What sort of things determine how something depends on something else? Functions, of course; functions in the most general set-theoretic sense, in which the domain of arguments and the range of values may consist of entities of any sort whatsoever, and in which it is not required that the function be specifiable by any simple rule. We have now found something to do at least part of what a meaning for a sentence, name, or common noun does: a function which yields as output an appropriate extension when given as input a package of the various factors on which the extension may depend. We will call such an input package of relevant factors an index; and we will call any function from indices to appropriate extensions for a sentence, name, or common noun an intension.⁵

A (Carnapian) intension,⁶ then, is a function from indices to appropriate extensions. To simplify matters in this introductory text, let us neglect all index dependence except for dependence on possible worlds. That is, we will neglect temporal dependence, speaker dependence, and what have you. For our limited purposes here, then, (Carnapian) intensions are functions from possible worlds to appropriate extensions. The intension of a sentence is a function from possible worlds to truth-values. The intension of a 1-place predicate is a function that maps possible worlds into characteristic functions of sets of individuals, etcetera. If you wonder about possible worlds, here is what David Lewis says about them:

The world we live in is a very inclusive thing. Every stick and every stone you have ever seen is part of it. And so are you and I. And so are the planet Earth, the solar system, the entire Milky Way, the remote galaxies we see through telescopes, and (if there are such things) all the bits of empty space between the stars and galaxies. There is nothing so far away from us as not to be part of our world. Anything at any distance at all is to be included. Likewise the world is inclusive in time. No long-gone ancient Romans, no long-gone pterodactyls, no long-gone primordial clouds of plasma are too far in the past, nor are the dead dark stars too far in the future, to be part of the same world. . . .

The way things are, at its most inclusive, means the way this entire world is. But things might have been different, in ever so many ways. This book of mine might have been finished on schedule. Or, had I not been such a commonsensical chap, I might be defending not only a plurality of possible worlds, but also a plurality of impossible worlds, whereof you speak truly by contradicting yourself. Or I might not have existed at all

– neither myself, nor any counterpart of me. Or there might never have been any people. Or the physical constants might have had somewhat different values, incompatible with the emergence of life. Or there might have been altogether different laws of nature; and instead of electrons and quarks, there might have been alien particles, without charge or mass or spin but with alien physical properties that nothing in this world shares. There are ever so many ways that a world might be: and one of these many ways is the way that this world is.⁷

12.3 An intensional semantics

In this section, we will develop a semantic system that allows denotations to be (Carnapian) intensions, and we will show that such a system solves the problem we ran into at the beginning of this chapter.

There are various intensional frameworks in the possible worlds tradition that you find in the literature. The framework we chose here for illustration is a conservative extension of the extensional semantics we have been working with all along. We start out with a recursive definition of an intensional system of semantic types (Montague's), which will be followed by a parallel definition of a typed system of semantic domains.

(1) Recursive definition of semantic types

- (a) e is a type.
- (b) t is a type.
- (c) If a and b are types, then <a,b> is a type.
- (d) If a is a type, then <s,a> is a type.
- (e) Nothing else is a type.

(2) Semantic domains

Let W be the set of all possible worlds. Associated with each possible world w is the domain of all individuals existing in w. Let D be the union of the domains of all possible worlds. That is, D contains all individuals existing in the actual world, but also all individuals existing in any of the merely possible worlds. It is the set of all possible individuals. The set of intensional domains is now defined as follows:

- (a) $D_e = D$
- (b) $D_t = \{0, 1\}$
- (c) If a and b are semantic types, then $D_{\langle a,b\rangle}$ is the set of all functions from D_a to D_b .
- (d) If a is a type, then $D_{s,a}$ is the set of all functions from W to D_a .

In addition to our familiar extensions, we now have intensions. The domain $D_{\epsilon_{s,t}}$ contains all functions from W to $\{0,1\}$, for example: that is, all characteristic functions of subsets of W. Possible world semanticists take such functions to be the formal construals of *propositions*. Construing propositions as characteristic functions of sets of possible worlds is natural, as emphasized in the following statement by Robert Stalnaker:

The explication of *proposition* given in formal semantics is based on a very homely intuition: when a statement is made, two things go into determining whether it is true or false. First, what did the statement say: what proposition was asserted? Second, what is the world like: does what was said correspond to it? What, we may ask, must a proposition be in order that this simple account be correct? It must be a rule, or a function, taking us from the way the world is into a truth value. But since our ideas about how the world is change, and since we may wish to consider the statement relative to hypothetical and imaginary situations, we want a function taking not just the actual state of the world, but various possible states of the world into truth values. Since there are two truth values, a proposition will be a way – any way – of dividing a set of possible states of the world into two parts: the ones that are ruled out by the truth of the proposition, and the ones that are not.⁹

Let us look at some examples of lexical entries. Following Montague, we will relativize the interpretation function to a possible world and an assignment function. As before, we can drop reference to an assignment when the choice of assignment doesn't matter, since we define for any possible world w, and any expression α :

$$(3) \quad \llbracket \alpha \rrbracket^{w} := \llbracket \alpha \rrbracket^{w,\varnothing}.$$

We now have:

(4) Names

For any possible world w:

 $[\![Jan]\!]^w = Jan.$

 $[\![\mathbf{Ann}]\!]^{\mathbf{w}} = \mathbf{Ann}.$

etc.

Following Saul Kripke, we treat proper names as *rigid designators*.¹⁰ Their reference is picked out in the actual world, and they denote the same individual that was so picked out in every possible world.¹¹ Proper names, then, differ from definite descriptions like **the coldest winter**, which may denote different winters in different possible worlds.

The extensions of predicates may vary depending on the circumstances as well:

(5) Easy predicates

For any possible world w: $[\![smoke]\!]^w = \lambda x \in D$. x smokes in w $[\![love]\!]^w = \lambda x \in D$. $[\![\lambda y \in D]\!]$ v loves x in w] $[\![cat]\!]^w = \lambda x \in D$. x is a cat in w etc.

Nothing exciting happens with quantifiers. Their extension does not depend on the circumstances, but for reasons of generality, we carry the world parameter along, as we did with proper names:

(6) Determiners

For any possible world w: $[\![every]\!]^w = \lambda f \in D_{<e,t>}$. $[\lambda g \in D_{<e,t>}$. for all x such that f(x) = 1, g(x) = 1] etc.

The fragment we have been building does not yet require any new composition rules. The ones we already have in place will do, except that the interpretation function depends now not just on an assignment, but also on a possible world. Both parameters have to be schlepped along as the interpretation machinery works its way through a given tree. Assignments, too, are what they used to be: partial functions from the set of natural numbers to D (but D has changed, of course).

Consider now the attitude verbs believe, know, hope, and so on. As a starting point, we will pursue an approach (in the spirit of Hintikka¹²) that has it that the content of an attitude can be characterized by a set of possible worlds: namely, those that are compatible with the attitude. Here is how David Lewis illustrates this rather simple idea:

The content of someone's knowledge of the world is given by his class of epistemically accessible worlds. These are the worlds that might, for all he knows, be his world; world W is one of them iff he knows nothing, either explicitly or implicitly, to rule out the hypothesis that W is the world where he lives. Likewise the content of someone's system of belief about the world (encompassing both belief that qualifies as knowledge and belief that fails to qualify) is given by his class of doxastically accessible worlds. World W is one of those iff he believes nothing, either explicitly or implicitly, to rule out the hypothesis that W is the world he lives.

Whatever is true at some epistemically or doxastically accessible world is epistemically or doxastically possible for him. It might be true, for all

he knows or for all he believes. He does not know or believe it to be false. Whatever is true throughout the epistemically or doxastically accessible worlds is epistemically or doxastically necessary; which is to say that he knows or believes it, perhaps explicitly or perhaps only implicitly.

Since only truths can be known, the knower's own world always must be among his epistemically accessible worlds. Not so for doxastic accessibility. If he is mistaken about anything, that is enough to prevent his own world from conforming perfectly to his system of belief.¹³

Lexical entries for attitude verbs will accordingly look as follows:

(7) Attitude verbs

For any possible world w:

[believe] $^{w} = \lambda p \in D_{\langle s,t \rangle}$. [$\lambda x \in D$. p(w') = 1, for all $w' \in W$ that are compatible with what x believes in w]

 $[\![know]\!]^w = \lambda p \in D_{s,t}$. $[\lambda x \in D \cdot p(w') = 1$, for all $w' \in W$ that are compatible with what x knows in w]

What a person believes, knows, or hopes can vary from one possible world to another. The world parameter in the lexical entries for attitude verbs matters, then, as it does for most other predicates. For example, we can't simply write "w is a world where Mary's hopes come true" unless we mean: "w is a world where those hopes come true that Mary has in the actual world". Otherwise, we must make explicit which world we are talking about; that is, we must write things of the form: "w' is a world where Mary's hopes in w come true".

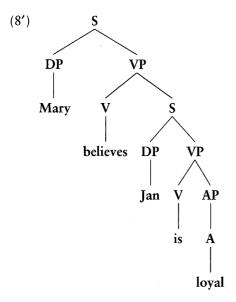
For Mary to believe that p it is *not* sufficient that p be true in *some* world that is compatible with what she believes. If her belief worlds include worlds where p is true as well as worlds where p is false, then she is agnostic as to whether or not p. Likewise for the other attitude verbs.

We can say that a possible world having such and such properties is compatible or incompatible with what Mary believes, or that w is a world where Mary's hopes come true. But never say things like "Mary hopes to be in w", "Mary believes that she is in w", "Mary knows that she is in w". Taken literally, these make sense only if Mary is so choosy that there is only one possible world that satisfies her desires, or so opinionated that just one possible world conforms to her beliefs, or omniscient. The same objection applies to locutions like "the world according to Mary", "the world of her dreams", "the world she thinks she lives in", etcetera. We talk this way when we are not semanticists, but it doesn't make literal sense within the present theoretical framework. If we don't want to commit ourselves to such unrealistic assumptions, we have

to express ourselves in a way that makes it clear that actual desires, beliefs, etcetera, pick out *sets* of worlds. For instance, we might write: "w is a world where Mary's hopes come true", "w is a world that conforms to everything Mary believes", "what Mary knows is true in w".

Attitude verbs denote functions that apply to propositions. But so far, our interpretation procedure doesn't yet deliver any propositions, nor intensions of any other kind. Look at what happens when we try to interpret sentence (8):

(8) Mary believes Jan is loyal.



We have for any possible world w:

- (a) [Mary [vpbelieves [sJan is loyal]]] = (FA)
- (b) [believes [$_s$ Jan is loyal]] w ([Mary] w) = (lexical entry Mary)
- (c) [believes [,Jan is loyal]] (Mary)

We are stuck. The denotation of the verb believes applies to a proposition: that is, an intension of type <s,t>. The interpretation system, however, provides only **[Jan is loyal]**^w, which is a denotation of type t, hence a mere truth-value: 1 if Jan is loyal in w, and 0 otherwise. Here is the proof:

By the definition of the $\lambda\text{-notation,}\ [\lambda x\in D\ .\ x\ is\ loyal\ in\ w]\ (Jan)=1\ iff\ Jan\ is\ loyal\ in\ w.$

To make sure that the lexical requirements of the verb believes can be met, we introduce an additional composition principle: Intensional Functional Application (IFA):¹⁴

(9) Intensional Functional Application (IFA)

If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then, for any possible world w and any assignment a, if $[\![\beta]\!]^{w,a}$ is a function whose domain contains $\lambda w'$. $[\![\gamma]\!]^{w',a}$, then $[\![\alpha]\!]^{w,a} = [\![\beta]\!]^{w,a} (\lambda w')$. $[\![\gamma]\!]^{w',a}$.

We can now continue our computation at the point where we got stuck:

- (c) [believes [sJan is loyal]] (Mary) = (by IFA)
- (d) $[believes]^w$ ($\lambda w'$. $[Jan is loyal]^{w'}$) (Mary) = (by previous computation)
- (e) [believes]^w (λw'. Jan is loyal in w')
 (Mary) = (by lexical entry believe)
- $\begin{array}{ll} (f) & [\lambda p \in D_{\scriptscriptstyle <\!s,t\!>} \; . \; [\lambda x \in D \; . \; p(w') = 1, \\ & \text{for all } w' \in W \; \text{that are compatible} \\ & \text{with what } x \; \text{believes in } w]] \; (\lambda w' \; . \\ & \text{Jan is loyal in } w') \; (Mary) \end{array} \; = (by \; definition \; of \; \lambda\text{-notation})$
- (g) $[\lambda x \in D \cdot [\lambda w']$. Jan is loyal in w'] (w') = 1, for all $w' \in W$ that are compatible with what x believes in w] (Mary)

= (by definition of λ -notation)

(h) $[\lambda x \in D]$. Jan is loyal in w', for all $w' \in W$ that are compatible with what x believes in w] (Mary).

Finally (again by definition of the λ -notation) we have:

 $[\lambda x \in D]$. Jan is loyal in w', for all $w' \in W$ that are compatible with what x believes in w] (Mary) = 1 iff Jan is loyal in w', for all $w' \in W$ that are compatible with what Mary believes in w.

We have arrived at the correct truth-conditions for sentence (8). (8) is true in the actual world iff Mary's actual beliefs exclude all possible worlds in which Jan is not loyal. In an analogous way, we obtain the right truth-conditions for (10):

(10) Mary believes Dick is deceitful.

(10) comes out true in the actual world iff there is no world that is compatible with Mary's actual beliefs in which Dick is not deceitful. Given an intensional semantics, we do not run into any difficulties any more when we assume that all of the sentences 11(a)–(d) might actually be true together:

- (11) (a) Jan is loyal.
 - (b) Dick is deceitful.
 - (c) Mary believes that Jan is loyal.
 - (d) Mary does not believe that Dick is deceitful.

The problem we started out with in this chapter is now gone. The solution is very much in the spirit of Frege. The usual denotations are extensions. But for nonextensional contexts, Intensional Functional Application allows a switch to intensions. The switch is triggered by particular lexical items – those that create nonextensional contexts. Whether a lexical item does or does not create a nonextensional context, then, is part of the information conveyed by its denotation, like any other information about selectional restrictions.

Exercise 1

Look at sentence (i):

- (i) Mary hopes that a plumber is available.
- (i) may mean that there is a particular plumber, say Mr French, whom Mary hopes will be available (*de re* interpretation of a **plumber**). Or else it may mean that she hopes that some plumber or other is available (*de dicto* interpretation of a **plumber**). It is customary to treat the ambiguity as a scope ambiguity. On the *de re* interpretation, a **plumber** is moved into the matrix clause. On the *de dicto* interpretation, a **plumber** can be interpreted *in situ*.
 - (a) Draw appropriate LFs for the two readings of (i).
 - (b) Add lexical entries to our fragment as needed. Treat **that** and **is** as semantically empty, but treat **a** as a quantifying determiner.
 - (c) Compute the truth-conditions for both LFs.

Exercise 2

Look at sentence (ii):

(ii) Lee is an alleged drug dealer from Springfield.

Draw an appropriate LF for (ii) and compute its truth-conditions. Add lexical entries to our fragment as needed. Treat **is** and **an** as semantically empty, and **drug dealer** as an unanalyzed common noun. The truth-conditions you want to end up with should imply that (ii) is true in the actual world just in case Lee is a drug dealer from Springfield in all those possible worlds that are compatible with the allegations in the actual world. Let's not fuss about the fact that allegations may vary from time to time and place to place, even in a single world. We chose to ignore that kind of dependence here to make things easier for all of us.

12.4 Limitations and prospects

Carnap¹⁵ insists that sentences embedded under attitude verbs are neither extensional nor intensional contexts. And he is right. Take two sentences that are true in the same possible worlds but do not have to be believed together. Here is an example due to John Bigelow:¹⁶

- (1) (a) Robin will win.
 - (b) Everyone who does not compete, or loses, will have done something Robin will not have done.
- (2) (a) Marian believes that Robin will win.
 - (b) Marian believes that everyone who does not compete, or loses, will have done something Robin will not have done.

It may take some time to figure this out, but (1a) and (1b) are true in exactly the same possible worlds. They express the same proposition, then. But if what we believe are propositions (as our analysis of attitude verbs assumes), anybody who believes (1a) should also believe (1b). This is not right. (2a) and (2b) can have different truth-values. We are in trouble again. Propositions are still not good enough as objects of beliefs and other attitudes.

Carnap proposed the concept of "intensional isomorphism" or "intensional structure" as a remedy.¹⁷ David Lewis follows up on this idea and identifies "meanings" with "semantically interpreted phrase markers minus their terminal nodes: finite ordered trees having at each node a category and an appropriate intension."¹⁸ A slightly simpler construct, "structured propositions", is proposed by Cresswell and von Stechow.¹⁹ There is no agreement on the issue yet. The good news is, however, that the uncertainty in the area of propositional attitudes does not seem to have a lot of repercussions on the way linguists do semantics every day. A slight change led us from an extensional system to an intensional one. The switch to a hyperintensional system should not be much more eventful. What we have learned about particular extensional or intensional phenomena should be adaptable to a new foundation without too much ado. Barbara Partee observes:

Many of the most fundamental foundational issues in formal semantics (and in semantics as a whole) remain open questions, and there may be even less work going on on them now than there was in the seventies; perhaps this is because there is more work by linguists and less by philosophers, so the empirical linguistic questions get most of the attention now.²⁰

The empirical linguistic questions have certainly been our main concern in this book.

Notes

- 1 At least of those ingredients of meaning that have to do with truth-conditions. There are other components of meaning that are not usually dealt with in semantics, but in pragmatics.
- 2 G. Frege, "Über Sinn und Bedeutung," trans. as "On Sense and Nomination," in A. P. Martinich (ed.), *The Philosophy of Language*, 2nd edn (New York and Oxford, Oxford University Press, 1990), pp. 190–202.
- 3 M. Dummett, Frege. Philosophy of Language, 2nd edn (Cambridge, Mass., Harvard University Press, 1981), p. 227.
- 4 R. Carnap, Meaning and Necessity. A Study in Semantics and Modal Logic (Chicago, University of Chicago Press, 1947, 1956). See also J. Hintikka, Knowledge and Belief (Ithaca, NY, Cornell University Press, 1962).
- 5 D. Lewis, "General Semantics," in D. Davidson and G. Harman (eds), Semantics of Natural Language (Dordrecht, Reidel, 1972), p. 174.
- 6 The term "Carnapian intension" is used by Lewis (ibid.).
- D. Lewis, On the Plurality of Worlds (Oxford, Basil Blackwell, 1986), pp. 1f. If you are interested in the foundations of possible world semantics, consult M. J. Loux (ed.), The Possible and the Actual. Readings in the Metaphysics of Modality (Ithaca, NY, Cornell University Press, 1979).

- Apart from the original works of D. Lewis, Montague, and Cresswell mentioned earlier, we recommend L. T. F. Gamut, Logic, Language and Meaning, vol. 2: Intensional Logic and Logical Grammar (Chicago, University of Chicago Press, 1991).
- R. C. Stalnaker, "Pragmatics," in Davidson and Harman (eds), Semantics, p. 381.
- S. A. Kripke, Naming and Necessity (Oxford, Basil Blackwell, 1980); J.-Y. Lerner and T. E. Zimmermann, "Eigennamen" (Proper Names), in A. von Stechow and D. Wunderlich (eds), Semantik/Semantics. An International Handbook of Contemporary Research (Berlin and New York, Walter de Gruyter, 1991), pp. 349-70.
- Scholars differ with respect to their views on trans-world identity of individuals. David Lewis has argued, for example, that individuals should not be assumed to exist in more than one possible world. Individuals may be related to very similar individuals in other possible worlds via a counterpart relationship. See D. K. Lewis, "Counterpart Theory and Quantified Modal Logic," Journal of Philosophy, 65 (1968), pp. 113-26; idem, Plurality of Worlds.
- Hintikka, Knowledge and Belief. 12
- Lewis, Plurality of Worlds, p. 27. 13
- Maria Bittner has proposed an analogous type adjustment rule (M. Bittner, "Cross-14 linguistic Semantics," Linguistics and Philosophy, 17/1 (1994), pp. 53-108).
- Carnap, Meaning and Necessity. 15
- Reported in M. J. Cresswell, Structured Meanings (Cambridge, Mass., MIT Press, 16 1985), p. 4.
- Carnap, Meaning and Necessity. 17
- 18
- Lewis, "General Semantics," p. 182. M. J. Cresswell and A. von Stechow, "De re Belief Generalized," Linguistics and 19 Philosophy, 5 (1982), pp. 503-35. See also Cresswell, Structured Meanings.
- B. H. Partee, "The Development of Formal Semantics in Linguistic Theory," in 20 S. Lappin (ed.), The Handbook of Contemporary Semantic Theory (Oxford, Basil Blackwell, 1996), p. 36.