

1 Quantifiers Denotations

- (1) a. a/the/some/every/no/... student
b. the/some/no/many/most/few/more than three/atmost two/... student
c. only John

What do quantifiers denote?

- (2) a. Only John
b. a/some student
c. every student
d. no student

$\llbracket \text{only John} \rrbracket \neq \llbracket \text{John} \rrbracket$

2 Entailment Patterns

2.1 Subset to Superset

- (3) a. John came yesterday morning.
b. John came yesterday.
- (4) a. At most one letter came yesterday morning.
b. At most one letter came yesterday.
- (5) a. $\alpha \text{ VP}_1$.
b. $\alpha \text{ VP}_2$.

$$(\llbracket VP_1 \rrbracket \subseteq \llbracket VP_2 \rrbracket)$$

• If $\llbracket \alpha \rrbracket \in D_e$, then (5b) **follows** from (5a).

• If $\llbracket \alpha \rrbracket \notin D_e$, then (5b) **may or may not** follow from (5a).

2.2 Contradiction

- (6) a. Stephin was born in Boston and Stephin was born outside Boston.
b. More than two linguists were born in Boston and more than two linguists were born outside Boston.
- (7) $\alpha \text{ VP}_1$ and $\alpha \text{ VP}_2$.
a. If $\llbracket VP_1 \rrbracket \cap \llbracket VP_2 \rrbracket = \phi$ and $\llbracket \alpha \rrbracket \in D_e$, then (7) is a **contradiction**.
b. If $\llbracket VP_1 \rrbracket \cap \llbracket VP_2 \rrbracket = \phi$ and $\llbracket \alpha \rrbracket \notin D_e$, then (7) may or may not be a **contradiction**.

2.3 Excluded Middle

- (8) a. I am over 30 years old, or I am under 40 years old.
b. Every person in this room is over 30 years old, or every person in this room is under 40 years old.
- (9) α VP₁ or α VP₂.
a. If $\llbracket VP_1 \rrbracket \cup \llbracket VP_2 \rrbracket = D_e$ and $\llbracket \alpha \rrbracket \in D_e$, then (9) is a **tautology**.
b. If $\llbracket VP_1 \rrbracket \cup \llbracket VP_2 \rrbracket = D_e$ and $\llbracket \alpha \rrbracket \notin D_e$, then (9) may or may not be a **tautology**.

3 Ambiguity and Structural Reordering

3.1 Effects of Structural Reordering

Certain structural reorderings do not affect the truth conditions of DP's that denote individuals, but may affect the truth conditions of other DP's.

- (10) Topicalization
a. I answered Problem 4.
b. Problem 4, I answered.
- (11) a. Buffy loves Angel.
b. Angel is such that Buffy loves him.
c. Buffy is such that she loves Angel.
- (12) Topicalization
a. Almost everybody answered at least one question.
b. At least one question, everybody answered.
- (13) a. Nobody saw more than one vampire.
b. More than one vampire is such that nobody saw him.
c. Nobody is such that he or she saw more than one vampire.

3.2 Ambiguity

Quantificational DP's display certain ambiguities that DP's that denote individuals do not.

- (14) a. It didn't snow on Christmas Day.
b. It didn't snow on more than two of these days.

4 Quantifiers as Sets of Individuals?

- (15) a. (Ann left promptly; Ann left.)
b. (Everyone left promptly; Everyone left.)
c. (Someone left promptly; Someone left.)
d. (Most people left promptly; Most people left.)
e. (No one left promptly; No one left.)
f. (Less than two people left promptly; Less than two people left.)

Possible Denotations:

- (16) a. $\llbracket \text{Ann} \rrbracket = \{\mathbf{Ann}\}$
b. $\llbracket \text{Everyone} \rrbracket = D_e$
c. $\llbracket \text{No one} \rrbracket = \phi$

Problems:

- (17) a. some, most
b. no, few, less than two

→ Overall, a bad idea.

5 Generalized Quantifiers

5.1 Everything and Nothing

- Instead of VP applying to the subject, the subject applies to the VP.

- (18) a. $\llbracket \text{nothing} \rrbracket = \lambda f \in D_{et}. \neg \exists x \in D_e. [f(x) = 1]$
 b. $\llbracket \text{everything} \rrbracket = \lambda f \in D_{et}. \forall x \in D_e. [f(x) = 1]$
 c. $\llbracket \text{something} \rrbracket = \lambda f \in D_{et}. \exists x \in D_e. [f(x) = 1]$

Type of $\llbracket \text{nothing} \rrbracket, \llbracket \text{everything} \rrbracket, \llbracket \text{something} \rrbracket$ is $(et)t$.

Handling the absent inference patterns:

- (19) a. Subset to Superset
 b. Contradiction
 c. Excluded Middle

5.2 Some, Every, and No

- (20) a. $\llbracket \text{no} \rrbracket = \lambda f \in D_{et}. [\lambda g \in D_{et}. \neg \exists x \in D_e. [f(x) = g(x) = 1]]$
 b. $\llbracket \text{some} \rrbracket = \lambda f \in D_{et}. [\lambda g \in D_{et}. \exists x \in D_e. [f(x) = g(x) = 1]]$
 c. $\llbracket \text{every} \rrbracket = \lambda f \in D_{et}. [\lambda g \in D_{et}. \forall x \in D_e. [f(x) \rightarrow g(x)]]$

Type of $\llbracket \text{no} \rrbracket, \llbracket \text{every} \rrbracket, \llbracket \text{some} \rrbracket$ is $(et)(et)t$.

6 Quantifiers as Relations between Sets

- (21) For any $A, B \subseteq D$:
- $(A, B) \in R_{every}$ iff $A \subseteq B$
 - $(A, B) \in R_{some}$ iff $A \cap B \neq \emptyset$
 - $(A, B) \in R_{no}$ iff $A \cap B = \emptyset$
 - $(A, B) \in R_{atleast2}$ iff $A \cap B \geq 2$
 - $(A, B) \in R_{most}$ iff $|A \cap B| > |A - B|$

- This relational meaning can be converted to an equivalent functional meaning.

6.1 Formal properties of Quantifiers

- (22) a. Reflexivity
 b. Irreflexivity
 c. Symmetry
 d. Antisymmetry
 e. Transitivity
 f. Conservativity
 g. Monotonicity

6.2 Conservativity

- (23) δ is conservative iff for all A, B : $(A, B) \in R_\delta$ iff $(A, A \cap B) \in R_\delta$.

- Conservativity is a putative universal:

- (24) a. All children sing. \iff All children are children that sing.
 b. Most children sing. \iff Most children are children that sing.
 c. No children sing. \iff No children are children that sing.

Some exceptions:

- (25) **only**:
- Only babies cry.
 - Only babies are babies that cry.
- (26)
- Many Scandinavians have won the Nobel Prize.
 - Many Scandinavians are Scandinavians who have won the Nobel Prize.

6.3 Extensionality

- Another putative universal.

Intuition: For the truth of $Q(A)(B)$, all that matters is the contents of A and B . The rest of the world is irrelevant.

- (27) **Extension**:
- For all $A, B \subseteq U$: if $Q_U(A, B)$ and $U \subseteq U'$ then $Q_{U'}(A, B)$.

- *only* is Extensional.

Putative counterexample:

- (28) There are many native speakers of Dutch.

6.4 Monotonicity

- (29)
- δ is left upward monotone: for all A, B, C : if $A \subseteq B$ and $(A, C) \in R_\delta$ then $(B, C) \in R_\delta$.
 - δ is left downward monotone: for all A, B, C : if $A \subseteq B$ and $(B, C) \in R_\delta$ then $(A, C) \in R_\delta$.
 - δ is right upward monotone: for all A, B, C : if $A \subseteq B$ and $(C, A) \in R_\delta$ then $(C, B) \in R_\delta$.
 - δ is right downward monotone: for all A, B, C : if $A \subseteq B$ and $(C, B) \in R_\delta$ then $(C, A) \in R_\delta$.

- Role in licensing the occurrence of Negative Polarity Items (NPIs) like *any*, *ever* etc.

6.5 Cardinal vs. Proportional Quantifiers

- (30) $Q(A)(B)$
- Q is **cardinal** if the truth conditions of $Q(A)(B)$ are based on the extension of $A \cap B$ alone.
 - Q is **proportional** if the truth conditions of $Q(A)(B)$ depend upon the extension of $A - B$.

Role in Existential Sentences.

Verification Procedures for Quantifiers

A difference between *some* and *no*.