24.903 Language & Structure III: Semantics and Pragmatics
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1 Semantic Relationships

1.1 Entailment and Implicature

(1) p *entails* $q =_{df}$

- whenever p is true, q is true
- the information that q conveys is contained in the information that p conveys
- a situation describable by p must also be a situation describable by q
- p *and not* q is a contradiction.

Two properties of entailments:
• Non-cancellabilty
• Reinforcement is redundant

Consider the difference between the relationship of (2a) and (2b) on the one hand, and the relationship between (3a) and (3b) on the other.

(2) a. Mary used to swim a mile daily.
b. Mary no longer swims a mile daily.

(3) a. After Hans painted the walls, Pete installed the cabinets.
b. Hans painted the walls.

Cancellability:

(4) a. In fact, she still does. (followup to (2a))
b. But Hans did not paint the walls. (followup to (3a)

Reinforcement:

(5) a. But she no longer does. (followup to (2a))
b. Hans painted the walls. (followup to (3a))

We can call implications that can be cancelled and reinforced *implicatures*.

Properties of *implicature*:
(i) calculability from contextual and common-sense assumptions
(ii) cancellability / defeasibility
(iii) reinforceability

1.2 Presupposition and Assertion

Presuppositions, like entailments, are a kind of implication. If A presupposes B, then A not only implies B but also implies that the truth of B is somehow taken for granted.

(6) a. Paul stopped smoking.
b. Presupposition: Paul used to smoke.

(7) a. Jenny also likes Ashton Kutcher.
b. Presupposition: Someone else likes Ashton Kutcher.

(8) a. It is Muriel who loves ABBA.
b. Presupposition: Someone loves ABBA.

Some other instances that involve presupposition: only, even, too, regret.

Tests for Presupposition:

(9) Presuppositions survive change in the sentential force:
a. Paul didn't stop smoking.
b. Did Paul stop smoking?
c. If Paul stops smoking, his health will improve.

What does it mean for a presupposition to survive?

(10) Complements of verbs like *surprise, regret*:
a. I regret / am surprised that Paul stopped smoking.
b. I regret / am surprised that Jenny also likes ABBA.

1.3 Ambiguity

- Lexical
  - Homophony: *bank, bug*,....
  - Homography: (*rein, rain, reign*), (*son, sun*),....
- Structural:
  Competent women and men hold all the good jobs in my department.
- Contextdepency:
  I am right, you are wrong.
  I am right here now.
- Vagueness:
  Many people came.
  John is tall.

• Polysemy vs. Lexical Ambiguity:

(11) a. **red** pen, **new** car
b. **bed**, **bed** of roses, river **bed**
c. **heavy** stone, **heavy** rain, **heavy** meal
d. 'The book inspired me.' vs. 'The book is on the table.'

• Interaction of lexical and structural ambiguity:

(12) a. They have written invitations.
b. Ambulance crews help dog bite victim.
c. What JLo dislikes is being ignored by the media.

Unclear cases:

(13) a. Some student admires every teacher.
b. Every student thinks that she is smart.
c. Someone likes pizza with anchovies.

2 Sets

2.1 Basics

A **set** is a group of objects represented as a unit. The objects may be of any type, including numbers, symbols, and even other sets. Sets can be described by listing their elements inside braces.

- SOMECUBES = $\{1, 8, 27, 64\}$
- SIMPSONS = {Marge, Bart, Lisa, Homer, Maggie}
- WHAT1 = {Marge, Bart, 8, 27}
- WHAT2 = {MIT, 2-151, WHAT1}

The ordering or repetition of elements does not matter.
$\{11, 13, 17, 19\} = \{13, 11, 19, 17\}$
$\{11, 13, 17, 19\} = \{11, 13, 13, 17, 17, 19\}$

Some notation:

- $\in$ denotes set-membership, $\notin$ denotes nonmembership.
- $\subseteq$ denotes subsethood.
  $A \subseteq B$ iff everything that is a member of A is a member of B
- $=$ denotes equality, $\neq$ denotes non-equality.
  $A = B$ iff $A \subseteq B$ and $B \subseteq A$
- $\subset$ denotes proper subsethood.
  A is a proper subset of B iff $A \subseteq B$ and $A \neq B$.

Two Special Sets:

- The **empty** set: has no members, and is represented as { } or ϕ.
- The **universal** set: contains the entire domain, and is represented by U.

Specifying Large or Infinite sets:

- Use ... to indicate **continue the sequence forever**.
  $\mathcal{N} = \{0, 1, 2, 3, \ldots\}$
  $\mathcal{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

- Give a description of the set:
  SetN $= \{n \mid$ rule about $n\}$
  Squares $= \{n \mid \exists m \in \mathcal{N}[n = m^2]\}$
  Squares-below-100 $= \{n \mid \exists m \in \mathcal{N}[n = m^2 \wedge n < 100]\}$

Subtleties of set notation:

(14)
a. {x: {y: y likes x} $= \phi$}
b. {y: {x: x likes y} $= \phi$}
c. {x: {y: x likes y} $= \phi$}
d. {y: {x: y likes x} $= \phi$}
e. {Iowa: Iowa is a midwestern state}
f. {x: x is a midwestern state}
g. {x: Iowa is a midwestern state}
h. {x: Florida is a midwestern state}
i. {x: x $\in$ {x: x $\neq$ 0}}
j. {x: x $\in$ {y: y $\neq$ 0}}

2.2 Operations on Sets

The following binary operations are defined on sets:

Union : $A \cup B$, contains everything that is in A **or** B.

Intersection : $A \cap B$, contains everything that is in A **and** in B.

Difference : $A - B$, contains everything that is in A but **not in** B.

The following unary operations are also defined on sets:

Cardinality : #(X), Card(X), the number of elements a set contains.

Power Set : 2^X, Pow(X), the set of all the subsets of X.

Set Complement : X' ($= U - X$)

2.3 Set-Theoretic Laws

Idempotency :
$X \cup X = X$, $X \cap X = X$

Commutativity :
$X \cup Y = Y \cup X$
reversing the order doesn't matter

Associativity :
$(X \cup Y) \cup Z = X \cup (Y \cup Z)$, $(X \cap Y) \cap Z = X \cap (Y \cap Z)$
the order in which we apply union / intersection does not matter

Distributivity :
$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$
$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$
Union distributes over intersection, and vice versa

Identity Laws :
$X \cup \phi = X$, $X \cap \phi = \phi$
$X \cup U = U$, $X \cap U = X$

Complement Laws :
$X \cup X' = U$, $X \cap X' = \phi$
$(X')' = X$, $X - Y = X \cap Y'$

de Morgan's Law :
$C - (A \cup B) = (C - A) \cap (C - B)$
$C - (A \cap B) = (C - A) \cup (C - B)$
if we substitute U, the universal set, for C, we get:
$(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$

3 Relations and Functions

3.1 Relations

The meanings of intransitive verbs and adjectives can be modeled by sets of individuals. What about transitive verbs and adjectives?

(15) a. Bill read Principia.
b. Jane is fond of Andy.
c. William is the son of Charles.

Intuitively, *read*, *fond of*, and *son of* denote relationship between two entities.

read represents pairs such that the first thing in the pair read the second thing in the pair. So the meaning of *read* can be thought of as the set of such pairs.

- note that unlike in sets, in a pair **order** matters. If Jane is fond of Andy, it does not follows that Andy is fond of Jane. Therefore these pairs are called **ordered pairs**.

There can be any number of elements in an ordered pair. An ordered pair with n elements is called an n-tuple. We need sets of 3-tuples to represent the meanings of ditransitive verbs.

(16) Paul gave the Interpol CD to Amy.

Two tuples are equal if each of their elements (in order) is equal:
$< x_1, x_2, \ldots, x_n >=< y_1, y_2, \ldots, y_n >$ iff $x_1 = y_1$ and . . . and $x_n = y_n$.

Tuples can be created from sets by applying the operation of **Cartesian Product** which is represented by $\times$, and defined as follows:

(17) $A \times B = \{< x, y > | x \in A, y \in B\}$

A relation R between a set A and a set B is always a subset of $A \times B$.

read is a relation between $PEOPLE$ and $BOOKS$ i.e.
$read \subseteq PEOPLE \times BOOKS$

3.2 Functions

Functions are a particular kind of relation - they have the following property:
A relation $R \subseteq A \times B$ is a function if for every x, y, z, if $< x, y >\in R$ and $< x, z >\in R$, then $y = z$.

x is called the argument of the function, and y the value.
R *applies* to x and *yields* y.
Alternatively, R *maps* x to y.

Important concepts:
• domain
• range
• partial function
• into vs. onto
• 1-1 (bijections) vs. many-one (vs. one-many)

Bijections can be used to show that two sets have the same cardinality.

3.3 Sets as Functions

Sets can be described by functions.

Consider the Simpson Nuclear Family =
{Marge, Homer, Lisa, Maggie, Bart }

We could describe this set by the function SNF, which when applied to all the people in the Simpsons World maps the members of the Simpsons Nuclear Family to 1 (or **Yes** or **True**), and everyone else to 0 (or **No** or **False**).

SNF(Marge) = SNF(Homer) = SNF(Maggie) = SNF(Lisa) = SNF(Bart) = 1
SNF(Maude) = SNF(Ned) = . . . = SNF(Itchy) = SNF(Scratchy) = 0

SNF is the membership function for the set Simpson Nuclear Family.
Such function are also called **characteristic** functions and characteristic functions can be used as convenient shorthand for sets.