

## 9. homework assignment; JAVA, Academic year 2016/2017; FER

Ovu domaću zadaću radite kao običan Eclipse Java projekt. Potrebne biblioteke koje se spominju smjestite u direktorij `lib` unutar projekta (napravite ga), i zatim ih dodajte u *Build Path*.

### Problem 1.

Radimo vrlo jednostavnu biblioteku koja nudi potporu za rad s nekoliko različitih matematičkih objekata: vektori, kompleksni brojevi te polinomi.

Napravite paket `hr.fer.zemris.math` i u njega smjestite modele vektora, kompleksnih brojeva i polinoma.

Napravite razred `Vector3` koji modelira neizmjenjiv trokomponentni vektor (sve operacije nad njime vraćaju nove objekte ovog tipa koji predstavljaju rezultat operacije). Javno sučelje ovog razreda specificirano je u nastavku.

```
public Vector3(double x, double y, double z) {...} // konstruktor

public double norm() {...} // norma vektora ("duljina")

public Vector3 normalized() {...} // normalizirani vektor

public Vector3 add(Vector3 other) {...} // zbrajanje

public Vector3 sub(Vector3 other) {...} // oduzimanje: ja minus drugi

public double dot(Vector3 other) {...} // skalarni produkt

public Vector3 cross(Vector3 other) {...} // vektorski produkt: ja i on

public Vector3 scale(double s) {...} // skaliranje zadanim faktorom

public double cosAngle(Vector3 other) {...} // kosinus kuta između mene i njega

public double getX() {...} // prva komponenta vektora

public double getY() {...} // druga komponenta vektora

public double getZ() {...} // treća komponenta vektora

public double[] toArray() {...} // pretvorba u polje s 3 elementa

public String toString() {...} // pretvorba u string
```

Primjer ispitnog programa:

```
public static void main(String[] args) {  
    Vector3 i = new Vector3(1,0,0);  
    Vector3 j = new Vector3(0,1,0);  
    Vector3 k = i.cross(j);  
  
    Vector3 l = k.add(j).scale(5);  
  
    Vector3 m = l.normalized();  
  
    System.out.println(i);  
    System.out.println(j);  
    System.out.println(k);  
    System.out.println(l);  
    System.out.println(l.norm());  
    System.out.println(m);  
    System.out.println(l.dot(j));  
    System.out.println(i.add(new Vector3(0,1,0)).cosAngle(l));  
}
```

Očekivani ispis:

```
(1.000000, 0.000000, 0.000000)  
(0.000000, 1.000000, 0.000000)  
(0.000000, 0.000000, 1.000000)  
(0.000000, 5.000000, 5.000000)  
7.0710678118654755  
(0.000000, 0.707107, 0.707107)  
5.0  
0.4999999999999999
```

U nastavku ćemo napraviti još model kompleksnog broja, te dva modela polinoma koji su zadani nad kompleksnim brojevima, i čiji su koeficijenti kompleksni brojevi. Sva tri modela moraju stvarati neizmjenjive objekte.

Napravite razred Complex koji modelira kompleksni broj, prema predlošku u nastavku.

```
public static class Complex {  
  
    ...  
  
    public static final Complex ZERO = new Complex(0,0);  
    public static final Complex ONE = new Complex(1,0);  
    public static final Complex ONE_NEG = new Complex(-1,0);  
    public static final Complex IM = new Complex(0,1);  
    public static final Complex IM_NEG = new Complex(0,-1);  
  
    public Complex() {...}  
  
    public Complex(double re, double im) {...}  
  
    // returns module of complex number  
    public double module() {...}  
  
    // returns this*c  
    public Complex multiply(Complex c) {...}  
  
    // returns this/c  
    public Complex divide(Complex c) {...}  
  
    // returns this+c  
    public Complex add(Complex c) {...}  
  
    // returns this-c  
    public Complex sub(Complex c) {...}  
  
    // returns -this  
    public Complex negate() {...}  
  
    // returns this^n, n is non-negative integer  
    public Complex power(int n) {...}  
  
    // returns n-th root of this, n is positive integer  
    public List<Complex> root(int n) {...}  
  
    @Override  
    public String toString() {...}  
}
```

Napravite razred `ComplexRootedPolynomial` koji modelira polinom nad kompleksnim brojevima, prema predlošku u nastavku. Radi se o polinomu  $f(z)$  oblika  $(z-z_1)*(z-z_2)*\dots*(z-z_n)$ , gdje su  $z_1$  do  $z_n$  njegove nultočke (a njih zadaje korisnik kroz konstruktor). Primjetite, radi se o polinomu  $n$ -tog stupnja (kada biste izmnožili zgrade). Sve nultočke zadaju se kao kompleksni brojevi, a i sam  $z$  je kompleksan broj. Metoda *apply* prima neki konkretan  $z$  i računa koju vrijednost ima polinom u toj točki.

```
public static class ComplexRootedPolynomial {

    // ...

    // constructor
    public ComplexRootedPolynomial(Complex ...roots) {...}

    // computes polynomial value at given point z
    public Complex apply(Complex z) {...}

    // converts this representation to ComplexPolynomial type
    public ComplexPolynomial toComplexPolynom() {...}

    @Override
    public String toString() {...}

    // finds index of closest root for given complex number z that is within
    // threshold; if there is no such root, returns -1
    public int indexOfClosestRootFor(Complex z, double threshold) {...}
}
```

Napravite razred `ComplexPolynomial` koji modelira polinom nad kompleksnim brojevima, prema predlošku u nastavku. Radi se o polinomu  $f(z)$  oblika  $z_n * z^n + z_{n-1} * z^{n-1} + \dots + z_2 * z^2 + z_1 * z + z_0$ , gdje su  $z_0$  do  $z_n$  koeficijenti koji pišu uz odgovarajuće potencije od  $z$  (i zadaje ih korisnik kroz konstruktor). Primjetite, radi se o polinomu  $n$ -tog stupnja (što još zovemo red – engl. *polinom order*). Svi koeficijenti zadaju se kao kompleksni brojevi, a i sam  $z$  je kompleksan broj. Metoda *apply* prima neki konkretan  $z$  i računa koju vrijednost ima polinom u toj točki.

```
public static class ComplexPolynomial {

    // ...

    // constructor
    public ComplexPolynomial(Complex ...factors) {...}

    // returns order of this polynomial; eg. For (7+2i)z^3+2z^2+5z+1 returns 3
    public short order() {...}

    // computes a new polynomial this*p
    public ComplexPolynomial multiply(ComplexPolynomial p) {...}

    // computes first derivative of this polynomial; for example, for
    // (7+2i)z^3+2z^2+5z+1 returns (21+6i)z^2+4z+5
    public ComplexPolynomial derive() {...}

    // computes polynomial value at given point z
    public Complex apply(Complex z) {...}

    @Override
    public String toString() {...}
}
```

## Problem 2.

We will consider another kind of fractal images: fractals derived from Newton-Raphson iteration. As you are surely aware, for about three-hundred years we know that each function that is  $k$ -times differentiable around a given point  $x_0$  can be approximated by a  $k$ -th order Taylor-polynomial:

$$f(x_0 + \varepsilon) = f(x_0) + f'(x_0)\varepsilon + \frac{1}{2!}f''(x_0)\varepsilon^2 + \frac{1}{3!}f'''(x_0)\varepsilon^3 + \dots$$

So let  $x_1$  be that point somewhere around the  $x_0$ :

$$x_1 = x_0 + \varepsilon$$

Substituting it into previously given formula we obtain:

$$f(x_1) = f(x_0) + f'(x_0)(x_1 - x_0) + \frac{1}{2!}f''(x_0)(x_1 - x_0)^2 + \frac{1}{3!}f'''(x_0)(x_1 - x_0)^3 + \dots$$

For approximation of function  $f$  we will restrict our self on linear approximation, so we can write:

$$f(x_1) \approx f(x_0) + f'(x_0)(x_1 - x_0)$$

Now, let us assume that we are interested in finding  $x_1$  for which our function is equal to zero, i.e. we are looking for  $x_1$  for which  $f(x_1) = 0$ . Plugging this into above approximation, we obtain:

$$0 = f(x_0) + f'(x_0)(x_1 - x_0)$$

and from there:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

However, since we used the approximation of  $f$ , it is quite possible that  $f(x_1)$  is not actually equal to zero; however, we hope that  $f(x_1)$  will be closer to zero than it was  $f(x_0)$ . So, if that is true, we can iteratively apply this expression to obtain better and better values for  $x$  for which  $f(x) = 0$ . So, we will use iterative expression:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

which is known as Newton-Raphson iteration.

For this homework we will consider complex polynomial functions. For example, let's consider the complex polynomial whose roots are  $+1$ ,  $-1$ ,  $i$  and  $-i$ :

$$f(z) = (z-1)(z+1)(z-i)(z+i) = z^4 - 1$$

After deriving we obtain:

$$f'(z) = 4z^3$$

It is easy to see that our function  $f$  becomes 0 for four distinct complex numbers  $z$ . However, we will pretend that we don't know those roots. Instead, we will start from some initial complex point  $c$  and plug it into our iterative expression:

$$z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)} = z_n - \frac{z^4 - 1}{4z^3} \quad \text{with} \quad z_0 = c.$$

We will generate iterations until we reach a predefined number of iterations (for example 16) or until module  $|z_{n+1} - z_n|$  becomes adequately small (for example, convergence threshold 1E-3). Once stopped, we will find the closest function root for final point  $z_n$ , and color the point  $c$  based on index of that root (let root indexes start from 1). However, if we stopped on a  $z_n$  that is further than predefined threshold from all roots, we will color the point  $c$  with a color associated with index 0.

For example, if the function roots are +1, -1,  $i$  and  $-i$ , if acceptable root-distance is 0.002, if convergence threshold equals 0.001 and if we stopped iterating after  $z_7 = -0.9995 + i0$  because  $z_7$  was closer to  $z_6 = -0.9991 + i0$  then convergence threshold, we will determine that  $z_7$  is closest to second function root (first is +1, second is -1, third is +i, fourth is -i) and that  $z_7$  is within predetermined root-distance (0.002) to -1, so we will color pixel  $c$  based on color associated with index 2.

We will proceed just as with Mandelbrot fractal:

```
for(y in y_min to y_max) {
  for(x in x_min to x_max) {
    c = map_to_complex_plain(x, y, x_min, x_max, y_min, y_max, re_min, re_max, im_min, im_max);
    zn = c;
    iter = 0;
    iterate {
      zn1 = zn - f(zn)/f'(zn);
      iter++;
    } while(|zn1-zn|>convergenceTreshold && iter<maxIter);
    index = findClosestRootIndex(zn1, rootTreshold);
    if(index==-1) { data[offset++]=0; } else { data[offset++]=index; }
  }
}
```

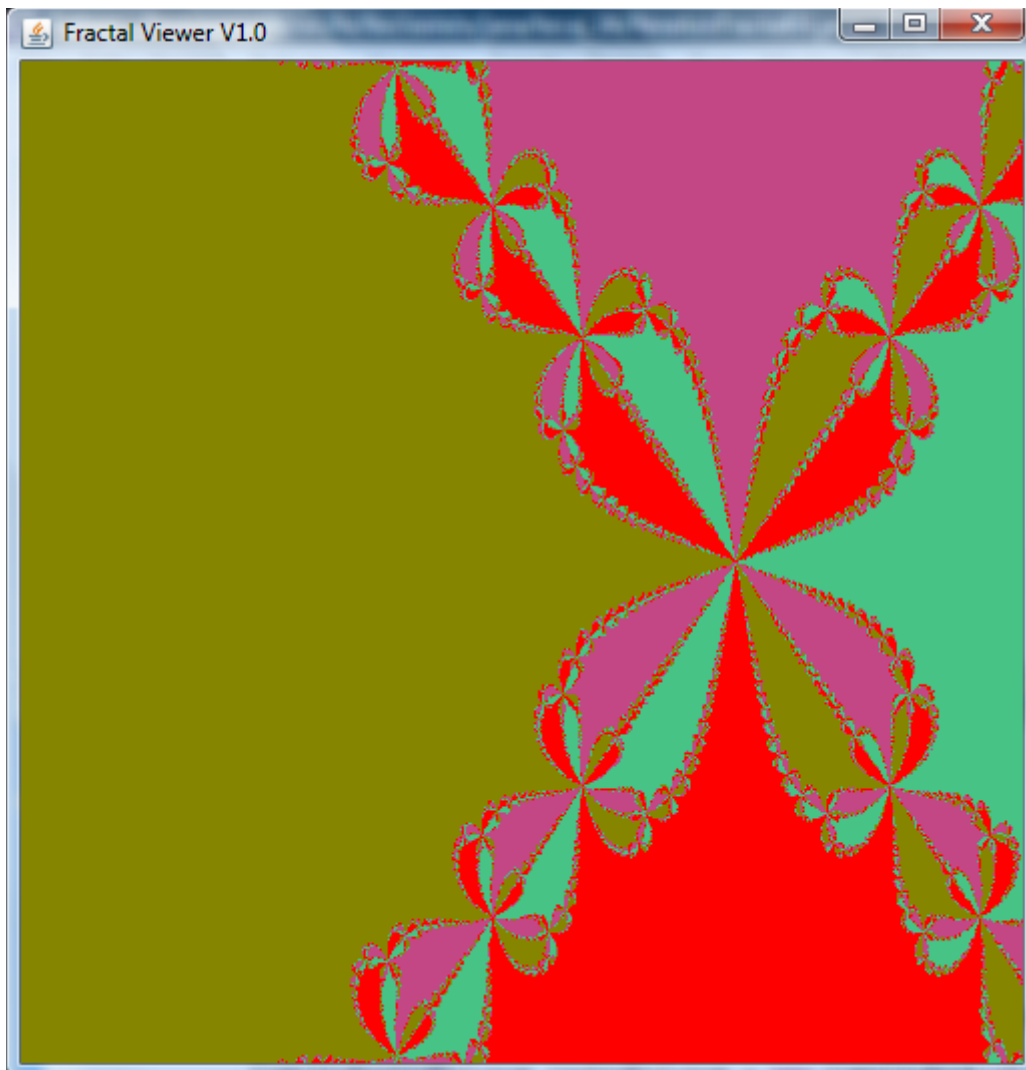
We use `data[]` array same way as we did for Mandelbrot fractal and the GUI component will handle the rest; the only difference here is that content of `data[]` array does not represent the speed of divergence but instead holds the indexes of roots in which observed complex point  $c$  has converged or 0 if no convergence to a root occurred. Another difference is that the upper limit to `data[i]` is number of roots, so we won't call observer with:

```
observer.acceptResult(data, (short)(m), requestNo);
```

but instead with:

```
observer.acceptResult(data, (short)(polynom.order()+1), requestNo);
```

If you completed this correct, for our first example with roots +1, -1, +i and -i you will get the following picture:



In order to solve this, you will have to download from Ferko repository `fractal-viewer-1.0.jar` and include it in *build-path* (save it into `lib` folder; *javadoc* is included as separate jar).

More verbose introduction to fractals based on Newton-Raphson iteration can be found at:  
<http://www.chiark.greenend.org.uk/~sgtatham/newton/>

## Details

Given the classes you developed in problem 1, the core of iteration loop can be written as:

```
Complex numerator = polynomial.apply(zn);  
Complex denominator = derived.apply(zn);  
Complex fraction = numerator.divide(denominator);  
Complex zn1 = zn.sub(fraction);  
module = zn1.sub(zn).module();  
zn = zn1;
```

Write a main program `hr.fer.zemris.java.fractals.Newton`. The program must ask user to enter roots as given below (observe the syntax used), and then it must start fractal viewer and display the fractal. In order to run this successfully, you will have to add classpath configuration argument in command line when starting java.

```
C:\somepath> java hr.fer.zemris.java.fractals.Newton
Welcome to Newton-Raphson iteration-based fractal viewer.
Please enter at least two roots, one root per line. Enter 'done' when done.
Root 1> 1
Root 2> -1 + i0
Root 3> i
Root 4> 0 - i1
Root 5> done
Image of fractal will appear shortly. Thank you.
```

(user inputs are shown in red)

General syntax for complex numbers is of form  $a+ib$  or  $a-ib$  where parts that are zero can be dropped, but not both (empty string is not legal complex number); for example, zero can be given as 0, i0, 0+i0, 0-i0. If there is 'i' present but no  $b$  is given, you must assume that  $b=1$ .

The implementation of `IFractalProducer` that you will supply must use parallelization to speed up the rendering. The range of  $y$ -s must be divided into  $8 * \text{numberOfAvailableProcessors}$  jobs. For running your jobs you must use `ExecutorService` based on `FixedThreadPool`, and you must collect your jobs by calling `get()` on provided `Future` objects. Do not create new `ExecutorService` for each call of method `produce`. Instead, create it in producer's constructor. Use a variant of `FixedThreadPool` which allows you to specify a custom `ThreadFactory` as last argument. Implement a `DaemonThreadFactory` that produces threads which have daemon flag set to `true` and pass an instance of this factory to the `newFixedThreadPool`; this way, your program won't hang once the GUI is closed.



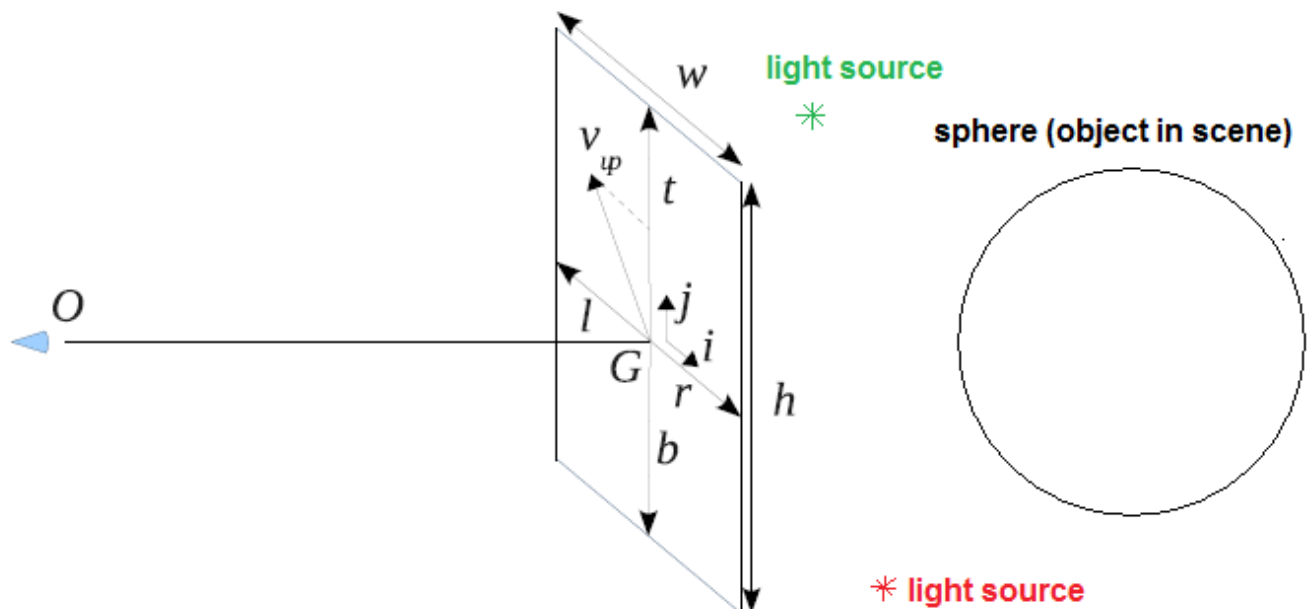
### Problem 3.

You will write a simplification of a ray-tracer for rendering of 3D scenes; don't worry – it's easy and fun. And also, we won't write a full-blown ray-tracer but only a ray-caster.

Please download from Ferko repository `raytracer-1.0.jar`; sources are available as separate jar. Save both jars in `lib` directory, add `raytracer-1.0.jar` to *build-path* and set `raytracer-1.0-sources.jar` as `raytracer-1.0.jar`'s sources file (right click, properties, ...). **Do not unpack raytracer-1.0-sources.jar** into your project. To better understand the needed theory, you are advised to download the book available at:

<http://java.zemris.fer.hr/nastava/irg/>

(version knjiga-0.1.2016-03-02.pdf) and read section 9.2 (Phong model, pages 231 to 236) and section 10.2 (Ray-casting algorithm, pages 241 to 244). To render an image using ray-casting algorithm, you start by defining which object are present in the 3D scene, where are you stationed (eye-position: O), where do you look at (view position: G) and in which direction is “up” (view-up approximation). See next image.



Now imagine that you have constructed a plane perpendicular to vector that connects the eye position (O) and the view point (G). In that plane you will create a 2D coordinate system, so you will have the x-axis (as indicated by vector  $i$  on the image) and the y-axis (as indicated by vector  $j$  on the image). If you only start with an eye-position and a view point, your y-axis can be arbitrarily placed in this plane (you could rotate it for any angle). To help us fix the direction of the y-axis, it is customary to specify another vector: the *view-up* vector which does not have to lay in the plane but it also must not be co-linear with G-O vector, so that a projection of this vector onto the plane exists. If this is true, then take a look at the projection of the view-up vector into the plane: we will use the normalized version of this projection to become our  $j$  vector and hence determine the orientation of y-axis.

Lets start calculating. Let:  $\vec{OG} = \frac{\vec{G} - \vec{O}}{\|\vec{G} - \vec{O}\|}$ , i.e. it is the normalized vector from  $\vec{O}$  to  $\vec{G}$ ; let

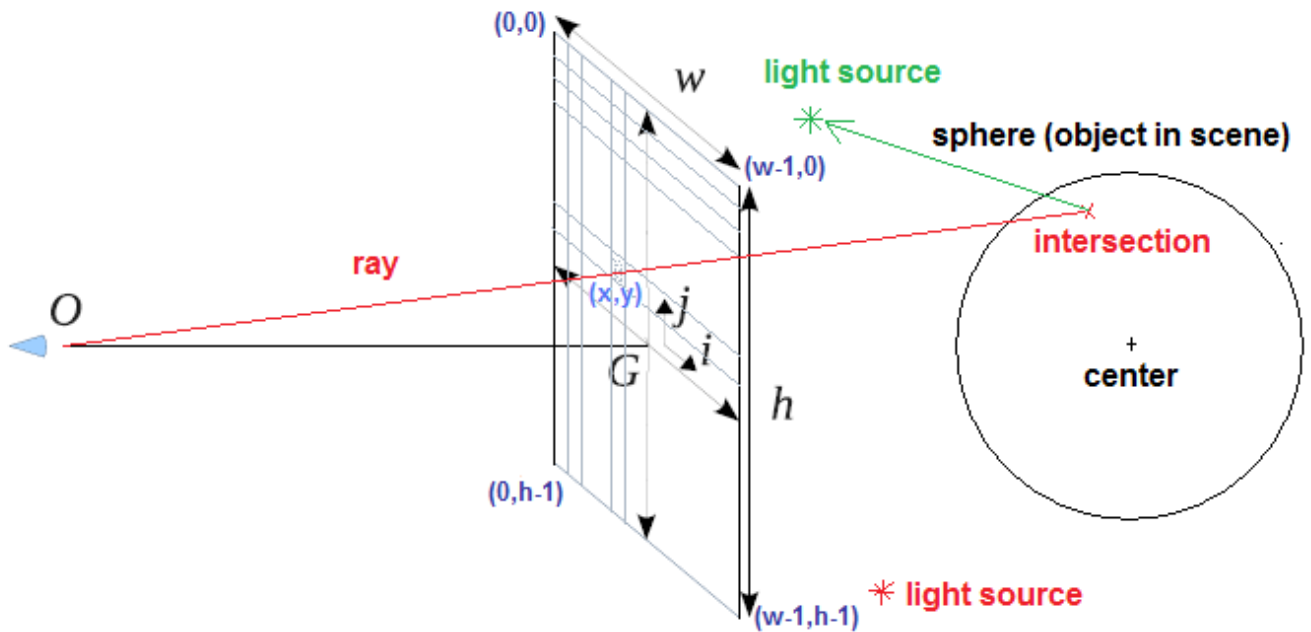
$\vec{VU}$  be normalized version of the view-up vector. Then we can obtain the  $\vec{j}'$  vector as follows:

$\vec{j}' = \vec{V}\vec{U}\vec{V} - \vec{O}\vec{G}(\vec{O}\vec{G} \cdot \vec{V}\vec{U}\vec{V})$  where  $\vec{O}\vec{G} \cdot \vec{V}\vec{U}\vec{V}$  is a scalar product. Define its normalized version to be:  $\vec{j} = \frac{\vec{j}'}{\|\vec{j}'\|}$ . Now we can calculate vector  $\vec{i}'$  which will determine the orientation of the x-axis as a cross product:  $\vec{i}' = \vec{O}\vec{G} \times \vec{j}$  and its normalized version  $\vec{i} = \frac{\vec{i}'}{\|\vec{i}'\|}$ .

Once we have determined where the plane is and what are the vectors determining our x-axis (i.e.  $\vec{i}$ ) and y-axis (i.e.  $\vec{j}$ ), we have to decide which part of this plane will be mapped to our screen so that we can determine where in this plane each screen-pixel is located (where is position (0,0), (0,1), etc.). We will assume it to be a rectangle going left from  $\vec{G}$  (i.e. in direction  $-\vec{i}$ ) for  $l$ , going right from  $\vec{G}$  (i.e. in direction  $\vec{i}$ ) for  $r$ , going up from  $\vec{G}$  (i.e. in direction  $\vec{j}$ ) for  $t$ , and finally going down from  $\vec{G}$  (i.e. in direction  $-\vec{j}$ ) for  $b$ . To simplify things further, let's assume that  $l=r=\frac{\text{horizontal}}{2}$  and  $t=b=\frac{\text{vertical}}{2}$  where we introduced two parameters: *horizontal* and *vertical*.

In provided libraries I have already prepared the class `Point3D` with implemented methods for calculation of scalar products, cross-products, vector normalization etc. so use it.

Now we will define final screen coordinate system, as shown in the next image.



We will define (0,0) to be the upper left point of our rectangular part of the plane; the x-axis will be oriented just as  $\vec{i}$  vector is, and the y-axis will be oriented opposite from  $\vec{j}$  vector. We can obtain the 3D coordinates of our upper-left corner as follows:

$$\text{corner} = \vec{G} - \frac{\text{horizontal}}{2} \cdot \vec{i} + \frac{\text{vertical}}{2} \cdot \vec{j}$$

Now for each  $x$  from 0 to  $w-1$  and for each  $y$  from 0 to  $h-1$  we can calculate the 3D position of the screen-pixel  $(x,y)$  in the plane as follows:

$$\text{point}_{xy} = \text{corner} + \frac{x}{w-1} \cdot \text{horizontal} \cdot \vec{i} - \frac{y}{h-1} \cdot \text{vertical} \cdot \vec{j}$$

And now it is simple: we define a ray of light which starts at  $\vec{O}$  and passes through  $\text{point}_{xy}$ . Then we

check if this ray which is specified by starting point  $\vec{O}$  and normalized directional vector

$$\vec{d} = \frac{\vec{point}_{xy} - \vec{O}}{\|\vec{point}_{xy} - \vec{O}\|}$$
 has any intersections with objects in scene! If an intersection is found, then that is

exactly what will determine the color of screen-pixel  $(x,y)$ . If no intersection is found, the pixel will be rendered black ( $r=g=b=0$ ). However, if an intersection is found, we must determine the color of the pixel. If multiple intersections are found, we must choose the closest one to eye-position since that is what the human observer will see. For coloring we will use Phong's model which assumes that there is one or more point-light-sources present in scene. In our example there are two light sources (one green and one red in the previous image). Each light source is specified with intensities of  $r$ ,  $g$  and  $b$  components it radiates.

Here is the pseudo code for the above described procedure:

```
for each pixel  $(x,y)$ 
  calculate ray  $r$  from eye-position to  $pixel_{xy}$ 
  determine closest intersection  $S$  of ray  $r$  and any object in the scene (in front of observer)
  if no  $S$  exists, color  $(x,y)$  with  $rgb(0,0,0)$  else use  $rgb(determineColorFor(S))$ 
```

The procedure `determineColorFor( $S$ )` is given by the following pseudocode:

```
set color =  $rgb(15,15,15)$  // i.e. ambient component
for each light source  $ls$ 
  define ray  $r'$  from  $ls.position$  to  $S$ 
  find closest intersection  $S'$  of  $r'$  and any objects in scene
  if  $S'$  exists and is closer to  $ls.position$  than  $S$ , skip this light source (it is obscured by that object!)
  else color += diffuse component + reflective component
```

## Details

Go through sources of `IrayTracerProducer`, `IrayTracerResultObserver`, `GraphicalObject`, `LightSource`, `Scene`, `Point3D`, `Ray` and `RayIntersection`. Create package `hr.fer.zemris.java.raytracer.model` in your homework and add class `Sphere`:

```
package hr.fer.zemris.java.raytracer.model;

public class Sphere extends GraphicalObject {
    ...

    public Sphere(Point3D center, double radius, double kdr, double kdg,
        double kdb, double krr, double krg, double krb, double krn) {
        ...
    }

    public RayIntersection findClosestRayIntersection(Ray ray) {
        ...
    }
}
```

and implement all that is missing. Until you do that, the method which is used to build the default scene will not work (`RayTracerViewer.createPredefinedScene()`). Coefficients  $kd^*$  determine the object parameters for diffuse component and  $kr^*$  for reflective components.

Write a main program `hr.fer.zemris.java.raytracer.RayCaster`. The basic structure of the program should look like this:

```
public static void main(String[] args) {
    RayTracerViewer.show(getIRayTracerProducer(),
        new Point3D(10,0,0),
        new Point3D(0,0,0),
        new Point3D(0,0,10),
        20, 20);
}

private static IRayTracerProducer getIRayTracerProducer() {
    return new IRayTracerProducer() {

        @Override
        public void produce(Point3D eye, Point3D view, Point3D viewUp,
            double horizontal, double vertical, int width, int height,
            long requestNo, IRayTracerResultObserver observer) {

            System.out.println("Započinjem izračune...");
            short[] red = new short[width*height];
            short[] green = new short[width*height];
            short[] blue = new short[width*height];

            Point3D zAxis = ...
            Point3D yAxis = ...
            Point3D xAxis = ...

            Point3D screenCorner = ...

            Scene scene = RayTracerViewer.createPredefinedScene();

            short[] rgb = new short[3];
            int offset = 0;
            for(int y = 0; y < height; y++) {
                for(int x = 0; x < width; x++) {
                    Point3D screenPoint = ...
                    Ray ray = Ray.fromPoints(eye, screenPoint);

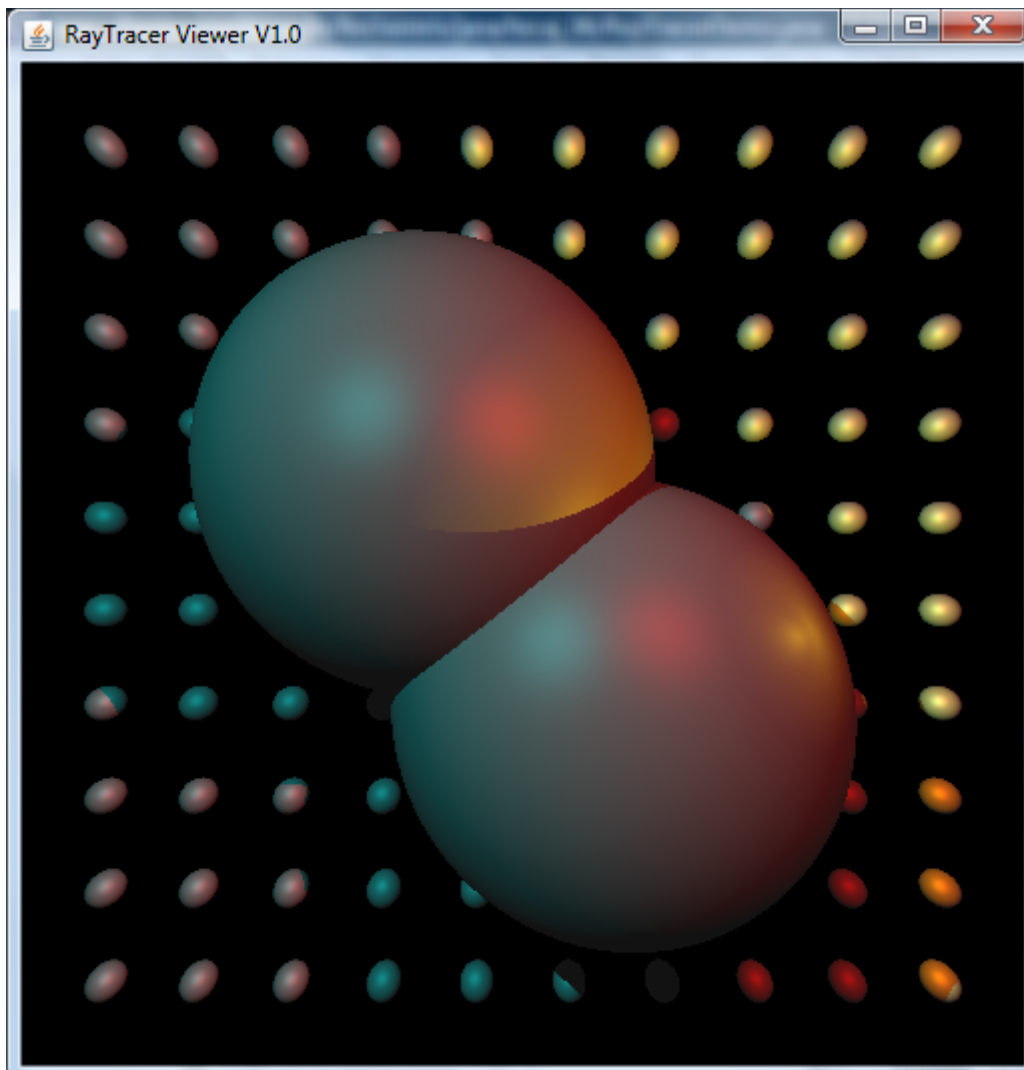
                    tracer(scene, ray, rgb);

                    red[offset] = rgb[0] > 255 ? 255 : rgb[0];
                    green[offset] = rgb[1] > 255 ? 255 : rgb[1];
                    blue[offset] = rgb[2] > 255 ? 255 : rgb[2];

                    offset++;
                }
            }

            System.out.println("Izračuni gotovi...");
            observer.acceptResult(red, green, blue, requestNo);
            System.out.println("Dojava gotova...");
        }
    };
}
```

Fill the missing parts! If you do this OK, you will get the following image.



Now if this goes OK, please observe that calculation of color for each pixel is independent from other pixels. Using this knowledge write a main program  
`hr.fer.zemris.java.raytracer.RayCasterParallel` which parallelizes the calculation using Fork-Join framework and `RecursiveAction`.

**Please note.** You can consult with your peers and exchange ideas about this homework *before* you start actual coding. Once you open you IDE and start coding, consultations with others (except with me) will be regarded as cheating. You can not use any of preexisting code or libraries for this homework (whether it is yours old code or someones else). You can use Java Collection Framework and other parts of Java covered by lectures; if unsure – e-mail me. Document your code!

**If you need any help, I have reserved a slot for consultations every day from Monday to Thursday at 1 PM with exception of Tuesday when the slot starts at 2 PM. Feel free to drop by my office.**

All source files must be written using UTF-8 encoding. All classes, methods and fields (public, private or otherwise) must have appropriate javadoc.

You are expected to write tests for Vector3 and Complex in problem 1. In order to do so, go in your Eclipse project build-path, Add library, JUnit 4. Create new source folder (test) and place tests in it.  
You are encouraged to write tests for other problems.

When your **complete** homework is done, pack it in zip archive with name `hw09-0000000000.zip` (replace zeros with your JMBAG). Upload this archive to Ferko before the deadline. **Do not forget to lock your upload** or upload will not be accepted. Deadline is May 5<sup>th</sup> 2017. at 07:00 AM.