The Carbon Tax as an Automatic Stabilizer in a Commodity-Producing Small Open Economy

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Abstract

In this paper, we evaluate the role of carbon taxes as automatic stabilizers in small open economies (SOEs) that specialize in the export of a single commodity, particularly those highly dependent on energy inputs for production. Specifically, we examine the carbon tax's ability to reduce the volatility of the real exchange rate and energy prices. This analysis is conducted through the lens of a DSGE model that incorporates an externality affecting GDP, originating from the burning of fossil fuels for energy generation. We assume this externality drives climate change, and the government, aiming to internalize these damages, imposes a Pigouvian tax on the energy sector. Our model is calibrated for the Chilean economy, which is highly specialized in copper production. The results show that the tax: (i) reduces energy volatility by 14% and energy price volatility by 10%, and (ii) lowers the variance of the real exchange rate by 1.8%. These stabilizing effects are robust to different shock specifications and the choice of model used to represent household consumption.

Keywords: Climate Change, Carbon Tax, CO2 Emissions, Real Exchange Rate, Optimal Taxation, DSGE

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1 Introduction

One of the most urgent global challenges today is climate change (Nordhaus, 2019). The Paris Agreement, signed in 2016, aims to limit the rise in global surface temperature to well below 2°C (3.6°F) above pre-industrial levels. To meet these objectives, countries are expected to reach net-zero carbon emissions by the middle of the 21st century. In line with these global efforts, the development of renewable energy and the pursuit of energy security have become key strategies for many nations. Energy security involves not only ensuring sufficient energy supply but also managing the volatility of energy quantities and prices. This issue is particularly relevant for small open economies that specialize in the export of energy-intensive commodities, as commodity shocks can disrupt energy security.

Energy security can be influenced by environmental policies aimed at achieving carbon neutrality. These economic policies seek to limit emissions by adjusting the incentives of economic agents, aligning them with the social costs and benefits of additional emissions (Timilsina, 2022). For instance, in small open economies, carbon pricing — through taxes on energy production that generates CO2 emissions — can discourage resource depletion by increasing extraction costs. Additionally, carbon taxes can serve as a buffer against commodity shocks by reducing both the volatility of commodity prices and energy prices, thereby enhancing energy security.

Thus, our main contribution is to quantify the effect of a carbon tax as an automatic stabilizer for enhancing energy security in a commodity-producing small open economy. This is possible because carbon taxes can act as automatic stabilizers by regulating energy consumption and prices in response to economic shocks. This stabilizer effect can be even stronger in small open economies that import oil, gas or other sources of energy to produce an exporting good. This is because the carbon tax, in addition to reduce the energy price volatility, can smooth real exchange rate fluctuations. To do so, we formulate a dynamic stochastic general equilibrium (DSGE) model with a climate change externality applied to the Chilean economy. The imposition of an optimal carbon tax seeks to control the level of emissions. One of the many benefits that a Pigouvian tax may carry is the ability to reduce the variance of the energy consumed, the energy price, and real exchange rate.

In our framework, we use a carbon taxes as an instrument for controlling the emissions levels. This is in line with papers that explicitly model the externality by using Dynamic Stochastic General Equilibrium models (DSGE). For example, Golosov et al. (2014), build a DSGE model that features a climate change externality that has a negative impact in the global economy productivity[1]. This idea is inspired by the work of Nordhaus and Boyer (2003), who includes the externality term in the economy production function. They assume that it takes the form of a quadratic cost term, mapping global temperatures into GDP damages. Instead, we follow Golosov et al. (2014) model by considering the externality directly as a function of CO2 stock damages. This, along with a simple process for the carbon cycle structure and log utility function in consumption allow to express the Pigouvian tax as a closed formula of exogenous parameters. These are, discounting, the externality damage and the carbon cycle process terms. Following this approach, we equalize the solutions of the central planner problem and the decentralized economy to obtain a similar formula for the carbon tax magnitude in our model.

The model treats Chile as a small open economy that exports a single commodity to the foreign markets. In this context, being a small economy implies that it cannot influence the international markets and therefore, acts as a price taker for the imported and exported goods. We have assumed that the exported good is copper as it is the most important commodity produced in Chile, accounting for half of the value of total exports. This is produced with energy inputs that are demanded from the firms producing the externality. Moreover, the presence of international trade defines an implicit real exchange rate that measures the relative price of a consumption basket between Chile and the rest of the economies.

In this regard, we contribute to the broader macroeconomic literature on optimally selected carbon taxes to improve welfare (Nordhaus, 2008, 2018; Golosov et al., 2014; Van der Ploeg and Rezai, 2021), as well as to the emerging literature on the effects of environmental policy on business cycles (Fischer and Springborn, 2011; Annicchiarico et al., 2021; Annicchiarico and Diluiso, 2019; Holladay et al., 2019; Economides and Xepapadeas, 2019). Additionally, we extend and complement Espinosa and Fornero (2014), who calibrates the welfare effects of a carbon tax using a DSGE model for the Chilean economy. Our results show that the carbon tax: (i) reduces emissions by roughly 8% and increases the energy price by 11%, (ii) is a welfare-improving policy, (iii) decreases energy volatility by 14% and price volatility by 10%, and (iv) reduces the variance of the real exchange rate by 1.8%.

As a robustness check, we extend our model by considering the environment as a non-renewable resource, following Van der Ploeg and Rezai (2021), and by allowing a proportion of consumption to be imported. In this context, the carbon tax can function as an automatic stabilizer in a small open economy.

[1]We have chosen to follow this approach by modeling the externality term in the aggregate production function. Alternatively, it could have been incorporated into the utility function; however, this would require additional assumptions regarding the specification and parameter values. Heutel (2012) demonstrated that this choice is not particularly significant, as his simulations using both specifications produced very similar results.

The rest of the paper is organized as follows: Section 2 is a literature review of environmental policies as automatic stabilizer. Section 3 presents the baseline model. Section 4 presents the calibration of key parameters and the baseline simulations. Section 5 presents two extensions for the baseline model. Section 6 presents a policy discussion on the main results, and section 7 concludes.

2 Review of environmental policies as automatic stabilizer

2.1 Environmental policies as automatics stabilizers

The definition of automatic stabilizers could include "any components of the government budget that act to offset fluctuations in effective demand by reducing taxes and increasing government spending in recession, and doing the opposite in expansion." (Auerbach and Feenberg, 2000). In this context a carbon tax can be understood as a countercyclical policy, similar to an income tax or corporate tax (Buettner and Fuest, 2010; Hüseyin and Ayşe, 2013; Kniesner and Ziliak, 2002). The feature of automatic stabilization of carbon regulation policies has been so recent that it has not yet been able to address the most pressing questions or has not yet been properly communicated to policymakers (Annicchiarico et al., 2021). The literature on business cycles and environmental policy started with Fischer and Springborn (2011) argue that Cap and trade policies are more countercyclical than carbon taxes. For instance, shocks that are detrimental to output reduces the demand for emissions lowering the permits prices implying a relief on firm's costs. Dissou and Karnizova (2016) finds that the countercyclical effect of carbon regulation policies depends on the type of shock. To do so, this paper uses a multisectorial model to show that both instruments generates similar effects on the business cycle when there is a shock in a non-energy sector. However, for a shock to the energy sector, cap and trade has a more countercyclical effect rather than carbon taxes but with a higher welfare cost. The literature has also examined the impact of environmental policy shocks on the business cycle. Indeed, Xiao et al. (2018) calibrate a DSGE model for China showing that the environmental policies are countercyclical. In particular, this paper simulate the effects of carbon taxes, emissions cap and emissions intensity shocks on the business cycle showing that emissions intensity shock will exert greater impacts than environmental tax rate shock and emissions cap shock.

2.2 Environmental policies, the open economy, and the dutch disease

In small open economies, one candidate to explain the volatility of growth in income per capita is the volatility of commodity prices. This includes not only oil, but also, copper or other mineral prices. Indeed, Blattman et al. (2007) show that countries that specialize in commodities with substantial price volatility have more volatility in their terms of trade, enjoy less foreign direct investment, and experience lower growth rates than countries that specialize in commodities with more stable prices or countries that are industrial leaders. In this regard, Van der Ploeg and Poelhekke (2009) includes direct effect of natural resource dependence on growth and, the indirect effect of natural resources on growth performance via volatility. This paper finds that the effect of natural resources on growth performance may well be positive. However, they quantify that the indirect effect of natural resources on growth via the volatility channel is negative. Indeed, the direct positive effect of resources on growth is swamped by the indirect negative effect through volatility. The relationship between natural resources extraction and uncertainty in the non-renewable natural resource demand is analyzed in the context of precautionary savings. Van der Ploeg (2010) shows that a government with prudent behavior depletes oil reserves even more aggressively and engages in additional precautionary saving financed by postponing spending and bringing taxes forward.

Also, in small open economies with a primary commodity exporter "the wealth increases following higher commodity prices or resource discoveries have a systematic impact on the sectoral allocation of resources. Booming demand, caused by higher wealth, leads to a shift of an economy's productive resources from tradable-goods sectors to non-tradable goods sectors. The squeeze of the tradable sector in this context has become known as the 'Dutch disease'." (Bruno and Sachs, 1982). To avoid this phenomenon, the traditional policy prescription is to intertemporally smooth the resource windfall. Indeed, adding volatility to the commodity prices precautionary buffers should be put in a stabilization fund (Van Der Ploeg, 2019).

One way to include volatility in the analysis of environmental policies as automatic stabilizers on open economy is by the use of variants of Environmental Dynamic Stochastic General Equilibrium (E-DSGE) models. E-DSGE has been used to analyze and quantify the cross-country spillovers from pollution policies and international environmental regulation adding volatility. Indeed, Annicchiarico and Diluiso (2019) use a two-country New Keynesian E-DSGE model to show that environmental regulations, such as carbon taxes or cap-and-trade systems, affect the international transmission of shocks. Carbon taxes amplify cross-border spillovers, while cap-and-trade systems reduce them. The channels of transmission—demand and competitiveness—depend on the environmental regime and exchange rate system. Economides and Xepapadeas (2019) focus on the impact of climate change on output and competitiveness in a small open economy. Their model shows that climate change reduces productivity, regardless of the exchange rate regime, leading to output losses and a worsening trade balance. Holladay et al. (2019) examine pollution taxes, cap-and-trade, and emissions targets in a small open economy model. Their findings suggest that cap-and-trade regulations can dampen business cycle fluctuations, especially during productivity shocks, by stabilizing trade flows. The cap-and-trade system reduces imports during recessions and limits the expansion of imports when foreign prices decline, thus mitigating business cycle effects on the trade balance.[2]

Given this, in the following section, we present a DSGE model for a small open economy that specializes in the export of a commodity sector that is highly energy-intensive. The objective is to quantify the role of the carbon tax as an automatic stabilizer. The main intuition we aim to convey is that, following a positive copper price shock, the mining sector demands more energy inputs to increase production, leading to higher energy prices. This increase in both copper and energy prices causes a real appreciation of the exchange rate. With the introduction of the carbon tax, the energy price rises further. In this scenario, the mining sector still increases its demand for energy inputs in response to the copper price shock; however, the higher energy price level means that the sector will demand less electricity than it would have in the absence of the tax. As a result, the upward pressure on prices is mitigated, leading to a smaller real exchange rate appreciation. Consequently, the carbon tax reduces both the volatility of the real exchange rate and the variance in energy use following the shock.

3 A DSGE model for carbon taxes in a small and open economy

In this section we present a highly stylized DSGE model that allows to understand the effects of carbon taxes in the Chilean economy. For this purpose, we have modeled the energy sector structure, while adding exports of a commodity that it's sold in the international markets. In addition, we add the Nordhaus and Boyer (2003) form of externality in the aggregate production, we follow Golosov et al. (2014) rule of CO2 taxation, and we follow Espinosa

[2]Regarding financial macroeconomic stability, Diluiso et al. (2021) show that financial regulation, which promotes the decarbonization of banks' balance sheets through capital requirements, can smooth the economic cycle by reducing the severity of financial crises but may also prolong the recovery phase. For a more detailed review on the financial effects of environmental policies see Daumas (2024)

and Fornero (2014) to model a carbon tax in an small economy. As our main focus is the real exchange rate, we introduce it in order to keep the trade balance in equilibrium.

The model consist of six key sectors: (i) A representative agent that reproduces the households decisions by maximizing an utility function subject to a budget constraint, (ii) energy firms that supply energy to the rest of the economy by importing fossil fuels and releasing CO2 into the atmosphere as a byproduct of the process , (iii) a copper sector that accounts for exports, (iv) a domestic sector that produces a non tradable good, (v) a government that set taxes on energy firms to correct the externality and, (vi) a real exchange rate that is consistent with an equilibrium level of the trade balance.

The representative agent represent households decisions and maximizes a utility function that depends on consumption of non tradable goods, c_t, residential electricity, e_{rt} and leisure, $1 - l_t$. The function is assumed to satisfy the Inada conditions and to be strictly concave in each of their arguments.

The agent is the owner of the firms and in each period maximizes the utility subject to the budget constraint. This states that the profits and rents from capital and labor are used for consumption of goods, electricity, and investment. The problem can be then summarized by the following equations:

$$\max_{c_t, e_{rt}, l_t, i_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t, e_{rt})$$

$$st\ c_t(1 + \tau_{c,t}) + p_t e_{rt} + i_t \leq r_t k_t + w_t l_t + \Pi_{gt} + \Pi_{ct} + \Pi_{et} + T_t \tag{1}$$

$$k_{t+1} = i_t + (1 - \delta) k_t \tag{2}$$

$$k_0 > 0 \text{ given,} \tag{3}$$

where the agent discount future streams with parameter β, the consumption of nontradable good is defined to be the *numèraire* and therefore, all other prices are defined in relative terms to it. There is a tax, τ_{ct}, on the consumption of these goods, while p_t denotes the residential electricity price.

The agent draw income from capital renting ($r_t k_t$) and supplying labor ($w_t l_t$) to the nontradable good sector firm. Investment moves according to the law of motion of capital, which depreciates at the rate δ. As households owns the firms, they collect profits from Π_{gt}, Π_{ct}, and Π_{et}, the good, copper and energy firms, respectively. Finally, the government set *lump sum* transfers, T_t every period.

The first order conditions yields the standard intertemporal Euler equation that allocates consumption over different periods:

$$U'_{c_t} = \beta \mathbb{E}_t \left[U'_{c_{t+1}} \frac{1+\tau_{c,t}}{1+\tau_{c,t+1}} \left(1 + r_{t+1} - \delta\right) \right]$$

Furthermore, the two intratemporal first order conditions relates consumption to labor and residential energy optimally allocation in every period :

$$\begin{aligned} \frac{U'_{c_{rt}}}{U'_{ert}} &= \frac{1+\tau_{c,t}}{p_t} \\ \frac{U'_{c_t}}{U'_{1-l_t}} &= \frac{1+\tau_{c,t}}{w_t} \end{aligned}$$

3.1 Energy firms

Energy firms maximizes profits by importing fossil fuels and using them to produce electricity. Total Energy, E_t is sold to good firms, copper firms and to households. To produce energy, it is necessary to burn the fossil fuels inputs, consisting of fuel, diesel or natural gas. During this process, CO2 is freed into the atmosphere, which ultimately has a negative impact in the rest of the economy. In particular, energy firms face the following problem:

$$\begin{aligned} \max_{M_t, E_t} \Pi_{et} &= (p_t - \tau_{et})E_t - p_t^m M_t \\ st\ E_t &= F_e(M_t) && (4) \\ \ln p_t^m &= (1-\rho_m)\ln \overline{p_t^m} + \rho_m \ln p_{t-1}^m + \epsilon_{mt}, \quad \epsilon_{mt} \sim (0, \sigma_m^2) && (5) \end{aligned}$$

where $F_e(\cdot)$ is a production function assumed to be a strictly concave. The energy production depends on the imported input M_t at a price of p_t^m. This is assumed to be exogenously determined in the rest of the world, consistent with the small open economy assumption, and follows an $AR(1)$ process.

The government levies a tax, τ_{et}, on energy sales, which produces an increase in energy prices, p_t, seeking to correct the negative effect of the externality.

The first order condition balances the value of the marginal productivity with the importing cost:

$$(p_t - \tau_{et})F'_{e,M_t} = p_t^m. \tag{6}$$

3.2 Copper sector

The representative firms in this sector maximize real profits by demanding electricity to produce copper. All the production is exported to the foreign markets.

Here, we assume that copper is the only exported commodity. Although this assumption is somewhat simplistic, it is standard in the literature (see, for example, Medina et al., 2007; Garcıa et al., 2019; Fornero and Kirchner, 2018), given that the copper sector accounts for approximately 10% of GDP and 40% of total exports. We only include energy in the production function to specifically account for the energy usage and associated pollution of the copper industry as the mining sector consumes about 34% of Chile's total energy production, and its contributions to national employment and capital average 2% and 14%, respectively, over the years of our calibration.[3] Once again, even though this assumption may seem simplistic, the literature has often modeled this sector as exogenous (Medina et al., 2007) or dependent on a single factor (Fornero and Kirchner, 2018). In addition, the optimal carbon tax is invariant on this assumption. With this, we can summarize the firm's problem as:

$$\begin{aligned}
\max_{e_{ct}, y_{ct}} \Pi_{ct} &= p_t^x y_{ct} - p_t e_{ct} \\
st \ y_{ct} &= F_c(e_{ct}) \\
\ln p_t^x &= (1-\rho_{px}) \ln \overline{p^x} + \rho_{px} \ln p_{t-1}^x + \epsilon_{px,t}, \quad \epsilon_{px,t} \sim (0, \sigma_{px}^2)
\end{aligned} \tag{7}$$

where $y_{ct} = F_c(\cdot)$ is a strictly concave production function in electricity inputs. We have assumed that the cooper real price p_t^x is exogenous an follows an $AR(1)$ process with an unconditional mean of $\overline{p^x}$. This assumption is in line with the small open economy literature as it takes the international prices as given[4]. The first order condition states that the value of the marginal productivity of energy equals the marginal costs, which in this case, is the energy price:

$$p_t^x F'_{c,e_{ct}} = p_t.$$

3.3 Domestic good's sector

The representative firm of the domestic good's sector maximize profits by hiring labor, renting capital and purchasing electricity to produce the non tradable consumption good.

The production function, $F_g(A_t, S_t, e_{gt}, l_t, k_t)$ depends negatively on the externality, S_t. As more electricity is demanded, more emissions of CO2 is released to the atmosphere, increasing the stock levels and ultimately, triggering a fall in output that dampens consumption and investment. Hence, aggregate GDP in this economy is the production made by the

[3]The source of this information is the Statistics Database of the Chilean Central Bank, and it is available here and here.

[4]Although Chile accounts for nearly 30% of world copper production and therefore, has some market power in the international copper price, we assume for simplicity that it is a pricer taker

domestic good sector. With this, the problem then becomes:

$$
\begin{aligned}
\max_{e_{gt},k_t,l_t} \Pi_{gt} &= y_{gt} - p_t e_{gt} - w_t l_t - r_t k_t \\
st\ y_{gt} &= F_g(A_t, S_t, e_{gt}, l_t, k_t) \\
S_t &= L(E_t) \qquad (8) \\
\ln A_t &= \rho_a \ln A_{t-1} + \epsilon_{at}, \quad \epsilon_{at} \sim (0, \sigma_a^2) \qquad (9)
\end{aligned}
$$

where y_{gt} is a strictly concave production function in electricity and labor. The technical change term A_t is assumed to follow an exogenous $AR(1)$ process. Labor wage is denoted by w_t and p_t is the energy price relative to the consumption price.

The function $L(\cdot)$ represents how total energy converts into CO2, which relates directly to the carbon cycle structure. This process describes the way in which CO2 released into the atmosphere increase stock levels after producing a unit of energy. Optimal conditions with respect to inputs equalize the value of the marginal product to their prices:

$$
\begin{aligned}
F'_{g,e_{gt}} &= p_t \qquad (10) \\
F'_{g,l_t} &= w_t \qquad (11) \\
F'_{g,k_t} &= r_t \qquad (12)
\end{aligned}
$$

3.4 Government

The government balances it's budget every period. This is, it taxes the nontradable consumption good and set taxes to energy firms. The aim of the latter tax is to correct the externality produced in the burning process of fossil fuels. Tax revenue is returned to households as *lump sum* transfers. Formally:

$$T_t = \tau_{c,t} c_t + \tau_{et} E_t$$

3.5 Real exchange rate

The real exchange rate (RER)[5] is the amount of local good needed to get a foreign one. A rise of the RER implies a real depreciation, meaning that more home goods are needed for a foreign one, or equivalently, local goods become cheaper. The conversely applies for the case of a real appreciation. Formally, the real exchange rate is defined as:

[5]From this point onward, we will use "RER" to refer to the real exchange rate.

$$RER_t = \frac{\epsilon_t P_t^*}{P_t}$$

Where ϵ_t is the nominal exchange rate and measures the amount of local currency needed to purchase a unit of foreign currency[6]. Prices P_t and P_t^* are the home and foreign indices respectively.

It is useful to express the RER in terms of tradable and non tradable goods. For this purpose, we can express each of the price indices as a geometric weighted average of tradable and nontradable components. In particular, assume:

$$\begin{aligned} P_t &= \left(P_t^t\right)^{1-\alpha} \left(P_t^{nt}\right)^{\alpha} \\ P_t^* &= \left(P_t^{t^*}\right)^{1-\alpha^*} \left(P_t^{nt^*}\right)^{\alpha^*} \end{aligned}$$

Where P_t^t and $P_t^{t^*}$ stands for the tradable prices in the home economy and abroad, while P_t^{nt} and $P_t^{nt^*}$ for the non tradable goods. The share of the non tradable component in the price index is captured by α and α^*, for the home and foreign economy respectively. This allow us to express the real exchange rate as:

$$RER_t = \left(\frac{\epsilon_t P_t^{t^*}}{P_t^t}\right) \left(\frac{P_t^t}{P_t^{nt}}\right)^{\alpha} \left(\frac{P_t^{nt^*}}{P_t^{t^*}}\right)^{\alpha^*}$$

In this regard, we follow Neary (1988) in the definition of the real exchange rate as "a fixed-weight index number of the relative prices of nontraded goods divided by a price index for traded goods". Instead of using an arithmetic average, we use a geometric average. Assuming that the law of one price holds for the tradable goods, we can get rid of the first term in the right hand side of the last equation. In effect, the nominal exchange rate adjust to ensure that the tradable prices are equalized between the local and the foreign economy. Defining $p_t^c = \frac{P_t^t}{P_t^{nt}}$ and $p_t^{c^*} = \frac{P_t^{t^*}}{P_t^{nt^*}}$, we can rewrite the RER as:

$$RER_t = \left(p_t^c\right)^{\alpha} \left(p_t^{c^*}\right)^{-\alpha^*}$$

The model involves four different prices, two endogenous and two exogenous. The first ones corresponds to the price of nontradable goods, normalized to one,[7] and the energy price, p_t. The other exogenous are the export and import price, p_t^x and p_t^m respectively. In this

[6]For the Chilean economy, it is measured in terms of Chilean pesos for U.S. dollar.

[7]Strictly speaking, since a tax was levied on the consumption of the nontradable good, its price is $1 + \tau_{c,t}$. For the simplicity of the exposure, we assume this as one.

context, they relate to the RER by:

$$p_t^c = \left(\frac{p_t^m}{p_t}\right) \text{ and } p_t^{c^*} = \left(\frac{p_t^x}{p_t^{nt^*}}\right)$$

Where $p_t^{nt^*}$ denotes the non tradable price in the foreign economy, which we will further assume that it is normalized to one.[8] Moreover, we assume that the proportion of nontradables in the price index in the home and the foreign economy are the same.[9] With these, the real exchange rate can be rewritten as:

$$RER_t = \left(\frac{p_t^x p_t}{p_t^m}\right)^{-\alpha}$$

Here, an improvement of the terms of trade, this is, the ratio of export to import prices, or an increase of the domestic energy prices, produces a real appreciation. Finally, it is important to note that as this economy is under financial autarky, in which Heathcote and Perri (2002) define as an environment where households have no access to international asset trade. Consequently, goods trade must be *quid pro quo*, meaning that the real exchange rate has to adjust to ensure that the trade balance is in equilibrium.

3.6 Equilibrium conditions

The following markets clear at any period:

$$\begin{aligned}
c_t + i_t + p_t^m M_t &= F_g(A_t, S_t, e_{gt}, k_t, l_t) + p_t^x F_c(e_{ct}) && (13)\\
E_t &= e_{gt} + e_{ct} + e_{rt} && (14)\\
k_t^s &= k_t^d && (15)\\
l_t^s &= l_t^d && (16)
\end{aligned}$$

The first equation states that aggregate output is equal to total consumption investment and imports. Aggregate production is divided into the economy output (the good firms sector) and the copper production. The second, third and fourth equation corresponds to energy, capital and labor market clearance conditions.[10]

[8] This assumption is innocuous since all that is needed is for it to remain constant. Hence, the level choice is irrelevant.

[9] Indeed, $\alpha = \alpha^*$.

[10] Please refer to the Appendix A.1 for a complete characterization of the decentralized model optimal conditions

3.7 The centralized problem

We now turn to the planning problem in order to derive the optimal tax for this economy. Following Golosov et al. (2014), we can adapt the optimal tax formula to this economy and arrive to an expression for the social cost of carbon. In effect, the Pigouvian tax allows to equalize the decentralized with the planning problem.[11]

The planner problem can be written as:

$$\max_{c_t, e_{gt}, e_{mt}, e_{rt}, E_t, S_t, k_{t+1}, k_t, l_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1-l_t, e_{rt})$$

subject to equations (22), (4), (5), (7), (8), (9), (13) and (14). As the planner maximizes over the resource constraint of the economy, accounts for the negative effect of the increased emissions in total GDP. In this context, equation (8) represents the carbon cycle structure through the function $S_t = L(E^t)$. Here, $E^t \equiv \{E_{-T}, E_{-T+1}, \ . \ . \ . \ , E_t\}$ represent the history of energy production and function $L(\cdot)$ captures how emissions relates to the stock of CO2 in the atmosphere.

3.8 Parametric Assumptions

In order to characterize the optimal tax in a better way, we make further assumptions about the functional forms of some of the key equations of the model:

Assumption 1 *The utility function is of constant relative risk aversion (CRRA) form and absolutely separable in it's arguments*

$$U_t(c_t, 1-l_t, e_{rt}) = \frac{c_t^{1-\theta_c}}{1-\theta_1} + \Omega_l \frac{(1-l_t)^{1-\theta_l}}{1-\theta_l} + \Omega_r \frac{e_{rt}^{1-\theta_r}}{1-\theta_r} \tag{17}$$

This general form for the utility function provides flexibility to replicate empirical regularities in steady state. In particular, we will assume that the utility function is logarithmic in consumption. This will allow to have a closed formula for the optimal tax.

Assumption 2 *The final goods firm's production function is equal to:*

$$F_g(A_t, S_t, e_{gt}, l_t, k_t) = (1 - D_t(S_t))\tilde{F}_g(A_t, e_{gt}, l_t, k_t)$$

Where $1 - D_t(S_t) = \exp\left(-\gamma[S_t - \bar{S}]\right)$, *and* $\tilde{F}_g(A_t, e_{gt}, l_t, k_t) = A_t e_{gt}^{\alpha_g} k_t^{\eta} l_t^{\nu}$.

[11] The welfare theorems states that, if the tax accounts for the social cost of carbon, then the planner and decentralized solutions must coincide.

The assumption of Cobb-Douglas production functions is rather standard in the literature. In addition, the A_t terms is a neutral technology shock and the externality, $1 - D_t$, is an exponential function that depends on the current stock of CO2 in the atmosphere. This assumption is widely used in the literature (see, for example, Golosov et al., 2014; Nordhaus, 2018; Van der Ploeg and Rezai, 2021). The intuition is that the damage function captures the relationship between the stock of carbon dioxide in the atmosphere and economic damages, measured as a percentage of final-good output. This relationship can be interpreted in two ways: either as a direct effect on overall production or as an effect on productivity. Notably, in Van der Ploeg and Rezai (2021), this term is incorporated into the process of total productivity of factors. Finally, γ measures the damage of a extra unit of CO2 emission in the atmosphere.

Assumption 3 *Mining and energy production functions are of the Cobb-Douglas form:*

$$\begin{aligned} F_c(e_{ct}) &= e_{ct}^{\alpha_c} \\ F_e(M_t) &= M_t^{\alpha_e} \end{aligned}$$

Both, the copper and energy sectors exhibit decreasing return to scale in their arguments and the parameters represent the input's share in the value of the production of each sector.

Assumption 4 *Following Engström (2012), we can express the depreciation structure of the carbon cycle, the $L(\cdot)$ function of equation (8), as:*

$$S_t = (1-\varphi)S_{t-1} + \varepsilon E_t = \sum_{i=0}^{t+T}(1-d_i)E_{t-i} = \sum_{i=0}^{t+T}\varepsilon(1-\varphi)^i E_{t-i} \tag{18}$$

The carbon cycle is a process in which the CO2 emissions is exchanged through the different reservoirs of the earth. This process is dynamic, as in every period, the fossil fuels that are burned for the energy generations, emits CO2 that enters into the atmosphere circulation and consequently, increase the concentration levels. These, build over time and are responsible for the climate change produced by the greenhouse effect.

The carbon process is determined by two key parameters. Firstly, φ, denotes the rate of CO2 removal from the atmosphere. Secondly, the airborne fraction, , ε, is the rate at which the emissions stay in the atmosphere.

This balance is crucially driven by two parameters: Firstly, φ, which denotes the rate of CO2 removal from the atmosphere. Secondly, the airborne fraction, ε, that determines the amount of emitted carbon dioxide that remain in the atmosphere.[12] Together, they determine how much of carbon emissions will be left in the atmosphere t periods ahead.[13]

3.9 Optimal tax characterization

We can further characterize the optimal tax by making use of assumptions 1 to 4:

$$\Lambda_t^s = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \frac{c_t}{c_{t+i}} \gamma Y_{t+i}(1-d_i)$$

We can divide by the GDP to express the tax as a fraction of the economy product[14]:

$$\hat{\Lambda}_t^s \equiv \frac{\Lambda_t^s}{Y_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \frac{c_t}{Y_t} \frac{Y_{t+i}}{c_{t+i}} \gamma(1-d_i) \tag{19}$$

If we assume constant saving rates[15], and replace (18) into (30), we get:

$$\hat{\Lambda}_t^s = \sum_{i=0}^{\infty} \beta^i \gamma(1-d_i) = \frac{\gamma\varepsilon}{1-\beta(1-\varphi)} \tag{20}$$

This formula allows to express the optimal tax as a function of only exogenous variables. To achieve this, we have assumed logarithmic utility function in consumption, a linear depreciation structure of the carbon cycle and constant saving rates. This formula slightly differs from the one derived by Golosov et al. (2014) primarily because of the depreciation structured assumed. Nevertheless, both taxes yields similar results. The optimal tax (as a percentage of GDP) depends on three aspects, discounting, damages and CO2 depreciation structure. The tax depends positively on the discount factor (β), which means that if agents care more about future streams, it is optimal to tax more today. It also has a direct relationship with the damage parameter (γ): higher damages increase the social cost of carbon and so, the tax is higher. Similarly, a larger value of the airborne fraction parameter (ε) imply that carbon dioxide stays more in the atmosphere and so, we tax more. Finally, bigger rates of CO2 removal (φ) reduce the amount of carbon in the atmosphere, lowering the optimal tax.

[12] The airborne fraction is calibrated as a constant parameter. This goes against some studies that claim that the airborne fraction is exhibiting an increasing trend. However, there are authors, like Knorr (2009), who argues that this trend is still statistically insignificant.

[13] The depreciation structure assumed in this model is different from the one used in Nordhaus (2008) or Golosov et al. (2014). This simpler approach delivers similar results as the ones that they report.

[14] In this economy, the GDP is described by the production in the Good's sector.

[15] This assumption is empirically sustained over long period of time. Additionally, this model is similar to the standard Neoclasical growth model in which the policy functions implies constant saving rates.

4 Simulated exercise for the Chilean economy

This section covers the calibration of the parameters present in the model. We have divided it into three parts. First, the calibration of the most standards parameters, for example, the ones in the utility and production functions. Second, the carbon cycle process which involves the CO2 removal rate and the airborne fraction parameters, and finally, the damage function, comprising the calibration of γ.

The sources of information consist of quarterly data between 1990 and 2017 from the Central Bank of Chile, National Statistic Institute (INE), Chilean Copper Commission (COCHILCO), National Energy Commission (CNE), and the Energy Information Administration (EIA). The last source is used to account for the level of CO2 emissions, useful for the calibration of the damage function, explained later on in this section.

Table 1 reports the values of some key parameters that are present in the model:

Table 1: Calibration of key parameters

Parameter	Value						Source
Discount factor	β	0.99					CBCh
Depreciation rate	δ	0.01					CBCh
Utility function parameters	θ_c	1.00	θ_l	1.14	θ_r	0.85	Own compilation
Firm product parameters	α_g	0.07	η	0.44	ν	0.49	CBCh
Mining product parameters	α_m	0.20					COCHILCO
Energy product parameters	α_e	0.98					CNE
Real exchange rate elasticity	α_e	0.12					Valdés and Délano (1998)
Technology shock	$\bar{A}$	1.00	ρ_a	0.83	σ_a	0.020	Own compilation
Copper price shock	$\bar{p^x}$	1.00	ρ_c	0.98	σ_a	0.038	Fornero and Kirchner (2018)
Imports price shock	$\bar{p^m}$	1.00	ρ_e	0.89	σ_e	0.139	Fornero and Kirchner (2018)

Note 1: Ω_l is chosen to match a value of $l = 1/3$ i steady state.
Note 2: Ω_l is chosen to replicate the share of residential energy into total energy produced
Quarterly data: 1990-2017
Source: Own compilation

From the table, we chose the values as follows: First, β is calibrated to match a quarterly rental rate of 1.01%, providing a value for the discount factor of $\beta = 0.99$. Second, the depreciation rate was chosen to match an investment to GDP ratio of 21% in steady state. Third, the parameters of the utility function were selected to ensure a logarithmic utility in consumption, target a Frisch elasticity of 2.3, and obtain consistent values for the price elasticity of demand for residential energy. This implies that both Ω_r and θ_r are calibrated to match the share of residential energy in total energy production, which is approximately 15%. Fourth, the production function parameters are calibrated according to the share of each of inputs in the value of the production, except for the energy one, which was set to match total exports in steady state. Fifth, the share of nontradables in the real exchange rate is consistent the study made by Valdés and Délano (1998) of the Chilean economy. Finally, for the AR(1) shocks calibration was done by running a regression to the cycle component of each series to estimate the standard deviation and persistent of each shock. Importantly, and following Fornero and Kirchner (2018), we normalize the steady state values of the copper and import prices to one.

This calibration allows to replicate some key ratios in steady state. Table 2 some of the aggregate variables with respect to GDP:

Table 2: Steady state ratios with respect to GDP

Consumption	Investment	Imports	Exports	Residential Energy	Copper Energy	Industrial Energy
79%	21%	10%	10%	1.5%	1.8%	6.6%

Source: Own compilation

To match the steady state ratios of the aggregate variable we used long periods of time with the information available at the Central Bank of Chile. The amount of consumption to GDP was fixed to 79% while the investment to outcome ratio, to 21%.[16] Turning to the tradable sector, the exports was set to 10% of the GDP, matching the share of copper production to GDP in the data. As we have imposed, that in steady state, the trade balance is in equilibrium, imports accounts for the same amount as exports. Although this might be considered a strong assumption, however, over long periods of time, the trade balance is nearly 0.4% of the product. Finally, energy ratios were calibrated with the information available at the CNE. The amount of residential energy demand accounts for 1.5%, which is similar in share to the one of the copper sector. The remaining is destined to industrial activities.

4.1 The carbon cycle

For the carbon cycle structure, we need to calibrate φ and ε. The first accounts for the rate at which carbon is absorbed by the atmosphere, terrestrial biosphere and different layers of the oceans. The second, is the proportion of human emitted CO2 that remains in the atmosphere.

[16] The consumption to GDP ratio in this model is higher than what it is reported in the National Accounts (NA). This is mainly because in the present model, the government doesn't purchase the consumption good and Instead, rebates all the lump sum taxes to the agents. Hence, the 79% already contains the 10% of government consumption to GDP ratio that is observed in the data.

To calibrate φ, we follow what it is concluded by the IPCC (2014), which states that in nearly 300 years of history, about 40% of the emissions have remained in the atmosphere.[17] Using this and noting that 300 years is equivalent to 1200 periods in our model, we can calibrate the parameter as $(1-\varphi)^{1200} = 0.25$. This gives a value of $\varphi = 0.076\%$. For the airborne fraction, we can rely on the study made by Knorr (2009). In his paper, he recommends to set ε equal to 0.43. This is a rather standard and accepted value in the literature.

4.2 The damage function

The damage function is a relationship that maps the stock levels of CO2 in the atmosphere, with the damages on GDP. For this purpose, we have assumed an exponential function in the production function in the good's sector. This approach is based on Golosov et al. (2014), which in turn, is an approximation to Nordhaus (2008) quadratic function relating global temperature levels to output drops.[18]

To calibrate the damage parameter, we need two points for the carbon concentration levels of the damage function: the baseline ($\bar{S}$) and the projected stock of CO2 (S_t). To get them, we first need to map CO2 stock levels to global temperatures. In the standard literature, the formula that link these two,[19] imply that a rise of 3 degree Celsius in global temperature, would double the CO2 concentration levels respect to the baseline. A direct application to Chile can be found in Cline (2007), who estimates that the temperature rise in the Chilean territory is in line with global temperature increases, in effect, 3 degrees Celcius. Following the formula, these quantities provides that the CO2 stock levels double the baseline, equivalently, $S_t = 2\bar{S}$.

To get these values, we need to choose the baseline levels. We will assume that they are

[17]There is a huge uncertainty of how to calibrate this parameter. For example, our procedure contrasts to what was concluded in the IPCC (2014) report. This states that: "*About half of a CO2 pulse to the atmosphere is removed over a timescale of 30 years; a further 30% is removed within a few centuries; and the remaining 20% will typically stay in the atmosphere for many thousands of years*". Other researchers calibrate this parameter following Archer (2005). He claims that "*..75% of an excess atmospheric carbon concentration has a mean lifetime of 300 year and the remaining 25% always stays forever*".

[18]Nordhaus (2008) assumes that the damage function takes the form of $1 - D_N(T_t) = \frac{1}{1+\theta_2 T_t^2}$, in which T is the mean global increase in temperature above the pre-industrial levels.

[19]The formula linking temperatures to CO2 carbon concentration is $T_t = T(S_t) = \kappa \ln\left(1 + \frac{S_t}{\bar{S}}\right) / \ln 2$. It is standard to assume κ to be 3 degrees Celcius, meaning that, a rise of 3 degree Celsius in global temperature, would double the CO2 concentration levels respect to the baseline. Although this estimation is rather standard in the literature, there's still no consensus on it's sensitivity. See Roe and Baker (2007) for further details.

a fraction of the steady state solution to the model without externality and carbon tax.[20] In particular, we will assume that this ratio is 50% of the steady state solution. This is consistent with the fact that, according to the EIA reported data,[21] since 1990, Chilean CO2 emissions has doubled (see Figure 1). For this purpose, we choose this benchmark to simulate the 1990 Chilean CO2 amount of emissions in the atmosphere. This is the year in which the Kyoto Protocol was agreed and the period where global awareness began to gain relevance.

Finally, to obtain a value for the damage parameter in the exponential function, we need to map these CO2 concentration levels to GDP drops. Recalling the study made by CEPAL, it states that at the end of the century, Chile might reduce its overall annual GDP between 0.78% and 1.1% due to the global warming impact.[22] We will assume a rather conservative GDP annual drop of 0.8%, which comprises a quarterly contraction of approximately a 0.2% of the product. We can use these quantities to calibrate the γ parameter using the production function of the good's sector. Taking logs and differentiating we get:

$$-\gamma[S_{t+1} - S_t] = \Delta\%Y_t$$

Considering S_t as the baseline levels, and S_{t+1} two times that quantity, imply that $S_{t+1} - S_t = \bar{S}$. Furthermore, the right hand side of the equation is in this case, -0.2%. With this calibration, the parameter becomes:

$$\gamma = \frac{0.2\%}{\bar{S}}$$

The value chosen for the baseline levels imply a damage parameter of $\gamma = 0.0018$. This implies that γ accounts for the damage of global emissions as this decrease in 0,8% of the Chilean GDP is due to climate change which is a global phenomenon. The implicit assumption made in our calibration is that Chilean emissions are a constant proportion of total emissions. With this, an increase in Chilean emissions is related to global emissions. This implies that under this assumption, the optimal carbon policy under international climate coordination with enforcement mechanisms should implies the same steady state solution.

[20]A model without externality can be interpreted as one in which either γ=0 or $S_t = \bar{S}$. The first case implies that the carbon tax zero by construction. Assuming that γ=0, the solution to the steady state CO2 levels depends on the parameters of the carbon cycle structure and the amount of energy production. Formally, the steady state solution corresponds to $S = \frac{\varepsilon \bar{E}}{\varphi}$.

[21]For more information, visit `www.eia.gov`.

[22]The estimation considers the direct impacts on GDP at a discount rate of 5% and under scenario A2 (one of the most severe of the global warming scenarios defined by the IPCC (2014). The scenario implies that average temperatures in Chile would rise in a range between 3 and 3.3 degrees Celsius by 2100.

4.3 Steady state results

It is important to remark that under production externalities like the one present in our model, nontrivial nonlinearities may arise in the solution concept. This is precisely the technical issue that Wen and Wu (2008) points out regarding the solution of DSGE models that exhibit this feature. In his paper he recommends to use a second order approximation to the steady state in order to capture the nonlinear term, a practice that we follow in this paper[23]. To make a comparison between the two scenarios, we take the economy without the carbon as the baseline. This means that when analyzing the results from the carbon tax model, they are read with respect to that benchmark. Table 3 shows the percent change of some of the key variables of the carbon tax model in steady state:

Table 3: Percentage deviations of the steady state solution

GDP	Consumption	Investment	Energy	Energy Price	Emissions	Taxes	Welfare	RER
2.4%	2.4%	2.5%	-7.5%	10.8%	-7.5%	7.3%	0.4%	-1.3%

Source: Own compilation

The carbon tax has been calibrated to be a fixed fraction of the GDP in steady state. This tax rate represents 15.8% of the energy price. The policy rises the tax collection by 7.3% and increases the energy price by 10.8%, which in turn, leads to a decreased demand that is distributed evenly across all sectors. As a result, the total amount of energy produced is reduced by 7.5%. As less energy is produced, the CO2 levels in the atmosphere decrease by 7.5%. This results into an increase of 2.4% in GDP, which translates into a rise of 2.4% in consumption, and 2.5% in investment.

As for the welfare measure, we follow Schmitt-Grohe (2005) recursive approach for the utility representation:

$$W_t = U_t(c_t, 1 - l_t, e_{rt}) + \beta \mathbb{E}_t(W_{t+1})$$

A priori, the welfare impact of the tax is ambiguous and depends on two forces. First, the tax has a direct effect in the economy by rising the energy price. This produces a drop in demand and production. The consumption drop, together with the fall of residential electricity demand, diminish welfare. Second, the reduction of the amount of energy traded has a midterm effect of moderating the CO2 emission levels. This, through the externality mechanism, has a positive impact in GDP, which leads to an increase of consumption and consequently, of welfare. The net effect depends on the relative weight of these two forces.

[23] To solve the model, we have made use of Dynare software with a second order approximation to the steady state

The steady state solution shows that welfare improved by an amount of 0.4%. The reason behind the small increment is that, even when consumption rises, there is a decline in the residential energy demand produced by the tax policy. In effect, the presence of energy demanded from households in the utility function, shrinks the overall measure of welfare as consumers have to pay more for electricity destined to residential purposes. The inclusion of this term makes our measure of welfare to be lower than other reported in the literature.

Finally, the real exchange rate appreciates by 1.3%, driven by the increase of the energy price. This implies that the home economy becomes more extensive with respect to the foreign and so, there is a loss of competitiveness in steady state. This result is consistent with most studies that analyze the macroeconomic effects of the carbon tax. For example, Siriwardana et al. (2011) reports a real appreciation of 0.75%[24] due to the carbon tax implemented during 2011 in Australia. On the other hand, McKibbin et al. (2011) and McKibbin et al. (2018) analyze the effect of a carbon tax in the U.S. economy. Both studies point out a real appreciation, while the latter, suggests a modest effect. With these, the RER effect from our model is in line with the international literature.

4.4 Analysis of Impulse and response functions

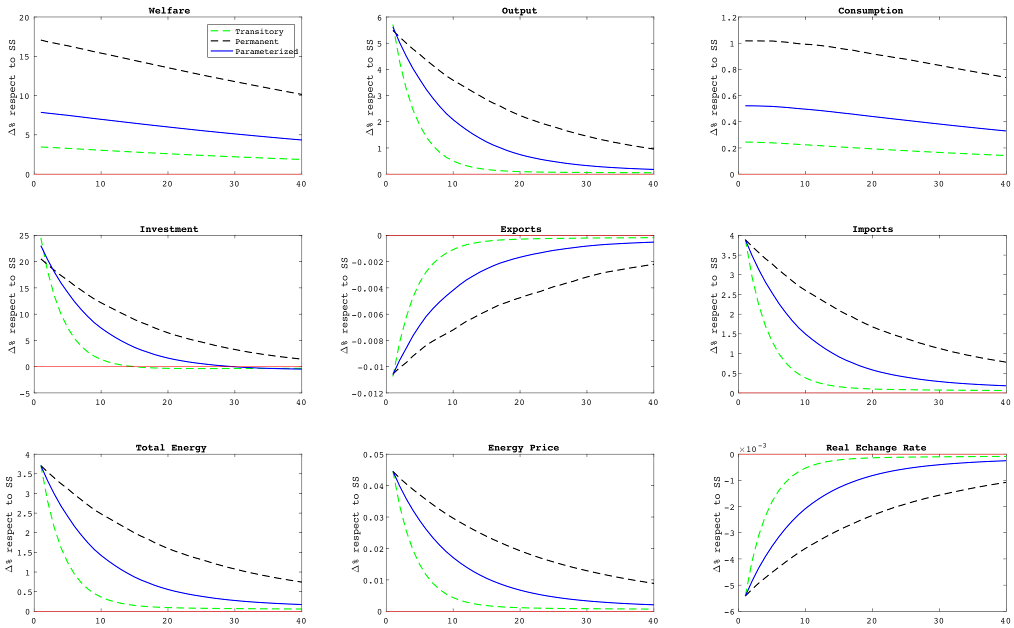
In this section, we carry out an impulse and response function analysis to understand the dynamics of some key variables of the carbon tax model. Two shocks are analyzed: a total factor productivity (TFP) and a copper price shock. The first one is standard in the literature, while the second is common in the Chilean economy (see e.g. Fornero and Kirchner, 2018 and Medina and Soto, 2007).

The Figure 1 illustrates the effect of a one standard deviation productivity shock[25] to the final goods firms:

[24] This effect results from a \$23 per ton of CO2 tax. Alternatively, a tax of \$15 generates a real appreciation of 0.39% and a value of \$30, a reduction of 1.05% of the real exchange rate

[25] See the Table 1 in the calibration section for details about the magnitude of each shock

Figure 1: Productivity shock



Source: Own compilation

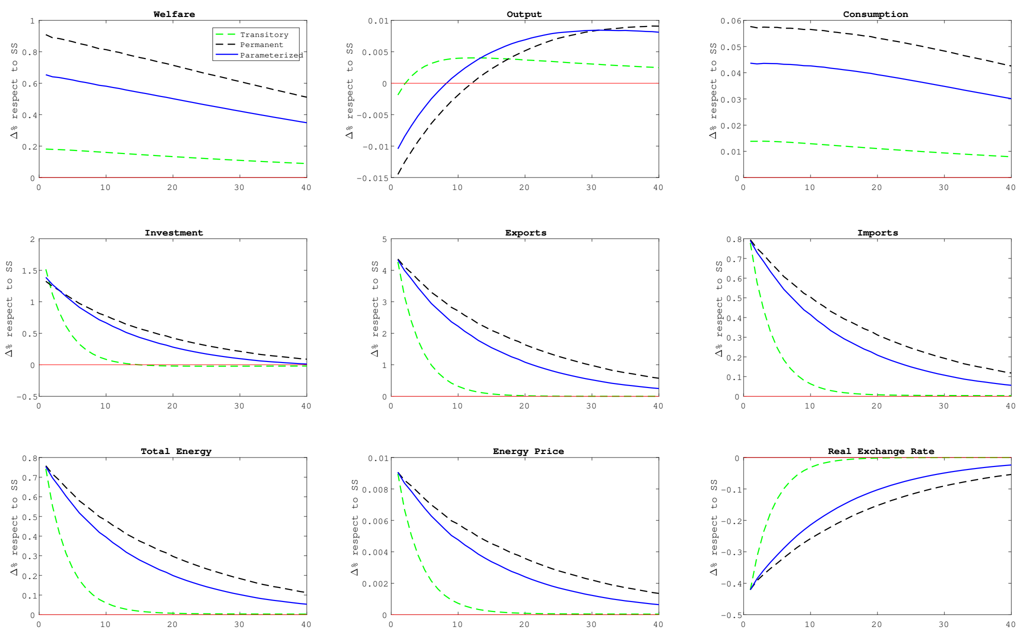
The TFP shock (the continuous blue line) in the final goods firms has an expansionary effect in the rest of the economy. The shock increases the productivity of all the inputs, leading to a rise in the labor and capital rents, which makes the representative agent better off. This makes labor to increase, as wages are higher, and investment to rise. The increased demand for energy inputs from industrial firm boosts the energy price and the amount of energy traded. To feed the higher electricity demand, imports have to rise, leading to an increase of the CO2 emissions. In addition, more energy traded increase the tax collection made by the government as the tax base is bigger.

A consequence of the higher energy price is that the marginal cost of inputs for the copper price firm is increased, reducing the commodity exports. In this context, the price level has a similar effect in the residential energy, by reducing the amount that it is demanded. In spite of this decline, the welfare levels rises because the consumption increment dominates over reduction of residential electricity and leisure in the utility function. Finally, the real exchange rate is marginally appreciated due to the domestic price effect.

Additionally, we also show the effect of a transitory and a permanent shock, denoted by a dashed green and black line respectively. When it is transitory, the effect tends to disappear quickly over time (10 years). The opposite happens when it is permanent, as the effect prevails in time. These results comprise for a couple of boundaries to what the model predicts.

Regarding the copper shock:

Figure 2: Copper price shock



Source: Own compilation

Figure 2 shows the effect a copper price shock has in the rest of the economy. First, it stimulated the production of the commodity and therefore, exports. The more energy demanded shifts the total amount of energy traded and the price upwards. The production of electricity adjust quickly and feeds the increased demand by importing fossil fuel and hence, emitting more CO2 in the process. This improves the tax collection as the tax base is higher.

The increase of the energy price has several consequences. First, it rises the marginal cost of the goods production that leads to a reduction of sales in the short run. This, together with the increase CO2 levels, explains the drops in output. The firm then substitutes the costly input by hiring more labor and renting capital, which stimulates investment. This explains the posterior recovery experienced after the drop of the first period. As the representative agent owns the firms, the loss from the goods producers is offset by the rents from the copper and energy sector. This income effect, explains the higher consumption. A second consequence of the larger energy costs, is that it declines the residential electricity consumption and consequently, affects welfare. However, the positive evolution that it shows, suggest that the consumption effect dominates over the residential demand decline. It is also important to note that, the permanent and transitory shocks have the expected evolution.

Finally, the real exchange rate appreciates by 0.4% respect to the steady state. The reason for this decline relies on two effects: the improvement of the terms of trade resulting from the shock, and the higher energy price. As for the limited effect of the copper price shock in the RER (i.e. less than 1%), it can be due to two factors. First, the RER elasticity parameter, α, is only of 12.4%. Second, is that total energy as a fraction of GDP is of only 10%, in which the copper sector accounts for only 1.8% of that magnitude. This implies that the increased demand doesn't rise the price enough to have a larger RER effect. These two factors together, explain the magnitude of the response from this variable to commodity shocks in our model.

4.5 Real exchange rate volatility

In this section we test the hypothesis whether the carbon tax helps to reduce the real exchange rate volatility. To do so, we calculate the variance of the simulated variables under each model when exposed to copper price shocks. Table 4 shows the percent change in the volatility of some key variables of the carbon tax model, respect to the baseline (without tax model):

Table 4: Percentage deviations of volatility under a copper price shock

Copper Energy	Energy	Energy Price	RER	Imports	Exports	CO2	Output	Welfare
-14.9%	-14.1%	-9.8%	-1.8%	-14.6%	-3.2%	-14.5%	-3.9%	-7.0%

Source: Own compilation

First, it is worth noting that in general, the overall volatility of the model is reduced respect to the baseline. When the economy is subject to a copper price shock, the variance of the copper energy demand is reduced by approximately 15%, while the energy traded drops by a similar amount. This impacts the energy price volatility, which is also reduced by an amount close to 10%. As the RER is a function of the energy price, it is affected by the changes in the variance of these variables. With this, the real exchange rate variance declines by 1.8%.

This result is driven by the following mechanism: under both models the economy is subject to a positive one standard deviation shock to the copper price. However, in the carbon tax model, the copper price is higher than the baseline model. As a result, copper firms demand less energy than they would have without the tax. This implies a reduction of the energy demand volatility for mining purposes. As a result, the pressure on the energy price is lower compared to the baseline, implying that it doesn't rise as much as without the tax. Consequently, the RER appreciates less and therefore, exhibits a lower variance. This result allow us to state that the carbon tax acts like an automatic stabilizer for the real exchange rate, as it moderates the cyclical fluctuations.

In addition, there are other variables that reduce their volatility as well. Imports and exports declined their variance by 14.6% and 3.2% respectively. As the total amount of CO2 is a function of the energy traded, it reduces its variance by almost 15%. The economy output is also moderated, as it depends on the variables described before,. Finally, the welfare measure, affected by the residential energy demand moderation, stabilizes by 7%. This suggests that the carbon tax helps to stabilize the economy as a whole when it is hit by copper price shocks. In brief, the moderation of the energy demand and its price produces a chain effect, as energy is a major input in all the productive sector, it affects output and welfare. The real exchange rate, as a function of the energy price, follows the same path as well.

To test the robustness of the RER result, we will simulate this variable under three different scenarios and calculate the volatility respect to the baseline model. These scenario are:

- A single one standard deviation positive copper price shock;
- Multiple copper price shocks each period; and
- Multiple shocks (copper price, imports price and TFP) each period.

Under each one of them, we will calculate the detrended simulated series, in particular, the real exchange rate and compute its variance. Table 5 reports this result, which again, are respect to the baseline model:

Table 5: Percent deviations of volatility of RER under each scenario

Single copper shock	Multiple copper shocks	Multiple shocks
-1.8%	-1.8%	-2.2%

Source: Own compilation

From the simulation, the real exchange variance under multiple copper price shocks remains the same as the single case shock. In contrast, when simulating the economy after multiple shocks from every source, the RER variance is slightly reduced, reaching 2.2%. This establishes that the real exchange rate volatility is robust to changes in the specification of the different scenarios. This is, the variance not only declines when the economy is subject to copper shocks, but also, when it is exposed to multiple shocks at the same time. The scope of this result is of big scale as it demonstrates that the carbon tax unrestrictedly reduces the volatility of the RER. Hence, the tax acts like an automatic stabilizer when the economy is subject to different shocks as it moderates the cyclical fluctuation of the real exchange rate.

5 Model extensions

In this section we present some model extensions. First, we extend the baseline model presented in Section 3 by including a imported consumption in the consumers' utility function. Second, we analytically and quantitativelly derive the carbon prices that achieve the Paris agreement.

5.1 Consumption of imported goods

We can extend our model by considering a tradable consumption good with some degree of substitution between the non-tradable consumption good. Thus, we assume that the consumer problem is given by:

$$\max_{c_t, c_t^m, e_{rt}, l_t, i_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t, c_t^m), 1 - l_t, e_{rt})$$

$$\begin{aligned} st \;\; (1+\tau_{1t})\,(c_t + p_t^m c_t^m) + p_t e_{rt} + i_t &\leq r_t k_t + w_t l_t + \Pi_{gt} + \Pi_{ct} + \Pi_{et} + T_t && (21) \\ k_{t+1} &= i_t + (1-\delta) k_t && (22) \\ k_0 &> 0 \;\; \text{given} && (23) \end{aligned}$$

That is, we assume that a portion of the consumption is imported (c_t^m). Particularly, we assume the utility function is fiven by:

$$\begin{aligned} U_t(A_t, 1-l_t, e_{rt}) &= \tfrac{A_t(c_t, c_t^m)^{1-\theta_c}}{1-\theta_c} + \Omega_l \tfrac{(1-l_t)^{1-\theta_l}}{1-\theta_l} + \Omega_r \tfrac{e_{rt}^{1-\theta_r}}{1-\theta_r} && (24) \\ A_t &= [a c_t^{1-1/\xi} + (1-a) c_t^{m^{1-1/\xi}}]^{\frac{1}{1-1/\xi}} && (25) \end{aligned}$$

Where we assume a CES aggregation function on both consumptions. The first order conditions of the consumer problem yields to:

$$\frac{a}{1-a}\left(\frac{c_t^m}{c_t}\right)^{1/\xi} = \frac{1}{p_t^m} \tag{26}$$

$$\frac{c_t^{-1/\xi}}{ac_t^{1-1/\xi} + (1+a)c_t^{m^{1-1/\xi}}}\frac{1}{1+\tau_{c,t}} = \beta\mathbb{E}_t\frac{c_{t+1}^{-1/\xi}}{ac_{t+1}^{1-1/\xi} + (1+a)c_{t+1}^{m^{1-1/\xi}}}\frac{1}{1+\tau_{c,t+1}}(1-\delta+r_{t+1}) \tag{27}$$

$$\Omega_l(1-l_t)^{-\theta_l} = \frac{ac_t^{-1/\xi}}{ac_t^{1-1/\xi} + (1+a)c_t^{m^{1-1/\xi}}}\frac{1}{1+\tau_{c,t}}w_t \tag{28}$$

$$\Omega_r e_{rt}^{-\theta_r} = \frac{ac_t^{-1/\xi}}{ac_t^{1-1/\xi} + (1+a)c_t^{m^{1-1/\xi}}}\frac{1}{1+\tau_{c,t}}p_t \tag{29}$$

We use $\xi = 1.12$ following Medina et al. (2007), and we calibrate a such that imported consumption represents 65% of domestic consumption c_t.

Next, we apply our extended model to compute the carbon tax, resulting in:

$$\hat{\Lambda}_t^s \equiv \frac{\Lambda_t^s}{Y_t} = \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i(1-\varphi)^i\gamma\varepsilon\left(\frac{c_t}{c_{t+i}}\right)^{1/\xi}\frac{ac_t^{1-1/\xi} + (1+a)c_t^{m^{1-1/\xi}}}{ac_{t+i}^{1-1/\xi} + (1+a)c_{t+i}^{m^{1-1/\xi}}}\frac{Y_{t+i}}{Y_t} \tag{30}$$

It is important to note that a more restrictive assumption is required to express the optimal tax as a proportion of GDP. Specifically, assuming constant saving rates and defining $\gamma_c \equiv \frac{c_t}{c_t^m}\ \forall t$, the optimal tax can be expressed as:

$$\hat{\Lambda}_t^s = \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i(1-\varphi)^i\gamma\varepsilon\left[\left(1+\frac{1-a}{a}\gamma_c^{-1+1/\xi}\right)^{-1} + \left(1+\gamma_c^{1-1/\xi}\frac{a}{1-a}\right)^{-1}\right] \tag{31}$$

$$= \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i(1-\varphi)^i\gamma\varepsilon \tag{32}$$

Thus, the optimal tax derived here is identical to the one obtained previously. This result indicates that the key assumption required to maintain the same optimal tax is that the ratio of imported to domestically produced goods remains constant over time.[26]

Regarding percentage deviations from the steady-state solution, this model changes the original results very little. The only significant difference is in the percentage change in welfare, which decreases from 0.4% to 0.3%. Similarly, the estimated volatilities remain largely unaffected, except for the volatility of imports, which now decreases by 11.6% (i.e.,

[26]Otherwise, it is necessary to account for the fact that, according to the first-order conditions, $\frac{c_{t+i}}{c_{t+i}^m}$ depends on p_{t+i}^m, where $\mathbb{E}_t\, p_{t+i}^m = \overline{p^m}^{1-\delta^i} p_t^{m\delta^i}\exp\left(\frac{\sigma^2}{2}\right)\frac{1-\delta^{2i}}{1-\delta^2}$.

this model reduces volatility less than before). Output volatility decreases by 3.5% (a smaller reduction than in the original model), while welfare volatility decreases by 7.2%, representing a greater reduction than previously observed. Finally, regarding RER volatility, there is a very slight increase; however, the values in Table 5 remain unchanged.

5.2 Carbon prices

We now derive a formula for the optimal carbon tax under a cap on temperature or cumulative emissions. This derivation builds on the work of Van der Ploeg and Rezai (2021), who demonstrates that carbon prices, in this context, follow a Hotelling rule.

To illustrate this, we introduce two minor modifications to our model, expressed through the following equations:

$$T_t = T_0 + \xi_1 S_t \tag{33}$$

$$1 - D_t(S_t) = \exp\left(-\xi_2 T_t\right) \tag{34}$$

where T_0 denotes the initial temperature, and ξ_1 represents the transient climate response to cumulative emissions (TCRE). This implies that temperature now impacts aggregate productivity, with productivity being a linear function of total emissions. In the next subsection, we discuss how this model applies specifically to Chile's emissions rather than global emissions.

Given this function, we now assume that a cap is imposed on total emissions, such that $S_t \leq \tilde{S}$, $\forall t$. This implies the existence of a carbon budget, which determines a Kuhn-Tucker condition. Note that if $\varphi = 0$ and $\varepsilon = 1$ (so that $S_t = S_{t-1} + E_t$), there are two possible cases: either the optimal trajectory never exhausts the carbon budget, meaning $\lim_{t\to\infty} S_t < \tilde{S}$, or the carbon budget is fully utilized, such that $\lim_{t\to\infty} S_t = \tilde{S}$. Otherwise, the condition would be binding at certain times and not at others.

Note that, once again, the planner's problem is defined as follows:

$$\max_{c_t, e_{gt}, e_{mt}, e_{rt}, E_t, S_t, k_{t+1}, k_t, l_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t, e_{rt})$$

subject to equations (22), (4), (5), (7), (8), (9), (13), (14) and $\omega_t(S_t - \tilde{S})$, where $S_t \leq \tilde{S}$ and $\omega_t \geq 0$.

We now derive the optimal carbon tax following Van der Ploeg and Rezai (2021), which determines it by tracing the Lagrange multiplier of S_t backward. Note that, for this problem as well as the previous one, $\lambda_t^s = -\varepsilon \lambda_t^{S_t}$, where $\lambda_t^{S_t}$ is the Lagrange multiplier associated with equation (8). Solving backward, we can then express:

$$\lambda_t^{S_t} = [\beta(1-\varphi)]^{-t}\lambda_0^{S_0} + \sum_{i=1}^{n} \frac{1}{\beta^i(1-\varphi)^i}\left(\frac{Y_{t-i}}{C_{t-i}}\xi_2\xi_1 - \omega_{t-i}\right) \tag{35}$$

Again, under the assumption of constant saving rates ($c_t = (1-s)Y_t$) and by setting a carbon price such that the constraint is never binding (i.e., $S_t < \tilde{S}\,\forall t$ or $\lim_{t\to\infty} S_t = \tilde{S}$), it follows that $\omega_t = 0\,\forall t$. Thus, the optimal carbon tax can be expressed as:

$$\hat{\Lambda}_t^s = \varepsilon\left(\xi_1\xi_2 - \lambda_0^{S_0}(1-s)\right)[\beta(1-\varphi)]^{-t} + \frac{\varepsilon\xi_1\xi_2}{1-\beta(1-\varphi)}. \tag{36}$$

Note that this expression depends on two terms: the term on the right corresponds to the usual component associated with an optimum without a binding temperature cap, while the term on the left represents a Hotelling path. To see this, recall that the Euler equation of consumption equalizes $\frac{c_{t+1}}{c_t} = \beta(1-\delta+r_{t+1})$. Under the assumption of constant saving rates, $\frac{Y_{t+1}}{Y_t}\frac{1}{\beta} = 1-\delta+r_{t+1}$. This mirrors the formula presented in Van der Ploeg and Rezai (2021), where, if $\varphi = 0$, the carbon price grows at a rate equal to the return on risky financial assets. If $\varphi > 0$, however, the growth rate is lower than that predicted by the Hotelling rule. This is intuitive, as a positive φ implies that the effect of energy contributes less to total emissions, which are not stable but subject to depletion.

5.3 Simulation of Carbon prices for Paris agreement

We now use the model from the previous subsection to set the carbon price necessary to limit warming to 1.5°C by the end of the century. This aligns with the 2015 Paris Climate Agreement, where countries committed to limiting global warming to below 1.5°C above pre-industrial levels. We use the 1.5°C threshold instead of 2°C because Chile's emissions alone would not cause the temperature to reach 2°C in the steady state.

It is also important to interpret how this model relates to Chilean emissions rather than global emissions. First, since global warming is inherently a global problem, the geographical origin of emissions is irrelevant. In this context, we follow Van der Ploeg and Rezai (2021), who set $\xi_1 = 0.002$, implying a transient climate response to cumulative emissions (TCRE) of 2°C per teratonne of carbon (TtC). Naturally, it is intuitive that Chilean emissions alone are insufficient to account for 2°C of global warming from pre-industrial levels.

Finally, and most importantly, for these results to be meaningful, we assume that other countries are making the necessary efforts to mitigate climate change, meaning that Chile's emissions are considered marginally relevant in this global context.

The calibration, as previously mentioned, follows Van der Ploeg and Rezai (2021), which sets the initial temperature at $T_0 = 1.3$. We set $\xi_2 = 0.9$ to ensure that $\xi_1\xi_2 = \gamma$, preserving the original calibration regarding the impact of cumulative emissions on GDP. Similarly, we set $s = 0.21$, while φ and ε retain their original values. This calibration implies the presence of CO_2 removal from the atmosphere, consistent with our initial assumptions.

Additionally, $\lambda_0^{S_0}$ is set such that the cap is reached by 2100. In our model, this implies a high value of $\hat{\Lambda}_t^s$ at 35.5%, which, in the steady state, corresponds to a reduction in cumulative emissions of 28.8%.

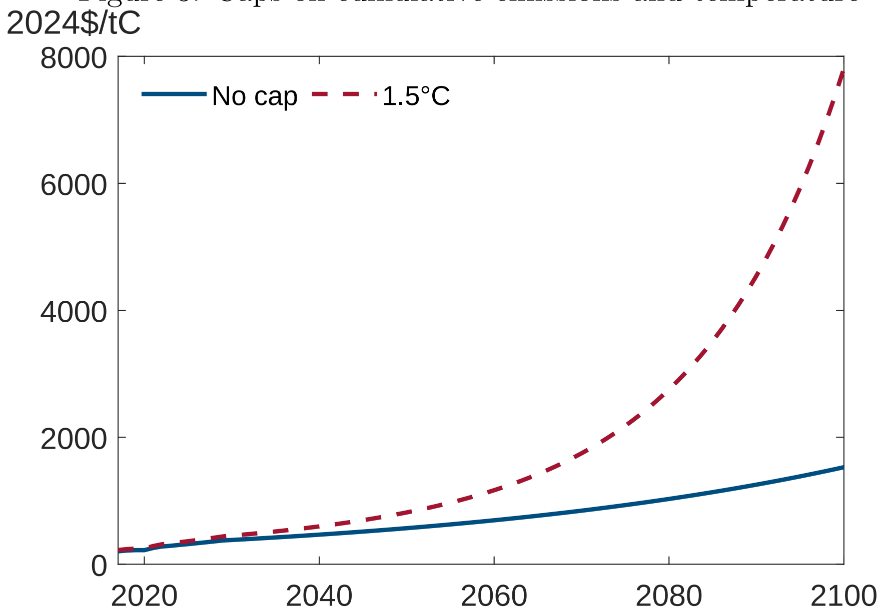
To express the carbon tax in terms of USD per ton of CO_2 emitted, we multiply $\hat{\Lambda}_t^s$ by the annual GDP, based on IMF projections, and divide by Chile's annual CO_2 emissions, which represent approximately 0.3% of global emissions. The results for the optimal tax, expressed in dollars per ton of carbon emissions, are presented in Table 6. These results are quantitatively high but fall within the range commonly found in the literature (see, for example, Golosov et al., 2014; Burke et al., 2015).

Table 6: Average optimal green taxes

Tax ($2024/tC)	2024	2025	2026	2027	2028	2029	2030
Baseline	304.20	317.07	330.92	345.05	359.53	374.55	382.04
Hotelling rule	345.64	362.27	380.25	398.85	418.16	438.43	450.18

Figure 3 below presents the time profiles of carbon taxes for both scenarios, assuming a 2% growth rate for the Chilean economy after 2029. As is typically the case, taxes increase exponentially when the cap is binding and are much flatter under the baseline scenario.

Figure 3: Caps on cumulative emissions and temperature



Note: Own compilation. The simulation uses IMF projections of Chilean GDP up to 2029, followed by an assumed 2% growth rate thereafter.

6 Policy discussion

In this section we discuss about the policy implications of our findings. First, we comment about the role of a carbon tax as an automatic stabilizer. Then, we comment on the amplification effect of carbon neutrality policies on the role of carbon taxes as automatic stabilizers.

6.1 Carbon tax and the business cycle

Our analysis shows that in a small, open economy specialized in commodity exports, a carbon tax can help smooth the business cycle. Additionally, implementing a carbon tax can enhance energy security by reducing the volatility of energy supply and prices, as it lessens the pass-through of commodity shocks to the energy market. Furthermore, the carbon tax can lower exchange rate fluctuations by decreasing the correlation between commodity shocks and the broader economy. These findings carry significant policy implications that warrant careful consideration.

Regarding energy security, in a small open economy, a significant portion of the electricity generated may be used to support commodity production. For instance, in Chile, 34% of the energy produced is consumed by the copper industry, which accounts for 15% of the country's total energy consumption. This implies that a positive commodity shock could put pressure on the energy market by increasing the cost of energy generation and raising emissions as fossil fuels or coal are used to meet the additional energy demand from the commodity sector. As a result, energy prices may rise not only for the commodity sector but across the entire electrical grid. This would lead to higher energy costs for households, particularly reducing energy security for low-income households. Since energy is an essential good with limited substitutes, such price increases could generate pressure for targeted electricity subsidies aimed at low-income households. In this context, a carbon tax, acting as an automatic stabilizer, can mitigate the impact of a commodity shock on energy security by reducing both the pass-through of the shock to the energy sector and its overall intensity.

The effect of a carbon tax in an open economy like Chile is that, when exposed to copper price shocks, the real exchange rate does not appreciate as much as it would without the tax. This could be particularly beneficial in reducing the likelihood of Dutch disease, a phenomenon that can be especially harmful for a country like Chile, which specializes in the export of a single commodity. With a smaller appreciation of the real exchange rate, the negative effects on the competitiveness of other sectors, such as manufacturing, are less pronounced. Additionally, during periods of positive shocks to the exporting commodity, revenues from the carbon tax can be used to smooth resource windfalls. For instance, part of the additional revenues generated by the shock could be allocated to precautionary savings in an international stabilization fund, as a measure to mitigate the impact of Dutch disease Van Der Ploeg (2019).

6.2 Carbon neutrality policies and its effect on the carbon tax stabilization

The Paris agreement creates a target for temperatures and for carbon emission. This implies an indirect effect on the automatic stabilization role of the optimal carbon taxes as there is a relationship between environmental targets and the capacity of the optimal carbon tax to function as a buffer for the economic cycle. The carbon price depends on whether carbon neutrality is a long-term goal and if the carbon emissions constraint is binding. In such a scenario, the carbon price follows the traditional Hotelling rule (Van der Ploeg and Rezai, 2021), implying that the policy instrument designed to regulate these prices—such as a carbon tax on the energy sector—would grow at a rate equal to the return on risky financial assets, which is generally higher than the economy's growth rate.

This suggests that commodity shocks can be even more effectively smoothed when such an optimal carbon tax is in place. As a result, the likelihood of Dutch disease decreases further, since the increase in wealth from large commodity shocks is more restrained. Consequently, the pressure on the non-tradable goods sector diminishes, reducing the incentive for a shift in productive resources from the tradable to the non-tradable sector.

7 Conclusions

In this document, we discussed how a carbon tax may act as an automatic stabilizer in a small open economy and contribute to energy security. Specifically, a carbon tax produces real appreciation, as the domestic economy becomes relatively more costly compared to foreign economies. It also affects the volatility of the real exchange rate (RER). When the economy experiences commodity shocks, the RER variance is lower in the model with the tax than in the model without it, and this finding is robust to changes in the sources of shocks. We test two scenarios: exposure to multiple shocks in copper prices and combined shocks, with results showing a 1.8% to 2.2% reduction in RER volatility in the tax model compared to the baseline. Furthermore, our findings indicate that the carbon tax reduces energy volatility by 14% and its price volatility by 10%. These results hold even when including tradable consumption in the model. A key policy implication is that a carbon tax can reduce the likelihood of Dutch disease, which is particularly relevant for Chile, given its heavy dependence on a single commodity export.

Future research could further investigate the role of carbon taxes as automatic stabilizers, for example, by modeling their impact on Dutch disease. Additionally, it would be valuable to study the stabilization effects of carbon taxes under heightened uncertainty, as they may help break the negative cycle between economic uncertainty and pollution (Lopez et al., 2022).

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A Appendix

A.1 Solution to the centralized model

When the planner's and the decentralized equilibrium match, the optimal tax takes the form of the social cost of carbon. To get an expression for this, we take the first order conditions of the planning problem with respect to c_t, E_t, e_{gt} and M_t:

$$\lambda_{1t} = U'_{c_t}, \tag{A.1}$$

$$\lambda_{2t} = -\lambda_t^s - \mu_t, \tag{A.2}$$

$$\lambda_{2t} = -\lambda_{1t} F'_{g,e_{gt}}, \tag{A.3}$$

$$\lambda_{1t} p_t^m = \mu_t F'_{e,M_t}. \tag{A.4}$$

In which λ_{1t}, λ_{2t} and μ_t are the Lagrange multipliers for restrictions (13),(14) and (4), respectively. The variable λ_t^s in (A.2) corresponds to the derivative of L with respect to E_t multiplied by the respective shadow price. By combining (A.1), (A.2), (A.3), and replacing into (A.4), we obtain an expression for the negative externality produced by the energy sector:

$$(F'_{g,e_{gt}} U'_{c_t} - \lambda_t^s) F'_{e,M_t} = U'_{c_t} p_t^m \tag{A.5}$$

Dividing by U_{c_t}, and denoting $\Lambda_t^s = \frac{\lambda_t^s}{U'_{c_t}}$, we get:

$$(F'_{g,e_{gt}} - \Lambda_t^s) F'_{e,M_t} = p_t^m \tag{A.6}$$

Equation (A.6), shows that in the first order condition of the energy firms, the value of the marginal productivity of energy inputs has to be corrected by the negative externality damage that it has in the goods nontradable sector.

To see the equivalence between both problems, we turn to the decentralized economy. In particular, we can get a similar expression to the one derived in the planning problem. For this purpose, replace the first order condition in the final goods firms with respect to energy into the equation (A.6), the first order condition of the energy sector:

$$(F'_{g,e_{gt}} - \tau_{et}) F'_{e,M_t} = p_t^m$$

From this, it is clear that the optimal tax must satisfy that $\tau_{et} = \Lambda_t^s$. To get a proper expression for Λ_t^s, we need to focus on the λ_t^s term first. It is important to note that it can be written as the expected present value of future damages:

$$\lambda_t^s = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \xi_{t+i} \frac{\partial S_{t+j}}{\partial E_t} \tag{A.7}$$

Where ξ_t is the shadow price associated with the restriction (8) and β is the discount factor. To simplify this expression, take the first order condition of the planning problem with respect to S_t to get:

$$\xi_t = -\lambda_{1t} \frac{\partial F_g}{\partial S_t} \tag{A.8}$$

Combining (A.8) with (A.1) yields:

$$\xi_t = -\lambda_{1t} \frac{\partial F_g}{\partial S_t}$$

Replacing this into (A.7) and considering $\lambda_t^s = \Lambda_t^s U'_{c_t}$, delivers:

$$\Lambda_t^s = -\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \frac{U'_{c_{t+i}}}{U'_{c_t}} \frac{\partial F_{g,t+i}}{\partial S_{t+i}} \frac{\partial S_{t+i}}{\partial E_t}$$

In which $\beta^i \left(\frac{U'_{c_{t+i}}}{U'_{c_t}}\right)$, the stochastic discount factor, is applied over the current and future damages in the domestic production produced by the increased levels of emissions. This expression captures the expected damages of the social of carbon and therefore, corresponds to the optimal tax.

A.2 Decentralized model optimal conditions

The first order conditions to the decentralized model are:

$$
\begin{aligned}
c_t + i_t + p_t^m M_t &= F_g(A_t, S_t, e_{gt}, k_t, l_t) + p_t^x F_c(e_{ct}) && \text{(A.9)}\\
RER_t &= \left(\frac{p_t^x p_t}{p_t^m}\right)^{-\alpha} && \text{(A.10)}\\
U'_{c_t} w_t &= U'_{1-l_t}(1+\tau_{ct}) && \text{(A.11)}\\
U'_{c_t} p_t &= U'_{e_{rt}}(1+\tau_{ct}) && \text{(A.12)}\\
U'_{c_t} &= \beta \mathbb{E}_t \left[U'_{c_{t+1}} \frac{1+\tau_{ct}}{1+\tau_{c,t+1}} (1 + r_{t+1} - \delta) \right] && \text{(A.13)}\\
k_{t+1} &= i_t + (1-\delta)k_t && \text{(A.14)}\\
p_t &= F'_{gt,e_{gt}} && \text{(A.15)}\\
r_t &= F'_{gt,k_t} && \text{(A.16)}\\
w_t &= F'_{gt,l_t} && \text{(A.17)}\\
p_t &= p_t^x F'_{mt,e_{mt}} && \text{(A.18)}\\
p_t^m &= F'_{e,M_t}(p_t - \tau_{et}) && \text{(A.19)}\\
S_t &= (1-\varphi)S_{t-1} + \varepsilon E_t && \text{(A.20)}\\
E_t &= F_{et}(M_t) && \text{(A.21)}\\
E_t &= e_{gt} + e_{m_t} + e_{rt} && \text{(A.22)}\\
\ln A_t &= \rho_a \ln A_t + \epsilon_{at} && \text{(A.23)}\\
\ln p_t^x &= (1-\rho_{px}) \ln \overline{p^x} + \rho_{px} \ln p_{t-1}^x + \epsilon_{px,t} && \text{(A.24)}\\
\ln p_t^m &= (1-\rho_m) \ln \overline{p_t^m} + \rho_m \ln p_{t-1}^m + \epsilon_{mt} && \text{(A.25)}
\end{aligned}
$$

These provide a solution for $\{c_t, k_t, i_t, l_t, S_t, RER_t, E_t, e_{rt}, e_{mt}, e_{et}, M_t, p_t, r_t, w_t, A_t, p_t^m, p_t^x\}_{t=0}^{\infty}$.