Pricing Following the Nominal Exchange Rate*

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Abstract

This paper studies how the informativeness of the exchange rate affects the sensitivity of prices to nominal depreciations. In an environment with imperfect information, the exchange rate is a signal of the state of the economy and thus relevant for pricing decisions, even if it does not affect costs. This "information channel" gives rise to a policy trade-off: a currency intervention that reduces the level of a depreciation also reduces exchange rate volatility and, therefore, increases its precision as a signal. This increases the elasticity of prices with respect to the exchange rate through the information channel. Overall, the effect of such currency interventions on inflation is ambiguous: on one hand they reduce the magnitude of the shock and, on the other, they increase the responsiveness of firms when adjusting their prices. To test the relevance of the information channel I use policy changes in Mexico to identify an increase in the precision of information provided by the Central Bank. I calibrate this increase in precision using data from a survey of private inflation forecasts. Holding everything else constant, this increase accounts for 4 out of a 12 point drop in the elasticity of prices with respect to the exchange rate.

Keywords: Exchange Rates, Incomplete Information, Aggregate Signals

JEL classications: E31, E58, F00

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1 Introduction

The relationship between the nominal exchange rate and inflation has been widely studied because of its potential implications for monetary policy. To do so, economists have focused on pass-through, which is the elasticity of prices with respect to the nominal exchange rate. This paper studies how pass-through changes with the informativeness of the exchange rate when agents use it as a signal of the state.

The main contribution of this paper is to introduce the *information channel* as a driver of the level of pass-through. The nominal exchange rate is observed more frequently and easily than other signals, such as inflation or economic activity indicators. Because of this informational feature, the nominal exchange rate is relevant for firms' individual pricing decisions, even if it does not affect production costs or the demand for their good.

Consider somebody operating a small barbershop whose only cost is a wage bill paid in domestic currency. Suppose she observes a nominal depreciation and knows that all domestic prices will remain the same. In this case she does not have any incentive to change her price as her nominal costs remain unchanged. Alternatively, suppose she does not know how all domestic prices will change. If she thinks the depreciation indicates domestic inflation,[1] then she will increase her prices in anticipation of a nominal wage increase.

Section 3 lays out an example of a closed economy that formalizes this pricing behavior. The exchange rate is defined as a noisy signal of the aggregate price level.[2] Pass-through is zero under perfect information and positive pass-through only arises with information frictions. Under incomplete information, agents also observe a noisy signal from the Central Bank about monetary policy. Signals are more or less relevant depending on how volatile they are (relative to the volatility of other signals). If the volatility of the nominal exchange rate is relatively high then it is a less informative signal and vice versa.

[1]For example, if she believes the law of one price holds then a nominal depreciation could be due to domestic inflation or a decrease in the foreign price level.

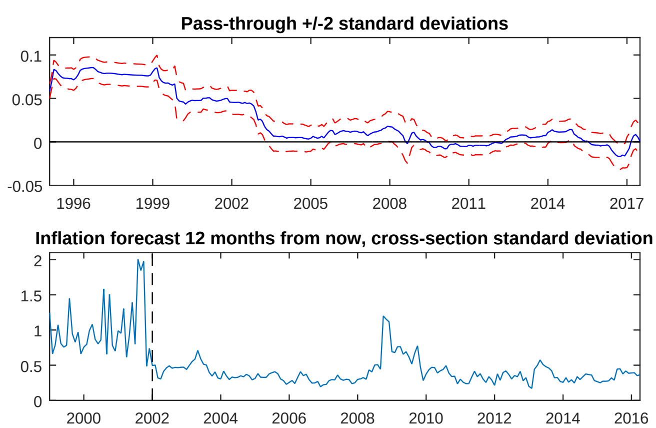
[2]This is a strong assumption that allows to simplify the exposition of the information channel. It is discussed in detail in Section 2.

Policy interventions that affect the level of currency shocks have a second order effect on prices by reducing the volatility of the exchange rate. In an extreme case the intervention fixes the exchange rate and its volatility becomes zero. Suppose that the objective of an intervention is to reduce the magnitude of a depreciation in order to decrease its effect on inflation. Under incomplete information, such intervention increases the elasticity of prices with respect to the exchange rate and, hence, the net effect of the intervention on inflation is ambiguous. Hereafter I refer to this ambiguity as the *currency intervention trade-off*.

Section 5 extends the one period model to nest what this paper calls the *costs channel*, as discussed by Taylor (2000). This channel focuses on the effect the nominal exchange rate has on prices directly through costs of imported inputs. If cost shocks are persistent then prices increase more and during more periods, which implies higher pass-through with higher persistence of nominal exchange rate shocks. This extension is used to compare the relevance of the information channel vis-à-vis the cost channel.

Section 6 calibrates the extended version of the model for the Mexican economy to analyze a decrease in pass-through in the early 2000's. This change has been documented by Capistran et al. (2011) and Cortes (2013), among others. The first chart in Figure 1 shows an estimate of pass-through for Mexico across time.[3]

Figure 1: Pass-through and inflation forecasts standard deviation



[3]By estimating the regression $\Delta_{12}\log CPI_t = \beta_0^s + \beta_1^s \Delta_{12}\log CPI_{t-1} + \beta_2^s \Delta_{12}\log ER_t + \varepsilon_t$ considering a moving sample s of 4 years using monthly data. The first chart in Figure 1 plots the estimates for β_2^s.

The second chart in Figure 1 illustrates how private inflation forecasts improved after three policy changes carried out by the Central Bank. The graph shows how the cross-sectional variance of private inflation forecasts decreased after 2002.[4] In the early 2000's the Central Bank carried out three policy changes which improved the information available to agents: it adopted an inflation-targeting regime with periodic announcements of its target, it increased the frequency of inflation reports from annual to quarterly and it increased the frequency of its objectives during open market operations.

Section 6 uses microprice data to estimate the average elasticity of prices (pass-through) with respect to foreign variations of the nominal exchange rate[5] for two periods (1995-2002 and 2003-2014); the estimated elasticities are 0.13 and 0.01, respectively. The calibration chooses the parameters in the extended model to target, among other moments, the estimated elasticity for the period 1995-2003. Finally, an improvement in the precision of the signal sent by the Central Bank in the model is identified with the data from the second chart of Figure 1.

Using the calibration for 1995-2003 but considering the policy changes (i.e. incresing the precision of the Central Bank's signal) the model yields an average elasticity of prices with respect to the exchange rate of 0.09. That is, the policy changes, through the information channel alone, explain 4 out of the 12 point drop in this elasticity. This is the main result of the paper.

The aforementioned policy changes improved the information regarding monetary policy that the Central Bank communicates. Given a more precise signal from the Central Bank the nominal exchange rate is less relevant for pricing decisions. This translates into a lower elasticity of prices with respect to the exchange rate.

2 The informativeness of the nominal exchange rate

In order to simplify the exposition and restrict attention to the information channel, the two models in this paper abstract from modeling why the nominal exchange rate is volatile and how it relates

[4] A similar change can be observed in the mean absolute error of these forecasts.

[5] That is, controlling for domestic variables.

to the aggregate price level. Instead, assume the nominal exchange rate is pinned down by the following general rule: $\log e_t = p_t + \eta_t$, where p_t is the log of the aggregate price level and η_t follows some stochastic process.

In a two country world with trade where the law of one price holds, $-\eta_t$ is the aggregate price level of the foreign country and the nominal exchange rate represents differences in inflation. The data shows that real exchange rates are volatile and persistent; thus define $\eta_t = \rho \eta_{t-1} - p_t^\star + \nu_t$. This is the interpretation η_t has in the remainder of this paper. Chari et al. (2002) show how sticky price models can produce the volatility and some of the persistence of real exchange rates observed in the data through trade of intermediate inputs.[6]

3 A one period model

There is an open economy with money and a nominal exchange rate pinned down by $\log e = p + \eta$, where $\eta \sim N\left(0, \frac{1}{\psi_\eta}\right)$. There is a subset of agents that consists of a household, a final good producer, and a continuum of intermediate goods producers who all take the rule that pins down the exchange rate as given. There is also a government that issues currency and an aggregate noisy signal about the currency's growth rate. Uncertainty arises due to productivity, monetary, information, and exchange rate shocks. Intermediate goods producers are monopolists that make pricing decisions under incomplete information. They observe the noisy signal of the monetary shock and a subset of prices, which includes the nominal exchange rate. All other agents have perfect information. Different levels of pass-through arise depending on the precision of the nominal exchange rate relative to the other signals available to monopolists.

[6] Alternatively, η_t could represent changes in the risk premium as discussed in Alvarez et al. (2007). In a similar paper, Alvarez et al. (2009) model exchange rate variations as a result of changes in the risk premium between domestic and foreign assets. Abstracting from trade and focusing on financial markets segmentation, they model changes in the risk premium driven by monetary policy.

Uncertainty, information and timing

The productivity shock affects intermediate producers. TFP for producer i is $\exp(A_i)$, which has an aggregate and an idiosyncratic component: $A_i = A + a_i$, $A \sim N\left(0, \frac{1}{\psi_A}\right)$, $a_i \sim N\left(0, \frac{1}{\psi_a}\right)$. There is a monetary shock such that the domestic money supply is $M = \exp(\mu) M_0$, where M_0 is some initial level of the money, $\mu = \bar{\mu} + \varepsilon$ and $\varepsilon \sim N\left(0, \frac{1}{\psi_\mu}\right)$. Intermediate producers cannot observe μ and instead observe a common noisy signal $\tilde{\mu} = \mu + \tilde{\varepsilon}$, with $\tilde{\varepsilon} \sim N\left(0, \frac{1}{\psi_{\tilde{\mu}}}\right)$. Finally, the external shock η, as defined previously, pins down the nominal exchange rate. All shocks are independent from each other.

The aggregate state of the economy is $s = \left(s^d, s^f\right)$. The domestic state, $s^d = (A, \mu, \tilde{\mu}, \mathscr{F}(a_i))$, consists of the aggregate productivity shock, the aggregate monetary shock, the common signal of monetary policy and a distribution of individual productivity shocks. The foreign state consists of the external shock $s^f = \eta$.

Each intermediate producer i observes an incomplete information set $\mathscr{I}^i(s) = \{A_i, W_i(s), e(s), \tilde{\mu}\}$ where W_i is the real wage monopolist i pays and $e(s)$ is the nominal exchange rate. Consumers and the final good producer observe the state and all prices. Intermediate producers satisfy the demand for their good given the price they set. At the beginning of the period shocks are realized and then all decisions are taken simultaneous.

Preferences and technology

There is a representative household with preferences for consumption of a final good, real money holdings, and labor. The household splits into a shopper and a continuum of workers. Each worker goes to an industry i and supplies labor in a competitive local market in exchange for a wage. The shopper makes decisions about consumption and holdings of real money balances. The household has access to a complete information set $\mathscr{I}^H(s) = \{s\}$, owns all the firms in the economy and solves:

$$\max_{\{C(s),M^d(s)/P(s),L(s)\}} \left\{ \gamma \log C(s) + (1-\gamma)\log\left[M^d(s)/P(s)\right] - \Psi \int_0^1 \frac{L_i(s)^{1+\zeta}}{1+\zeta} \right\}$$

$$s.t. \quad P(s)C(s) + M^d(s) \leq P(s)\int_0^1 W_i(s)L_i(s) + \int_0^1 \pi_i(s)\,di + M_0 + T(s)$$

$$M_0 = 1$$

where $W_i(s)$ is the real wage in industry i, $\pi_i(s)$ are the nominal profits of intermediate producer i, and $T(s)$ are money transfers from the government such that $T(s) = (\exp(\mu)-1)M_0$. From the first-order conditions of the household's problem we get:

$$\frac{1-\gamma}{\gamma}C(s)P(s) = M^d(s) \tag{1}$$

$$\Psi L_i(s)^{\zeta} = \gamma \frac{W_i(s)}{C(s)} \tag{2}$$

where (1) is a standard quantity equation and (2) is a continuum of intratemporal conditions that equate the marginal disutility for worker i of supplying one extra unit of labor to its marginal contribution to the household's utility. On the production side, there is a competitive final good producer that transforms intermediate goods $(Y_i)_{i\in[0,1]}$ into a final consumption good Y using the following technology:

$$Y = \left(\int_0^1 Y_i^{\theta}\,di\right)^{\frac{1}{\theta}}$$

This producer has access to a complete information set $\mathscr{I}^F(s) = \{s\}$, solves:

$$\max_{\left\{(Y_i(s))_{i\in[0,1]}\right\}} P(s)Y(s) - \int_0^1 P_i(s)Y_i(s)\,di$$

and makes zero profits. From this problem we get that the demand for intermediate good Y_i^d is:

$$Y_i^d(s) = \left(\frac{P(s)}{P_i(s)}\right)^{\frac{1}{1-\theta}} Y(s) \tag{3}$$

and from the technology and the zero profit condition we get the aggregate price level is:

$$P(s)=\left[\int_0^1 P_i(s)^{\frac{\theta}{\theta-1}}\,di\right]^{\frac{\theta-1}{\theta}} \tag{4}$$

Intermediate goods are produced by monopolists with access to a linear technology:

$$Y_i=\exp(A_i)\,L_i \tag{5}$$

and to an incomplete information set $\mathscr{I}^i(s)=\{A_i,W_i(s),e(s),\tilde{\mu}\}$. Monopolist i chooses its price to maximize its expected profits given its available information:

$$\max_{P_i(\mathscr{I}^i)}\sum_s\Gamma\left(s|\mathscr{I}^i\right)\left[P_i\left(\mathscr{I}^i\right)Y_i^d(s)-P(s)W_i(s)L_i(s)\right]$$
$$s.t.\quad Y_i^d(s)=\exp(A_i)\,L_i(s)$$

where $\Gamma\left(s|\mathscr{I}^i\right)$ is the probability of state s conditional on observing information set $\mathscr{I}^i(s)$. From the solution of this problem we get that the price is:

$$P_i\left(\mathscr{I}^i(s)\right)=\frac{W_i(s)}{\exp(A_i)}\frac{\mathbb{E}\left[P(s)^{\frac{2-\theta}{1-\theta}}Y(s)\,|\mathscr{I}^i(s)\right]}{\theta\mathbb{E}\left[P(s)^{\frac{1}{1-\theta}}Y(s)\,|\mathscr{I}^i(s)\right]} \tag{6}$$

which is a markup ($\frac{1-\theta}{\theta}$) above the expected marginal cost given the available information $\mathscr{I}^i(s)$.[7]

In equilibrium, real wages and the aggregate price level depend on the aggregate productivity and monetary shocks. Hence, the nominal exchange rate depends on these same shocks plus the external shock. Each intermediate producer observes four signals $\{A_i,W_i(s),e(s),\tilde{\mu}\}$ and uses them to infer five unobserved state variables $(A,a_i,\mu,\tilde{\varepsilon},\eta)$.

[7] Note that, under perfect information, this is the standard monopolistic competition pricing policy $P_i(s)=\frac{P(s)W_i(s)}{\theta\exp(A_i)}$.

4 Equilibrium and pass-through

DEFINITION 1. An *equilibrium* is allocations, prices, and cross-sectional distributions of intermediate goods and prices, $\mathscr{F}_p$ and $\mathscr{F}_y$, such that: (i) given the prices, the allocation of consumption, money holdings and labor solves the household's problem, (ii) given the prices, the allocation of final good production and intermediate inputs solves the final good firm's problem, (iii) given information set $\mathscr{I}^i(s)$, intermediate producer i's price solves the monopolist's problem, (iv) all markets clear, and (v) the aggregation of industry prices and production with $\mathscr{F}_p$ and $\mathscr{F}_y$ is consistent with the aggregate price level and product.

DEFINITION 2. *Pass-through* in this model is the elasticity of a price with respect to the exchange rate given a foreign shock. In the case of the aggregate price level, pass-through is:

$$PT(s) = \frac{\Delta\%P}{\Delta\%e} = \frac{\frac{d}{ds^f}P(s)}{\frac{d}{ds^f}e(s)}\frac{e(s)}{P(s)}$$

In this paper, pass-through abstracts from any changes in the nominal exchange rate through changes in domestic prices caused by domestic shocks, such as a productivity shock. The reason for this simplification is to focus only on the cases when the exchange rate has a causal effect on domestic prices.[8]

Linear equilibrium

To solve for the equilibrium assume the intermediate producers' prior beliefs about A and μ are normally distributed, with means 0 and $\bar{\mu}$, and variances $\frac{1}{\psi_A}$ and $\frac{1}{\psi_\mu}$, respectively. Also, assume that their posterior beliefs about the distributions of A and μ are also normal. Denote the means of these distributions $\mathbb{E}_i[A]$ and $\mathbb{E}_i[\mu]$, and their variances $Var_i(A)$ and $Var_i(\mu)$, respectively. Finally, guess that the cross-sectional distributions of log prices and production, p_i and y_i, are normal.[9] Under these assumptions, the logs of equations (1) through (6) and the definition of the exchange

[8] Without solving for the equilibrium it is easy to show that the above definition using the differentials with respect to the domestic state is always equal to 1.

[9] Which is the case in a symmetric linear equilibrium as defined below.

rate are:

$$\text{quantity equation:} \quad \log\left(\frac{\gamma}{1-\gamma}\right)+\mu = p+y \tag{7}$$

$$\text{intratemporal conditions:} \quad \log(\Psi)+\zeta l_i = \log(\gamma)+w_i-y \tag{8}$$

$$\text{intermediate demand:} \quad y_i = \frac{1}{1-\theta}p - \frac{1}{1-\theta}p_i + y \tag{9}$$

$$\text{aggregate price:} \quad p = \int_0^1 p_i + \frac{\theta}{\theta-1}\frac{1}{2}\left[\int_0^1 p_i^2 di - \left(\int_0^1 p_i di\right)^2\right] \tag{10}$$

$$\text{production technology:} \quad y_i = A + a_i + l_i \tag{11}$$

$$\begin{aligned}\text{optimal pricing:} \quad p_i = {} & \log\left(\frac{1}{\theta}\right) + w_i - A - a_i + \mathbb{E}_i[p] \\ & + \frac{1}{2}Var_i\left(\frac{2-\theta}{1-\theta}p+y\right) - \frac{1}{2}Var_i\left(\frac{1}{1-\theta}p+y\right)\end{aligned} \tag{12}$$

$$\text{definition of exchange rate} \quad \log e = p + \eta \tag{13}$$

Hereafter I restrict attention to a symmetric equilibrium.[10]

DEFINITION 3. A *symmetric linear equilibrium* is allocations y, $(y_i, l_i)_{i\in[0,1]}$ and prices $\log e$, p, $(p_i, w_i)_{i\in[0,1]}$ such that: (i) the allocations and prices are linear functions of the state variables, (ii) posterior moments are conditional on all available information, meaning $\mathbb{E}_i[f(x)] = \mathbb{E}\left[f(x)|\mathscr{I}^i(s)\right]$, and (iii) the allocations and prices satisfy (7) through (13) (i.e. allocations are optimal and prices clear all markets).

PROPOSITION 1. A *linear equilibrium* exists and is unique for a generic set of parameters.

This is an important feature of the model. Its main goal is to show how the equilibrium level of pass-through changes as the precision of exogenous information changes (i.e. as $\psi_{\tilde{\mu}}$ changes). PROPOSITION 1 guarantees that this comparison can be done for a generic set of $\psi_{\tilde{\mu}}$.

The proof of PROPOSITION 1 follows in three steps. The first is to note that the posterior moments of p and y in equilibrium are linear expressions of the posterior moments of μ and A (this follows from rearranging the above expressions and taking expectations). The second step

[10] As discussed in Amador and Weill (2010), not too many results have dealt with the existence of nonlinear equilibria in economies with asymmetric information.

is to show that the posterior moments of μ and A are linear expressions of the signals $\tilde{\mu}$, A_i, w_i and $\log e$ (this is briefly discussed below and follows from the assumption of Gaussian independent shocks). From the first two steps we get that equations (7) through (13) define a linear system of seven equations with seven unknowns. For a generic set of parameter values the system has a unique solution.[11] This completes the proof.

Perfect information benchmark

Under perfect information (12) can be written as $p_i = \log\left(\frac{1}{\theta}\right) + p + w_i - A - a_i$ and the aggregate price in equilibrium is $P(s) = \exp(\mu - A + \mathscr{K}_0)$. Pass-through as defined before is 0 since the price does not depend on the foreign state. This result follows directly from the assumption that the exchange rate is disconnected from all the agents in the model.

As in the barbershop example highlighted in the introduction, a change in the foreign state moves the nominal exchange rate but is meaningless to all the agents in the model since it does not have cost or demand implications. Hence, under perfect information individual monopolists do not adjust their individual price due to a depreciation, which implies pass-through is zero.

Incomplete information

The characterization of the incomplete information equilibrium is exposed in detail in the Appendix. Here I will focus on two key intermediate steps that help give intuition about how agents

[11] This comes from the fact that the constants in the system depend only on parameters and that the set of non-singular matrices is generic.

use information and how it affects aggregates. First, rearrange (7) through (13) to get:

$$\begin{aligned}
w_i &= \mathscr{A}_0 + \mathscr{A}_1\mu + \mathscr{A}_2 A + \mathscr{A}_3 a_i + \mathscr{A}_4 \mathbb{E}\left[\mu|\mathscr{I}^i(s)\right] + \mathscr{A}_5 \mathbb{E}\left[A|\mathscr{I}^i(s)\right] \\
&\quad + \mathscr{A}_6 \int_0^1 \mathbb{E}\left[\mu|\mathscr{I}^j(s)\right] dj + \mathscr{A}_7 \int_0^1 \mathbb{E}\left[A|\mathscr{I}^j(s)\right] dj \\
p &= \mathscr{B}_0 + \mathscr{B}_1\mu + \mathscr{B}_2 A + \mathscr{B}_3 \int_0^1 \mathbb{E}\left[\mu|\mathscr{I}^j(s)\right] dj + \mathscr{B}_4 \int_0^1 \mathbb{E}\left[A|\mathscr{I}^j(s)\right] dj \\
\log e &= \mathscr{B}_0 + \mathscr{B}_1\mu + \mathscr{B}_2 A + \mathscr{B}_3 \int_0^1 \mathbb{E}\left[\mu|\mathscr{I}^j(s)\right] dj + \mathscr{B}_4 \int_0^1 \mathbb{E}\left[A|\mathscr{I}^j(s)\right] dj - \eta
\end{aligned}$$

Domestic prices do not depend directly on the foreign state, but only through the effect the nominal exchange rate has on expectations. Recall that monopolist i observes $\mathscr{I}^i(s) = \{A_i, w_i, \log e, \tilde{\mu}\}$. Rearrange the information set to get conditionally independent signals for μ:

$$\begin{aligned}
\tilde{\mu} &= \mu + \tilde{\varepsilon} \\
\tilde{A}_{i,\mu} &= \mu + \frac{1}{\mathscr{D}_1} a_i \\
\tilde{w}_{i,\mu} &= \mu + \frac{1}{\mathscr{D}_2} A \\
\log \tilde{e}_\mu &= \mu + \frac{1}{\mathscr{D}_3} \varepsilon^\star
\end{aligned}$$

where $\tilde{A}_{i,\mu}$, $\tilde{w}_{i,\mu}$ and $\log \tilde{e}_\mu$ are linear expressions of variables that are observable to monopolist i. Note that these include the its own posterior expectations as well as the aggregate posterior expectations of all other monopolists. In a symmetric equilibrium the pricing policy function of all monopolists is the same, so an individual monopolist i can, in equilibrium, correctly calculate the integral of all agents expectations.[12] Given the above rearrangement, the posterior expectation is a

[12]The rearranged signals $\tilde{w}_{i,\mu}$ and $\log \tilde{e}_\mu$ are linear expressions of expectations and aggregate expectations. Integrating over i, this posterior expectation (and doing the equivalent for A) defines a linear system on aggregate expectations $\int_0^1 \mathbb{E}\left[\mu|\mathscr{I}^j(s)\right] dj$ and $\int_0^1 \mathbb{E}\left[A|\mathscr{I}^j(s)\right] dj$. The solution of this linear system is aggregate expectations that depend linearly on $\tilde{\mu}$, w_i and $\log e$. Plugging back into the posterior expectations for monopolist i defines a linear system on individual expectations, the solution to this system is individual expectations that are also linear functions of $\tilde{\mu}$, w_i and $\log e$. These derivations are detailed in the Appendix on the proof of PROPOSITION 2.

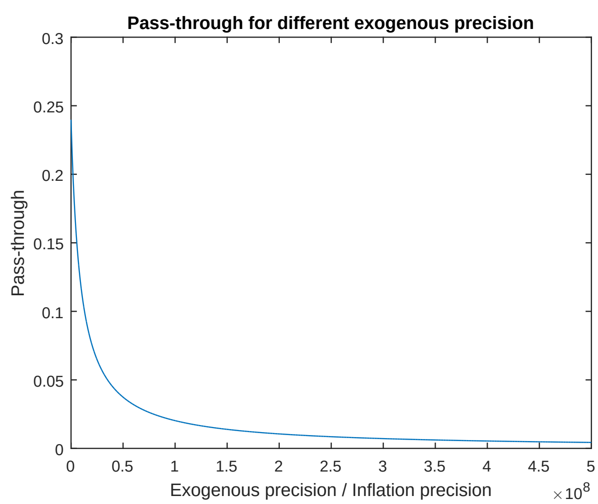
weighted sum of these signals and the posterior variance is a constant:

$$\mathbb{E}\left[\mu|\mathscr{I}^i\left(s^t\right)\right] = \frac{\psi_\mu \bar{\mu} + \psi_{\tilde{\mu}} \tilde{\mu} + \mathscr{D}_1^2 \psi_a \tilde{A}_{i,\mu} + \mathscr{D}_2^2 \psi_A \tilde{w}_{i,\mu} + \mathscr{D}_3^2 \psi_\eta \log \tilde{e}_\mu}{\psi_\mu + \psi_{\tilde{\mu}} + \mathscr{D}_1^2 \psi_a + \mathscr{D}_2^2 \psi_A + \mathscr{D}_3^2 \psi_\eta}$$

$$Var\left(\mu|\mathscr{I}^i\left(s^t\right)\right) = \frac{1}{\psi_\mu + \psi_{\tilde{\mu}} + \mathscr{D}_1^2 \psi_a + \mathscr{D}_2^2 \psi_A + \mathscr{D}_3^2 \psi_\eta}$$

Note that the weights of these signals are increasing in their relative precision. For instance, if $\psi_{\tilde{\mu}}$ increases then the weight of $\tilde{\mu}$ increases, which means it becomes a more informative signal about μ. Note that if $\psi_{\tilde{\mu}} \rightarrow \infty$ then μ becomes observable and its weight converges to 1. If μ becomes observable then A becomes observable as well through wages and the nominal exchange rate becomes irrelevant to determine individual prices. Figure 2 illustrates the convergence of pass-through to the perfect information benchmark as $\psi_{\tilde{\mu}} \rightarrow \infty$. On the horizontal axis the plot has the precision of the Central Bank's signal relative to the precision of the prior $\psi_{\tilde{\mu}}/\psi_\mu$ ("inflation precision").[13]

Figure 2: Pass-through for different exogenous precision $\psi_{\tilde{\mu}}$

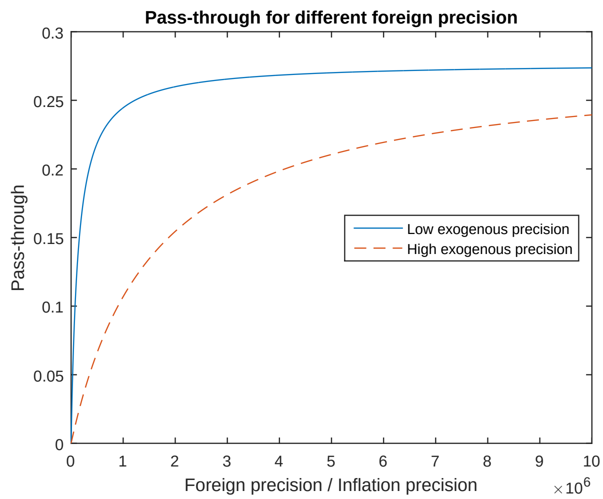


Now, define ψ_η as the foreign precision of the nominal exchange rate. If this foreign precision increases (i.e. foreign exchange rate volatility decreases), then the weight of the nominal exchange rate increases, making it a more relevant signal for price decisions and increasing pass-through.

[13] Figures 2 and 3 are generated using the parameter values from the calibration detailed in Section 6.

Figure 3 illustrates this effect for different levels of the variance of the foreign shock relative to inflation variance.

Figure 3: Pass-through for different relative variance



These two figures illustrate the information channel: the precision of the nominal exchange rate relative to the precision of all the other signals in the economy determines the level of pass-through. Figure 3 illustrate the currency intervention trade-off discussed in the introduction: pass-through is decreasing in the foreign volatility of the exchange rate.

As an example, suppose pass-through is 0.1 (i.e. ψ_η/ψ_μ is approximately 1 and $\psi_{\tilde{\mu}}$ is consistent with the dashed line) and that the Central Bank observes a 10% depreciation. Without intervention, the shock would increase inflation in 1%. If a currency intervention reduces the depreciation to 7% and increases ψ_η/ψ_μ to 6 then the net effect on inflation would be of 1.4%.

5 Infinite periods and persistent RER shocks

This section extends the model from Section 3 to infinite periods. The main purpose of this extension is to be able to compare the effects on pass-through of two channels: information and costs. To do so, I add three additional features: infinite periods, imported inputs for intermediate producers, and persistent real exchange rate shocks.

Time is discrete and runs forever. Define the nominal exchange rate such that the real exchange rate in period t is volatile and persistent. There are different trade frictions that can deliver volatile and persistent real exchange rates.[14] For simplicity I abstract from modeling said frictions and instead take as given the following exogenous process for the real exchange rate: $RER_t = \exp(\eta_t)$, where $\eta_t = \rho\eta_{t-1} + \nu_t$, $0 < \rho < 1$, and $\nu_t \sim N\left(0, \frac{1}{\psi_\eta}\right)$.

The other shocks are analogous to those in the one period model: TFP in period t for intermediate producer i is $\exp(A_{i,t})$ where $A_{i,t} = A_t + a_{i,t}$, $A_t \sim N\left(0, \frac{1}{\psi_A}\right)$, $a_{i,t} \sim N\left(0, \frac{1}{\psi_a}\right)$. The domestic money supply in period t is $M_t = \exp(\mu_t) M_{t-1}$ where $\mu_t = \bar{\mu} + \varepsilon_t$ and $\varepsilon_t \sim N\left(0, \frac{1}{\psi_\mu}\right)$. Intermediate producers observe the aggregate exogenous $\tilde{\mu}_t = \mu_t + \tilde{\varepsilon}_t$, with $\tilde{\varepsilon}_t \sim N\left(0, \frac{1}{\psi_{\tilde{\mu}}}\right)$. All shocks are independent from each other and across time.

The aggregate state of the economy in period t is now $s_t = \left(s_t^d, s_t^f\right)$. The domestic state is $s_t^d = \left(A_t, \mu_t, \tilde{\mu}_t, \eta_t, \mathscr{F}_t(a_{i,t})\right)$ and the foreign state is η_t. Let $s^t = \left(s_t, s^{t-1}\right)$ be the history of states up to period t.

At the beginning of each period all shocks are realized. Then all agents observe their information sets and make decisions simultaneously. Intermediate goods producers observe incomplete information set $\mathscr{I}^i(s^t)$ and make pricing decisions. Consumers and the final good producer observe the state and prices and make consumption and production decisions, respectively. At the end of the period all agents observe the state.[15]

The household's problem is now:

$$\max_{\left\{C(s^t), M^d(s^t)/P(s^t), L(s^t)\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \Gamma(s^t) \left\{ \gamma \log C(s^t) + (1-\gamma)\log\left[M^d(s^t)/P(s^t)\right] - \Psi \int_0^1 \frac{L_i(s^t)^{1+\zeta}}{1+\zeta} di \right\}$$

$$s.t. \quad P(s^t) C(s^t) + M^d(s^t) \le P(s^t) \int_0^1 W_i(s^t) L_i(s^t) di + \int_0^1 \pi_i(s^t) di + M^d(s^{t-1}) + T(s^t)$$

$$M_0 = 1$$

[14] See Chari et al. (2002) for a detailed discussion of these frictions.

[15] This assumption simplifies the use of information for intermediate producers. As in the one period model, at each period t it is sufficient to characterize the posterior moments of η_t, A_t and μ_t. Without this assumption it would be also necessary to update the posterior moments of all previous realizations of η_t with the new information. Because η_t is persistent, signals at period t are informative about all previous realizations of the shock which are relevant for the current posterior distribution.

where $0 < \beta < 1$ is the discount factor and $\Gamma(s^t)$ is the probability of the history of states s^t. From the first-order conditions of this problem we get the following Euler Equation for money and intratemporal condition:

$$\frac{\gamma}{1-\gamma} C\left(s^t\right)^{-1} P\left(s^t\right)^{-1} = M\left(s^t\right)^{-1} + \beta \mathbb{E}\left[\frac{\gamma}{1-\gamma} C\left(s^{t+1}\right)^{-1} P\left(s^{t+1}\right)^{-1} | s^t\right]$$

$$\Psi L_i\left(s^t\right)^{\zeta} = W_i\left(s^t\right) \gamma C\left(s^t\right)^{-1} \tag{14}$$

Iterating on the Euler Equation one can get:

$$\frac{\gamma}{1-\gamma} C\left(s^t\right)^{-1} P\left(s^t\right)^{-1} = \sum_{k=t}^{t+T} \beta^{k-t} \mathbb{E}\left[M^d\left(s^k\right)^{-1} | s^t\right] + \beta^T \mathbb{E}\left[M^d\left(s^{t+T}\right)^{-1} | s^t\right] \tag{15}$$

PROPOSITION 2: If $\left\{M^d\left(s^{t+h}\right)\right\}_{h=0}^{\infty}$ belongs to an equilibrium allocation then, in the limit, we have $\lim_{T\to\infty} \beta^T \mathbb{E}\left[M^d\left(s^{t+T}\right)^{-1} | s^t\right] = 0$.

The proof for PROPOSITION 2 can be found in the Appendix. It applies a similar argument as Obstfeld and Rogoff (1983) and Amador and Weill (2010) to rule out explosive solutions for money holdings. Given that $\mu_t - \bar{\mu}$ follows a white noise process, it can easily be shown that the limit of the right hand side of (15) as $T \to \infty$ is well defined.[16] So we can rewrite the equation as:

$$\frac{\gamma}{1-\gamma} Y\left(s^t\right)^{-1} P\left(s^t\right)^{-1} = M\left(s^t\right)^{-1} \frac{\exp\left(\bar{\mu} + \frac{1}{2}\frac{1}{\psi_\mu}\right)}{\exp\left(\bar{\mu} + \frac{1}{2}\frac{1}{\psi_\mu}\right) - \beta} \tag{16}$$

for all t, s^t (here I also used the market clearing conditions for the final good and for money).

The problem of the final producer is static at each history of states s^t. The aggregation technol-

[16]This structure for the process of μ_t is crucial for tractability of the solution. The model can be further extended to have μ_t follow a more general AR(1) process. One can still show that the infinite sum is well defined in this case; however it no longer has a closed form solution, so for exposition purposes, this paper does not consider this case.

ogy, demand equations and aggregate price are analogous:

$$Y\left(s^t\right)=\left(\int_0^1\left[Y_i\left(s^t\right)\right]^{\theta}di\right)^{\frac{1}{\theta}}$$

$$Y_i^d\left(s^t\right)=\left(\frac{P\left(s^t\right)}{P_i\left(\mathscr{I}^i\left(s^t\right)\right)}\right)^{\frac{1}{1-\theta}}Y\left(s^t\right) \tag{17}$$

$$P\left(s^t\right)=\left[\int_0^1 P_i\left(s^t\right)^{\frac{\theta}{\theta-1}}di\right]^{\frac{\theta-1}{\theta}} \tag{18}$$

The intermediate producers have access to a Cobb-Douglas technology:

$$Y_i=A_tQ_i^{\delta}L_i^{1-\delta}$$

where Q_i is an imported input that cannot be produced domestically. Denote the price of Q_i as P^Q and for simplicity assume it is constant. Now, intermediate monopolist i observes $\mathscr{I}^i\left(s^t\right)=\left\{s^{t-1},W_i\left(s^t\right),P^Q,e\left(s^t\right),\tilde{\mu}_t\right\}$ and solves:

$$\max_{P_i\left(\mathscr{I}^i\left(s^t\right)\right)}\sum_{s^t}\Gamma\left(s^t|\mathscr{I}^i\left(s^t\right)\right)\left[P_i\left(\mathscr{I}^i\left(s^t\right)\right)Y_i^d\left(s^t\right)-P\left(s^t\right)W\left(s^t\right)L_i\left(s^t\right)-e\left(s^t\right)P^QQ_i\left(s^t\right)\right]$$

$$s.t.\quad Y_i^d\left(s^t\right)=A_tQ_i\left(s^t\right)^{\delta}L_i\left(s^t\right)^{1-\delta}$$

and thus the optimal price is:

$$P_i\left(\mathscr{I}^i\left(s^t\right)\right)=\frac{1}{\delta^{\delta}\left(1-\delta\right)^{1-\delta}}\left(\frac{P^T}{P^{\star}}\right)^{\delta}W_i\left(s^t\right)^{1-\delta}\frac{\mathbb{E}\left[\frac{1}{\exp\left(A_{i,t}\right)}\exp\left(\delta\eta_t\right)P\left(s^t\right)^{\frac{2-\theta}{1-\theta}}Y\left(s^t\right)|\mathscr{I}^i\left(s^t\right)\right]}{\theta\mathbb{E}\left[P\left(s^t\right)^{\frac{1}{1-\theta}}Y\left(s^t\right)|\mathscr{I}^i\left(s^t\right)\right]} \tag{19}$$

and the demand of labor and imported input:

$$L_i^d\left(s^t\right)=\frac{Y_i^d\left(s^t\right)}{\exp\left(A_{i,t}\right)}\left(\frac{e\left(s^t\right)P^Q}{P\left(s^t\right)W_i\left(s^t\right)}\frac{1-\delta}{\delta}\right)^{\delta} \tag{20}$$

$$Q_i^d\left(s^t\right)=\frac{Y_i^d\left(s^t\right)}{\exp\left(A_{i,t}\right)}\left(\frac{P\left(s^t\right)W_i\left(s^t\right)}{e\left(s^t\right)P^Q}\frac{\delta}{1-\delta}\right)^{1-\delta} \tag{21}$$

Note that if we take the limit as $\delta \to 0$ we get that the technology goes to the linear technology from (5) and the optimal price (14) approaches (6). Now the nominal exchange rate does affect costs directly, so even under perfect information an exogenous variation in the exchange rate (a change in η_t) will imply positive pass-through. Since it is a persistent shock, exogenous depreciations have an effect on contemporaneous and future price increases.

Finally, the nominal exchange rate is pinned down by:

$$\frac{e\left(s^t\right)P^{\star}}{P\left(s^t\right)} = \exp\left(\eta_t\right) \tag{22}$$

Linear equilibrium

Equations (14) and (16) through (22) define a static system and the definition of equilibrium is analogous to the one period version.

To characterize the equilibrium, as before, assume the prior beliefs that intermediate producers have about the distributions of A_t, η_t and μ_t are Normal with means 0, $\rho\eta_{t-1}$ and $\bar{\mu}$ and variances $\frac{1}{\psi_A}$, $\left(\frac{1}{1-\rho^2}\frac{1}{\psi_\eta}\right)$ and $\frac{1}{\psi_\mu}$, respectively. Also, assume that their posterior beliefs are normal with means $\mathbb{E}_i\left[A_t\right]$, $\mathbb{E}_i\left[\eta_t\right]$ and $\mathbb{E}_i\left[\mu_t\right]$ and variances $Var_i\left(A_t\right)$, $Var_i\left(\eta_t\right)$ and $Var_i\left(\mu_t\right)$, respectively. Finally, assume that the cross-sectional distributions of log prices and production, $p_{i,t}$ and $y_{i,t}$, are normal.

Given these assumptions rewrite equations (14) and (16) through (22) in log linear form:

$$\text{quantity equation:} \quad \log\left(\frac{\gamma}{1-\gamma}\frac{1}{\tilde{\beta}}\right)+\mu_t = y_t + p_t \tag{23}$$

$$\text{intratemporal conditions:} \quad \log(\psi)+\zeta l_{i,t} = \log(\gamma)+w_{i,t}-y_t \tag{24}$$

$$\text{intermediate demand:} \quad y_{i,t} = \frac{1}{1-\theta}p_t - \frac{1}{1-\theta}p_{i,t}+y_t \tag{25}$$

$$\text{aggregate price:} \quad p_t = \int_0^1 p_{i,t} + \frac{\theta}{\theta-1}\frac{1}{2}\left[\int_0^1 p_{i,t}^2 di - \left(\int_0^1 p_{i,t} di\right)^2\right] \tag{26}$$

$$\text{labor demand:} \quad l_{i,t} = y_{i,t} - A_{i,t} + \delta\eta_t - \delta w_{i,t} + \delta\log\left(\frac{1-\delta}{\delta}\frac{P^Q}{P^\star}\right) \tag{27}$$

$$\text{imports demand:} \quad q_{i,t} = y_{i,t} - A_{i,t} - (1-\delta)\eta_t + (1-\delta)w_{i,t} + (1-\delta)\log\left(\frac{\delta}{1-\delta}\frac{P^\star}{P^Q}\right) \tag{28}$$

$$\text{optimal pricing:} \quad p_{i,t} = \log\tilde{\theta} + (1-\delta)w_{i,t} - A_{i,t} + \mathbb{E}_i[\delta\eta_t + p_t] \tag{29}$$

$$+\frac{1}{2}Var_i\left(\delta\eta_t + \frac{2-\theta}{1-\theta}p_t + y_t\right) - \frac{1}{2}Var_i\left(\frac{1}{1-\theta}p_t + y_t\right)$$

$$\text{definition of exchange rate:} \quad \log e_t + p^\star = p_t + \eta_t \tag{30}$$

where $\tilde{\beta} = \frac{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)}{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)-\beta}$ and $\log\tilde{\theta} = \log\left(\frac{1}{\delta^\delta(1-\delta)^{1-\delta}}\frac{1}{\theta}\left(\frac{P^T}{P^\star}\right)^\delta\right)$. Now, the optimal pricing equation has additional terms related to the moments of the real exchange rate shock. Equations (23) through (30) define a static linear system for the equilibrium variables. DEFINITION 6 and PROPOSITION 3 below follow immediately:

DEFINITION 4. A *symmetric linear equilibrium* is allocations $y_t, (y_{i,t}, l_{i,t}, q_{i,t})_{i\in[0,1]}$ and prices $\log e_t, p_t, (w_{i,t}, p_{i,t})_{i\in[0,1]}$ such that: (i) the allocations and prices are linear functions of the state variables, (ii) posterior moments are conditional on all available information, that is $\mathbb{E}_i[f(x)] = \mathbb{E}\left[f(x)\,|\,\mathscr{I}^i(s)\right]$, and (iii) the allocations and prices satisfy (23) through (30) (i.e. allocations are optimal and prices clear all markets).

PROPOSITION 3. A *linear equilibrium* exists and is unique for a generic set of parameters.

The proof of PROPOSITION 3 is analogous to the proof of PROPOSITION 1.

The nominal exchange rate affects prices in this economy through two channels. The first is the information channel discussed in the previous section. The second is the cost channel mentioned in the introduction: intermediate producers now use an imported input through which the exchange rate affects their costs. This implies that pass-through in the perfect information benchmark is no longer zero and instead is equal to δ, which is the expenditure share on imported inputs. Given the persistence of the exchange rate process, the effect of a one time shock lingers in subsequent periods through this cost channel.

Pass-through

Having multiple periods allows one to analyze the following three concepts of pass-through:

DEFINITION 5. *Short-term pass-through*:

$$PT\left(s^t\right) = \frac{\frac{d}{ds^f}P\left(s^t\right)}{\frac{d}{ds^f}e\left(s^t\right)}\frac{e\left(s^t\right)}{P\left(s^t\right)}$$

DEFINITION 6. *Medium-term pass-through*:

$$PT\left(s^{t+h}|s^t\right) = \frac{\frac{d}{ds_t^f}P\left(s^{t+h}\right)}{\frac{d}{ds_t^f}e\left(s^t\right)}\frac{e\left(s^t\right)}{P\left(s^{t+h}\right)}$$

DEFINITION 7. *Long-term accumulated pass-through*:

$$PT_\infty\left(s^t\right) = \sum_{h=0}^{\infty} PT\left(s^{t+h}|s^t\right)$$

Medium-term pass-through is the effect of the nominal exchange rate on future prices. These will be directly affected by the persistence of the real exchange rate shocks. Finally, the long-term accumulated pass-through measures the total effect of a single exogenous innovation on the exchange rate at a given period t. Having these temporal distinctions of pass-through helps disentangle the relevance of the information and the persistence channels. It also allows one to quantify their relative importance.

In general, it can be shown that medium-term and long-term pass-through are $\rho^h PT\left(s^t\right)$ and $\frac{1}{1-\rho}PT\left(s^t\right)$, respectively. Regardless of the presence of information frictions, the nominal exchange rate shocks affects future prices only through the persistence of the real exchange rate shock. In a richer version of the model in which monetary shocks were persistent as well, future prices would also be affected through the consumer's dynamic decisions about money holdings, since these would depend on present and future expected prices. In this case, both the persistence and information channels as defined in this paper would still be present, but their disentanglement would be more complicated to show.

Perfect information benchmark

Under perfect information, the aggregate price in equilibrium is $P\left(s^{t}\right)=\exp\left(\mu_{t}-\frac{1}{1-\delta}A_{t}+\frac{\delta}{1-\delta}\eta_{t}+\mathscr{K}_{0}\right)$. Short-term pass-through is δ, which is the optimal expenditure share on the imported good. This is an intuitive and expected result since under perfect information the information channel is not present and prices are just affected by the exchange rate through costs.

Incomplete information

As in the one period case, we can rearrange the equations that characterize the symmetric linear equilibrium to get a linear system for individual and aggregate expectations.[17] The most important difference is that now prices depend directly on the foreign state through costs and indirectly through expectations:

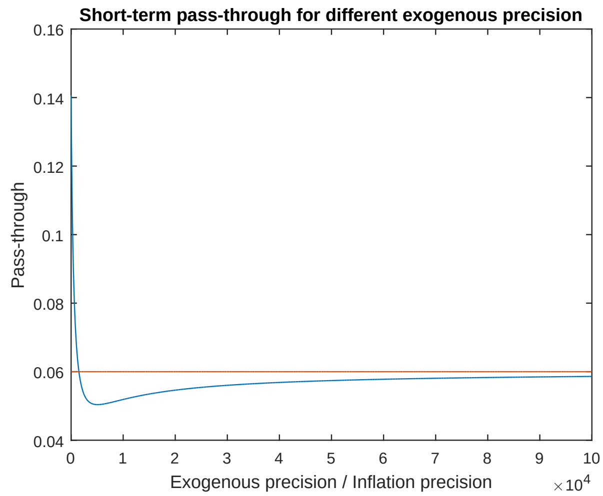
$$p_{t}=\mathscr{C}_{0}+\mathscr{C}_{1}\mu_{t}+\mathscr{C}_{2}A_{t}+\mathscr{C}_{3}\eta_{t}+\mathscr{C}_{4}\int_{0}^{1}\mathbb{E}\left[\mu_{t}|\mathscr{I}^{j}\left(s^{t}\right)\right]dj+\mathscr{C}_{5}\int_{0}^{1}\mathbb{E}\left[A_{t}|\mathscr{I}^{j}\left(s^{t}\right)\right]dj+\mathscr{C}_{6}\int_{0}^{1}\mathbb{E}\left[\eta_{t}|\mathscr{I}^{j}\left(s^{t}\right)\right]dj$$

As in the one period model pass-through converges to the perfect information benchmark as $\psi_{\tilde{\mu}}\rightarrow\infty$. Figure 4 shows the convergence of equilibrium pass-through to the perfect information benchmark for $\delta=0.06$.[18]

[17] See the appendix for the detailed derivation and solution of equillibriium.

[18] Figures 4 and 5 are generated using the parameter calibration detailed in Section 6.

Figure 4: Pass-through for different exogenous precision $\psi_{\tilde{\mu}}$



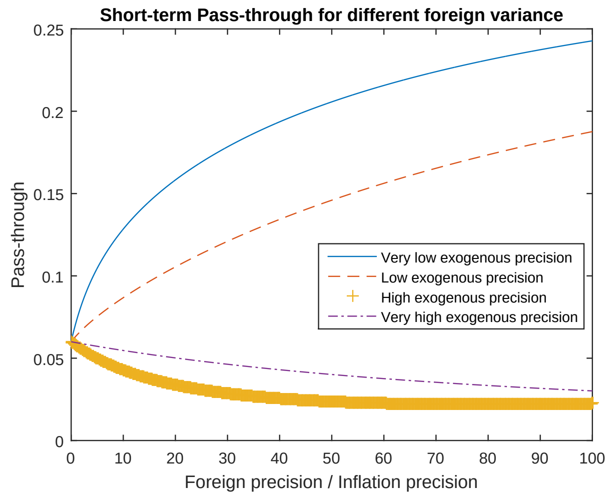
In this case, pass-through is lower than the benchmark before converging. This is because now the information friction poses a wedge on labor and imported input expenditure shares:

$$L^s = (1-\delta)\frac{1}{(1-\delta)+\delta\frac{\mathbb{E}[P(s^t)|\mathscr{I}^i(s^t)]}{P(s^t)}}$$

$$Q^s = \delta\frac{1}{\delta+(1-\delta)\frac{P(s^t)}{\mathbb{E}[P(s^t)|\mathscr{I}^i(s^t)]}}$$

This wedge depends on whether monopolists overestimate or underestimate prices, which in turn depends on how well informed they are. Thus, the information friction also affects the level of the direct channel discussed before, which explains why pass-through can be lower than the benchmark. The behavior of this wedge also affects the currency intervention trade-off, as Figure 5 illustrates:

Figure 5: Currency intervention trade-off



The trade-off exists if the precision of the signal from the Central Bank is too low (i.e. if it is in the decreasing region of Figure 4). For high enough precision the trade-off is no longer relevant and the elasticity becomes less responsive to changes in the exchange rate precision (hence the flatter curves).

6 Data, calibration and main results

Micro-price data and pass-through

Pass-through is estimated using confidential micro-price monthly data for the period 1995-2014. This is the same data used to construct the CPI in Mexico and is publicly available for 2011 onward. Each data point is the observed price for a generic product in a specific city. There are 282 generic products observed in 46 cities in Mexico. Using a similar specification as that in Kochen and Samano (2016), the estimation considers the following econometric model:

$$\Delta p_t^s = \beta_0 + \beta_1 \Delta_c e_t^s + \beta_2 \Delta_c \log y_t + u_t \tag{31}$$

where s denotes the generic product, Δp_t^s is the log price change of product s at time t, $\Delta_c e_t^s$ is the cumulative log change in the Mexican peso-US dollar nominal exchange rate during the same price spell and $\Delta_c \log y_t$ is the cumulative log change in the monthly economic activity indicator IGAE. This last variable is used to control for variations caused by domestic factors. Under this specification, short-term pass-through (as defined previously in the model) is identified by β_1. Hence, long-term pass-through is $\frac{1}{1-\rho}\beta_1$, where ρ is the persistence of the process for η_t, which is estimated below.

The estimation is carried out for two separate periods: 1995-2002 and 2003-2014 with estimates $\beta_1^{95-02} = 0.13$ and $\beta_1^{03-14} = 0.01$. For both one can reject the null of $\beta = 0$ at the 1% confidence level.

Parameter calibration

Table 1 summarizes the calibration of the parameters in the model. All but two parameters are identified by moments other than the elasticity of prices estimated above. The two parameters set to match the level of pass-through are $\zeta = 1.95$ and $\theta = 0.29$, which imply labor elasticity of 0.58 and a markup of 240%.[19]

[19] As a reference for these numbers, Chari et al. (2002) set equivalent parameters to target a labor supply elasticity of 0.5 and a markup of 11% in the US.

Table 1: Calibrated parameters

Parameter	Value		Identification
	1995-2002	2003-2014	
β	0.94		Annual interest rate of 6%
γ	0.94		As in Chari, Kehoe
Ψ	10		and McGrattan (2002)
ζ	1.33		Set to match pass-through of 0.13
θ	0.38		between 1995 and 2002
δ	0.06		$C_{imported}/C_{Total}$ for 1995 - 2002
ρ	0.89		$\Delta_{12}\log RER_t = \rho\Delta_{12}\log RER_{t-1} + \nu_t$ for 1995 - 2002
ψ_η	371		Variance of ν_t
$\psi_{\tilde{\mu}}$	8,656	57,986	Cross-sectional variance of inflation forecasts
ψ_μ	152		Annual inflation variance
ψ_A	1,088		Variance of annual *GDP* growth
ψ_a	0.045		Cross-sectional price variance
$\bar{\mu}$	0.18		monthly inflation for Mexico

The discount factor β is to be consistent with an annual interest rate of 6%. I set γ and Ψ as in Chari, Kehoe and McGrattan (2002). I set δ to match the share of imported goods in consumption expenditure. I set $\bar{\mu}$ to match average annual inflation in 1995-2002. I choose ψ_μ to match an annual inflation standard deviation of 0.12. I choose ψ_A to match a variance of annual GDP growth of 0.04. Finally, I choose ψ_a to match the cross-sectional variance of log price changes observed in the data of 3.

To characterize the process for η_t, recall that it is defined as $\exp(\eta_t) = \frac{e_t P_t^\star}{P_t}$. I use monthly data for the nominal exchange rate (pesos/USD), the CPI for the U.S. and Mexico, to construct a time series for η_t and estimate the AR(1) model $\Delta_{12}\log RER_t = \rho\Delta_{12}\log RER_{t-1} + \nu_t$. This yields a value of $\rho = 0.89$ and a standard deviation of ν_t of 0.051.

Policy changes and main results

Table 2 summarizes three policies that affect the information available to agents. The column of communication shows the Central Bank's policy for reporting all of its activities to the public. The signaling column shows the Central Bank's daily operation policy, through which it sends signals to the financial markets. These three signaling policies are described in detail in the documents regarding the Central Bank's operational objectives.[20] This change can be summarized as going from less to more frequent short run updates regarding how the Central Bank will operate to achieve its targets. The objective column specifies the Central Bank's objective and term through which it aims to comply with its Constitutional mandate of price stability. Before 2001 the Central Bank targeted short term growth rates for the money supply. After 2001 the Central Bank adopted an inflation-targeting regime and started announcing multi-annual inflation targets as opposed to the shorter term money growth rate targets.

Table 2: Policy changes affecting agents' information

<table>
<tr><th>Period</th><th>Communication</th><th>Signaling</th><th>Objective</th></tr>
<tr><td>Jan 1995 - Dec 1999</td><td>Annual reports</td><td rowspan="3">Accumulated Balances</td><td rowspan="2">Money supply growth, short term</td></tr>
<tr><td>Jan 2000 - Dec 2000</td><td rowspan="4">Quarterly reports</td></tr>
<tr><td>Jan 2001 - Sep 2003</td><td rowspan="3">Inflation targeting, long term</td></tr>
<tr><td>Oct 2003 - Jan 2008</td><td>Daily Balances</td></tr>
<tr><td>Feb 2008 - present</td><td>Interest rate objective</td></tr>
</table>

To calibrate the precision of the aggregate signal, $\psi_{\tilde{\mu}}$, I use data from the Survey of Expectations of Specialists in Economics from the Private Sector[21] elaborated by the Bank of Mexico. This survey is conducted on a monthly basis since January 1999. On the first day of each month, 31 experts (on average) are asked about their expectations regarding current and future macroeconomic variables. I use data about their responses to the question "What will inflation be 12 months from now?" I set the precision parameter $\psi_{\tilde{\mu}}$ to match the average cross-sectional variance of annual inflation forecasts for the two periods of interest.

Table 3 has the main result of this paper. It compares the estimated short and long term pass-through with that implied by the model calibration. By construction, model pass-through is the same for the period 1995-2003. Given that, the calibration for 2003-2014 considers only an improvement in the available information (that is, it only changes the value of $\psi_{\tilde{\mu}}$). This means that the policy changes, through the information channel alone, explain approximately one third of the reduction in short and long term pass-through.

[20] Objetivos Operacionales del Banco de México.

[21] In Spanish is the Encuesta sobre las Expectativas de los Especialistas en Economía del Sector Privado (EEEESP)

Table 3: Pass-through before and after policy changes

Period	Short-term		Long-term	
	Data	Model	Data	Model
1995 - 2003	0.13	0.13	0.65	0.65
2003 - 2014	0.01	0.09	0.05	0.45
Change	-0.12	-0.04	-0.60	-0.20

Currency intervention trade-off

The data is rich enough to estimate pass-through in (31) distinguishing by product, city, product-city and across time.[22] Monthly estimates of β_1 are used to test how relevant the trade-off for currency interventions is. Table 4 below shows the results of running the regression $\beta_{1,t} = \gamma_0 + \gamma_1 Var_t\left(\Delta_{1\text{ day}} \log ER\right) + v_t$ where $Var_t\left(\Delta_{1\text{ day}} \log ER\right)$ is the variance of the daily exchange rate log change during month t.

Table 4: Pass-through and ER volatility

1995-2002		
Coefficient	Estimate	p-value
γ_0	0.2	0.0000
γ_1	-6.7	0.0302
R^2	0.02	

2003-2016		
Coefficient	Estimate	p-value
γ_0	0.03	0.0722
γ_1	-6.7	0.0552
R^2	0.02	

A negative value of γ_1 means that prices are more sensitive to movements of the nominal exchange rate in months when the latter is less volatile, thus suggesting the existence of the currency intervention trade-off. During the period 1995-2016 the average of $Var_t\left(\Delta_{1\text{ day}} \log ER\right)$ was 0.00004 and its standard deviation was 0.00017. A decrease in monthly exchange rate variance of one standard deviation increases pass-through in 0.0012.

Note from Table 1 that the ratio $\psi_{\tilde{\mu}}/\psi_{\mu}$ goes from 56 to 380, both values are in the region in which the model predicts that the currency intervention trade-off exists (that is, when pass-through is decreasing in the precision of the Central Bank's signal). Figure 6 illustrates this point, it is the same as Figure 4 but for values of $\psi_{\tilde{\mu}}/\psi_{\mu}$ less than 500.

[22]These estimations are part of a work in progress that would improve the robustness of the results in this paper.

Figure 6: Pass-through for different exogenous precision $\psi_{\tilde{\mu}}$

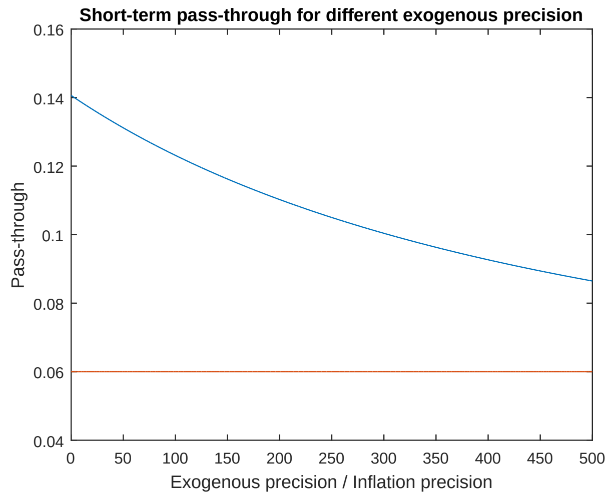
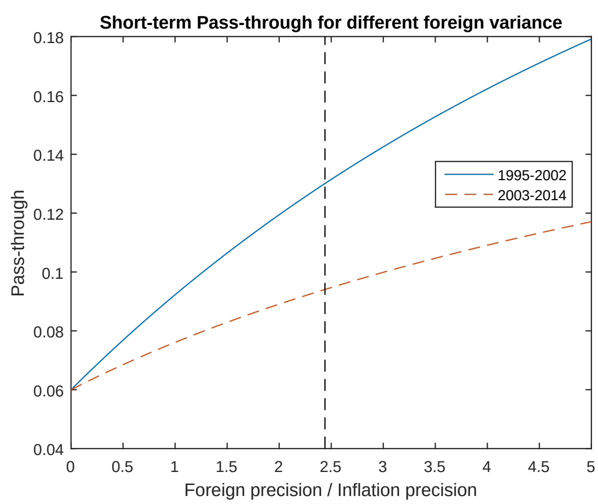


Figure 7 is the same as Figure 5 using the calibrated values for $\psi_{\tilde{\mu}}/\psi_{\mu}$ of 56 and 380 for the periods 1995-2002 and 2003-2014, respectively. The vertical line is at the calibrated value for ψ_{η}/ψ_{μ}. This Figure shows how the model predicts that the improvement in the available information implied by the policies is not enough by itself to eliminate the currency intervention trade-off, which is consistent with the implication of the estimates in Table 4.

Figure 7: Currency intervention trade-off



7 Conclusions

The informativeness of the nominal exchange rate affects the sensitivity of prices to the exchange rate, even if it does not affect demand and costs directly. This information channel implies a trade-off for currency interventions when the precision of other available information is relatively low. Given this trade-off, a currency intervention intended to decrease inflation after an exchange rate shock could end up increasing it depending on how much the depreciation reduces the volatility of the exchange rate.

Taking advantage of policy changes in Mexico as a case study, I find that the information channel explains approximately one third of the observed decrease in the average elasticity of prices. I also find that before and after the policy change, the policy intervention trade-off exists.

Further research on this topic would include a general equilibrium model in which the nominal exchange rate (as well as its relation to the price level) is modeled endogenously. Also, throughout this paper I assumed agents do not internalize the fact that the a change in the volatility of the exchange rate could be caused by an action by the Central Bank. Allowing agents internalize the strategic actions of the Central Bank is a theoretical extension that would enrich the results of this paper.

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8 Appendix

Ruling out explosive equilibria and simplifying the Euler Equation

Recall that from F.O.C.s we have:

$$\frac{\gamma}{1-\gamma}C\left(s^t\right)^{-1}P\left(s^t\right)^{-1}=\sum_{k=t}^{t+T}\beta^{k-t}\mathbb{E}\left[M\left(s^k\right)^{-1}|s^t\right]+\beta^T\mathbb{E}\left[M\left(s^{t+T}\right)^{-1}|s^t\right]$$

First, note that it must be the case that:

$$\beta^T\mathbb{E}\left[M\left(s^{t+T}\right)^{-1}|s^t\right]\geq 0$$

for all t, T. Hence, we have:

$$\lim_{T\to\infty}\beta^T\mathbb{E}\left[M\left(s^{t+T}\right)^{-1}|s^t\right]\geq 0$$

Now, note that given the process for μ_t we can exactly calculate for any $t\geq 0$, $k>0$:

$$\begin{aligned}
\mathbb{E}\left[M\left(s^{t+k}\right)^{-1}|s^t\right] &= \mathbb{E}\left[\left(\left\{\prod_{h=1}^{k}\exp\left(\mu_{t+k}\right)\right\}M\left(s^t\right)\right)^{-1}|s^t\right] \\
&= \mathbb{E}\left[\left(\left\{\exp\left(\sum_{h=1}^{k}\left[\bar{\mu}+\varepsilon_{t+k}\right]\right)\right\}M\left(s^t\right)\right)^{-1}|s^t\right] \\
&= \mathbb{E}\left[\exp\left(-\sum_{h=1}^{k}\varepsilon_{t+k}\right)|s^t\right]\exp\left(k\bar{\mu}\right)^{-1}M\left(s^t\right)^{-1} \\
&= \exp\left(-\sum_{h=1}^{k}\left[\mathbb{E}\left[\varepsilon_{t+k}|s^t\right]+\frac{1}{2}Var\left(\varepsilon_{t+k}|s^t\right)\right]\right)\exp\left(k\bar{\mu}\right)M\left(s^t\right)^{-1} \\
&= \exp\left(\frac{1}{2}\frac{1}{\psi_\mu}k\right)^{-1}\exp\left(k\bar{\mu}\right)M\left(s^t\right)^{-1} \\
&= \exp\left(k\left[\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right]\right)^{-1}M\left(s^t\right)^{-1}
\end{aligned}$$

So, we can use this to show that the following sum is well defined:

$$\begin{aligned}
\sum_{k=t}^{\infty} \beta^{k-t} \mathbb{E}\left[M\left(s^{k}\right)^{-1} | s^{t}\right] &= \sum_{h=0}^{\infty} \beta^{h} \mathbb{E}\left[M\left(s^{t+h}\right)^{-1} | s^{t}\right] \\
&= \sum_{h=0}^{\infty} \beta^{h} \exp\left(h\left[\bar{\mu} + \frac{1}{2}\frac{1}{\psi_{\mu}}\right]\right)^{-1} M\left(s^{t}\right)^{-1} \\
&= M\left(s^{t}\right)^{-1} \sum_{h=0}^{\infty} \left(\frac{\beta}{\exp\left(\bar{\mu} + \frac{1}{2}\frac{1}{\psi_{\mu}}\right)}\right)^{h} \\
&= M\left(s^{t}\right)^{-1} \frac{1}{1 - \frac{\beta}{\exp\left(\bar{\mu} + \frac{1}{2}\frac{1}{\psi_{\mu}}\right)}} \\
&= M\left(s^{t}\right)^{-1} \frac{\exp\left(\bar{\mu} + \frac{1}{2}\frac{1}{\psi_{\mu}}\right)}{\exp\left(\bar{\mu} + \frac{1}{2}\frac{1}{\psi_{\mu}}\right) - \beta}
\end{aligned}$$

Now, consider an arbitrary history s^t for which, in equilibrium, the state is such that:

$$\lim_{T \to \infty} \beta^{T} \mathbb{E}\left[M\left(s^{t+T}\right)^{-1} | s^{t}\right] > 0$$

Then we have that:

$$\begin{aligned}
\frac{\gamma}{1-\gamma} C\left(s^{t}\right)^{-1} P\left(s^{t}\right)^{-1} &> \sum_{k=t}^{\infty} \beta^{k-t} \mathbb{E}\left[M\left(s^{k}\right)^{-1} | s^{t}\right] \\
\iff u_{c}\left(C\left(s^{t}\right)\right) P\left(s^{t}\right)^{-1} &> \sum_{k=t}^{\infty} \beta^{k-t} \mathbb{E}\left[v_{m}\left(M\left(s^{k}\right)/P\left(s^{k}\right)\right)/P\left(s^{k}\right) | s^{t}\right]
\end{aligned}$$

where we are using $u(C) = \gamma \log C$ and $v(M/P) = (1-\gamma)\log(M/P)$. Note that this cannot be an equilibrium since there is some positive number η such that it is feasible for the household to increase consumption of the good at time t by η and reduce money holdings by a fraction of η in all subsequent periods. This deviation would be strictly preferred given the above inequality, so in equilibrium it must be the case that:

$$\lim_{T \to \infty} \beta^{T} \mathbb{E}\left[M\left(s^{t+T}\right)^{-1} | s^{t}\right] = 0$$

and thus:

$$\frac{\gamma}{1-\gamma}C\left(s^t\right)^{-1}P\left(s^t\right)^{-1}=\sum_{k=0}^{\infty}\beta^k\mathbb{E}\left[M\left(s^{t+k}\right)^{-1}|s^t\right]$$
$$=M\left(s^t\right)^{-1}\frac{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)}{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)-\beta}$$

Characterization of the linear equilibrium

All derivations shown here are for the more general model presented in Section 4. Taking logs from the equations that characterize the equilibrium with asymmetric information we get:

$$\log\left(\frac{\gamma}{1-\gamma}\frac{1}{\tilde{\beta}}\right)+m_t=c_t+p_t$$

$$\log e_t=\eta_t+p_t-p_t^\star$$

$$\log\left(\frac{\psi}{\gamma}\right)+\zeta l_{i,t}=w_{i,t}-c_t$$

$$y_{i,t}=\frac{1}{1-\theta}p_t-\frac{1}{1-\theta}p_{i,t}+y_t$$

$$y_t=\int_0^1 y_{i,t}di+\frac{1}{2}\theta\left[\int_0^1 y_{i,t}^2di-\left(\int_0^1 y_{i,t}di\right)^2\right]$$

$$l_{i,t}=y_{i,t}-A_{i,t}+\delta\eta_t-\delta w_{i,t}+\delta\log\left(\frac{1-\delta}{\delta}\right)$$

$$q_{i,t}=y_{i,t}-A_{i,t}-(1-\delta)\eta_t+(1-\delta)w_{i,t}+(1-\delta)\log\left(\frac{\delta}{1-\delta}\right)$$

$$p_{i,t}=\log\left(\frac{1}{\delta^\delta(1-\delta)^{1-\delta}}\frac{1}{\theta}\right)+(1-\delta)w_{i,t}+\mathbb{E}\left[\delta\eta_t-\log A_t+p_t|\mathscr{I}^i\left(s^t\right)\right]$$
$$+\frac{1}{2}Var\left(\delta\eta_t-\log A_t+\frac{2-\theta}{1-\theta}p_t+y_t|\mathscr{I}^i\left(s^t\right)\right)-\frac{1}{2}Var\left(\frac{1}{1-\theta}p_t+y_t|\mathscr{I}^i\left(s^t\right)\right)$$

$$p_t=\int_0^1 p_{i,t}di+\frac{1}{2}\frac{\theta}{\theta-1}\left[\int_0^1 p_{i,t}^2di-\left(\int_0^1 p_{i,t}di\right)^2\right]$$

$$\log e_t=p_t-p_t^\star$$

Assume the solution is linear functions of the state, which yields that the variances are constants:

$$p_{i,t}=\mathscr{A}_{-1}+(1-\delta)w_{i,t}-\mathbb{E}\left[\log A_t|\mathscr{I}^i\left(s^t\right)\right]+\delta\mathbb{E}\left[\eta_t|\mathscr{I}^i\left(s^t\right)\right]+\mathbb{E}\left[p_t|\mathscr{I}^i\left(s^t\right)\right]$$

Rearranging we can rewrite local wages and the aggregate price level as linear functions of state variables and posterior expectations:

$$p_t = \mathscr{G}_0 + \mathscr{G}_1 \mu_t + \mathscr{G}_2 A_t + \mathscr{G}_3 \eta_t + \mathscr{G}_4 \int_0^1 \mathbb{E}\left[\mu_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{G}_5 \int_0^1 \mathbb{E}\left[A_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{G}_6 \int_0^1 \mathbb{E}\left[\eta_t | \mathscr{I}^j\left(s^t\right)\right] dj$$

$$\begin{aligned} w_{i,t} = {} & \mathscr{J}_0 + \mathscr{J}_1 \mu_t + \mathscr{J}_2 A_t + \mathscr{J}_3 a_{i,t} + \mathscr{J}_4 \eta_t + \mathscr{J}_5 \mathbb{E}\left[\mu_t | \mathscr{I}^i\left(s^t\right)\right] + \mathscr{J}_6 \mathbb{E}\left[\log A_t | \mathscr{I}^i\left(s^t\right)\right] + \mathscr{J}_7 \mathbb{E}\left[\eta_t | \mathscr{I}^i\left(s^t\right)\right] \\ & + \mathscr{J}_8 \int_0^1 \mathbb{E}\left[\mu_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{J}_9 \int_0^1 \mathbb{E}\left[A_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{J}_{10} \int_0^1 \mathbb{E}\left[\eta_t | \mathscr{I}^j\left(s^t\right)\right] dj \end{aligned}$$

Conditional expectations

The information set of agent i is:

$$\begin{aligned} \tilde{\mu}_t &= \mu_t + \tilde{\varepsilon}_t \\ A_{i,t} &= A_t + a_{i,t} \\ w_{i,t} &= \mathscr{J}_0 + \mathscr{J}_1 \mu_t + \mathscr{J}_2 A_t + \mathscr{J}_3 a_{i,t} + \mathscr{J}_4 \eta_t + \mathscr{J}_5 \mathbb{E}\left[\mu_t | \mathscr{I}^i\left(s^t\right)\right] + \mathscr{J}_6 \mathbb{E}\left[\log A_t | \mathscr{I}^i\left(s^t\right)\right] + \mathscr{J}_7 \mathbb{E}\left[\eta_t | \mathscr{I}^i\left(s^t\right)\right] \\ & \quad + \mathscr{J}_8 \int_0^1 \mathbb{E}\left[\mu_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{J}_9 \int_0^1 \mathbb{E}\left[A_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{J}_{10} \int_0^1 \mathbb{E}\left[\eta_t | \mathscr{I}^j\left(s^t\right)\right] dj \\ \log e_t &= \mathscr{G}_0 + \mathscr{G}_1 \mu_t + \mathscr{G}_2 A_t + \left(1 + \mathscr{G}_3\right) \eta_t + \mathscr{G}_4 \int_0^1 \mathbb{E}\left[\mu_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{G}_5 \int_0^1 \mathbb{E}\left[A_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{G}_6 \int_0^1 \mathbb{E}\left[\eta_t | \mathscr{I}^j\left(s^t\right)\right] dj - \log p^\star \end{aligned}$$

Rearranging we get observed variables on LHSs and state variables on RHSs:

$$\begin{aligned} \tilde{\mu}_t &= \mu_t + \tilde{\varepsilon}_t \\ A_{i,t} &= A_t + a_{i,t} \\ \tilde{w}_i = - \mathscr{J}_0 + w_{i,t} - \mathscr{J}_5 \mathbb{E}\left[\mu_t | \mathscr{I}^i\left(s^t\right)\right] - \mathscr{J}_6 \mathbb{E}\left[\log A_t | \mathscr{I}^i\left(s^t\right)\right] - \mathscr{J}_7 \mathbb{E}\left[\eta_t | \mathscr{I}^i\left(s^t\right)\right] & \\ - \mathscr{J}_8 \int_0^1 \mathbb{E}\left[\mu_t | \mathscr{I}^j\left(s^t\right)\right] dj - \mathscr{J}_9 \int_0^1 \mathbb{E}\left[A_t | \mathscr{I}^j\left(s^t\right)\right] dj - \mathscr{J}_{10} \int_0^1 \mathbb{E}\left[\eta_t | \mathscr{I}^j\left(s^t\right)\right] dj &= \mathscr{J}_1 \mu_t + \mathscr{J}_2 A_t + \mathscr{J}_3 a_{i,t} + \mathscr{J}_4 \eta_t \\ \log \tilde{e} = \left[\log p^\star - \mathscr{G}_0\right] + \log e_t - \mathscr{G}_4 \int_0^1 \mathbb{E}\left[\mu_t | \mathscr{I}^j\left(s^t\right)\right] dj & \\ - \mathscr{G}_5 \int_0^1 \mathbb{E}\left[A_t | \mathscr{I}^j\left(s^t\right)\right] dj - \mathscr{G}_6 \int_0^1 \mathbb{E}\left[\eta_t | \mathscr{I}^j\left(s^t\right)\right] dj &= \mathscr{G}_1 \mu_t + \mathscr{G}_2 A_t + \left(1 + \mathscr{G}_3\right) \eta_t \end{aligned}$$

- Rearrange to get conditionally independent signals for μ_t:

$$\begin{aligned}
\tilde{\mu}_t &= \mu_t + \tilde{\varepsilon}_t \\
\tilde{A}_{i,\mu} &= \mu_t - \frac{(\mathscr{J}_2 - \mathscr{J}_3)(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_2}{\mathscr{J}_1(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_1} a_{i,t} \\
\tilde{w}_{i,\mu} &= \mu_t + \frac{(\mathscr{J}_2 - \mathscr{J}_3)(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_2}{\mathscr{J}_1(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_1} A_t \\
\log\tilde{e}_\mu &= \mu_t + \frac{[(\mathscr{J}_2 - \mathscr{J}_3)(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_2](1+\mathscr{G}_3)}{\mathscr{G}_1[(\mathscr{J}_2 - \mathscr{J}_3)(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_2] - \mathscr{G}_2[\mathscr{J}_1(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_1]}\eta_t
\end{aligned}$$

  so the conditional expectation is standard for normal conjugate priors.

- Do the same for A_t:

$$\begin{aligned}
\tilde{\mu}_A &= A_t - \frac{\mathscr{J}_1(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_1}{(\mathscr{J}_2 - \mathscr{J}_3)(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_2}\tilde{\varepsilon}_t \\
A_{i,t} &= A_t + a_{i,t} \\
\tilde{w}_{i,A} &= A_t + \frac{\mathscr{J}_1(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_1}{(\mathscr{J}_2 - \mathscr{J}_3)(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_2}\varepsilon_t \\
\log\tilde{e}_A &= A_t + \frac{[\mathscr{J}_1(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_1](1+\mathscr{G}_3)}{\mathscr{G}_2[\mathscr{J}_1(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_1] - \mathscr{G}_1[(\mathscr{J}_2 - \mathscr{J}_3)(1+\mathscr{G}_3) - \mathscr{J}_4\mathscr{G}_2]}\eta_t
\end{aligned}$$

- Do the same for η_t:

$$\begin{aligned}
\tilde{\mu}_\eta &= \eta_t - \frac{([\mathscr{J}_2 - \mathscr{J}_3]\mathscr{G}_1 - \mathscr{J}_1\mathscr{G}_2)\mathscr{G}_1}{(1+\mathscr{G}_3)\{[\mathscr{J}_2 - \mathscr{J}_3]\mathscr{G}_1 - \mathscr{J}_1\mathscr{G}_2\} - \mathscr{G}_2[\mathscr{J}_4\mathscr{G}_1 - \mathscr{J}_1(1+\mathscr{G}_3)]}\tilde{\varepsilon}_t \\
\tilde{A}_{i,\eta} &= \eta_t - \frac{[\mathscr{J}_2 - \mathscr{J}_3]\mathscr{G}_1 - \mathscr{J}_1\mathscr{G}_2}{\mathscr{J}_4\mathscr{G}_1 - \mathscr{J}_1(1+\mathscr{G}_3)} a_{i,t} \\
\tilde{w}_{i,\eta} &= \eta_t + \frac{[\mathscr{J}_2 - \mathscr{J}_3]\mathscr{G}_1 - \mathscr{J}_1\mathscr{G}_2}{\mathscr{J}_4\mathscr{G}_1 - \mathscr{J}_1(1+\mathscr{G}_3)} A_t \\
\log\tilde{e}_\eta &= \eta_t + \frac{([\mathscr{J}_2 - \mathscr{J}_3]\mathscr{G}_1 - \mathscr{J}_1\mathscr{G}_2)\mathscr{G}_1}{(1+\mathscr{G}_3)\{[\mathscr{J}_2 - \mathscr{J}_3]\mathscr{G}_1 - \mathscr{J}_1\mathscr{G}_2\} - \mathscr{G}_2[\mathscr{J}_4\mathscr{G}_1 - \mathscr{J}_1(1+\mathscr{G}_3)]}\varepsilon_t
\end{aligned}$$

Rename the weights to get:

$$\begin{aligned}
\mathbb{E}\left[\mu_t | \mathscr{I}^i\left(s^t\right)\right] &= \alpha_{0,\mu}\bar{\mu} + \alpha_{1,\mu}\tilde{\mu}_t + \alpha_{2,\mu}\tilde{A}_{i,\mu} + \alpha_{3,\mu}\tilde{w}_{i,\mu} + \alpha_{4,\mu}\log\tilde{e}_\mu \\
\mathbb{E}\left[A_t | \mathscr{I}^i\left(s^t\right)\right] &= \alpha_{1,A}\tilde{\mu}_A + \alpha_{2,A}A_{i,t} + \alpha_{3,A}\tilde{w}_{i,A} + \alpha_{4,A}\log\tilde{e}_A \\
\mathbb{E}\left[\eta_t | \mathscr{I}^i\left(s^t\right)\right] &= \alpha_{0,\eta}\rho\eta_{t-1} + \alpha_{1,\eta}\tilde{\mu}_\eta + \alpha_{2,\eta}\tilde{A}_{i,\eta} + \alpha_{3,\eta}\tilde{w}_{i,\eta} + \alpha_{4,\eta}\log\tilde{e}_\eta
\end{aligned}$$

Rearranging and renaming constants we get the following system:

$$
\begin{aligned}
\mathbb{E}\left[\mu_t|\mathscr{I}^i\left(s^t\right)\right] &= \mathscr{Y}_0+\mathscr{Y}_1\tilde{\mu}_t+\mathscr{Y}_2A_{i,t}+\mathscr{Y}_3w_{i,t}+\mathscr{Y}_4\log e_t+\mathscr{Y}_5\mathbb{E}\left[\mu_t|\mathscr{I}^i\left(s^t\right)\right]+\mathscr{Y}_6\mathbb{E}\left[\log A_t|\mathscr{I}^i\left(s^t\right)\right]\\
&\quad+\mathscr{Y}_7\mathbb{E}\left[\eta_t|\mathscr{I}^i\left(s^t\right)\right]+\mathscr{Y}_8\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj+\mathscr{Y}_9\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj+\mathscr{Y}_{10}\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj\\
\mathbb{E}\left[A_t|\mathscr{I}^i\left(s^t\right)\right] &= \mathscr{Z}_0+\mathscr{Z}_1\tilde{\mu}_t+\mathscr{Z}_2A_{i,t}+\mathscr{Z}_3w_{i,t}+\mathscr{Z}_4\log e_t+\mathscr{Z}_5\mathbb{E}\left[\mu_t|\mathscr{I}^i\left(s^t\right)\right]+\mathscr{Z}_6\mathbb{E}\left[\log A_t|\mathscr{I}^i\left(s^t\right)\right]\\
&\quad+\mathscr{Z}_7\mathbb{E}\left[\eta_t|\mathscr{I}^i\left(s^t\right)\right]+\mathscr{Z}_8\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj+\mathscr{Z}_9\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj+\mathscr{Z}_{10}\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj\\
\mathbb{E}\left[\eta_t|\mathscr{I}^i\left(s^t\right)\right] &= \tilde{\mathscr{A}_0}+\tilde{\mathscr{A}_1}\tilde{\mu}_t+\tilde{\mathscr{A}_2}A_{i,t}+\tilde{\mathscr{A}_3}w_{i,t}+\tilde{\mathscr{A}_4}\log e_t+\tilde{\mathscr{A}_5}\mathbb{E}\left[\mu_t|\mathscr{I}^i\left(s^t\right)\right]+\tilde{\mathscr{A}_6}\mathbb{E}\left[\log A_t|\mathscr{I}^i\left(s^t\right)\right]\\
&\quad+\tilde{\mathscr{A}_7}\mathbb{E}\left[\eta_t|\mathscr{I}^i\left(s^t\right)\right]+\tilde{\mathscr{A}_8}\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj+\tilde{\mathscr{A}_9}\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj+\tilde{\mathscr{A}}_{10}\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj
\end{aligned}
$$

Integrate, rearrange and rename constants to get the system on aggregate expectations:

$$
\begin{aligned}
\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj &= \tilde{\mathscr{B}}_0+\tilde{\mathscr{B}}_1\tilde{\mu}_t+\tilde{\mathscr{B}}_2A_t+\tilde{\mathscr{B}}_3\int_0^1 w_{j,t}dj+\tilde{\mathscr{B}}_4\log e_t+\tilde{\mathscr{B}}_5\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj+\tilde{\mathscr{B}}_6\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj\\
\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj &= \tilde{\mathscr{C}_0}+\tilde{\mathscr{C}_1}\tilde{\mu}_t+\tilde{\mathscr{C}_2}A_t+\tilde{\mathscr{C}_3}\int_0^1 w_{j,t}dj+\tilde{\mathscr{C}_4}\log e_t+\tilde{\mathscr{C}_5}\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj+\tilde{\mathscr{C}_6}\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj\\
\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj &= \tilde{\mathscr{D}}_0+\tilde{\mathscr{D}}_1\tilde{\mu}_t+\tilde{\mathscr{D}}_2A_t+\tilde{\mathscr{D}}_3\int_0^1 w_{j,t}dj+\tilde{\mathscr{D}}_4\log e_t+\tilde{\mathscr{D}}_5\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj+\tilde{\mathscr{D}}_6\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj
\end{aligned}
$$

Recall wages are:

$$
\begin{aligned}
w_{i,t} &= \mathscr{J}_0+\mathscr{J}_1\mu_t+\mathscr{J}_2A_t+\mathscr{J}_3a_{i,t}+\mathscr{J}_4\eta_t+\mathscr{J}_5\mathbb{E}\left[\mu_t|\mathscr{I}^i\left(s^t\right)\right]+\mathscr{J}_6\mathbb{E}\left[\log A_t|\mathscr{I}^i\left(s^t\right)\right]+\mathscr{J}_7\mathbb{E}\left[\eta_t|\mathscr{I}^i\left(s^t\right)\right]\\
&\quad+\mathscr{J}_8\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj+\mathscr{J}_9\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj+\mathscr{J}_{10}\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj
\end{aligned}
$$

The integral is then:

$$
\begin{aligned}
\int_0^1 w_{j,t}dj &= \mathscr{J}_0+\mathscr{J}_1\mu_t+\mathscr{J}_2A_t+\mathscr{J}_4\eta_t\\
&\quad+\left[\mathscr{J}_5+\mathscr{J}_8\right]\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj+\left[\mathscr{J}_6+\mathscr{J}_9\right]\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj+\left[\mathscr{J}_7+\mathscr{J}_{10}\right]\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj
\end{aligned}
$$

Plugging in and renaming the constants:

$$
\begin{aligned}
\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj &= \tilde{\mathscr{E}_0}+\tilde{\mathscr{E}_1}\mu_t+\tilde{\mathscr{E}_2}\tilde{\mu}_t+\tilde{\mathscr{E}_3}A_t+\tilde{\mathscr{E}_4}\eta_t+\tilde{\mathscr{E}_5}\log e_t+\tilde{\mathscr{E}_6}\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj+\tilde{\mathscr{E}_7}\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj\\
\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj &= \tilde{\mathscr{F}_0}+\tilde{\mathscr{F}_1}\mu_t+\tilde{\mathscr{F}_2}\tilde{\mu}_t+\tilde{\mathscr{F}_3}A_t+\tilde{\mathscr{F}_4}\eta_t+\tilde{\mathscr{F}_5}\log e_t+\tilde{\mathscr{F}_6}\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj+\tilde{\mathscr{F}_7}\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj\\
\int_0^1\mathbb{E}\left[\eta_t|\mathscr{I}^j\left(s^t\right)\right]dj &= \tilde{\mathscr{G}_0}+\tilde{\mathscr{G}_1}\mu_t+\tilde{\mathscr{G}_2}\tilde{\mu}_t+\tilde{\mathscr{G}_3}A_t+\tilde{\mathscr{G}_4}\eta_t+\tilde{\mathscr{G}_5}\log e_t+\tilde{\mathscr{G}_6}\int_0^1\mathbb{E}\left[\mu_t|\mathscr{I}^j\left(s^t\right)\right]dj+\tilde{\mathscr{G}_7}\int_0^1\mathbb{E}\left[A_t|\mathscr{I}^j\left(s^t\right)\right]dj
\end{aligned}
$$

Solving the system:

$$\int_0^1 \mathbb{E}\left[\mu_t | \mathscr{I}^j\left(s^t\right)\right] dj = \tilde{\mathscr{K}_0} + \tilde{\mathscr{K}_1}\mu_t + \tilde{\mathscr{K}_2}\tilde{\mu}_t + \tilde{\mathscr{K}_3}A_t + \tilde{\mathscr{K}_4}\eta_t + \tilde{\mathscr{K}_5}\log e_t$$
$$\int_0^1 \mathbb{E}\left[A_t | \mathscr{I}^j\left(s^t\right)\right] dj = \tilde{\mathscr{L}_0} + \tilde{\mathscr{L}_1}\mu_t + \tilde{\mathscr{L}_2}\tilde{\mu}_t + \tilde{\mathscr{L}_3}A_t + \tilde{\mathscr{L}_4}\eta_t + \tilde{\mathscr{L}_5}\log e_t$$
$$\int_0^1 \mathbb{E}\left[\eta_t | \mathscr{I}^j\left(s^t\right)\right] dj = \tilde{\mathscr{M}_0} + \tilde{\mathscr{M}_1}\mu_t + \tilde{\mathscr{M}_2}\tilde{\mu}_t + \tilde{\mathscr{M}_3}A_t + \tilde{\mathscr{M}_4}\eta_t + \tilde{\mathscr{M}_5}\log e_t$$

Which can be plugged in to solve the system for individual expectations.

Recall the aggregate price only depends on aggregate expectations:

$$p_t = \mathscr{G}_0 + \mathscr{G}_1\mu_t + \mathscr{G}_2 A_t + \mathscr{G}_3\eta_t + \mathscr{G}_4\int_0^1 \mathbb{E}\left[\mu_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{G}_5\int_0^1 \mathbb{E}\left[A_t | \mathscr{I}^j\left(s^t\right)\right] dj + \mathscr{G}_6\int_0^1 \mathbb{E}\left[\eta_t | \mathscr{I}^j\left(s^t\right)\right] dj$$

Plugging in:

$$\begin{aligned} p_t &= \left[\mathscr{G}_0 + \mathscr{G}_4\tilde{\mathscr{K}_0} + \mathscr{G}_5\tilde{\mathscr{L}_0} + \mathscr{G}_6\tilde{\mathscr{M}_0}\right] + \left[\mathscr{G}_1 + \mathscr{G}_4\tilde{\mathscr{K}_1} + \mathscr{G}_5\tilde{\mathscr{L}_1} + \mathscr{G}_6\tilde{\mathscr{M}_1}\right]\mu_t + \left[\mathscr{G}_4\tilde{\mathscr{K}_2} + \mathscr{G}_5\tilde{\mathscr{L}_2} + \mathscr{G}_6\tilde{\mathscr{M}_2}\right]\tilde{\mu}_t \\ &+ \left[\mathscr{G}_2 + \mathscr{G}_4\tilde{\mathscr{K}_3} + \mathscr{G}_5\tilde{\mathscr{L}_3} + \mathscr{G}_6\tilde{\mathscr{M}_3}\right]A_t + \left[\mathscr{G}_3 + \mathscr{G}_4\tilde{\mathscr{K}_4} + \mathscr{G}_5\tilde{\mathscr{L}_4} + \mathscr{G}_6\tilde{\mathscr{M}_4}\right]\eta_t + \left[\mathscr{G}_4\tilde{\mathscr{K}_5} + \mathscr{G}_5\tilde{\mathscr{L}_5} + \mathscr{G}_6\tilde{\mathscr{M}_5}\right]\log e_t \\ &= \tilde{\mathscr{N}_0} + \tilde{\mathscr{N}_1}\mu_t + \tilde{\mathscr{N}_2}\tilde{\mu}_t + \tilde{\mathscr{N}_3}A_t + \tilde{\mathscr{N}_4}\eta_t + \tilde{\mathscr{N}_5}\log e_t \end{aligned}$$

Plugging in for $\log e_t = \eta_t + p_t - p^\star$,

$$\begin{aligned} p_t &= \tilde{\mathscr{N}_0} + \tilde{\mathscr{N}_1}\mu_t + \tilde{\mathscr{N}_2}\tilde{\mu}_t + \tilde{\mathscr{N}_3}A_t + \tilde{\mathscr{N}_4}\eta_t + \tilde{\mathscr{N}_5}\log e_t \\ p_t &= \frac{\tilde{\mathscr{N}_0} - \tilde{\mathscr{N}_5}p^\star}{1 - \tilde{\mathscr{N}_5}} + \frac{\tilde{\mathscr{N}_1}}{1 - \tilde{\mathscr{N}_5}}\mu_t + \frac{\tilde{\mathscr{N}_2}}{1 - \tilde{\mathscr{N}_5}}\tilde{\mu}_t + \frac{\tilde{\mathscr{N}_3}}{1 - \tilde{\mathscr{N}_5}}A_t + \frac{\tilde{\mathscr{N}_4} + \tilde{\mathscr{N}_5}}{1 - \tilde{\mathscr{N}_5}}\eta_t \\ &= \tilde{\mathscr{O}_0} + \tilde{\mathscr{O}_1}\mu_t + \tilde{\mathscr{O}_2}\tilde{\mu}_t + \tilde{\mathscr{O}_3}A_t + \tilde{\mathscr{O}_4}\eta_t \end{aligned}$$