Default Depressions*

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January 2026

Abstract

Prolonged sovereign default episodes are followed by deep economic depressions that persist well after their resolution. Short defaults do not have this scarring effect. I develop a novel theory that explains this difference along with the standard cyclical behavior of sovereign risk. It incorporates elements from the "big push" development literature that generate demand complementarities and increasing returns to scale in a unique-equilibrium setting. Default disrupts technology adoption and long defaults act as a "big pull" force that takes the economy from a "rich" to a "poor" state. These differentiated effects of long and short defaults generate a bimodal ergodic distribution, and the economy fluctuates around two regimes: "normal times" and a "default trap". In the default trap capital and technology adoption are lower, defaults are more frequent, and spreads are higher and more volatile than in normal times.

Keywords: Sovereign default, Economic depressions.

JEL Codes: F34, F41, O11

*For useful comments and discussions I thank Laura Alfaro, Fernando Arce, Joao Ayres, Tamon Asonuma, Satyajit Chatterjee, Hal Cole, Thomas Drechsel, Stelios Fourakis, Grey Gordon, Emilio Gutierrez, Tobey Kass, Tim Kehoe, Jennifer La'O, Juan Pablo Nicolini, Tom Sargent, Mathieu Taschereau-Dumouchel, Nicholas Trachter, Miguel Zerecero, and participants at various seminars and conferences. All errors are my own.

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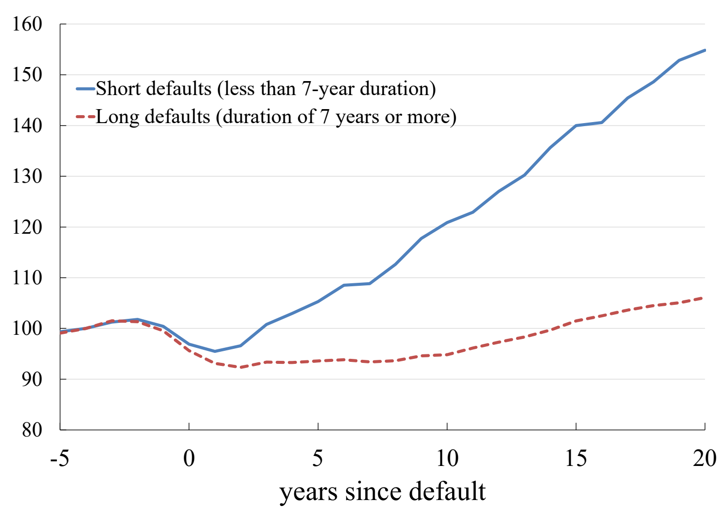
1 Introduction

A frequently discussed idea regarding the difficulties of industrialization relates to the concept of the "big push", introduced by Rosenstein-Rodan (1943) and more recently developed by Murphy, Shleifer, and Vishny (1989) and many others. According to this idea, adoption of increasing returns technologies (industrialization) by a particular sector may only be profitable if other sectors do so. The evident policy implication in such an environment is for the government to act as a coordinating force to give the economy the needed big push toward industrialization. An important question, however, is whether governments have incentives to do so (or to do it right) when political economy frictions may generate misalignment between the incentives of governments and its constituents. This paper studies the big push in the shadow of a set of such frictions that have also been widely studied in the literature: those that drive sovereign borrowing and default.

Figure 1: Average real GDP per capita around short and long defaults

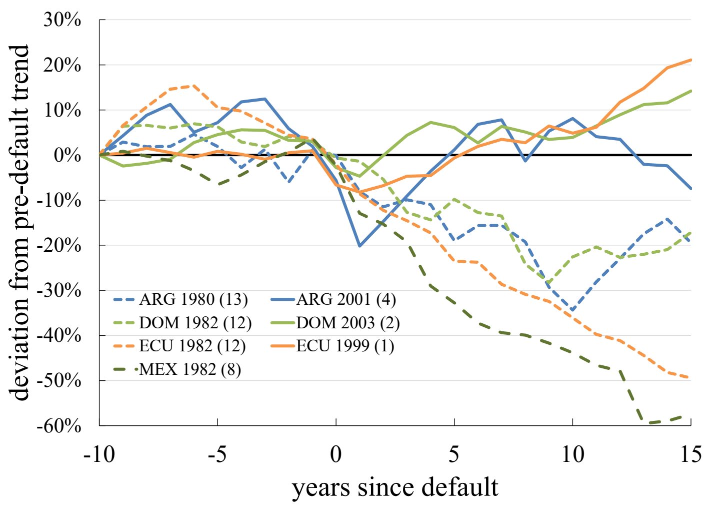


Real GDP per capita is measured in local currency units and normalized so that the year 4 before each default event is equal to 100. The data include 64 default events between 1975 and 2015.

Sovereign defaults are large and disruptive events, but some are more disruptive than others. Long-lasting defaults have scarring effects on GDP that are not proportionally observed for shorter ones. Figure 1 illustrates this by sorting 64 default episodes into two groups: long defaults with a duration above the median of 7 years, and short defaults with a duration below the median. The data are the average paths of GDP per capita, with period 0 denoting the year of default.

Both groups experience a contraction in GDP around the default event. The recoveries, however, are significantly different. Output quickly bounces back to the pre-default trend in the case of short defaults, while it remains depressed following long defaults. Farah-Yacoub, Graf von Luckner, and Reinhart (2024) use synthetic control methods to document robust evidence that long defaults have large long-lasting negative effects on output and other development variables that short defaults do not. One could argue that countries that experience long defaults are also countries with chronically lower productivity growth rates, which would explain a slower recovery after a downturn. Figure 2 shows that this is not necessarily the case by comparing three pairs of long and short defaults: 1980 and 2001 in Argentina, 1982 and 2003 in the Dominican Republic, and 1982 and 1999 in Ecuador. The labels include the year of default and in parenthesis its duration in years.

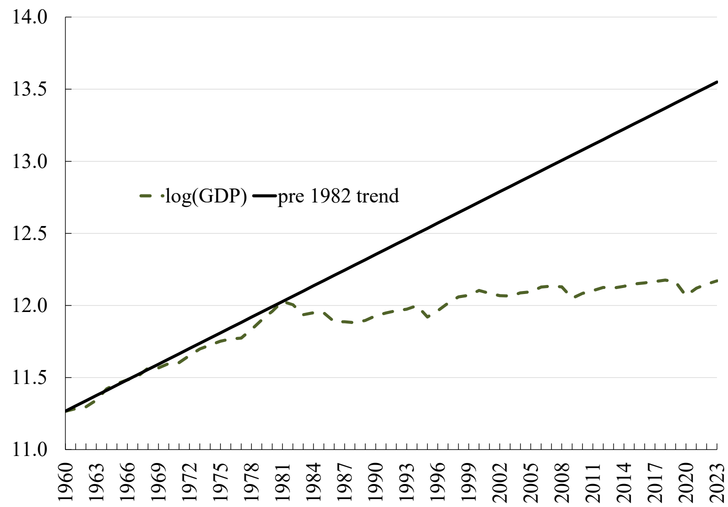
Figure 2: Deviation from trend around short and long defaults



Each line corresponds to log-deviations of real GDP per capita from its pre-default trend. The pre-default trend is computed as a linear trend using the ten years prior to each event. The country labels correspond to Argentina (ARG), the Dominican Republic (DOM), Ecuador (ECU), and Mexico (MEX). The year of the default is stated after the country's code and the duration of each default is in parenthesis.

The comparison of these three pairs is very stark because it shows that, for the same country, a different default duration is associated with very different long-lasting effects on income. The graph also includes the case of the 1982 Mexican default, which is the most dramatic in this group and one of the great depressions of the twentieth century studied in Kehoe and Prescott (2007) (see also Bergoeing, Kehoe, Kehoe, and Soto (2002)). The gap between GDP per capita and its pre-default trend continued to widen years after the default was resolved. In fact, the Mexican economy never recovered from this crisis, as can be seen in Figure 3.

Figure 3: Mexico GDP per capita and pre-1982 trend



The data are the natural logarithm of Mexican real GDP per capita and its pre-default trend. The pre-default trend is computed as a linear trend between 1960 and 1981.

Most models of sovereign default (see Arellano (2008) and the subsequent literature) are not suited to study these long-lasting scarring effects because they study default risk over the business cycle, with output fluctuating around a stable long-run mean. This is also true for richer models with endogenous production (see Mendoza and Yue (2012)) and capital accumulation (see Park (2017); Gordon and Guerron-Quintana (2018); Arellano, Bai, and Mihalache (2018)) in which business cycles are driven by stationary productivity shocks. Default in these models may have large effects on output and capital accumulation may make these effects persistent, but once a default is resolved the economy eventually reverts back to its long-run mean. An important exception is earlier work by Aguiar and Gopinath (2006) and Aguiar and Gopinath (2007), who show that models with permanent shocks to trend growth are better suited to explain the business cycle of emerging economies.

The main contribution of this paper is to develop a novel model of sovereign default that can accommodate both the cyclical behavior that is studied in the literature and the large scarring effects of long defaults. At the core of the model is a mechanism through which years after the resolution of a default total factor productivity (TFP) may remain depressed below its pre-default mean (an endogenous mechanism akin to the exogenous shocks to trend growth in Aguiar and Gopinath (2006) and Aguiar and Gopinath (2007)). This TFP-driven theory of default depressions is consistent with the findings in Kehoe and Prescott (2007), who show that long-run changes in TFP are crucial for accounting for economic depressions. Moreover, the likelihood of a default depression occurring in the model is increasing in the length of the default event, as suggested by the above data.

I develop a sovereign default model with production and incorporate some elements from the big push. This literature analyzes the idea of how, in the context of imperfect competition and aggregate demand spillovers, a big push into industrialization can move an economy from a "bad" (poor) to a "good" (rich) equilibrium. The model builds on these ideas but features a unique dynamic equilibrium with multiple steady states, similar to the work by Schaal and Taschereau-Dumouchel (2024). In the ergodic distribution the economy fluctuates around—and ocassionally switches between—a regime with a high degree of industrialization and another with a low degree. Sovereign defaults trigger a temporary productivity decline and long-lasting ones act as a "big pull" force that can pull the economy down from a rich, highly industrialized state, to a poor, less industrialized one. Short defaults are less likely to have this scarring effect. On one hand, this potential big pull endogenously increases the cost of defaulting when the degree of industrialization is high, allowing the economy to sustain high levels of debt with minimal default risk. On the other hand, the cost of defaulting is relatively low for the economy in the poor region with low industrialization. Since more frequent default events in this region reduce the likelihood of re-industrializing, I refer to this region in the ergodic distribution as a default trap.

The model has a small open economy in which a sovereign government makes borrowing, investment, and default decisions without commitment. The government is relatively more impatient than its constituent households, which reflects the presence of political economy frictions and biases the government's decisions toward debt accumulation to front-load consumption. This is the standard sovereign-default block of the model. The novelty is on the production side, which features a continuum of monopolistically competitive firms whose productivity is heterogeneous in two dimensions. One dimension is permanent and firm-specific. The other is the result of firms' choices to operate a traditional technology or to pay a fixed cost to access a modern, more productive technology. When capital is scarce, only the most productive firms adopt the modern technology. As capital becomes more abundant, production costs decrease and more firms find it profitable to adopt the modern technology. These assumptions generate increasing returns to scale for certain levels of capital and demand complementarities in the sense that, for some firms, the decision to adopt the modern technology depends on how many other firms are doing so. As in the model in Buera, Hopenhayn, Shin, and Trachter (2021), ex-ante firm heterogeneity generates a big-push region, where a small increase in the capital stock has a large effect on technology adoption (in their model the main focus is on firm-level distortions and the effects of reducing them through structural reforms). There are other more stable regions, with significantly lower or higher levels of capital, in which changes to the aggregate stock only have moderate effects on adoption. This duality is crucial for the model to generate the bimodal ergodic distribution described above. Another advantage from the ex-ante heterogeneity of firms is that it guarantees a unique production equilibrium under certain conditions on parameters, which simplifies the analysis by ruling out production multiplicity due to coordination failures.

The only source of risk in the model is an aggregate productivity shock that is exogenously penalized during periods of default, which is a standard assumption in the literature. While in default, the government is in financial autarky and regains access with a constant probability. This exogenous default resolution is standard in the literature, and, more importantly, it allows me to perform experiments of different default durations without them interacting with other endogenous variables. Why some defaults last longer than others, as well as what are the sources of the default penalty on productivity, are interesting open questions in the literature that are beyond the scope of this paper (for a more detailed treatment of these questions see Benjamin and Wright (2009); Bocola (2016); Dvorkin, Sanchez, Sapriza, and Yurdagul (2021); and Arellano, Bai, and Bocola (2017)). While the frictions in the economy could interact with the drivers of default duration in a richer environment (like an environment with debt renegotiation), the main mechanisms introduced in this paper would still be present. In the interest of clarity, I abstract from such interactions and simplify the duration of default events.

I calibrate the model to Argentina using the subsample of the ergodic distribution in the default trap to target data moments. Default events in the model feature significantly different long-lasting effects on GDP and production decisions. After short defaults, GDP and technology adoption quickly bounce back and GDP remains close to its pre-default level in the long-run. After a long default GDP also bounces back up but to a lower level than before the event. Moreover, GDP remains significantly below its pre-default level several years after the event has been resolved.

These differentiated effects of default events generate a bimodal ergodic distribution, which shows that the economy fluctuates around two regimes: normal times and the default trap described above. In normal times the capital stock is relatively high, the fraction of firms that operate the modern technology is close to one, default risk is low, and sovereign spreads are low and feature low variance. When the economy is in the default trap, both the capital stock and technology adoption are significantly lower. In addition, defaults are more frequent and spreads are higher and more volatile. Under the benchmark calibration, the median of the fraction of firms adopting the modern technology in the ergodic distribution is 0.07 and there is full adoption only 30 percent of the time. This indicates that the default trap is more common, which illustrates the absence of a big push force to counter the big pull of lengthy defaults.

Unlike in most of the big push literature, there is not much scope for the government to intervene and improve economic outcomes in this model. This is because of the big pull that results from the government's relative impatience and its inability to commit to future policy. This, along with the assumption that the government can choose all aggregate allocations directly, makes any analysis regarding domestic policy redundant. Instead, I use the model to analyze the effects of foreign aid implemented after a long default of ten years. I evaluate the welfare and re-industrialization implications of three types of aid: an unconditional transfer that can be freely allocated by the government, debt relief, and a capital injection. I define "time to development" as the expected value of the number of years that it would take the economy to reach a given level of industrialization (measured as the fraction of firms adopting the modern technology) from a given starting point. There are two main findings from this exercise. First, the government prefers aid in the form of a transfer, while households are better off with a capital injection. This is because the capital injection reduces both default risk and the expected time to exit the default trap, which are future benefits that are more heavily discounted by the government. Second, debt relief not only implies the smallest increase in welfare for both the government and the household, but has the smallest effect on time to development. These results make a strong case against debt relief in favor of aid in the form of targeted investment programs.

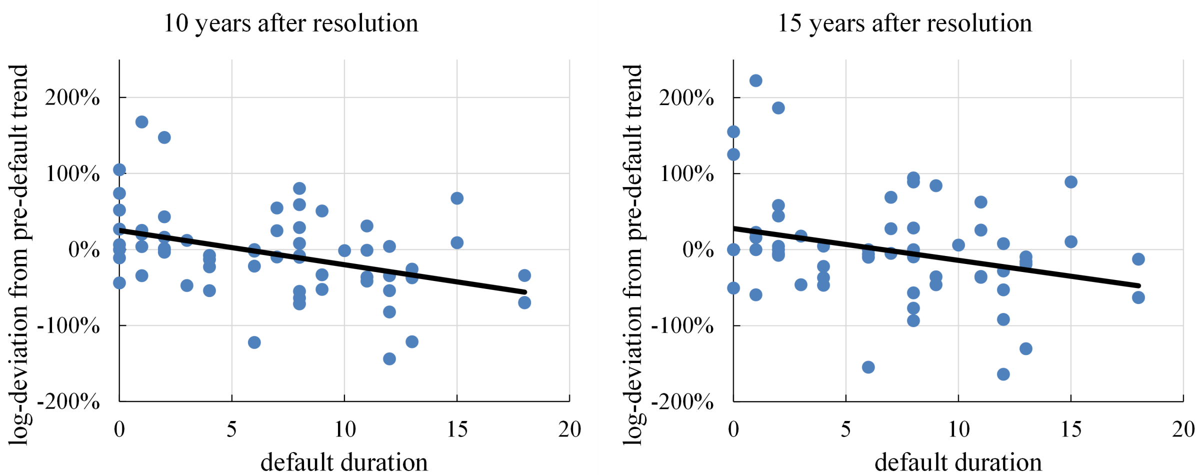
Layout.—Section 2 presents additional motivating evidence on the scarring effect of long defaults, Section 3 presents the model and discusses the main novel mechanisms, Section 4 describes the calibration exercise and presents the main results, Section 5 concludes.

2 Default depressions and TFP

This Section presents two exercises that illustrate the relationship between the length of default episodes and the long-run evolution of GDP per capita and TFP. The first exercise presents additional evidence that longer defaults are associated with GDP being significantly below trend years after their resolution, which is the main motivating observation from Figure 1. The second exercise shows that the evolution and recovery of TFP around default events also varies substantially within the two groups. This relationship is at the core of the theoretical mechanism in the model presented in the following section.

I analyze 64 default events in 54 different countries reported in Laeven and Valencia (2018). These data report the year of sovereign defaults and the year of their resolution, which is what I use as the duration variable (Appendix A lists all 64 default events and the relevant dates). For the first exercise, I take real GDP per-capita expressed in constant local currency units and compute linear trends using the ten years that precede a default. Then, I compute the log-deviation from this trend in each of the years after default. Figure 4 shows two scatter plots relating the duration of each default episode to the log-deviation of GDP from the pre-default trend 10 and 15 years after their resolution.

Figure 4: Default duration and GDP deviation from long-run trend



Each observation corresponds to a default episode. The horizontal axis is the number of years it took for the default to be resolved. The vertical axes are log-deviations of GDP, measured in local currency units, from its trend in the 10 years prior to default. The left panel shows the log-deviation of GDP 10 years after resolution, and the right panel shows it 15 years after.

There is a clear negative correlation between default duration and GDP relative to its pre-default trend several years after the episode was resolved. I then estimate the following regression

$$\Delta y_{ijt} = \psi_0 + \psi_1 d_i + \overrightarrow{\psi} X, \tag{1}$$

where Δy_{it} is the log-deviation of GDP in country j from its pre-default trend t years after the resolution of default episode i, d_i is the duration (in years) of default episode i, and X is a vector of additional controls. The controls include a dummy variable for default events that last less than a year. This ensures that the results are not driven by the large positive observations of Δy_{ijt} in Figure 4. I also include a dummy variable for "surprise" defaults, which takes a value of 1 if the economy was growing in the two years prior to default. I include a dummy variable for defaults in the 1980s, which constitute 34 of the 64 observations. Defaults in the 1980s were much longer than defaults in more recent years (see Gelos, Sahay, and Sandleirs (2011)) and extended all over the world. As pointed out by Almeida, Esquivel, Kehoe, and Nicolini (2024), the 1980s featured the most widespread sovereign debt crisis in history and was accompanied by abnormally large real interest rates due to monetary tightening in the United States. Finally, I include an interaction between the 1980s dummy variable and the duration of default events. Table 1 reports the estimated coefficients for $t = 10$ and $t = 15$.

Table 1: Effect of default length on real GDP per capita after default episode ended

	10 years after resolution		15 years after resolution	
variable	(1)	(2)	(3)	(4)
duration	-0.0467***	-0.0546***	-0.0518***	-0.0568**
	(0.0132)	(0.0195)	(0.0192)	(0.0265)
duration=0		0.0291		0.511
		(0.252)		(0.467)
surprise default		-0.00833		-0.0267
		(0.143)		(0.210)
1980s		-0.245		-0.322
		(0.295)		(0.414)
1980s*duration		0.0304		0.0404
		(0.0326)		(0.0447)
constant	0.268**	0.310	0.386**	0.389
	(0.114)	(0.192)	(0.174)	(0.278)
Observations	61	61	55	55

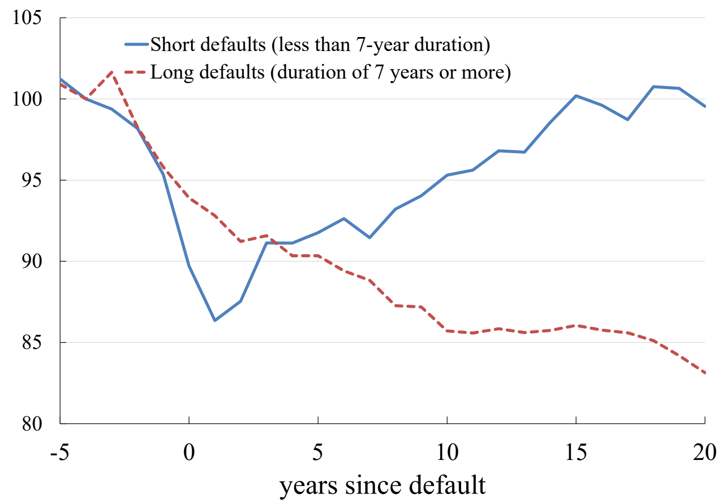
Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Each observation corresponds to a default episode. The dependent variable is the log-deviation of GDP from its pre-default trend measured 10 and 15 years after the default episode was resolved.

The parameter of interest ψ_1 is negative and statistically different from zero, with and without additional controls. In fact, none of the controls are significant, which suggests that the negative relationship illustrated in Figure 4 is quite robust. Defaults, even short ones, have a long-lasting scarring effect on economic performance. The coefficients in the first row of columns (1) and (2) show that, following a default episode that is resolved within one year, GDP per-capita remains roughly 5 percent below its pre-default trend ten years after its resolution. Moreover, each additional year that the episode takes to resolve increases this difference by 5 percentage points, on average. The fact that the coefficients in columns (3) and (4) have a very similar magnitude indicates that GDP per-capita in these economies is evolving on a different path rather than slowly converging back to their pre-default trend (which could be the case if the coefficients in columns (3) and (4) were smaller than those in columns (1) and (2)). These results are consistent with the findings in Farah-Yacoub, Graf von Luckner, and Reinhart (2024), who use a synthetic controls method to document similarly persistent effects of sovereign defaults on GDP and a significantly larger scarring effect for longer defaults.

Figure 5 presents the average evolution of TFP indices around short and long defaults. These indices are constructed based on TFP growth rates calculated by the World Bank following the Penn World Table (PWT) 9.1 methodology (see Dieppe (2021)).

Figure 5: TFP around short and long defaults



Country indices are constructed based on human capital-adjusted TFP growth rates calculated by the World Bank (indicator code WB-ASPD-DTFP). All indices are normalized so that the year 4 before each default event is equal to 100. The data include the 51 default events between 1980 and 2015 for which there are TFP data available.

For both groups, TFP is already declining in the years before a default and, interestingly, the average decline is sharper for short default events. The long-run evolution of TFP, however, differs significantly after default. For short defaults, TFP eventually recovers to its pre-default levels. The average duration within the short-default group is 2.6 years, which is around when the average TFP index starts recovering. After the 7-year mark (when all the short defaults in the sample have been resolved) the average TFP index accelerates its recovery, reaching its pre-default level at around year 15. The average duration of long defaults is 11.3 years, which is around when the average TFP index begins to level off. However, unlike with the short-defaults, the average TFP index does not show any signs of recovery even after all episodes had been resolved (in periods 19 and 20), and it even begins to decline once again during these later years. (The longest episode is the Peruvian default in 1978, which was not resolved until 18 years later with the Brady-Bonds restructuring in 1996.).

The different paths of TFP depending on the duration of default episodes is at the core of the theoretical mechanism presented in the following section. As mentioned before, this paper does not attempt to claim that the duration of a default episode is the sole driving force of the later evolution of GDP or that the theory of industrialization presented below is the main driver of TFP in the data. To this effect, TFP in the model is comprised of an endogenous component associated with the theory of industrialization to be analyzed and an exogenous component meant to capture other factors affecting TFP, including frictions that hamper it during default that have been studied in the literature (see Mendoza and Yue (2012), Bocola (2016), and Arellano, Bai, and Bocola (2017)).

3 Model

There is a small open economy populated by a representative household, a government, and a continuum of firms. The household has preferences for consumption of a final good, supplies labor in a competitive market, and is excluded from international financial markets. The government has access to financial markets, and makes borrowing and investment decisions on behalf of the household. The government cannot commit to future policies and discounts the future at a higher rate than the household, which can be modeled as a result of political economy frictions (see for instance Persson and Svensson (1989), Alesina and Tabellini (1990), and Aguiar and Amador (2011)). As is standard in the sovereign default literature, the economy suffers an exogenous productivity cost while in default and is excluded from financial markets for a random number of periods. The key innovation is in the production side of the economy, which is laid out in detail below.

3.1 Environment

Technology.—There is a competitive firm that produces a final tradable good using technology

$$Y_t = \left(\int_0^1 (y_{it})^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where y_{it} are differentiated non-tradable varieties of intermediate goods, and $\epsilon > 1$ is the elasticity of substitution across varieties. There is a measure one of monopolistically competitive firms $i \in [0, 1]$ that produce the intermediate goods using capital k_{it} and labor ℓ_{it}. Each intermediate good producer is characterized by an individual productivity level $z_i \geq 1$, which is fixed across time. These productivity levels are distributed according to a Pareto distribution with CDF $G(z_i) = 1 - z_i^{-\xi}$, where ξ is the tail parameter. Each period, the productivity of all firms is scaled by an aggregate productivity shock z_t, which follows a standard Markov process. This aggregate shock is the only source of risk in the model. At the beginning of each period firms choose whether to produce using a "traditional" technology

$$y_{it}^T = z_t z_i z_T k_{it}^{\alpha} \ell_{it}^{1-\alpha}, \tag{3}$$

with $\alpha \in (0,1)$, or pay a fixed cost f to operate a "modern" technology

$$y_{it}^M = z_t z_i z_M k_{it}^{\alpha} \ell_{it}^{1-\alpha}, \tag{4}$$

where $z_T < z_M = 1$ indicates that the modern technology is more productive. The fixed cost f is expressed in terms of the final good and has to be paid each period that the firm chooses to operate the modern technology.

Preferences and capital accumulation.—The household has preferences for consumption of the final good and labor represented by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_H^t \frac{\left(c_t - \frac{L_t^{1+\nu}}{1+\nu} \right)^{1-\sigma}}{1-\sigma} \right], \tag{5}$$

where $\beta_H \in (0,1)$ is the discount factor of the household, $\sigma > 0$ is the coefficient of constant relative risk aversion, and $1/\nu$ is the Frisch elasticity of labor supply. This formulation removes the wealth effect on the labor supply, preventing it from sharply rising when TFP is low or when consumption drops, which are responses that are at odds with the data (see Greenwood, Hercowitz, and Huffman (1988)). This is particularly important during default events, which are characterized by low TFP and consumption in the model. The household owns all firms and capital in the economy and rents labor and capital to the intermediate firms for a wage w_t and a rental rate r_t, respectively. The law of motion of capital is

$$K_{t+1} = I_t + (1-\delta) K_t, \tag{6}$$

where $\delta \in (0,1)$ is the depreciation rate and I_t is investment. The final good is used for investment, but there is a capital adjustment cost $\Psi(K_t, K_{t+1}) = \frac{\phi}{2}\frac{(K_{t+1}-K_t)^2}{K_t}$. This assumption, which is standard in open-economy business cycle models, prevents investment from being significantly more volatile in the model than in the data and prevents the sovereign from sharply depleting the capital stock to pay foreign debt in periods of distress, as discussed by Gordon and Guerron-Quintana (2018). As mentioned above, the government directly makes investment decisions on behalf of the household. The discussion in Subsection 3.3 below elucidates the importance of this assumption in ruling out potential sources of equilibrium multiplicity which, while interesting, are beyond the scope of this paper.

Borrowing and default.—The government issues long-term debt in international financial markets and cannot commit to repay it. Following the literature (see Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012)), I assume that debt matures at a constant rate $\gamma \in (0,1)$. The law of motion of debt is

$$B_{t+1} = i_{bt} + (1-\gamma)B_t, \tag{7}$$

where i_{bt} is current debt issuance that is sold to competitive risk-neutral international investors for a price q_t. If the government defaults then it is excluded from financial markets and gets readmitted with probability θ and all debt forgiven. While in default, aggregate productivity is $z_D(z_t) = z_t - \max\{0, d_0 z_t + d_1 z_t^2\}$, with $d_0 < 0 < d_1$. This formulation was introduced by Chatterjee and Eyigungor (2012) in a pure exchange economy and has been widely implemented in production economies as a cost to TFP (see Gordon and Guerron-Quintana (2018) and Arellano, Bai, and Mihalache (2018)). What is important is that it makes the cost of defaulting large in "good times" when productivity is high and small or zero in "bad times", which is essential for default events and default risk to be countercyclical. Mendoza and Yue (2012) show how such an asymmetric cost of defaulting can emerge in a production economy similar to the one laid out here as a result of firms losing access to working-capital to finance purchases of certain intermediate inputs. While the above environment can be extended to feature a similarly endogenous cost, this extension would come at the expense of clarity of the novel mechanism that generates a depression after a lengthy default, which is discussed in detail below.

3.2 Aggregate production

This section characterizes the choices of individual firms and shows how increasing returns to scale arise as a result of the technology assumptions. Moreover, since all firm choices are static, this section characterizes aggregate output as a function of only the productivity shock z_t and the aggregate capital stock K_t. Crucially, the combination of (z_t, K_t) determines the mass of firms that choose to adopt the modern technology, which is a key driver of depressions following a lengthy default episode in the model.

Operating profits.—The final demand for variety i is

$$y_{it} = \left(\frac{P_t}{p_{it}}\right)^{\epsilon} Y_t, \tag{8}$$

where Y_t is output of the final good and P_t is its corresponding price index

$$P_t = \left(\int_0^1 (p_{it})^{1-\epsilon}\, di\right)^{\frac{1}{1-\epsilon}}. \tag{9}$$

Given a production plan y, cost minimization for firm i operating technology $j \in \{T, M\}$ yields the total cost function

$$C_{it}^{j}(y) = \frac{\mu_t}{z_{it}^{j}} y, \tag{10}$$

where $\mu_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^{\alpha}$ and z_{it}^{j} is firm i's productivity in period t, which depends on its individual level z_i, on the current realization of the aggregate shock z_t, and on whether the firm chose to operate the traditional technology $z_{it}^{T} = z_t z_i z_T$ or the modern (M) one $z_{it}^{M} = z_t z_i$. Given a technology choice $j \in \{T, M\}$, operating profits of firm i in period t are

$$\pi_{it}^{Oj} = \max_{y} \left\{ p_{it}(y)\, y - C_{it}^{j}(y) \right\}, \tag{11}$$

where $p_{it}(y)$ is the inverse demand function implied by (8). The solution to the problem in (11) implies the standard choice of monopoly pricing

$$p_{it}^{j} = \frac{\epsilon}{\epsilon - 1} \frac{\mu_t}{z_{it}^{j}}, \tag{12}$$

charging a markup $\frac{\epsilon}{\epsilon-1}$ over the firm's marginal cost μ_t/z_{it}^j. Plugging (12) into (8) and using the assumption that firms satisfy their demand we get that operating profits are

$$\pi_{it}^{Oj} = \frac{1}{\epsilon}\left(\frac{\epsilon-1}{\epsilon}\frac{z_{it}^j}{\mu_t}\right)^{\epsilon-1}(P_t)^{\epsilon}Y_t. \tag{13}$$

Technology adoption.—A firm i chooses to adopt the modern technology if

$$\pi_{it}^{OM} - P_t f \geq \pi_{it}^{OT}, \tag{14}$$

that is, if the additional profits of operating the modern technology more than compensate for the fixed cost of adoption. Note that, all else equal, firms with high z_i are more profitable, so the assumption that z_i are distributed Pareto implies that there is always a positive (albeit potentially small) mass of firms that adopts the modern technology. In addition, note that firms with high z_{it}^j produce more and charge lower prices. This implies that total output Y_t increases as the mass of modern firms increases. This in turn increases demand for all varieties (equation (8)) making it more attractive for low z_i firms to adopt the modern technology.

This virtuous cycle of adoption in which some firms adopt the modern technology as other firms do so is what the development literature highlights as "the big push". Some of the work in this literature introduces such a mechanism in environments with identical firms, which gives rise to multiplicity of equilibra and generates a scope for government intervention as a coordinating force. In this model, instead, the assumptions regarding ex ante heterogeneity of firms guarantee uniqueness with respect to the adoption decision, which removes the need for coordination via government intervention. This simplifies the analysis on the interactions between increasing returns and default events, which is the main focus of the paper. Appendix (B) shows that if $(\epsilon-1) > \frac{1+\nu}{\alpha+\nu}$ then the adoption decision is uniquely characterized by a cutoff $z_i = z_t^*$ such that equation (14) holds with equality. Under this assumption, the mass of firms adopting the modern technology $m_t = \left(\frac{1}{z_t^*}\right)^{\xi}$ is increasing in the aggregate shock z_t and in the aggregate stock of capital K_t.

TFP and final output.—Define total factor productivity as

$$A_t = \left(\int_0^1 \left(z_{it}^{j(i,t)}\right)^{\epsilon-1} di\right)^{\frac{1}{\epsilon-1}}, \tag{15}$$

where $j(i,t) \in \{T, M\}$ is firm i's adoption choice in period t. The assumption that z_i follow a Pareto distribution with tail parameter $\xi > \epsilon - 1$ implies that

$$A_t = A\left(z_t, z_t^*\right) = z_t \left(\frac{\xi}{1+\xi-\epsilon}\left[1+\left(\hat{z}^{\epsilon-1}-1\right)(m_t)^{\frac{1+\xi-\epsilon}{\xi}}\right]\right)^{\frac{1}{\epsilon-1}}, \tag{16}$$

where z_t is the aggregate shock and m_t is the mass of firms adopting the modern technology. The condition $\xi > \epsilon - 1$ implies that the tail of the Pareto distribution cannot be too heavy (i.e. the mass of extremely productive firms should be small, and smaller for higher elasticity of substitution ϵ). The Pareto-distribution assumption allows for a closed-form characterization of A_t, and significantly simplifies the analysis and computation of equilibrium. This assumption, however, is not necessary to generate the endogenous increasing returns to scale in aggregate output. Assuming that z_i are distributed log-normal, for instance, would work similarly at the expense of having to compute A_t numerically (with a corresponding bound on the mass at the tail of the distribution).

From the market clearing conditions for production factors and using individual firm's optimality conditions we can derive total output as a function of aggregates and TFP:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{17}$$

where L_t is the aggregate labor supply. Finally, with GHH preferences the labor supply dos not depend on the current consumption level, which makes it a static decision that can be written as a function of A_t and K_t:

$$L_t = \left[(1-\alpha)\frac{\epsilon-1}{\epsilon}A_t K_t^{\alpha}\right]^{\frac{1}{\alpha+\nu}}. \tag{18}$$

This implies that final output is also a function of z_t and K_t only and we can define

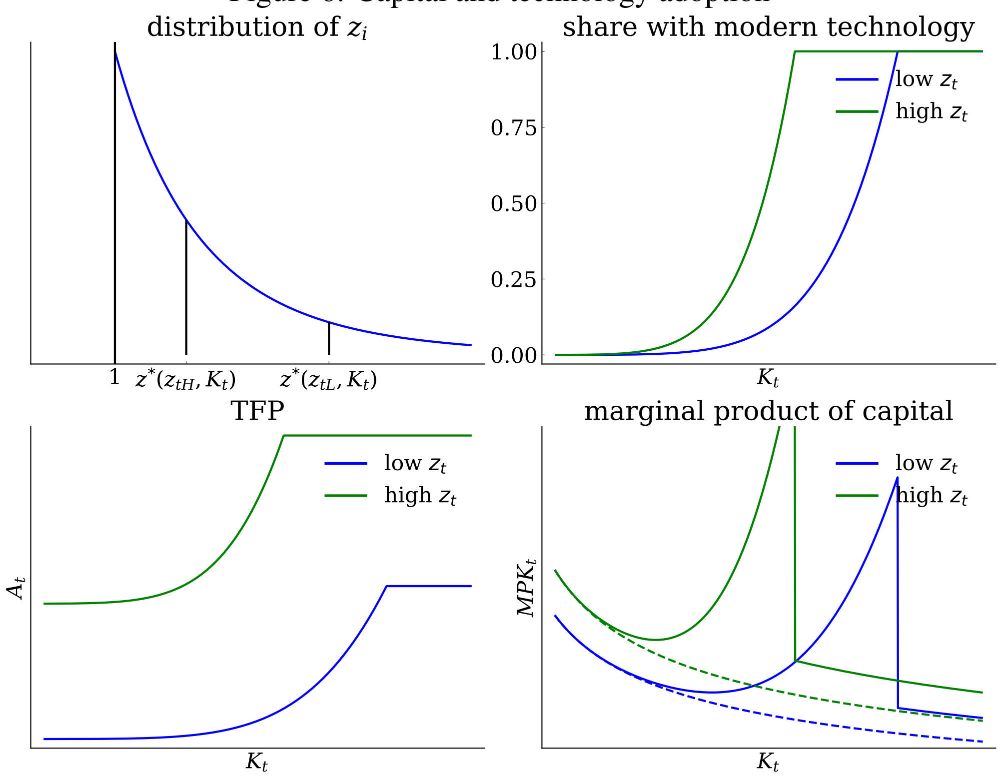
$$F\left(z_t, K_t\right) = \left[(1-\alpha)\frac{\epsilon-1}{\epsilon}\right]^{\frac{1-\alpha}{\alpha+\nu}}\left(A_t K_t^{\alpha}\right)^{\frac{1+\nu}{\alpha+\nu}} - P_t m_t f, \tag{19}$$

which is aggregate output net of total adoption costs (see Appendix (B) for the full derivation of these equations). These characterizations imply that the productivity cutoff implicitly defined as the solution to equation 14 is only a function of z_t and K_t

$$z_t^* = z^*(z_t, K_t) \tag{20}$$

Increasing returns to scale.—Figure 6 illustrates how capital has increasing returns to scale in some regions of the state space. The top-left panel shows the distribution of individual firm productivities z_i. There is a large mass of firms with levels of z_i close to 1, which is the lowest value, and a small and decreasing mass of firms with high z_i. The figure also shows two productivity cutoff values corresponding to a high and low realization of the shock $z^*(z_{tH}, K_t)$ and $z^*(z_{tL}, K_t)$ for a given level of capital K_t. For the given state, all firms with productivity z_i to the right of the cutoff adopt the modern technology. As mentioned before, since the support of the Pareto distribution is $[1, \infty)$ there is always a positive mass of adopting firms. This mass, however, is lower for low realizations of z_t and close to zero for low enough levels of capital K_t, as illustrated by the top-right panel of Figure 6.

Figure 6: Capital and technology adoption



The top-right panel also shows how, for a large enough level of capital K_t all firms adopt the modern technology $m_t = 1$. The bottom-left panel shows the corresponding values of TFP A_t for high and low z_t and different levels of capital. As suggested by the behavior of m_t, TFP is low and close to being flat for low levels of capital, rapidly increasing for medium levels, and high and flat for high levels. These dynamis have important implications for the marginal product of capital MPK_t, which is shown on the bottom-right panel. For low enough levels of capital, MPK_t is decreasing in K_t, as is standard with a Cobb-Douglas production function. Once capital reaches a level at which adoption starts to quickly increase then the marginal product of capital becomes increasing as the increase in A_t more than compensates for the decreasing returns to scale in the production function. Once capital reaches the level at which all firms adopt the modern technology $m_t = 1$, the MPK_t function features a discontinuity and becomes decreasing, but still higher than if A_t had remained constant at its level with zero adoption (illustrated by the continuing dashed lines).

3.3 Recursive formulation and equilibrium

The state of the economy is (B, K, z), where B is the debt level, K is the aggregate capital stock, and z is the aggregate productivity shock (z_t in equations (3) and (4)). At the beginning of a period in good financial standing, the value of the government is:

$$V(B,K,z) = \max_{D\in\{0,1\}} \left\{ DV^D(K,z) + (1-D)V^P(B,K,z) \right\}, \tag{21}$$

where V^D and V^P are the values of defaulting and repaying, respectively, and D is the default choice. If the government decides to repay $D = 0$, the value is

$$V^P(B,K,z) = \max_{\{B',K',c\}} \{u(c, L(z,K)) + \beta\mathbb{E}[V(B',K',z')]\} \tag{22}$$

$$s.t. \qquad c + K' + \Psi(K,K') + \gamma B \leq F(z,K) + (1-\delta)K + q(B',K',z)[B' - (1-\gamma)B]$$

where $\beta < \beta_{HH}$ is the government's discount factor, q is the price schedule for government debt (defined below), and L and F are the functions defined in the previous subsection. If the government defaults $D = 1$, the value is

$$V^D(K,z) = \max_{\{K',c\}} \left\{ u\left(c, L\left(z_D(z), K\right)\right) + \beta\theta\mathbb{E}\left[V\left(0,K',z'\right)\right] + \beta\left(1-\theta\right)\mathbb{E}\left[V^D\left(K',z'\right)\right] \right\} \quad (23)$$

$$s.t. \qquad c + K' + \Psi\left(K,K'\right) \leq F\left(z_D(z), K\right) + \left(1-\delta\right)K,$$

where $z_D(z) = z - \max\left\{0, d_0 z + d_1 z^2\right\} \leq z$ is aggregate productivity in default.

Equilibrium.—An equilibrium is value and policy functions in default V^D, K^D, c^D; value and policy functions in repayment V, V^P, D, B^P, K^P, c^P; and a price schedule q such that: (i) given the price schedule, the value and policy functions solve the system of Bellman equations (21), (22), and (23); and (ii) the price schedule q satisfies

$$q\left(B',K',z\right) = \frac{\mathbb{E}\left[\left(1 - D\left(B',K',z'\right)\right)\left(\gamma + \left(1-\gamma\right)q\left(B'',K'',z'\right)\right)\right]}{1+r}, \quad (24)$$

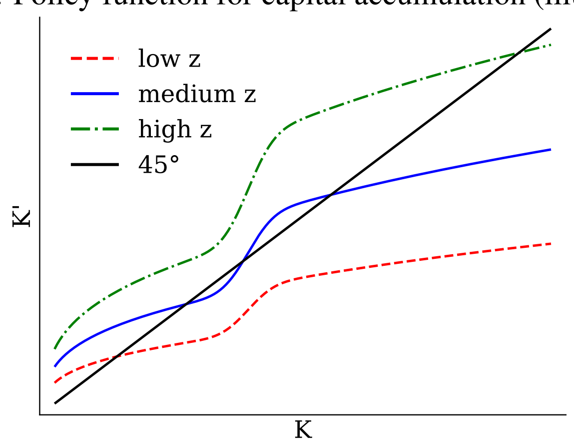
where $B'' = B^P\left(B',K',z'\right)$, $K'' = K^P\left(B',K',z'\right)$, and r is the international risk-free interest rate.

Uniqueness.—Before proceeding with the analysis it is important to discuss whether this dynamic equilibrium is unique or, if not, what is the scope for multiplicity. First, the conditions $\xi > \epsilon - 1$ and $(\epsilon - 1) > \frac{1+\nu}{\alpha+\nu}$ guarantee that there is a unique cutoff for technology adoption, which allows us to define $F\left(z_t, K_t\right)$ in equation (19). Under these assumptions the model does not feature the classic multiplicity that gives rise to coordination failures in the big push literature, but still retains the complementarity of firms' technology adoption, which is an important mechanism from the literature. Then, the dynamic model collapses to a standard sovereign default model with production. Because the aggregate production function features increasing returns to capital there is a possibility for multiple solutions to the Euler equation for capital, such as in the model studied in Schaal and Taschereau-Dumouchel (2025). This is an important challenge in their paper because they study a decentralized environment in which competitive households make the consumption-savings decision taking prices as given. I circumvent this problem by assuming that the domestic government makes the aggregate consumption-savings choice directly. That is, the government chooses $\left(K', B'\right)$ to solve the problems in (22) and (23), taking into account how these choices affect prices and future returns (this solution can be decentralized with appropriate investment subsidies like the ones discussed in Gordon and Guerron-Quintana (2018), and corrects underinvestment of the type studied in Esquivel (2024)). If the Euler equation for capital were to have multiple solutions for a given state then the government would choose the one that yields the higher value, so the set of parameterizations under which this problem would have multiple solutions is non-generic. Finally, as shown by Aguiar and Amador (2020), sovereign default models with long-term debt may feature multiple equilibria because creditor beliefs regarding future borrowing can be self-fulfilling. To this respect, I characterize the equilibrium that results from the limit of a finite-horizon economy, following Hatchondo and Martinez (2009) and the literature thereafter.

3.4 The "Big Pull" effect of default

To analyze the dynamic implications of the production structure, consider the deterministic closed-economy version of the environment laid out above. Figure 7 illustrates the shape that the policy function for capital accumulation takes for different fixed values of the aggregate productivity shock z. For a high enough z (the green dashed-dotted line) there is a unique steady state with a high level of capital. Similarly, for a low-enough z (the red dashed line) there is a unique steady state with low capital.

Figure 7: Policy function for capital accumulation (illustrative)



The most interesting case is illustrated by the solid blue line, which features three steady-state levels of capital. The lowest and highest stead-state levels are stable, while the middle one is unstable in the sense that an epsilon-deviation from it would put the economy's capital level on a dynamic path toward one of the other steady states.

In an economy with productivity shocks, different realizations of z shift similar-looking policy functions up and down. Starting at a high enough level, capital fluctuates around the high-capital steady state for small enough shocks, but a large (and persistent) negative shock to z may push it into a trajectory toward the low-capital steady state. The converse is true for a low starting point: capital fluctuates around the low steady state and a large persistent shock may push it toward the high steady state.

This somewhat symmetric (and low-frequency) transition between fluctuations around the high and low steady states breaks in the economy with endogenous default. This is because default, which is associated with low realizations of z, triggers an additional productivity reduction through the exogenous default penalty. A lengthy default makes low values of z more persistent because high subsequent realizations are hindered by it. When default happens in the region of the state space with high levels of capital it has a "big pull" effect on technology adoption. When the economy is in the low-capital region there is no corresponding positive event that would have a "big push" effect similar to the pull-down that default has when capital is high.

4 Quantitative analysis

I use value function iteration to solve for the equilibrium using the formulas derived in Subsection 3.2 to compute aggregate output and the labor supply as functions of the state. Following Hatchondo, Martinez, and Sapriza (2010), I compute the limit of the finite-horizon version of the economy. I jointly solve for investment and borrowing decisions using a non-linear optimization routine, and approximate value functions and the price schedule using linear interpolation. To compute expectations over the productivity shock I use a Gauss-Legendre quadrature.

4.1 Calibration

I calibrate the model to Argentina, which is a widely studied example in the sovereign default literature. There are two sets of parameters: the first (summarized in Table 2) is calibrated directly and the second (summarized in Table 3) is chosen so that moments generated by model simulations match their data counterparts. A period in the model is one quarter. The relative risk aversion parameter is $\sigma = 2$ and the risk-free rate $r^* = 0.01$, which are standard values in the business cycle literature. Following Mendoza and Yue (2012), I set $\nu = 0.455$ so that the Frisch elasticity is equal to 2.2.

Table 2: Parameters calibrated directly from the data

Parameter		Value	Source
Relative risk aversion	σ	2	Standard value
Risk-free rate	r^*	0.01	Standard value
Labor disutility parameter	ν	0.455	Mendoza and Yue (2012)
Debt duration	γ	0.05	Chatterjee and Eyigungor (2012)
Probability of reentry	θ	0.0625	Gelos, Sahay, and Sandleirs (2011)
Capital share	α	0.33	Standard value
Elasticity of substitution	ϵ	2.92	Schaal and Taschereau-Dumouchel (2024)
Depreciation rate	δ	0.05	Standard value
Mean of productivity shock	μ_z	0.768	High-capital steady state $y^H_{ss} = 1$
Persistence of productivity	ρ	0.95	Gordon and Guerron-Quintana (2018)
Variance of productivity	σ^2	0.017	Gordon and Guerron-Quintana (2018)
Pareto curvature	ξ	20.5	high-capital steady state with $m_t = 1$
Fixed cost	f	0.04	low-capital steady state with $m_t < 0.1$
Productivity of traditional technology	z_T	0.898	$y^L_{ss}/y^H_{ss} = 0.60$

I set the debt duration parameter to $\gamma = 0.05$ to match the maturity information for Argentina reported in Broner, Lorenzoni, and Schmukler (2013), as done by Chatterjee and Eyigungor (2012). I set the probability of reentry to financial markets to $\theta = 0.0625$ for an average duration in autarky of 16 quarters, which is the median duration of default events documented by Gelos, Sahay, and Sandleirs (2011). On the production side, I set the elasticity of substitution to $\epsilon = 2.92$, following Schaal and Taschereau-Dumouchel (2024) who calibrate a similar model of heterogeneous firms. This is close to the value of 3, which is standard in the literature (see the discussion in Hsieh and Klenow (2009)). The capital share $\alpha = 0.33$ and the capital depreciation rate $\delta = 0.05$ are standard values. The aggregate productivity shock follows an AR(1) process

$$\log z_t = (1-\rho)\log \mu_z + \rho \log z_{t-1} + \sigma_z \epsilon_t, \tag{25}$$

where the persistence $\rho = 0.95$ and variance $\sigma^2 = 0.017$ are taken from Gordon and Guerron-Quintana (2018), who calibrate them for Argentina using a Cobb-Douglas production technology. I set the mean value of the shock $\mu_z = 0.768$ so that output $y^H_{ss} = 1$ in the deterministic steady-state in which all firms adopt the modern technology. Let y^L_{ss} be the level of output in the hypothetical steady state in which no firms adopt the modern technology, I set the productivity level of the traditional techology $z_T = 0.898$ so that $y^L_{ss} = 0.6$, which is the average drop of output from trend observed in depressions that follow lengthy defaults. I jointly choose the Pareto tail parameter $\xi = 20.5$ and the fixed cost of adoption $f = 0.04$ so that $m_t = 1$ at the steady-state level of capital with full adoption k^H_{ss} and so that $m_t < 0.1$ at the hypothetical steady-state level of capital with no adoption k^L_{ss}. These choices will allow the ergodic distribution to feature long periods of high and low technology adoption.

Finally, I set the discount factor $\beta = 0.9697$, the parameters governing the default penalty to productivity $d_0 = -0.4826$ and $d_1 = 0.5882$, and the capital adjustment cost $\phi = 3.1395$, to jointly match average spreads of 0.08, an average debt-to-GDP ratio of 0.31, a default probability of 0.03, and a volatility of investment of 12.8. I interpret the Argentinean data as being generated by an economy that is fluctuating around a low-adoption long-run steady state, which is consistent with high default risk and low GDP in the model. To that end, the targeted moments in the model are calculated conditional on the fraction of firms adopting the modern technology being below its long-run median $m_t \leq 0.17$. Table 3 summarizes this calibration exercise.

Table 3: Parameters calibrated simulating the model

Parameter		Value	Moment	Data	Model, low adoption
discount factor	β	0.9697	default probability	0.03	0.032
default cost	d_0	-0.4826	$Av\,(r - r^*)$	0.08	0.08
default cost	d_1	0.5882	$Av\left(\frac{b}{gdp}\right)$	0.31	0.31
capital adjustment cost	ϕ	3.1395	σ_i	12.8	11.5

To compute the model moments I draw 100 samples of 11,000 periods and drop the first 1,000. Since the targeted moments are conditional on low adoption, I also drop all periods in which the mass of adopters is below its unconditional median. Investment data is HP-Filtered with a smoothing parameter of 1,600. In the data, debt corresponds to External debt stocks, public and publicly guaranteed from the World Development Indicators database. All model moments are conditional on the economy being in good financial standing.

Table 4 compares other business cycle statistics from the model to their data counterparts. Column (2) in Table 4 reports moments conditional on m_t being below its median, as in the calibration exercise. Column (3) reports them conditional on m_t being above its median and Column (4) reports them conditional on full adoption $m_t = 1$.

Table 4: Business cycle statistics

moment	(1) data	(2) low adoption	(3) high adoption	(4) full adoption
			model	
Default frequency	0.03	0.016	0.014	0.008
Av. spreads	0.08	0.08	0.05	0.04
Std. spreads	0.04	0.12	0.06	0.03
debt-to-GDP ratio	0.31	0.31	0.38	0.38
Capital stock (total)	n.a.	2.64	4.24	4.71
σ_c/σ_y	1.23	1.01	0.99	0.87
σ_i/σ_y	2.65	2.24	2.06	1.64
$\sigma_{TB/y}$	2.34	4.38	4.02	3.23
Cor (spreads, y)	-0.79	-0.17	-0.35	-0.48
$Cor\,(TB/y, y)$	-0.68	-0.11	-0.13	-0.16

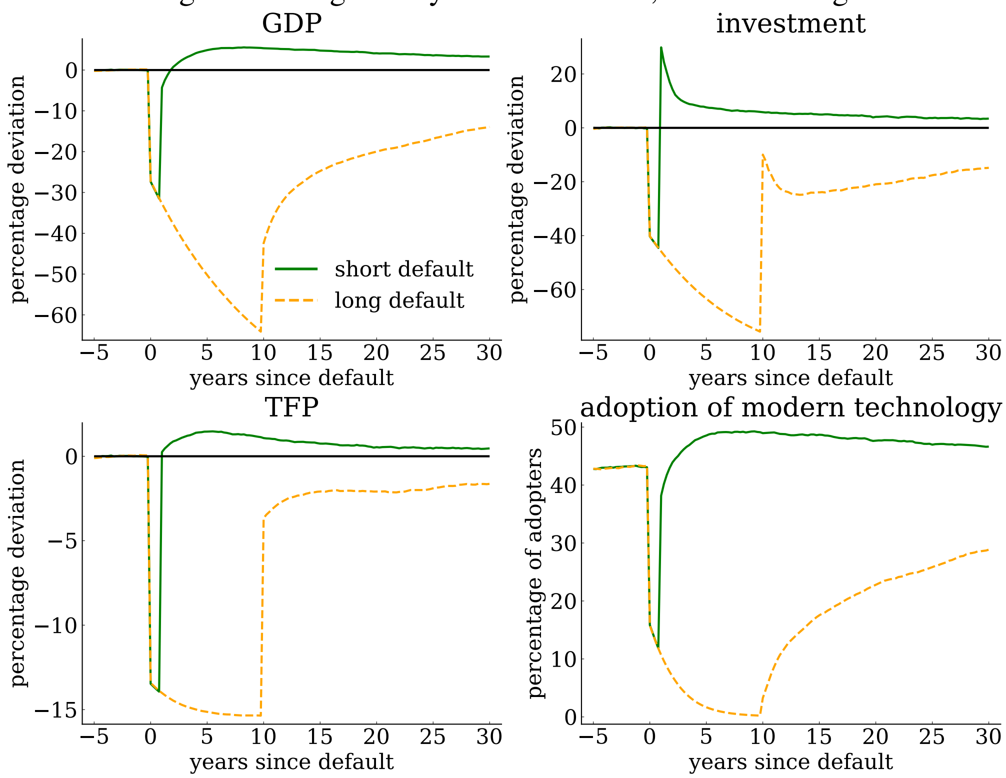
To compute the model moments I draw 100 samples of 11,000 periods and drop the first 1,000. Column (2) drops all periods in which the mass of adopters is above its unconditional median, Column (3) drops all periods in which the mass of adopters is below the median, and Column (4) drops all the periods in which the mass of adopters is not 1. Except for spreads and the debt-to-GDP ratio, all variables are HP-Filtered with a smoothing parameter of 1,600. In the data, debt corresponds to External debt stocks, public and publicly guaranteed from the World Development Indicators database. All model moments are conditional on the economy being in good financial standing.

The model does a good job of replicating long-run business cycle statistics of Argentina during low adoption periods, including moments that were not directly targeted by the calibration exercise. Columns (3) and (4) show how periods of high adoption are associated with higher capital accumulation, lower default risk, and lower spreads, which allow the economy to support higher debt levels (it is worth noting that, while the median adoption level is well below 1, there is full adoption 34 percent of the time). In contrast, the region of the ergodic distribution with low adoption features spreads that are significantly higher and more volatile, the aggregate capital stock is roughly half of what it is in periods with high adoption, and the economy is not able to sustain as much debt. Once the economy gets pulled down to this part of the ergodic distribution, more frequent defaults (even if they are short) make transitioning back to normal times less likely. This point is explored further in the following subsections.

4.2 Default depressions

To analyze the differentiated effects of short and long defaults in the model I perform the following two exercises. First, I consider a random draw from the ergodic distribution and exogenously shift the economy to the default state, regardless of whether the government would have chosen to default or not. I then impose default durations of one year and ten years, and continue the simulation for 30 years after the induced default. Figure 8 shows the average of 10,000 of the paths for each default duration (one year duration is the green solid line and ten year duration is the yellow dashed line), between five years before and 30 years after the default began.

Figure 8: Exogenously induced default, short and long

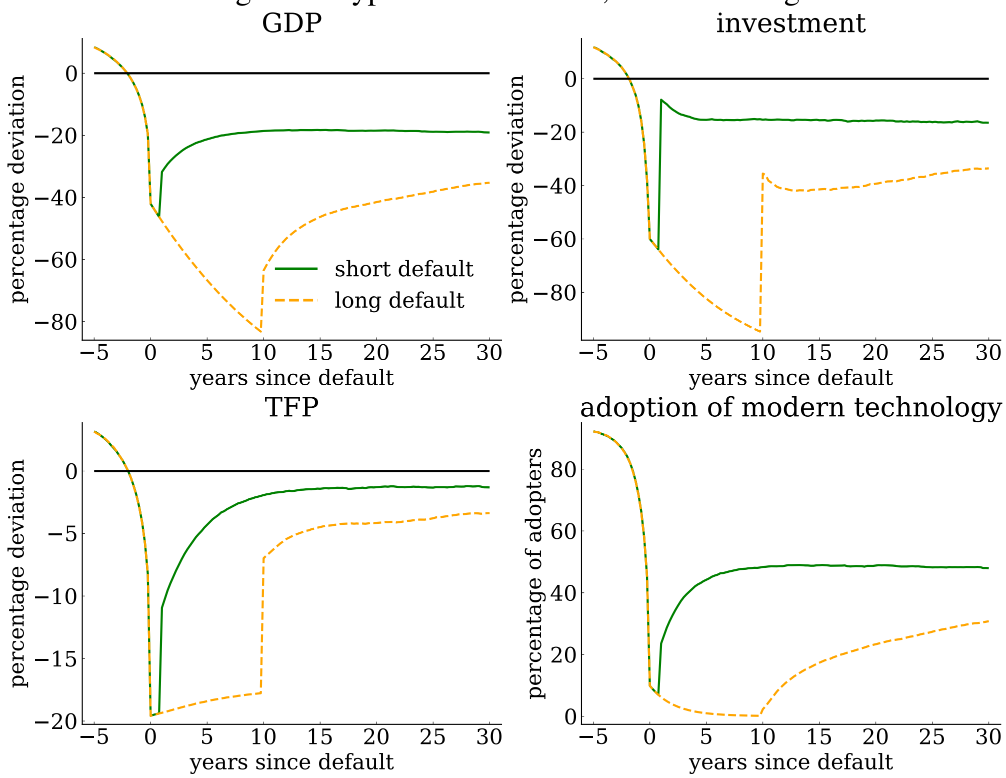


Each line is the average of 10,000 paths for each default duration (one year duration is the green solid line and ten year duration is the yellow dashed line). The paths are centered around an exogenously induced default in year $t = 0$.

The average drop in GDP is initially the same, but its recovery in the long-run is significantly different, just like in the data presented in Figure 1 in the introduction. The top-left panel shows that GDP bounces back after the short default, driven by the recovery in technology adoption that increases TFP. GDP after the long default event also recovers but to a much lower level. Moreover, GDP in this case remains well below its pre-default level 20 years after the default crisis ended. This is because the stock of capital falls so much that technology adoption does not recover after the crisis ends and TFP remains at a much lower level.

Figure 9 does a similar exercise for the typical default in the model that happens following periods with full-adoption. That is, I now select defaults that happen endogenously in the model following periods with full adoption and impose the same two alternative durations. As is standard in these models, default episodes are preceded by a sequence of bad shock realizations, which explain why GDP, investment, TFP, and technology adoption are already dropping in the years prior.

Figure 9: Typical default event, short and long



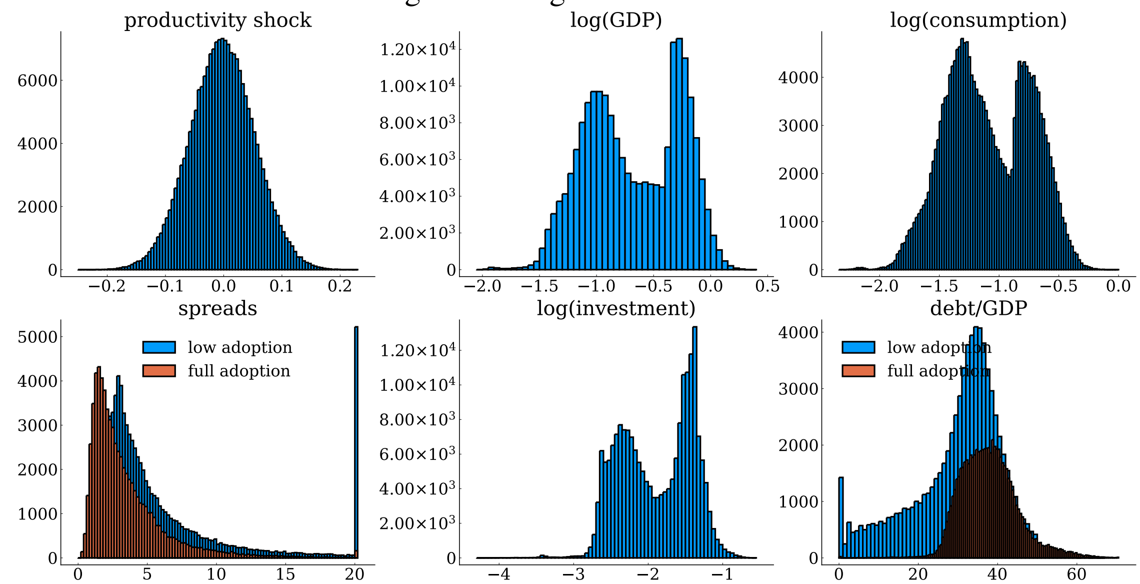
Each line is the average of 10,000 paths for each default duration (one year duration is the green solid line and ten year duration is the yellow dashed line). The paths are centered around an endogenous default in year $t = 0$ that followed a period with full technology adoption.

As before, the duration of the default episodes matters. GDP, TFP, investment, and adoption all bounce back to a higher level in the economy with the short default episode than in the economy with the long default. The long default has a differentiated and long-lasting negative effect that remains 20 years after the event was resolved. In general, average adoption of the modern technology does not fully recover after default, increasing the vulnerability of the economy to future negative shocks even after short defaults.

4.3 The default trap

As suggested by Table 4 and consistent with the previous analysis regarding two stable steady states, Figure 10 shows that the ergodic distribution of the endogenous variables in the model is bimodal. It presents histograms for the natural logarithm of the productivity shock (relative to its mean), the natural logarighm of GDP, consumption, and investment from 200,000 periods simulated from the model. It also presents two conditional histograms for the debt-to-GDP ratio and spreads: one with full adoption ($m_t = 1$) and one with adoption below the median (the histograms for spreads feature an overflow bin to the right for values ≥ 20).

Figure 10: Ergodic distribution



Each histogram is constructed from a sample of 200,000 periods simulated from the ergodic distribution of the model. The panels for spreads and debt each present two conditional histograms: one with full adoption ($m_t = 1$) and one with adoption below the median (the histograms for spreads feature an overflow bin to the right for values ≥ 20).

The histograms for GDP, consumption, and investment are not only bimodal, but accumulate more mass on the lower end of the distribution. As can be seen in columns (3) and (4) of Table 4, when capital is abundant technology adoption is high and defaults are infrequent. This makes the cost of defaulting relatively high because there is a probability that, if the default lasts long enough, the economy will be pulled down to the "poor" side of the ergodic distribution. With a relatively high cost of defaulting the economy sustains higher average levels of debt and lower spreads, illustrated by the orange histograms in the bottom-right and bottom-left panels, respectively.

When capital is scarce (column (2) in Table 4), this component of the cost of defaulting is absent, making default more attractive and spreads higher. The spreads distribution of the low-adoption periods is to the right of that of the high adoption and has a fatter tail. In addition, the government has an additional incentive to accumulate debt rapidly besides its preference for front-loading consumption. When the stock of capital is close to the region with increasing returns, borrowing to invest becomes attractive despite the large cost from high spreads because the economy may get into a path toward the "rich" side of the ergodic distribution. This is a gamble that may not pay off given the high probability of a set-back due to a default along the way. This creates the default trap in the model.

4.4 Foreign aid

The model studied in this paper does not have much scope for the government to intervene and improve economic outcomes. In particular, there is no scope for a big push by increasing borrowing from abroad to boost demand and industrialization. The domestic government already has instruments to select all aggregate allocations, and the big pull around the default trap stems from the government's inability to commit to future policy and its bias toward front-loading consumption.

I now explore the effects of foreign aid following a long default of 10 years. The main objective is to evaluate the welfare and development implications of different types of aid. The experiment considers a one-time transfer of resources a from an international entity in the form of a non-defaultable perpetuity that has to be serviced every period at a subsidized interest rate $r_a < r$. The resource constraint in repayment after receiving aid is now

$$c + K' + \Psi(K, K') + \gamma B + r_a a \leq F(z, K) + (1-\delta) K + q(B', K', z) \left[B' - (1-\gamma) B\right],$$

and in default it is

$$c + K' + \Psi(K, K') + r_a a \leq F(z_D(z), K) + (1-\delta) K.$$

I consider three types of aid: an unconditional transfer a that can be freely allocated by the government, debt relief so that $B' = B - a$, and a capital injection so that $K' = K + a$. I compute welfare gains in consumption equivalent units for the household and the government. To measure the effect on development, I compute the following "time to development" variable

$$\mathcal{T}_0(\tilde{m}) = \mathbb{E}\left[\text{years until } m_t = \tilde{m} \,|\, (z_0, K_0, B_0)\right]$$

which is the expected number of years from period 0, given a state (z_0, K_0, B_0), before the economy reaches a mass $\tilde{m}$ of firms that adopts the modern technology. I set a to be equal to 15 percent of average GDP, $r_a = r/2$, and $\beta_{HH} = \frac{1}{1+r}$. I compute the time to development for $\tilde{m} = 0.5$ and for full adoption $\tilde{m} = 1.0$. Table 5 presents the results.

Table 5: Effects of foreign aid on welfare and development

case		welfare gains government (1)	welfare gains household (2)	m_0 (3)	time to development $\tilde{m} = 0.5$ (4)	time to development $\tilde{m} = 1.0$ (5)
1 year after default resolution	no aid	0.00	0.00	0.11	42	52
	debt relief	3.81	2.22	0.11	44	54
	capital injection	3.82	5.06	0.11	37	47
	transfer	5.08	3.36	0.11	36	46
5 years after default resolution	no aid	0.00	0.00	0.17	38	47
	debt relief	3.98	2.38	0.17	35	45
	capital injection	4.24	5.38	0.17	33	42
	transfer	5.51	3.54	0.17	33	42

The first four rows present the results of implementing the aid program one year after the default has been resolved, and the next four correspond to implementing the program 5 years later. Column (3) shows that one year after the resolution of the default episode 11 percent of firms adopt the modern technology, while 17 percent do so five years later. In both cases welfare gains for the government are the highest if the program is an unconditional transfer. This would be the optimal policy if the government were benevolent, since it is a non-distortionary lump-sum transfer. Note, however, that the household strictly prefers the capital injection because it reduces default risk and the expected time to exit the default trap, both of which are future benefits that are more heavily discounted by the impatient government. Both the government and the household prefer capital injections over debt relief, regardless of the timing of aid. Moreover, debt relief not only implies the smallest increase in welfare, but has the smallest effect on pushing the economy out of the default trap. These results make a strong case against debt relief in favor of aid in the form of targeted investment programs.

5 Conclusion

Motivated by the effects of long defaults in the data, this paper presents a novel theory of default-induced depressions. In the model, long default episodes pull the economy down to a default trap in which the capital stock and technology adoption are lower, defaults are more frequent, and spreads are higher and more volatile than in normal times. Besides these novel dynamics, the model features all the standard business cycle properties of emerging economies that face sovereign risk, as all the workhorse models in this literature. The model allows for the evaluation of different forms of foreign aid and makes a strong case against debt relief, favoring aid in the form of capital injections instead. This is because capital injections relax the resource constraint and push the economy closer to a development path, while debt relief only achieves the former. This difference is stronger under political economy frictions that make the government relatively more impatient than its constituent households.

The model sheds light on a plausible mechanism through which an emerging (or even an advanced) economy may become trapped in a low-income and high-default risk regime. The results from the model and the motivating evidence stress the importance of understanding the sources of lengthy default episodes and different policies to prevent them. It is tempting to conclude that the model makes a strong case for swift resolutions to sovereign crises through the intervention of international financial institutions. Moreover, one may conclude that this case is even stronger for crises in rich economies, who stand a lot more to lose by falling into a default trap. The effects of such interventions, however, are not innocuous because of the potential ex ante effects they may have on borrowing and default incentives. This scope for moral hazard would also have significant consequences on the regime switching dynamics introduced in this paper. Better understanding the effects of such interventions and other similar policy proposals is an exciting avenue for future research.

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A Data on default events

Table 6 summarizes the relevant data on default events from Laeven and Valencia (2018).

Table 6: Default events

Country	Default year	Default resolution	Duration
Albania	1990	1992	2
Argentina	1980	1993	13
Argentina	2001	2005	4
Argentina	2014	2016	2
Bulgaria	1990	1994	4
Belize	2007	2007	0
Bolivia	1980	1992	12
Brazil	1983	1994	11
Chile	1983	1990	7
Cote d'Ivoire	1984	1997	13
Cote d'Ivoire	2001	2009	8
Cote d'Ivoire	2010	2010	0
Cameroon	1989	1992	3
Congo, Dem. Rep.	1976	1989	13
Congo, Rep.	1986	1992	6
Costa Rica	1981	1990	9
Cyprus	2013	2013	0
Dominica	2002	2004	2
Dominican Republic	1982	1994	12
Dominican Republic	2003	2005	2
Ecuador	1982	1994	12
Ecuador	1999	2000	1
Ecuador	2008	2014	6
Egypt, Arab Rep.	1984	1992	8
Gabon	1986	1994	8
Guinea	1985	1992	7
Gambia, The	1986	1988	2
Greece	2012	2012	0
Grenada	2004	2005	1
Guyana	1982	1992	10
Honduras	1981	1992	11
Indonesia	1999	2002	3

Table 7: Default events (continued)

Country	Default year	Default resolution	Duration
Iran, Islamic Rep.	1992	1994	2
Jamaica	1978	1990	12
Jamaica	2010	2010	0
Jordan	1989	1993	4
Morocco	1983	1990	7
Moldova	2002	2002	0
Madagascar	1981	1992	11
Mexico	1982	1990	8
Malawi	1982	1988	6
Niger	1983	1991	8
Nigeria	1983	1992	9
Nicaragua	1980	1995	15
Panama	1983	1996	13
Peru	1978	1996	18
Philippines	1983	1992	9
Paraguay	1982	1994	12
Russian Federation	1998	2000	2
Sudan	1979	1985	6
Senegal	1981	1996	15
Sierra Leone	1977	1995	18
Seychelles	2008	2009	1
Togo	1979	1997	18
Trinidad and Tobago	1989	1989	0
Turkiye	1978	1982	4
Tanzania	1984	1992	8
Ukraine	1998	1999	1
Ukraine	2015	2015	0
Uruguay	1983	1991	8
Uruguay	2002	2003	1
Venezuela, RB	1982	1990	8
South Africa	1985	1993	8
Zambia	1983	1994	11

B Characterization of the adoption cutoff

B.1 Problem of a firm

- Given a production plan y_{it}, the cost minimization problem of the firm is

$$\min_{k_{it},l_{it}} r_t k_{it} + w_t l_{it}$$
$$s.t. \qquad y_{it} \leq z_{it} k_{it}^{\alpha} l_{it}^{1-\alpha}$$

the F.O.C.s are

$$r_t = \eta_{it} \alpha \frac{z_{it} k_{it}^{\alpha} l_{it}^{1-\alpha}}{k_{it}}$$
$$w_t = \eta_{it} (1-\alpha) \frac{z_{it} k_{it}^{\alpha} l_{it}^{1-\alpha}}{l_{it}}$$

where $\eta_{i,t}$ is the multiplier of the constraint. Assume the constraint binds so $\eta_{it} > 0$, rearranging we get

$$\frac{k_{it}}{l_{it}} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}$$

plugging into the constraint we get

$$l_{it} = \left(\frac{1-\alpha}{\alpha} \frac{r_t}{w_t} \right)^{\alpha} \frac{y_{it}}{z_{it}}$$
$$k_{it} = \left(\frac{\alpha}{1-\alpha} \frac{w_t}{r_t} \right)^{1-\alpha} \frac{y_{it}}{z_{it}}$$

so the cost function is

$$C_{it}(y_{it}) = \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t}{\alpha} \right)^{\alpha} \frac{y_{it}}{z_{it}}$$
$$= \frac{\mu_t}{z_{it}} y_{it}$$

where $\mu_t = \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t}{\alpha} \right)^{\alpha}$

- Operating profits are

$$\pi_{it}^{O} = \max_{y_{it}} p_{it}\left(y_{it}\right) y_{it} - \frac{\mu_t}{z_{it}} y_{it}$$

using the demand curve the F.O.C. implies

$$\begin{aligned} p_{it} &= \frac{\epsilon}{\epsilon-1}\frac{\mu_t}{z_{it}} \\ y_{it} &= \left(\frac{P_t}{p_{it}}\right)^{\epsilon} Y_t \end{aligned}$$

price is a markup $\frac{\epsilon}{\epsilon-1}$ over marginal cost μ_t/z_{it}. Operating profits are then

$$\begin{aligned} \pi_{it}^{O} &= \frac{1}{\epsilon-1}\frac{\mu_t}{z_{it}} y_{it} \\ &= \frac{1}{\epsilon}\left(\frac{\epsilon-1}{\epsilon}\frac{z_{it}}{\mu_t}\right)^{1-\epsilon} \left(P_t\right)^{\epsilon} Y_t. \end{aligned}$$

B.2 Price of final good and aggregate productivity

- The price of the final good satisfies

$$\begin{aligned} P_t^{1-\epsilon} &= \int_0^1 \left(p_{it}\right)^{1-\epsilon} di \\ &= \int_0^1 \left(\frac{\epsilon}{\epsilon-1}\frac{\mu_t}{z_{it}}\right)^{1-\epsilon} di \\ &= \left(\frac{\epsilon-1}{\epsilon}\frac{1}{\mu_t}\right)^{\epsilon-1} \int_0^1 \left(z_{it}\right)^{\epsilon-1} di \end{aligned}$$

define a productivity aggregator as

$$A_t = \left(\int_0^1 \left(z_{it}\right)^{\epsilon-1} di\right)^{\frac{1}{\epsilon-1}}$$

then the price of the final good is

$$P_t = \frac{\epsilon}{\epsilon-1}\frac{\mu_t}{A_t}$$

- Note that

$$\int_0^1 \left(z_{it}\right)^{\epsilon-1} di = \int_1^{\infty} \left(z_{it}\right)^{\epsilon-1} dF\left(z_i\right)$$

let the break-even cutoff productivity to operate the modern technology be z_t^*, then

$$\int_0^1 (z_{it})^{\epsilon-1}\, di = \int_1^{z_t^*} (z_{it})^{\epsilon-1}\, dF(z_i) + \int_{z_t^*}^{\infty} (z_{it})^{\epsilon-1}\, dF(z_i)$$

recall that z_i are distributed Pareto, so

$$\begin{aligned} F(z_i) &= 1 - z_i^{-\xi} \\ dF(z_i) &= \xi z_i^{-\xi-1} \end{aligned}$$

plugging in and rearranging

$$\int_0^1 (z_{it})^{\epsilon-1}\, di = \xi\left(\hat{z}z_t\right)^{\epsilon-1} \int_1^{z_t^*} (z_i)^{\epsilon-\xi-2}\, dz_i + \xi\left(z_t\right)^{\epsilon-1} \int_{z_t^*}^{\infty} (z_i)^{\epsilon-\xi-2}\, dz_i$$

which implies that the integrals are

$$\begin{aligned} \int_1^{z_t^*} (z_i)^{\epsilon-\xi-2}\, dz_i &= \frac{(z_i)^{\epsilon-\xi-1}}{\epsilon-\xi-1}\Big|_1^{z_t^*} \\ &= \frac{1}{1+\xi-\epsilon}\left[1-\left(\frac{1}{z_t^*}\right)^{1+\xi-\epsilon}\right] \\ \int_{z_t^*}^{\infty} (z_i)^{\epsilon-1}\, dF(z_i) &= \frac{(z_i)^{\epsilon-\xi-1}}{\epsilon-\xi-1}\Big|_{z_t^*}^{\infty} \\ &= \frac{1}{1+\xi-\epsilon}\left(\frac{1}{z_t^*}\right)^{1+\xi-\epsilon} \end{aligned}$$

and note that the second is well defined iff $0 < 1+\xi-\epsilon$. Plugging in and rearranging

$$\int_0^1 (z_{it})^{\epsilon-1}\, di = (z_t)^{\epsilon-1}\, \frac{\xi}{1+\xi-\epsilon}\left[z_T^{\epsilon-1} + \left(1-z_T^{\epsilon-1}\right)\left(\frac{1}{z_t^*}\right)^{1+\xi-\epsilon}\right]$$

- So the productivity aggregator is

$$A_t = A\left(z_t^*\right) = z_t\left(\frac{\xi}{1+\xi-\epsilon}\left[z_T^{\epsilon-1} + \left(1-z_T^{\epsilon-1}\right)\left(\frac{1}{z_t^*}\right)^{1+\xi-\epsilon}\right]\right)^{\frac{1}{\epsilon-1}}$$

note that in the limit where no firm adopts the modern technology we get

$$\lim_{z_t^* \to \infty} A\left(z_t^*\right) = z_t z_T \left(\frac{\xi}{1+\xi-\epsilon}\right)^{\frac{1}{\epsilon-1}}$$

and in the limit where all firms adopt the modern technology we get

$$\lim_{z_t^* \to 1} A\left(z_t^*\right) = z_t \left(\frac{\xi}{1+\xi-\epsilon}\right)^{\frac{1}{\epsilon-1}}$$

B.3 Aggregate production

- Taking stock, we know:

$$\begin{aligned}
l_{it} &= \left(\frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right)^{\alpha}\frac{y_{it}}{z_{it}} \\
k_{it} &= \left(\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{1-\alpha}\frac{y_{it}}{z_{it}} \\
\mu_t &= \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t}{\alpha}\right)^{\alpha} \\
p_{it} &= \frac{\epsilon}{\epsilon-1}\frac{\mu_t}{z_{it}} \\
y_{it} &= \left(\frac{P_t}{p_{it}}\right)^{\epsilon} Y_t \\
p_{it}y_{it} &= \left(p_{it}\right)^{1-\epsilon}\left(P_t\right)^{\epsilon} Y_t \\
A_t &= \left(\int_0^1 \left(z_{it}\right)^{\epsilon-1} di\right)^{\frac{1}{\epsilon-1}} \\
P_t &= \frac{\epsilon}{\epsilon-1}\frac{\mu_t}{A_t}
\end{aligned}$$

- Combining individual factor demands with optimal production plans and rearranging we get

$$\begin{aligned}
l_{it} &= \frac{1-\alpha}{w_t}\frac{\epsilon-1}{\epsilon}\left(p_{it}\right)^{1-\epsilon}\left(P_t\right)^{\epsilon} Y_t \\
k_{it} &= \frac{\alpha}{r_t}\frac{\epsilon-1}{\epsilon}\left(p_{it}\right)^{1-\epsilon}\left(P_t\right)^{\epsilon} Y_t
\end{aligned}$$

integrating both sides over all firms i

$$\int_0^1 l_{it} di = \int_0^1 \frac{1-\alpha}{w_t} \frac{\epsilon - 1}{\epsilon} (p_{it})^{1-\epsilon} (P_t)^{\epsilon} Y_t di$$
$$\int_0^1 k_{it} di = \int_0^1 \frac{\alpha}{r_t} \frac{\epsilon - 1}{\epsilon} (p_{it})^{1-\epsilon} (P_t)^{\epsilon} Y_t di$$

simplifying and using market clearing

$$L_t = \frac{1-\alpha}{w_t} \frac{\epsilon - 1}{\epsilon} P_t Y_t$$
$$K_t = \frac{\alpha}{r_t} \frac{\epsilon - 1}{\epsilon} P_t Y_t$$

so we get the aggregate capital-to-labor ratio is

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} = \frac{k_{it}}{l_{it}} \qquad \forall i$$

- Production of firm i is

$$y_{it} = \left(\frac{P_t}{p_{it}} \right)^{\epsilon} Y_t$$

plugging in for p_{it} and P_t we get

$$y_{it} = \left(\frac{\frac{\epsilon}{\epsilon-1} \frac{\mu_t}{A_t}}{\frac{\epsilon}{\epsilon-1} \frac{\mu_t}{z_{it}}} \right)^{\epsilon} Y_t$$
$$= \left(\frac{z_{it}}{A_t} \right)^{\epsilon} Y_t$$

plugging in for the production function and rearranging

$$\left(\frac{k_{it}}{l_{it}} \right)^{\alpha} l_{it} = (z_{it})^{\epsilon - 1} \left(\frac{1}{A_t} \right)^{\epsilon} Y_t$$

plugging in for the aggregate capital-to-labor ratio

$$\left(\frac{K_t}{L_t} \right)^{\alpha} l_{it} = (z_{it})^{\epsilon - 1} \left(\frac{1}{A_t} \right)^{\epsilon} Y_t$$

integrating over i and rearranging we get that aggregate production is

$$Y_t = A_t \left(K_t\right)^{\alpha} \left(L_t\right)^{1-\alpha}$$

B.4 Labor supply

- Let households have GHH preferences, then

$$\sum_{t=0}^{\infty} \beta^t \frac{\left(c - \frac{L^{1+\nu}}{1+\nu}\right)^{1-\sigma}}{1-\sigma}$$

the F.O.C. with respect to labor implies

$$L^{\nu} = w_t$$

recall the capital-to-labor ratio is

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}$$

plugging in and rearranging

$$L^{1+\nu} = \frac{1-\alpha}{\alpha} K_t r_t$$

- Now, recall from the individual capital demand we had

$$k_{it} = \frac{\alpha}{r_t} \frac{\epsilon - 1}{\epsilon} \left(p_{it}\right)^{1-\epsilon} \left(P_t\right)^{\epsilon} Y_t$$

integrating over i and using market clearing and the definition of P_t we get

$$K_t = \frac{\alpha}{r_t} \frac{\epsilon - 1}{\epsilon} P_t Y_t$$

normalizing $P_t = 1$, plugging in for Y_t and rearranging we get

$$K_t r_t = \alpha \frac{\epsilon - 1}{\epsilon} A_t \left(K_t\right)^{\alpha} \left(L_t\right)^{1-\alpha}$$

- Plugging into the household's optimality condition from labor supply we get

$$L_t = \left[(1-\alpha) \frac{\epsilon - 1}{\epsilon} A_t (K_t)^\alpha \right]^{\frac{1}{\alpha+\nu}}$$

the labor supply L_t as a function of the state (z_t, K_t) and of the cutoff z_t^* (note A_t depends on the cutoff)

- So output is

$$\begin{aligned} Y_t &= A_t (K_t)^\alpha (L_t)^{1-\alpha} \\ &= \left[(1-\alpha) \frac{\epsilon - 1}{\epsilon} \right]^{\frac{1-\alpha}{\alpha+\nu}} (A_t)^{\frac{1+\nu}{\alpha+\nu}} (K_t)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)} \end{aligned}$$

B.5 Adoption cutoff

- Operating profits of a firm i are

$$\pi_{it}^O = \frac{1}{\epsilon - 1} \frac{\mu_t}{z_{it}} y_{it}$$

plugging in for $y_{it} = \left(\frac{P_t}{p_{it}}\right)^\epsilon Y_t$ and using the normalization $P_t = 1$ we get

$$\pi_{it}^O = \frac{1}{\epsilon - 1} \frac{\mu_t}{z_{it}} \left(\frac{1}{p_{it}} \right)^\epsilon Y_t$$

plugging in for $p_{it} = \frac{\epsilon}{\epsilon-1} \frac{\mu_t}{z_{it}}$ we get

$$\pi_{it}^O = \frac{1}{\epsilon} \left(\frac{\epsilon - 1}{\epsilon} \frac{z_{it}}{\mu_t} \right)^{\epsilon-1} Y_t$$

- The productivity cutoff z_t^* satisfies

$$\pi_{it}^{OT} = \pi_{it}^{OM} - f_m (1 + r_t)$$

plugging in for π_{it}^{OT}

$$\frac{1}{\epsilon}\left(\frac{\epsilon-1}{\epsilon}\frac{z_t z_t^* z_T}{\mu_t}\right)^{\epsilon-1} Y_t = \frac{1}{\epsilon}\left(\frac{\epsilon-1}{\epsilon}\frac{z_t z_t^*}{\mu_t}\right)^{\epsilon-1} Y_t - f_m\left(1+r_t\right)$$

rearranging

$$\left(\frac{\epsilon-1}{\epsilon}\frac{z_t z_t^*}{\mu_t}\right)^{\epsilon-1}\left[1-\left(z_T\right)^{\epsilon-1}\right] = \frac{\epsilon f_m\left(1+r_t\right)}{Y_t}$$

plugging in for $Y_t = \left[(1-\alpha)\frac{\epsilon-1}{\epsilon}\right]^{\frac{1-\alpha}{\alpha+\nu}}\left(A_t\right)^{\frac{1+\nu}{\alpha+\nu}}\left(K_t\right)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)}$ and rearranging:

$$\left(z_t^*\right)^{\epsilon-1} = \frac{\epsilon f_m\left(1+r_t\right)}{\left(A_t\right)^{\frac{1+\nu}{\alpha+\nu}}(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}\left(K_t\right)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)}\left[1-\left(z_T\right)^{\epsilon-1}\right]}\left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}+(\epsilon-1)}\left(\frac{\mu_t}{z_t}\right)^{\epsilon-1}$$

- Note that μ_t is a function of A_t:

$$\mu_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t}{\alpha}\right)^{\alpha}$$
$$w_t = \frac{1-\alpha}{\alpha}\frac{K_t}{L_t}r_t$$
$$r_t = \alpha\frac{\epsilon-1}{\epsilon}A_t\left(K_t\right)^{\alpha-1}\left(L_t\right)^{1-\alpha}$$
$$L_t = \left[(1-\alpha)\frac{\epsilon-1}{\epsilon}A_t\left(K_t\right)^{\alpha}\right]^{\frac{1}{\alpha+\nu}}$$

plugging in and rearranging

$$\mu_t = \frac{\epsilon-1}{\epsilon}A_t$$

- Plugging for μ_t and rearranging we get

$$\left(z_t^*\right)^{\epsilon-1} = \frac{\epsilon f_m\left(1+r_t\right)}{(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}\left(K_t\right)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)}\left[1-\left(z_T\right)^{\epsilon-1}\right]}\left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(\frac{1}{z_t}\right)^{\epsilon-1}\left(A_t\right)^{(\epsilon-1)-\frac{1+\nu}{\alpha+\nu}}$$

- Note that A_t is decreasing in z_t^*. If we assume $(\epsilon-1) > \frac{1+\nu}{\alpha+\nu}$ then the RHS is decreasing in z_t^*. As $z_t^* \to \infty$ we get

$$\infty > \frac{\epsilon f_m\left(1+r_t\right)}{(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}\left(K_t\right)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)}\left[1-\left(z_T\right)^{\epsilon-1}\right]}\left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(\frac{1}{z_t}\right)^{\frac{1+\nu}{\alpha+\nu}}\left(z_T\left(\frac{\xi}{1+\xi-\epsilon}\right)^{\frac{1}{\epsilon-1}}\right)^{(\epsilon-1)-\frac{1+\nu}{\alpha+\nu}}$$

so the LHS is higher with high enough cutoff z_t^*. This means that there's always a positive mass of firms adopting the modern technology. To have an interior cutoff we need

$$1 < \frac{\epsilon f_m (1+r_t)}{(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}} (K_t)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)} \left[1-(z_T)^{\epsilon-1}\right]} \left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}} \left(\frac{1}{z_t}\right)^{\frac{1+\nu}{\alpha+\nu}} \left(\left(\frac{\xi}{1+\xi-\epsilon}\right)^{\frac{1}{\epsilon-1}}\right)^{(\epsilon-1)-\frac{1+\nu}{\alpha+\nu}}$$

otherwise we get the corner solution of full adoption $z_t^* = 1$. This could happen with a large enough stock of capital or a large enough productivity shock z_t.

- If the above inequality does not hold then the equation

$$\left(z_t^*\right)^{\epsilon-1} = \frac{\epsilon f_m (1+r_t)}{(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}} (K_t)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)} \left[1-(z_T)^{\epsilon-1}\right]} \left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}} \left(\frac{1}{z_t}\right)^{\epsilon-1} \left(A\left(z_t^*\right)\right)^{(\epsilon-1)-\frac{1+\nu}{\alpha+\nu}}$$

pins down $z_t^* \in (1,\infty)$. Once we know z_t^* we know A_t and all other variables.