Sovereign Risk and Dutch Disease*

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Abstract

This paper studies how, in the presence of default risk, the Dutch disease amplifies an inefficiency in the sectoral allocation of capital. I develop a sovereign default model with production of two consumption goods, a tradable and a non-tradable, and endowments of natural resources. Goods are produced with capital, which has to be allocated into each sector one period in advance. Households allocate capital without internalizing how this affects the interest rate on the government debt. I show that default incentives are stronger when there is more capital in the non-traded sector. This gives rise to a disagreement between households and a benevolent government regarding sectoral investment, which can be amplified when endowments of natural resources are larger. I show that the efficient allocation can be decentralized with a tax on capital income from the non-traded sector, which is akin to sterilization policies like accumulation of international reserves during commodity windfalls. I present empirical evidence that supports the main implications of the model.

Keywords: Sovereign default, Dutch disease, real exchange rates

JEL Codes: F34, F41, H3, H63

*This paper builds on the second chapter of my PhD dissertation.

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1 Introduction

The Dutch disease refers to the wealth effect that commodity booms have on the sectoral allocation of production factors: with an increase in commodity income, production factors are reallocated to non-traded sectors, which is supported by an appreciation of the real exchange rate.[1] There is a large literature on the effects that the Dutch disease has on economic growth, which inspired the concept of "natural resource course" coined by Auty (1993).[2] However, the relation between the Dutch disease and sovereign default risk have been less studied. This paper studies an environment in which, in the presence of default risk, the Dutch disease could amplify an inefficiency in the sectoral allocation of capital that directly affects the borrowing terms that the government faces.

I develop a two-period model of a small open economy populated by a continuum of households and a benevolent government. Households have preferences for consumption of a composite aggregate of two intermediate goods: one tradable with the rest of the world and another non-tradable. Each good is produced using capital, which has to be allocated into each sector one period in advance, and before the realization of a common productivity shock. Households own all the capital in the economy and choose its sectoral allocation. The government, on the other hand, issues non-contingent debt denominated in terms of the tradable good. The debt is due in the second period and the government cannot commit to repay it. International lenders purchase the debt in the first period and consider the government's incentives to default when pricing it, which depend on both the amount of debt issued and the aggregate sectoral allocation of capital.

I show that default incentives are stronger when there is relatively more capital allocated in the non-traded sector. The inefficiency in the model arises from the fact that households do not internalize how their sectoral allocation of capital affects the government's ability to borrow. In a competitive equilibrium, household's choices are such that the expected return of capital in both sectors is equalized. However, I show that this allocation is Pareto-dominated by a first-best alternative in which more capital is allocated to the traded sector because this allows the economy to borrow more under better terms. I also show that this first-best allocation can be decentralized as

[1] Higher revenue from commodity exports increases the demand for all consumption goods. This income effect raises the price of non-traded goods, which causes an appreciation of the real exchange rate. This appreciation makes imports of other traded goods relatively cheap and thus induces a reallocation of production factors into the production of non-traded goods which cannot be imported. The term was first used in 1977 by The Economist to describe this phenomenon in the Dutch economy after the discovery of natural gas reserves in 1959.

[2] Sachs and Warner (1995) document that countries with large natural resource wealth grow more slowly.

a competitive equilibrium with an appropriate tax to capital returns in the non-traded sector. This tax is proportional to the desired level of borrowing and inversely proportional to the expected real exchange rate depreciation. I argue that exchange rate sterilization policies, such as the accumulation of international reserves, work in the same direction as the optimal tax since they tame the appreciation of the domestic currency and, thus, lower the required tax.

The economy may also be endowed with some perishable amount of natural resources that can be sold in international markets. This endowment provides a source of tradable income that is a perfect substitute for production of the tradable consumption good. The larger this endowment is, the lower the incentives to allocate capital in the tradable sector (this is the Dutch disease in the model). Given this, large endowments of natural resources could amplify the inefficiency highlighted above and a larger tax may be required to implement the first-best allocation. Because of the stronger incentives to invest in the non-tradable sector, the capital allocation in the tradable sector falls and, with it, the government faces higher interest rates for any debt level it may issue.

I then present empirical evidence for the two main implications of the model. I perform reduced-form estimations of a linear model of sovereign spreads as a function of debt, international reserves, and average rents from natural resources as a country-specific shifter. I find that this shifter is positive, which indicates that resource-rich countries face more stringent borrowing terms, on average. This result is robust for different measures of government debt and sovereign spreads. I then show that international reserves have a strong positive correlation to rents from natural resources, which I interpret as evidence of central banks implementing sterilization policies in an attempt to reduce the inefficient effects of the Dutch disease on the allocation of capital.

Related literature.—This paper is related to the strand of literature that studies the Dutch disease and its relation to production and real exchange rates. Corden and Neary (1982) developed the benchmark model to analyze the reallocation of production factors and the process of de-industrialization. More recently, Benigno and Fornaro (2013) present a model that features episodes of abundant access to foreign capital coupled with weak productivity growth. They show that periods of large capital inflows, triggered by a fall in the interest rate, may result in inefficient outcomes in the presence of productivity externalities in the tradable sector. Alberola and Benigno (2017) study an environment in which the Dutch disease delays a commodity exporter's convergence to the world technological frontier because of the presence of an externality in dynamic productivity gains in the manufacturing sector. Ayres, Hevia, and Nicolini (2020) document that there is a strong and robust co-movement between the real exchange rates of Germany, Japan and the UK against the US dollar and a handful of primary commodities that are widely traded internationally.

This paper is also related to the literature that studies sovereign default risk and its relation to the production structure of the economy and commodity exports. Arellano, Bai, and Mihalache (2018) document how sovereign debt crises have disproportionately negative effects on non-traded sectors. They develop a model with capital, production in two sectors, and one period debt. The two-period model in Section 2 resembles a simplified version of their infinite horizon model. The two key differences are that I introduce exogenous endowments of commodities and, more importantly, that I analyze the inefficency that arises from different agents choosing capital allocations and debt, while in their framework all allocations are chosen by a benevolent government. Hamann, Mendoza, and Restrepo-Echavarria (2020) study the relation between oil exports, proved oil reserves, and sovereign risk. They document that sovereign risk is lower when oil production increases, but higher when reserves increase. Similarly, Esquivel (2021) documents that sovereign interest rate spreads increase substantially following news of giant oil field discoveries. Both of these papers also develop models in which a benevolent government makes all production and borrowing decisions in a centralized fashion. In a recent paper, Galli (2021) studies an environment with fiscal policy and private capital accumulation. In his environment, multiple equilibria exist where the expectations of lenders are self-fulfilling. In the bad equilibrium, pessimistic beliefs about investment make borrowing more costly. The government responds by increasing taxation, which depresses investment and makes these beliefs self-fulfilling. There are two key differences between his paper and this. First, multiplicity of equilibria is central to his analysis, while for this paper what is central is the pecuniary externality from the sectoral allocation of capital, which can be studied more clearly in an environment with a unique equilibrium. Second, borrowing terms in his framework depend on the absolute level of capital, while in this paper they depend on the sectoral allocation of capital. To make this point clearer, I fix the level of capital to turn the focus on the role of its sectoral allocation.

Layout.—Section 2 presents the environment of the model, defines equilibrium and efficiency, and discusses the decentralization of the efficient allocation; Section 3 shows discusses how large commodity endowments amplify the inefficiency through the Dutch disease and presents a numerical exercise to illustrate the main theoretical results; Section 4 presents the empirical analysis; and Section 5 concludes.

2 Model

There is a small open economy with a continuum of identical households and a benevolent government. Time is discrete and there are two periods $t = 0, 1$.

Preferences and technology.—There is a final consumption good that is produced by a competitive firm with technology $Y(c_N, c_T) = \left[\omega^{\frac{1}{\eta}} c_N^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} c_T^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$, where $\eta > 0$, $\omega \in (0,1)$ and c_T and c_N are intermediate traded and non-traded goods, respectively. This good cannot be traded with the rest of the world. The intermediate goods are produced by competitive firms using technologies $y_T = zK_T^{\alpha_T}$ and $y_N = zK_N^{\alpha_N}$; where $\alpha_T, \alpha_N \in (0,1)$, $z \in [\underline{z}, \bar{z}]$ is a common productivity shock with CDF $F(z)$, and K_T and K_N are the amounts of capital rented by firms in each sector. There is a fixed amount of capital $\bar{K}$ in the economy that is owned by the households. Capital can be allocated in the two intermediate sectors, but this allocation has to be made one period in advance. Households have preferences for consumption of the final good in each period represented by $u(c_0) + \beta \mathbb{E}[u(c_1)]$, where $\beta \in (0,1)$, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and $\sigma > 0$. Each household owns the same amount of capital $\bar{k}$ and chooses in period 0 how much capital to allocate into each sector for period 1, subject to $k_{T,1} + k_{N,1} \leq \bar{k}$. The initial allocations $k_{T,0}$ and $k_{N,0}$ are given.

Commodity goods.—In each period, households are endowed with some quantity of a perishable commodity good $y_{C,t} \geq 0$ that can be sold in international markets for a relative price $p_{C,t}$ (all prices are in terms of the traded intermediate good). The results of this paper hold regardless of which agent (households or government) control this endowment or whether it is costly to extract it. The results are also robust to whether $p_{C,t}$ is fixed or volatile, so for simplicity I will assume that $p_{C,t} = 1$. What is essential is that the available income from this commodity—like natural resources such as minerals or oil underground—is given and cannot be directly affected by the agents in the economy.[3]

[3]There is a vast literature about natural-resource extraction in which production of commodity goods is chosen by the agents through capital and/or labor allocations into these sectors (see Arezki, Ramey, and Sheng (2017), Hamann, Mendoza, and Restrepo-Echavarria (2020), and Esquivel (2021) for more references). What is common in these

Government debt and default.—There is a benevolent government that can issue non-contingent debt B_1 in period 0 that is due in period 1 and cannot commit to repay it. The debt is denominated in terms of the traded nummeraire good T and purchased at a price q by risk-neutral international lenders that behave competitively and have deep pockets. Lenders have access to a risk-free bond that pays interest r^*, which reflects their opportunity cost. At the beginning of period 1, the government observes z_1 and the capital allocations chosen by the households, $K_{T,1}$ and $K_{N,1}$, and decides whether to repay or default. If the government defaults it does not pay anything to the international lenders but productivity is penalized $z_D(z_1) \leq z_1$. I assume that z_D is continuously differentiable over $[\underline{z}, \bar{z}]$ and that $\frac{\partial z_D}{\partial z} \leq \frac{z_D}{z}$.[4]

Timing in period 0.—At the beginning of period 0 the government decides how much debt B_1 to issue. The government takes as given how this borrowing decision will affect the choices that households and firms make. The government also takes as given the price schedule q from the lenders' demand for its bonds. Once the government chooses B_1, households and firms make their individual choices taking all prices as given, as well as the aggregate allocations of capital, $K_{T,1}$ and $K_{N,1}$. Finally, lenders observe B_1, $K_{T,1}$, and $K_{N,1}$ and purchase the government bonds. This timing assumption rules out the multiplicity of equilibria studied by Galli (2021). In that environment, lenders price the government debt before investment occurs, which makes their expectations about capital in the second period self-fulfilling. Here, lenders make their pricing decisions after observing the actions of both the households and the government. [5]

Firms.—All firms behave competitively and maximize profits. From the maximization problem of the final good producer we get the demands for intermediate goods are:

$$c_{N,t} = \omega \left(\frac{P_t}{p_{N,t}} \right)^{\eta} Y_t \tag{1}$$

$$c_{T,t} = (1-\omega)\,(P_t)^{\eta}\, Y_t \tag{2}$$

frameworks is an exogenous endowment of natural resources that is exploited, which is the crucial feature that is highlighted in this paper. The abstraction from the intensity of extraction is made for simplicity.

[4]This condition will be used in the proof of **Lemma 1** below. It is satisfied by many common assumptions for this cost, like when productivity in default is proportional $z_D = \phi z$ with $\phi \in (0,1)$. This assumption is also satisfied by the cost function used by Chatterjee and Eyigungor (2012) in an endowment economy, in which productivity in default would be $z_D = z - \max\left\{0, d_0 z + d_1 z^2\right\}$ with $d_0 < 0 < d_1$ and $\frac{z_D}{z} = 1 - d_0 - d_1 z \geq \frac{\partial z_D}{\partial z}$.

[5]Cole and Kehoe (2000) study the case in which lenders price the bonds before observing government borrowing, which is another source of multiplicity that I rule out with this timing assumption.

where $p_{N,t}$ is the relative price of the non-traded intermediate. Since the production function features constant-returns to scale, I assume that the firm makes zero profits and $P_t = \left[\omega p_{N,t}^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}}$ is the price index reflecting the cost of the final consumption good. From the maximization problem of the intermediate good producers we get that the rental rates of capital in each sector are:

$$r_{N,t} = p_{N,t}\alpha_N z_t \left(K_{N,t}\right)^{\alpha_N - 1} \tag{3}$$

$$r_{T,t} = \alpha_T z_t \left(K_{T,t}\right)^{\alpha_T - 1} \tag{4}$$

for a given capital allocation and productivity shock.

Households.—Each household owns the same amount of capital $\bar{k}$. Since all households are identical, they do not trade capital with each other and I assume capital cannot be sold to foreigners. The problem of a representative household is:

$$\begin{aligned}
&\max_{k_{N,1},k_{T,1}} u(c_0) + \beta\mathbb{E}\left[u(c_1)\right] \qquad (5)\\
s.t.\quad & P_0 c_0 = r_{N,0}k_{N,0} + r_{T,0}k_{T,0} + y_{C,0} + \Pi_0 + G_0\\
& P_1 c_1 = r_{N,1}k_{N,1} + r_{T,0}k_{T,1} + y_{C,1} + \Pi_1 + G_1\\
k_{N,1} + k_{T,1} &\leq \bar{k}
\end{aligned}$$

where Π_t are the profits of the intermediate goods firms and G_t is a government transfer. In period 0, the transfer is $G_0 = q_0 B_1 - B_0$. If the government defaults in period 1 then $G_1 = 0$, and if the government repays $G_1 = -B_1$.[6] The expectation integrates over $F(z)$ and the households have some beliefs about the aggregate capital allocation and the government's default choice. Subsection 2.2 reformulates this problem in recursive form and discusses these beliefs in detail.

2.1 Static allocations

The dynamic choices in the model are borrowing and the sectoral allocation of capital. These are the key objects of interest to expose the inherent inefficiency and its relation to the endowment

[6] Note that, if the government owned the natural resources then $y_{C,0}$ and $y_{C,1}$ would be on the government's budget constraint, instead of the households. However, since it is a given endowment, the government would just include them in the lump-sum transfers, which would yield an isomorphic problem.

of commodities. This subsection uses optimality conditions from the firms and market clearing conditions for goods to characterize all other allocations and prices as functions of these dynamic allocations, which will extremely simplify notation.

Let $\lambda_t \in (0,1)$ be the share of $\bar{k}$ that a representative household allocates to the traded sector T for period t, and Λ_t the share of aggregate capital allocated in T. The aggregate state of the economy in a given period is (z_t, x_t), where $x_t = (\Lambda_t, B_t)$. I will treat z_0, λ_0, Λ_0, B_0, $y_{C,0}$, and $y_{C,1}$ as fixed parameters. For simplicity, I assume that the legacy debt B_0 is low enough so that the government would not want to default on it in $t = 0$.

Consumption in period 0.—Since λ_0 and Λ_0 are given, aggregate consumption in period 0 is given by:

$$C_0(x_1) = Y\left(y_N\left(z_0, (1-\Lambda_0)\bar{K}\right), y_T\left(z_0, \Lambda_0\bar{K}\right) + y_{C,0} + qB_1 - B_0\right) \tag{6}$$

where q is the price of newly issued debt. As discussed in the next subsection, q is a function of both Λ_1 and B_1. Note that consumption in $t = 0$ can only increase through borrowing. Since borrowing terms only depend on aggregate variables, households behave taking c_0 as given.

Consumption in period 1.—Given an aggregate state (z,x), the demand for the intermediate traded good if the government decides to repay is $c_T^P(z,x) = z(\Lambda\bar{K})^{\alpha_T} + y_{C,1} - B$ and the demand for the non-traded good is $c_N^P(z,x) = z((1-\Lambda)\bar{K})^{\alpha_N}$. Similarly, if the government defaults these demands are $c_T^D(z,x) = z_D(z)(\Lambda\bar{K})^{\alpha_T} + y_{C,1}$ and $c_N^D(z,x) = z_D(z)((1-\Lambda)\bar{K})^{\alpha_N}$, respectively. It is clear that in default consumption of the non-traded good drops because of the productivity penalty, but consumption of the traded good may increase because there is no debt to repay. Given these equations, aggregate consumption in repayment is $C^P(z,x) = Y\left(c_N^P(z,x), c_T^P(z,x)\right)$ and in default $C^D(z,x) = Y\left(c_N^D(z,x), c_T^D(z,x)\right)$.

Prices in period 1.—From equations 1 and 2 we get that the price of the non-traded intermediate in repayment is $p_N^P(z,x) = \left(\frac{\omega}{1-\omega}\frac{c_T^P(z,x)}{c_N^P(z,x)}\right)^{\frac{1}{\eta}}$ and in default it is $p_N^D(z,x) = \left(\frac{\omega}{1-\omega}\frac{c_T^D(z,x)}{c_N^D(z,x)}\right)^{\frac{1}{\eta}}$. These imply that the price index of the final consumption good is $P^P(z,x) = \left[\omega\left(p_N^P(z,x)\right)^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}}$ in repayment and $P^D(z,x) = \left[\omega\left(p_N^D(z,x)\right)^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}}$ in default. Finally, from the optimality conditions for the intermediate firms' problems we get that the rental rates of capital in repayment are $r_T^P(z,x) = \alpha_T z(\Lambda\bar{K})^{\alpha_T - 1}$ and $r_N^P(z,x) = p_N^P(z,x)\alpha_N z((1-\Lambda)\bar{K})^{\alpha_N - 1}$ in the traded and non-traded sectors, respectively. Similarly, these rental rates in default are $r_T^D(z,x) =$

$\alpha_T z_D(z)(\Lambda\bar{K})^{\alpha_T-1}$ and $r_N^D(z,x) = p_N^D(z,x)\,\alpha_N z_D(z)\left((1-\Lambda)\bar{K}\right)^{\alpha_N-1}$.

2.2 Decentralized equilibrium

This subsection takes the functions defined in the previous subsection as primitives and uses them to define the equilibrium concept and the problems of the government and a representative household.

Default decision.—At the beginning of period 1, the government observes (z,x) and solves the default problem:

$$\max\left\{V^P(z,x), V^D(z,x)\right\} \tag{7}$$

where the value of repayment is $V^P(z,x) = u\left(C^P(z,x)\right)$ and the value of default is $V^D(s_1,x_1) = u\left(C^D(z,x)\right)$. Given x, the default set is characterized by a cutoff value $z^*(x)$ such that:

$$u\left(C^P(z^*(x),x)\right) = u\left(C^D(z^*(x),x)\right) \tag{8}$$

Household's problem.—In period 0, households make their decisions after the government has issued B_1. As mentioned before, households behave as if their actions do not affect consumption in $t=0$, since it can only change with improved borrowing terms which households take as given. Given this, the problem of a representative household is:

$$\max_{\lambda_1} \int_{\underline{z}}^{z^*(x)} \beta u\left(c^D\right) dF(z) + \int_{z^*(x)}^{\bar{z}} \beta u\left(c^P\right) dF(z) \tag{9}$$

$$\begin{aligned} s.t. \quad P^D c^D &= \left[r_N^D(1-\lambda_1) + r_T^D\lambda_1\right]\bar{k} + y_{C,1} + \Pi^D \\ P^P c^P &= \left[r_N^P(1-\lambda_1) + r_T^P\lambda_1\right]\bar{k} + y_{C,1} + \Pi^P - B_1 \\ \Lambda_1 &= \Gamma_H(B_1) \end{aligned}$$

where $x = (\Lambda_1, B_1)$, Π^D and Π^P are profits made by all firms in default and repayment, respectively, all prices depend on (z,x_1) as defined in Subsection 2.1, and Γ_H is the household's belief about the aggregate capital allocation for $t=1$. Denote the policy function of a representative household as $\lambda^*(B)$.

Debt issuance.—At the beginning of period 0, the government chooses debt issuance B_1 to solve:

$$\max_{B_1} u\left(C_0\left(x\right)\right)+\beta\int_{\underline{z}}^{z^*(x)}u\left(C^D\left(z,x\right)\right)dF\left(z\right)+\beta\int_{z^*(x)}^{\bar{z}}u\left(C^P\left(z,x\right)\right)dF\left(z\right) \tag{10}$$
$$s.t.\quad \Lambda_1=\Gamma_G\left(B_1\right)$$

where C_0, C^D and C^P depend on (z,x_1) as defined in Subsection 2.1, and Γ_G is the government's belief about the aggregate capital allocation for $t=1$. Unlike the representative household, the government does internalize how its choice of B_1 affects the capital allocation for next period through Γ_G. Denote the solution to the maximization problem as B^*.

DEFINITION 1: (Decentralized Equilibrium) A decentralized equilibrium is a policy function for the households $\lambda^*\left(B\right)$, a debt issuance for the government B^*, a price schedule $q\left(x\right)$, and beliefs $\Gamma_H\left(B\right)$ and $\Gamma_G\left(B\right)$ such that: (i) given q, Γ_H, and Γ_G, the policy function λ^* and the allocation B^* solve the household's and government's problems, respectively; (ii) the beliefs are consistent $\Gamma_G\left(B\right)=\Gamma_H\left(B\right)=\lambda^*\left(B\right)$, and (iii) the price schedule q satisfies the lenders' no-arbitrage condition:

$$q\left(x\right)=\frac{1-F\left(z^*\left(x\right)\right)}{1+r^*} \tag{11}$$

DEFINITION 2: (Equilibrium Allocation) An equilibrium allocation is $\tilde{x}=\left(\tilde{\Lambda},\tilde{B}\right)$ such that $\tilde{B}=B^*$ and $\tilde{\Lambda}=\lambda^*\left(B^*\right)$.

2.3 Efficiency

Given a state (z,x) in period 1, a benevolent social planner would face the same default problem as the government in 7 (and, hence, the same price schedule in period 0). However, in period 0 the planner simultaneously chooses Λ_1 and B_1 to solve:

$$\max_{x} u\left(C_0\left(x\right)\right)+\beta\int_{\underline{z}}^{z^*(x)}u\left(C^D\left(z,x\right)\right)dF\left(z\right)+\beta\int_{z^*(x)}^{\bar{z}}u\left(C^P\left(z,x\right)\right)dF\left(z\right) \tag{12}$$

where the key difference is that the planner chooses Λ_1 directly, as opposed to the government who can only indirectly affect it through its choice of B_1.

DEFINITION 3: (Efficient Allocation) An allocation $\hat{x}=\left(\hat{\Lambda}_1,\hat{B}_1\right)$ is efficient if it solves the social planner's problem in 12.

2.4 Decentralization

As can be seen in equations 8 and 11, the borrowing terms depend on the amount of debt issued B_1 and on the capital allocation between the two intermediate sectors, summarized by Λ_1.[7] Given a level of debt issuance $\tilde{B}_1$,the capital allocation in the decentralized equilibrium $\tilde{\Lambda}$ is pinned down by the Euler equation of a representative household:

$$\mathbb{E}\left[\beta u'\left(\tilde{C}_1\right)\frac{\left(\tilde{r}_{T,1}-\tilde{r}_{N,1}\right)\bar{K}}{\tilde{P}_1}\right]=0 \tag{13}$$

where, to ease notation, "tildes" indicate prices and allocations consistent with the decentralized equilibrium.[8] Equation 13 can be interpreted as a no-arbitrage condition in which the expected returns to allocating capital in either sector are equated. On the other hand, the Euler equation for Λ_1 from the social planner's problem is:

$$\mathbb{E}\left[\beta u'\left(\hat{C}_1\right)\frac{\left(\hat{r}_{T,1}-\hat{r}_{N,1}\right)\bar{K}}{\hat{P}_1}\right]+u'\left(\hat{C}_0\right)\frac{\hat{\partial q}}{\partial\Lambda}\frac{\hat{B}_1}{\hat{P}_0}=0 \tag{14}$$

where I use "hats" for variables that correspond to the efficient allocation. Equation 14 shows that the planner also takes into account how Λ_1 affects the borrowing terms and consumption in period 0.[9]

Lemma 1: If $\eta<1$, then the default set is shrinking in Λ_1. That is, $\frac{\partial z^*(x)}{\partial\Lambda}\leq 0$.

Proof: See Appendix B.

Lemma 1 implies that borrowing terms improve with more capital allocated to the traded sector,

[7] These terms would also depend on the level of capital installed for $t=1$, as studied by Gordon and Guerron-Quintana (2018), Arellano, Bai, and Mihalache (2018), Galli (2021), and Esquivel (2021). In order to highlight the role of the sectoral allocation, I assume that the total amount of capital installed in the economy is fixed.

[8] Formally, this expression is $\int_{\underline{z}}^{z^*(\tilde{x})}\beta u'\left(C^D(z,\tilde{x})\right)\frac{r_T^D(z,\tilde{x})-r_N^D(z,\tilde{x})}{P^D(z,\tilde{x})}dF(z)+\int_{z^*(\tilde{x})}^{\bar{z}}\beta u\left(C^P(z,\tilde{x})\right)\frac{r_T^P(z,\tilde{x})-r_N^P(z,\tilde{x})}{P^P(z,\tilde{x})}dF(z)=0.$ See Appendix A for the formal derivation of this equation.

[9] Formally, the only price in the social planner's problem is q. See Appendix A for the formal derivation of this equation. It is easy to see that the formal expression can be rewritten as equation 14 where $\hat{r}_{T,1}$, $\hat{r}_{N,1}$, $\hat{P}_1$, and $\hat{P}_0$ are the prices, in default or repayment, depending on the sate, defined in Subsection 2.1 evaluated at the efficient allocation. This is due to the fact that there is no inefficiency in the static allocation of resources.

since

$$\frac{\partial q}{\partial \Lambda} = -\frac{f\left(z^*\left(x\right)\right)}{1+r^*}\frac{\partial z^*\left(x\right)}{\partial \Lambda} \geq 0$$

where f is the PDF of z. The sufficient condition for this to hold is that traded and non-traded intermediates are "sufficiently complements", that is, that the elasticity of substitution η is less than 1. The range of estimates for this elasticity is between 0.4 and 0.83, so this sufficient condition would hold in any standard parametrization.[10]

PROPOSITION 1: (*Misallocation*) For any given level of debt issuance B_1, households in the decentralized equilibrium overinvest in the non-traded sector and underinvest in the traded one.

Proof: See Appendix C.

From LEMMA 1 it follows that the second term in equation 14 is positive for any $\hat{B}_1 \geq 0$, so with the planner's optimal allocation we get:

$$\mathbb{E}\left[\beta u'\left(\hat{C}_1\right)\frac{\left(\hat{r}_{T,1}-\hat{r}_{N,1}\right)\bar{K}}{\hat{P}_1}\right] \leq 0$$

where we can interpret the second term in equation 14 as a "wedge" that represents the degree of disagreement for the capital allocation between the social planner and the private sector. Intuitively, the planner foregoes higher expected returns in the non-traded sector in period 1 in order to have a higher ability to borrow in period 0 (q is increasing in Λ). Households do not internalize this effect on the borrowing terms, so they choose to continue to allocate capital in the non-traded sector until the expected returns are equated, as indicated by equation 13. An immediate corollary of PROPOSITION 1 is that the equilibrium allocation is not efficient.

PROPOSITION 2: The government can implement the efficient allocation as a decentralized equilibrium by imposing the following tax on returns to investment in the non-traded sector:

$$\tau^* = \frac{u'\left(\hat{C}_0\right)\frac{\widehat{\partial q}}{\partial \Lambda}\frac{\hat{B}_1}{\hat{P}_0}}{\mathbb{E}\left[\beta u'\left(\hat{C}_1\right)\frac{\hat{r}_{N,1}\bar{K}}{\hat{P}_1}\right]} \tag{15}$$

Proof: Obvious after multiplying $\tilde{r}_{N,1}$ by $\left(1-\tau_N^*\right)$ in equation 13.

If we define the real exchange rate as the price of the traded good in terms of the domestic

[10]See Stockman and Tesar (1995), Mendoza (2005), and Bianchi (2011).

final consumption good $\xi = \frac{1}{P}$, then the optimal tax is inversely proportional to the expected real depreciation $\frac{\xi_1}{\xi_0}$ and proportional to the desired borrowing level $\hat{B}_1$. Also note that, absent default risk $\tau^* = 0$ since $q = \frac{1}{1+r^*}$ would be constant and $\frac{\partial q}{\partial \Lambda} = 0$. This implies that the inefficiency presented in this section only arises in the presence of default risk.

3 The Dutch disease

The Dutch disease refers to the wealth effect that commodity booms have on the sectoral allocation of production factors: following an increase in commodity income, production factors are reallocated to non-traded sectors, which is supported by an appreciation of the real exchange rate.

3.1 Inefficiency of the Dutch disease

LEMMA 2: (*Dutch disease*) The shares $\tilde{\Lambda}_1$ and $\hat{\Lambda}_1$ of capital allocated in the traded sector by the households and the social planner, respectively, are decreasing in the commodity endowment for period 1 $y_{C,1}$.

Proof: It is easy to see from the prices defined in Subsection 2.1 that the real exchange rate appreciates if the commodity endowment y_C increases. This implies that, for any Λ_1 and B_1, the return to capital in the non-traded sector is also increasing in y_C, while the return to capital in the traded sector does not depend directly on y_C. Intuitively, as commodity income for $t = 1$ increases, incentives to invest in the non-traded sector are stronger for both households and the planner.

CONJECTURE 1: (*Inefficiency of the Dutch disease*) The tax that implements the efficient allocation τ^* is increasing in $y_{C,1}$.

From LEMMA 2 we get that with larger y_C the expected depreciation is smaller, which lowers the denominator in equation 15. A larger endowment of commodities in $t = 1$ increases the incentives to invest in the non-traded sector, due to the expected strong exchange rate. This is true for both the planner and the households, but the planner still considers the trade-off between borrowing terms and returns in the non-traded sector, so these stronger incentives potentially amplify the disagreement between the private sector and the social planner. In addition, as long as $\beta < \frac{1}{1+r^*}$, a larger commodity endowment for period 1 increases the desire to borrow in order to front-load consumption, which also increases τ^* and the size of the "wedge" in equation 14. The following subsection shows a numerical example in which, under a standard calibration, the Dutch disease amplifies the degree of inefficiency. This result seems to hold quantitatively for a wide set of standard parametrizations.

3.2 Numerical exercise

CONJECTURE 1 suggests that economies with default risk have a greater misallocation of capital when they receive larger commodity windfalls. To illustrate the above results, I consider the calibration summarized in Table 1, which takes parameter values from the sovereign default literature.

Table 1: Calibration

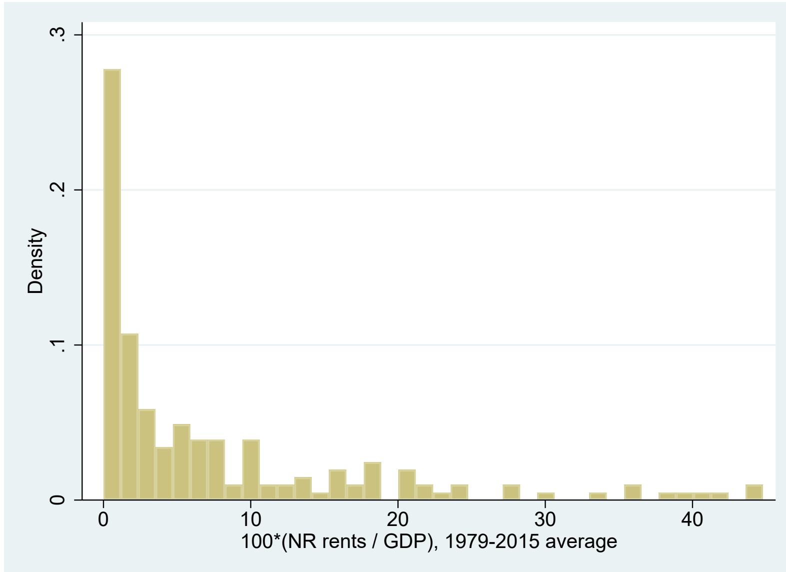
Parameter	Value	Parameter	Value
σ	2	β	0.88
r^*	0.04	$\bar{K}$	1
η	0.83	ω	0.6
α_N	0.33	α_T	0.33
d_0	-0.16	d_1	0.22
μ_z	1	σ_z	0.02
$\underline{z}$	0.85	$\bar{z}$	1.15
$\lambda_0 = \Lambda_0$	0.39	$y_{C,0}$	0
z_0	1	B_0	0

I assume that the productivity shock z has a truncated log-normal distribution over $[\underline{z}, \bar{z}]$ with mean 0 and standard deviation $\sigma_z = 0.02$, which is a standard value in the literature. I take the productivity cost in default from Chatterjee and Eyigungor (2012), so that $z_D(z) = z - \max\left\{0, d_0 z + d_1 z^2\right\}$ with $d_0 < 0 < d_1$. Note that z_D is continuously differentiable over $[\underline{z}, \bar{z}]$ as long as $-d_0/d_1 < \underline{z}$ (which is true for the selected calibration). I take the values for β, d_0, and d_1 from Esquivel (2021), who considers a yearly calibration. The values for σ, r^*, η, ω, α_T, and α_N are standard in the literature, and $\bar{K} = 1$ is a normalization. I set initial $y_{C,0} = 0$, $B_0 = 0$, and choose $\lambda_0 = \Lambda_0$ to maximize output given $z_0 = 1$ and no government transfer.

The purpose of this numerical exercise is to illustrate the inefficiency of the Dutch disease by comparing the efficient and equilibrium allocations for different values of the commodity endowment in $t = 1$. I will use total natural resource rents as a data counterpart for $y_{C,1}$ (see Subsection 4.1 for a full description of this variable). For each country with available data, I compute the average of total natural resource rents as a percentage of GDP from 1979 to 2015. Figure 1 shows the histogram of this average for all countries. These rents range from 0 to up to 43 percent of

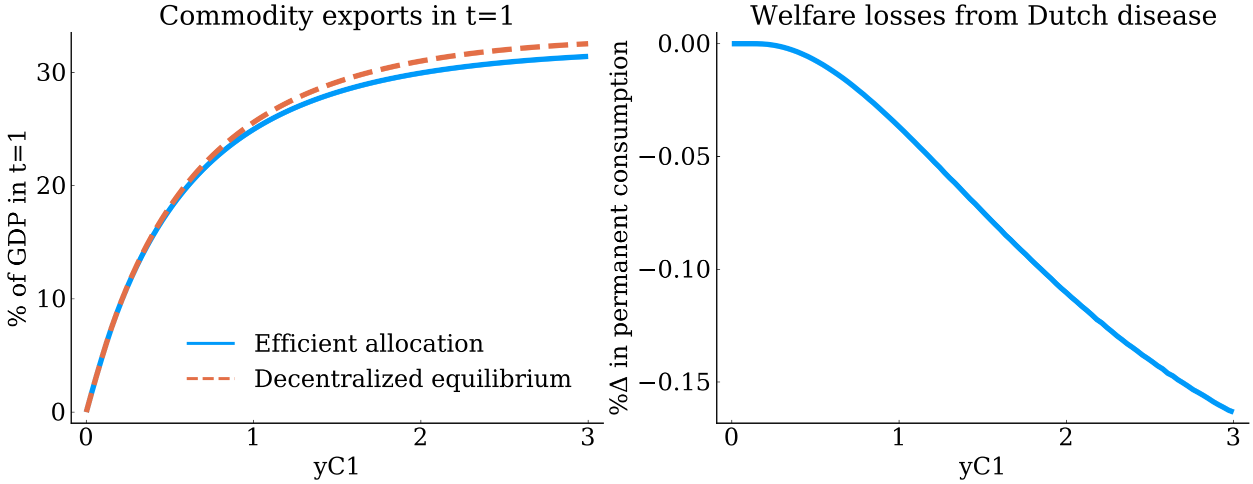
GDP.

Figure 1: Distribution of average natural resource rents as a percentage of GDP



The left panel of Figure 2 shows $100 * \frac{y_{C,1}}{P_1 Y_1}$ for different values of $y_{C,1}$, which are rents from natural resources as a fraction of GDP in period 1 in the model. The right panel shows welfare losses from the Dutch disease in consumption equivalent units.

Figure 2: Commodity exports and welfare losses from Dutch disease



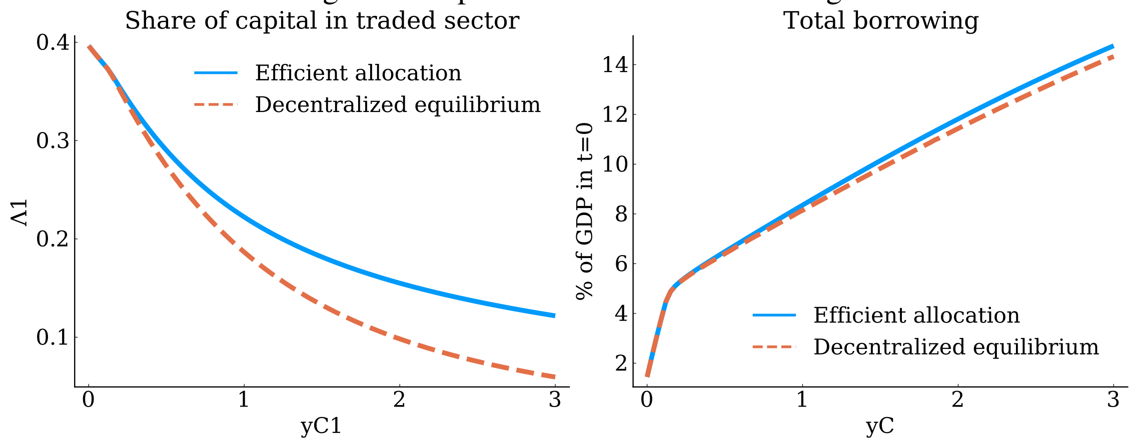
Let $(\hat{c}_0, \hat{c}_1)$ be consumption in periods 0 and 1 calculated with the efficient allocation and $(\tilde{c}_0, \tilde{c}_1)$ consumption calculated with the allocation from the decentralized equilibrium. Then welfare losses χ are defined as the solution to:

$$u((1+\chi)\hat{c}_0) + \beta \mathbb{E}[u((1+\chi)\hat{c}_1)] = u(\tilde{c}_0) + \beta \mathbb{E}[u(\tilde{c}_1)]$$

which is the fraction of first-best consumption that the households lose ($\chi \leq 0$) from being in the decentralized economy. As suggested by CONJECTURE 1, losses are increasing in $y_{C,1}$.

The left panel of Figure 3 compares the share of capital Λ_1 allocated in the traded sector in equilibrium to the share from the efficient allocation. Consistent with the Dutch disease (LEMMA 2), this share decreases as the commodity endowment $y_{C,1}$ increases. Moreover, as suggested by CONJECTURE 1, this share is lower in the equilibrium allocation and the difference between the two increases with $y_{C,1}$.

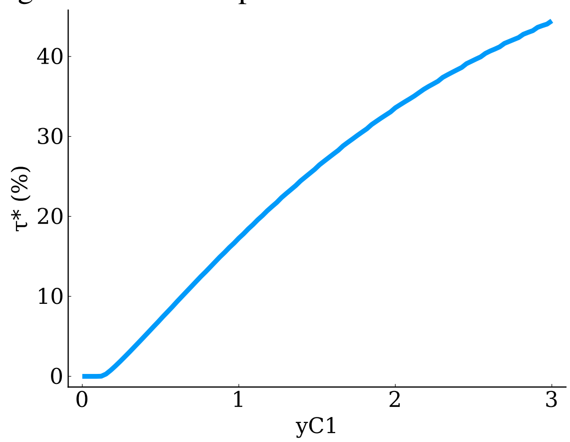
Figure 3: Capital allocation and borrowing



The right panel shows total borrowing is increasing in $y_{C,1}$. Since $\beta(1+r^*)<1$, the agents in the economy are relatively impatient and desire to front-load consumption. Larger $y_{C,1}$ increases permanent income, which increases desired consumption in $t=0$. More importantly, this figure shows how the social planner is able to borrow more because the choice of $\hat{\Lambda}_1$ implies more favorable borrowing terms.

Finally, Figure 4 shows the tax τ^* that would implement the efficient allocation as a function of $y_{C,1}$. As indicated by CONJECTURE 1, this tax is increasing in the commodity endowment for $t=1$.

Figure 4: Tax to implement efficient allocation



Recall that this tax is inversely proportional to the expected depreciation of the real exchange rate. Higher income from commodities in $t = 1$ appreciates the real exchange rate, which calls for a higher tax to implement the efficient allocation. Sterilization policies—such as accumulation of international reserves during commodity windfalls—tame this real appreciation and effectively act as a tax on returns to investment in the non-traded sector. The following section presents evidence that the empirical relations between commodity endowments, default risk, and accumulation of international reserves are consistent with the implications of the model.

4 Empirical analysis

This section makes two empirical points. The first is that the Dutch disease is associated with higher default risk. In order to show this, I present evidence that resource-rich countries face more stringent borrowing terms and appreciated real exchange rates following commodity windfalls. The second point is that central banks accumulate more international reserves during commodity windfalls in order to prevent over-appreciation of the real exchange rate, which is akin to the relation suggested by CONJECTURE 1 between τ^* and commodity endowments.

4.1 Data description

Unless indicated otherwise, all data are yearly and taken from The World Bank (2021) and the International Monetary Fund (2021). I consider all countries with available data for the years 1979–2015.

I use two measures of default risk. The first is the interest rate spreads from JP Morgan's Emerging Markets Bonds Index (EMBI), which are widely used in the literature. These data are available for 37 countries starting no earlier than 1993.[11] As an alternative measure, I use the Institutional Investor Index (*III*) to construct measures of spreads for other countries for which sovereign bonds spread data are not available. The *III* is a measure of sovereign risk that was published biannually by the Institutional Investor magazine. It measures country risk by aggregating into an index a collection of risk-related variables that are related to investing in a foreign country, including political risk, exchange rate risk, economic risk, sovereign risk and transfer risk. The *III* takes values between 0 and 100, where 100 indicates lowest risk and 0 the most risk. To assess the effect that the *III* has on sovereign spreads, I estimate the following econometric model:

$$\ln\left(\text{spread}_{i,t}\right) = \gamma_0 + \gamma_1 \ln\left(III_{i,t}\right) + \kappa_i + \mu_t + \varepsilon_{i,t} \tag{16}$$

where κ_i are country fixed effects, μ_t are year fixed effects, $III_{i,t}$ is the average index for country i in year t, and $\varepsilon_{i,t}$ is the error term.[12] I then use equation (16) and *III* data to construct time-series of spreads for all countries.

I use data on total natural resource rents as a fraction of GDP. Natural resource rents are calculated as the difference between the price of a commodity and the average cost of producing it. These unit rents are then multiplied by the physical quantities that countries extract to determine the rents for each commodity. Total natural resource rents are the sum of oil rents, natural gas rents, coal rents, other mineral rents, and forest rents.

I use two measures of foreign debt: total external debt stocks and central government debt, both as a fraction of GDP. The former includes both private and public debt, while the latter includes only government debt but is available for a smaller set of countries. I use international reserves excluding gold as a fraction of GDP and a measure of the real exchange rate described below.

[11] The 37 countries are: Argentina, Belize, Brazil, Bulgaria, Chile, China, Colombia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Ghana, Hungary, Indonesia, Iraq, Jamaica, Kazakhstan, Republic of Korea, Lebanon, Malaysia, Mexico, Pakistan, Panama, Peru, Philippines, Poland, Russian Federation, Serbia, South Africa, Sri Lanka, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, and Vietnam.

[12] The estimated coefficients are

$$\ln\left(\text{spread}_{i,t}\right) = \underset{(0.629)}{8.791} - \underset{(0.177)}{1.958} \ln\left(III_{i,t}\right)$$

where the numbers in parenthesis are clustered standard errors. The *III* is significant at the 0.01 level and the $R^2 = 0.64$.

4.2 Default risk and natural resources

First, to show the effect that being a commodity exporter has on borrowing terms I estimate the following panel regression:

$$s_{i,t} = \beta_0 + \beta_1 \overline{NR}_i + \beta_2 100 * \frac{debt_{i,t}}{GDP_{i,t}} + \beta_3 100 * \frac{reserves_{i,t}}{GDP_{i,t}} + \mu_t + u_{i,t} \tag{17}$$

where subscripts i refer to countries and t to years, $s_{i,t}$ are interest rate spreads, $\overline{NR}_i$ is the average natural resource rents as a percentage of GDP for country i over the available time period, μ_t are year fixed effects, and $u_{i,t}$ is the error term. Table 2 summarizes the estimation results for different measures of spreads and government debt.

Table 2: Commodity exporters and default risk

	(1) EMBI	(2) EMBI	(3) Constructed EMBI	(4) Constructed EMBI
Av (NR rents / GDP)	0.128**	0.137	0.208**	0.926***
	(0.0605)	(0.125)	(0.0804)	(0.281)
Reserves / GDP	-0.124***	-0.132**	-0.360***	-0.0853***
	(0.0375)	(0.0481)	(0.0358)	(0.0285)
Total Debt / GDP	0.0678*		0.167***	
	(0.0332)		(0.0237)	
Gov Debt / GDP		0.0442**		0.122***
		(0.0198)		(0.0380)
Constant	4.330**	3.882***	4.438***	-5.040**
	(1.513)	(0.627)	(0.975)	(1.829)
Year FE	Yes	Yes	Yes	Yes
Observations	520	246	2,645	1,033
Number of countries	43	31	105	84
R-squared	0.267	0.307	0.216	0.292

Robust standard errors in parenthesis based on Driscoll and Kraay (1998).
*** p<0.01, ** p<0.05, * p<0.1

The first row shows that the estimates of β_1 are positive and statistically different from 0 (except for column (2), which has the least number of observations). The variable $\overline{NR}_i$ is a country-specific shifter and the positive sign of β_1 indicates that countries for which natural resource rents are relatively large face higher default risk. The estimates in column (4) indicate that a 1 percent higher share of rents from commodities on GDP implies that average government spreads are 92 basis points higher.

4.3 Real exchange rates and reserve accumulation

To explore the relation between accumulation of international reserves and commodity windfalls I estimate the following regression:

$$\ln\left(100*\frac{reserves_{i,t}}{GDP_{i,t}}\right)=\chi_0+\chi_1\ln\left(100*\frac{NR_{i,t}}{GDP_{i,t}}\right)+\kappa_i+\mu_t+v_{i,t} \tag{18}$$

where the dependent variable is the natural logarithm of international reserves as a percentage of GDP, $NR_{i,t}$ are rents from natural resources in country i in year t, κ_i are country fixed effects, μ_t are year fixed effects, and $v_{i,t}$ is the error term. Table 3 reports the estimated coefficients.

Table 3: Relation between reserves and commodity windfalls

	(1)
	Reserves
$\ln\left(100*\frac{NR_{i,t}}{GDP_{i,t}}\right)$	0.117***
	(0.0333)
Constant	1.635***
	(0.0380)
Year FE	Yes
Country FE	Yes
Observations	5,044
Number of countries	160
R-squared	0.183

Robust standard errors in parenthesis based on Driscoll and Kraay (1998).
*** p<0.01, ** p<0.05, * p<0.1

There is a significant positive relation between rents from natural resources and international reserves, suggesting that in the presence of commodity windfalls, central banks increase their reserve accumulation. Finally, to analyze the effect that natural resources and reserve acummulation have on real exchange rates, I estimate the following regression:

$$\ln(rer_{i,t})=\rho\ln(rer_{i,t-1})+\phi_1\left(100*\frac{NR_{i,t}}{GDP_{i,t}}\right)+\phi_2\Delta_{t,t-1}\left(100*\frac{reserves_{i,t}}{GDP_{i,t}}\right)+\kappa_i+\mu_t+\varepsilon_{i,t} \tag{19}$$

where κ_i are country fixed effects, μ_t are year fixed effects, $\Delta_{t,t-1}x_t$ indicates the change of variable x from $t-1$ to t, and $rer_{i,t}$ is the real exchange rate of country i in year t *vis-a-vis* the US dollar.[13]

[13] I compute $rer_{i,t}=\frac{e_{i,t}P_{US,t}}{P_{i,t}}$ where $e_{i,t}$ is the nominal exchange rate (amount of currency from country i per US

Table 4 reports the estimated coefficients from equation (19).

Table 4: Effect of reserves and natural resources on the real exchange rate

	(1) Real Exchange Rate
$\ln\left(rer_{i,t-1}\right)$	0.909***
	(0.0272)
$\left(100 * \frac{NR_{i,t}}{GDP_{i,t}}\right)$	-0.00597**
	(0.00284)
$\Delta_{t,t-1}\left(100 * \frac{reserves_{i,t}}{GDP_{i,t}}\right)$	0.00203**
	(0.000833)
Constant	0.280***
	(0.0945)
Year FE	Yes
Country FE	Yes
Observations	3,980
Number of countries	158
R-squared	0.919

Robust standard errors in parenthesis based on Driscoll and Kraay (1998).
*** p<0.01, ** p<0.05, * p<0.1

The sign of $\phi_1 < 0$ indicates that the real exchange rate appreciates when rents from natural resources increase, which is consistent with the prices defined in Subsection 2.1. The sign of $\phi_2 > 0$ indicates that the real exchange rate depreciates when reserve accumulation increases. Intuitively, the central bank increases the domestic demand for foreign currency in order to limit the effects of higher supply from higher commodity exports. This positive relation suggests that increasing the accumulation of international reserves tames the appreciation of the real exchange rate during commodity windfalls, which in turn decreases the distortion from the Dutch disease suggested by the model.

5 Conclusion

This paper presented an environment with production in traded and non-traded sectors in which, in the presence of default risk, atomistic households allocate higher than optimal amounts of capital in the non-traded sector. This misallocation of capital is a result of the private sector failing to

dollar), $P_{US,t}$ is the US GDP deflator in year t and $P_{i,t}$ is country i's GDP deflator.

internalize how its capital-allocation decisions affect ex-post default incentives and ex-ante borrowing terms. In addition, this misallocation is more severe in the presence of alternative sources of tradable income, such as large endowments of natural resources.

The efficient allocation can be decentralized as a competitive equilibrium with an appropriate tax to capital income in the non-traded sector. This tax is proportional to the desired borrowing level and invesrsly proportional to the expected appreciation of real exchange rate that results from a commodity windfall. Sterilization policies such as accumulation of international reserves during commodity windfalls have effects that are consistent with the optimal tax: ex-post they depreciate the real exchange rate and reduce the realized return to capital in non-traded sectors; ex-ante they reduce the incentives to overinvest in non-traded sectors, which reduces the capital misallocation. The empirical evidence supports the three main implications from the model: (i) "resource-rich" economies face higher default risk, which is reflected in higher interest rate spreads, and (ii) the accumulation of international reserves increases during commodity windfalls.

An immediate avenue for future research is the development of richer quantitative models of default risk with traded and non-traded production to inform about appropriate policy responses to commodity windfalls.

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A Euler equations for capital

A.1 Households

The problem of a representative household is:

$$\max_{\lambda_1} \int_{\underline{z}}^{z^*(x_1)} \beta u\left(c_1^D\right) dF(z) + \int_{z^*(x_1)}^{\bar{z}} \beta u(c_1)\, dF(z)$$

$$\begin{aligned} s.t. \quad P^D c_1^D &= \left[r_N^D (1-\lambda_1) + r_T^D \lambda_1\right] \bar{k} + y_{C,1} + \Pi_1^D \\ P^P c_1 &= \left[r_N^P (1-\lambda_1) + r_T^P \lambda_1\right] \bar{k} + y_{C,1} + \Pi_1^P - B_1 \\ \Lambda_1 &= \Gamma_H\left(B_1; y_{C,1}\right) \end{aligned}$$

where the rental rates of capital are:

$$\begin{aligned} r_N^D &= \left(\frac{\omega}{1-\omega} \frac{z_D(z)\left(\Lambda_1 \bar{K}\right)^{\alpha_T} + y_{C,1}}{z_D(z)\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} \alpha_N z_D(z)\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N - 1} \\ r_N^P &= \left(\frac{\omega}{1-\omega} \frac{z\left(\Lambda_1 \bar{K}\right)^{\alpha_T} + y_{C,1} - B_1}{z\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} \alpha_N z\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N - 1} \\ r_T^D &= \alpha_T z_D(z)\left[\Lambda_1 \bar{K}\right]^{\alpha_T - 1} \\ r_T^P &= \alpha_T z\left[\Lambda_1 \bar{K}\right]^{\alpha_T - 1} \end{aligned}$$

and are taken as given by the household, as well as the prices:

$$\begin{aligned} P^D &= \left[\omega\left(p_N^D\right)^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}} \\ p_N^D &= \left(\frac{\omega}{1-\omega} \frac{z_D(z)\left(\Lambda_1 \bar{K}\right)^{\alpha_T} + y_{C,1}}{z_D(z)\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} \\ P^P &= \left[\omega\left(p_N^P\right)^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}} \\ p_N^P &= \left(\frac{\omega}{1-\omega} \frac{z\left(\Lambda_1 \bar{K}\right)^{\alpha_T} + y_{C,1} - B_1}{z\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} \end{aligned}$$

which all only depend on the aggregate state in period 1. The first-order condition of this problem is:

$$0=\int_{\underline{z}}^{z^*(x_1)} u'\left(c_1^D\right)\frac{\partial c_1^D}{\partial \lambda_1}dF\left(z\right)+\int_{z^*(x_1)}^{\bar{z}} u'\left(c_1\right)\frac{\partial c_1}{\partial \lambda_1}dF\left(z\right)$$

where

$$\frac{\partial c_1^D}{\partial \lambda_1}=\frac{r_T^D-r_N^D}{P^D}\bar{K}$$
$$\frac{\partial c_1}{\partial \lambda_1}=\frac{r_T-r_N}{P}\bar{K}$$

do not depend on λ_1, only on $\Lambda_!$.

A.2 Social planner

The problem of the social planner is:

$$\max_{\Lambda_1,B_1} u\left(C_0\right)+\beta\int_{\underline{z}}^{z^*(x_1)} u\left(C^D\right)dF\left(z\right)+\beta\int_{z^*(x_1)}^{\bar{z}} u\left(C\right)dF\left(z\right)$$

using Leibniz integral rule we get that the first order condition for Λ_1 is:

$$u'\left(C_0\right)\frac{\partial C_0}{\partial \Lambda_1}+\beta\int_{\underline{z}}^{z^*(x_1)} u'\left(C_1^D\right)\frac{\partial C^D}{\partial \Lambda}dF\left(z\right)+\beta\int_{z^*(x_1)}^{\bar{z}} u'\left(C_1^P\right)\frac{\partial C^P}{\partial \Lambda}dF\left(z\right)=0$$

It is easy to see from the firm's problems in the decentralized equilibrium that $\frac{\partial C^P}{\partial \Lambda}=\frac{r_T^P-r_N^P}{P^P}\bar{K}$ and $\frac{\partial C^D}{\partial \Lambda}=\frac{r_T^D-r_N^D}{P^D}\bar{K}$. The derivative of consumption in period 0 with respect to Λ_1 is:

$$\frac{\partial C_0}{\partial \Lambda_1}=\frac{\partial Y}{\partial c_T}\frac{\partial q}{\partial \Lambda_1}B_1$$

where $\frac{\partial q}{\partial \Lambda_1}$ is the derivative of the price of bonds with respect to Λ_1. It is easy to see from equation (8) that q is continuously differentiable over $[\underline{z},\bar{z}]$ as long as z_D is continuously differentiable over $[\underline{z},\bar{z}]$ and F is smooth.

B Proof of Lemma 1

Lemma 1: If $\eta < 1$ and $\frac{\partial z_D}{\partial z} \leq \frac{z_D}{z}$, then the default set is shrinking in Λ_1. That is, $\frac{\partial z^*(x;y_C)}{\partial \Lambda} \leq 0$.

Proof: From fully differentiating equation (8) with respect to Λ we get:

$$\frac{\partial z^*(x_1;y_C)}{\partial \Lambda} = -\frac{\frac{\partial C^P}{\partial \Lambda_1} - \frac{\partial C^D}{\partial \Lambda_1}}{\frac{\partial C^P}{\partial z} - \frac{\partial C^D}{\partial z}} \tag{20}$$

where $C^P = c\left(\left(z^*(x_1;y_C), d=0, y_{C,1}\right), x_1, G=-B_1\right)$ and $C^D = c\left(\left(z^*(x_1;y_C), d=1, y_{C,1}\right), x_1, G=0\right)$. The proof proceeds by first showing separately that the numerator and denominator in (20) are both positive. Then the results follows.

Remark L1.1: The numerator is positive.

We have that the derivatives on the numerator of (20) are:

$$\begin{aligned}
\frac{\partial C^P}{\partial \Lambda} &= \frac{\partial Y}{\partial c_N}\frac{\partial y_N}{\partial \Lambda} + \frac{\partial Y}{\partial c_T}\frac{\partial y_T}{\partial \Lambda} \\
&= \frac{1}{P(z^*,x_1)}\left[\frac{\alpha_T z^*(\Lambda_1\bar{K})^{\alpha_T}}{\Lambda_1} - p_N(z^*,x_1)\frac{\alpha_N z^*((1-\Lambda_1)\bar{K})^{\alpha_N}}{(1-\Lambda_1)}\right] \\
\frac{\partial C^D}{\partial \Lambda} &= \frac{1}{P^D(z^*,x_1)}\left[\frac{\alpha_T z_D^*(\Lambda_1\bar{K})^{\alpha_T}}{\Lambda_1} - p_N^D(z^*,x_1)\frac{\alpha_N z_D^*((1-\Lambda_1)\bar{K})^{\alpha_N}}{(1-\Lambda_1)}\right]
\end{aligned}$$

where $z_D^* = z_D(z^*)$. Rearranging we can rewrite the numerator of (20) as:

$$\begin{aligned}
\frac{\partial C^P}{\partial \Lambda_1} - \frac{\partial C^D}{\partial \Lambda_1} = &\left[\frac{z^*}{P(z^*,x_1)} - \frac{z_D^*}{P^D(z^*,x_1)}\right]\frac{\alpha_T(\Lambda_1\bar{K})^{\alpha_T}}{\Lambda_1} \\
&- \left[z^*\frac{p_N(z^*,x_1)}{P(z^*,x_1)} - z_D^*\frac{p_N^D(z^*,x_1)}{P^D(z^*,x_1)}\right]\frac{\alpha_N((1-\Lambda_1)\bar{K})^{\alpha_N}}{(1-\Lambda_1)}
\end{aligned} \tag{21}$$

Remark L1.1.1: $p_N \leq p_N^D$ for for any non-negative debt level:

$$\begin{aligned}\frac{p_N}{p_N^D} &= \left(\frac{z\left(\Lambda\bar{K}\right)^{\alpha_T} + y_C - B}{z_D\left(\Lambda\bar{K}\right)^{\alpha_T} + y_C}\frac{z_D}{z}\right)^{\frac{1}{\eta}} \\ &= \left(\frac{z_D\left(\Lambda\bar{K}\right)^{\alpha_T} + \frac{z_D}{z}\left(y_C - B\right)}{z_D\left(\Lambda\bar{K}\right)^{\alpha_T} + y_C}\right)^{\frac{1}{\eta}} \\ &\leq \left(\frac{z_D\left(\Lambda\bar{K}\right)^{\alpha_T} + \frac{z_D}{z}y_C}{z_D\left(\Lambda\bar{K}\right)^{\alpha_T} + y_C}\right)^{\frac{1}{\eta}} \leq 1\end{aligned}$$

which implies that $P \leq P^D$ and this in turn implies that $\left(\frac{1}{P}z - \frac{1}{P^D}z_D\right) \geq 0$

Remark L1.1.2: evaluated at exactly $z^*\left(x; y_C\right)$ we know from equation (8) that $C^P = C^D$ and so we get:

$$\begin{aligned}\frac{p_N}{P} &= \left(\frac{\omega C^P}{z^*\left(\left(1-\Lambda_1\right)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} \\ &= \left(\frac{\omega C^D}{z^*\left(\left(1-\Lambda_1\right)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} \\ &\leq \left(\frac{\omega C^D}{z_D^*\left(\left(1-\Lambda_1\right)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} = \frac{p_N^D}{P^D}\end{aligned}$$

moreover, the above expression yields that at exactly $z^*\left(x; y_C\right)$:

$$\frac{p_N^D}{P^D} = \left(\frac{z^*}{z_D^*}\right)^{\frac{1}{\eta}}\frac{p_N}{P}$$

which implies that

$$\frac{p_N}{P}z^* - \frac{p_N^D}{P^D}z_D^* = \frac{p_N}{P}z^*\left(1 - \left(\frac{z^*}{z_D^*}\right)^{\frac{1-\eta}{\eta}}\right) \leq 0$$

where the inequality follows from $\eta < 1$.

Remark L1.1 then follows from remarks L1.1.1 and L1.1.2.

Remark L1.2: The denominator in (20) is positive.

We have that the derivatives on the denominator of (20) are:

$$\frac{\partial C^P}{\partial z} = \frac{p_N(z^*,x_1)\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}{P(z^*,x_1)} + \frac{(\Lambda_1\bar{K})^{\alpha_T}}{P(z^*,x_1)}$$
$$\frac{\partial C^D}{\partial z} = \left[\frac{p_N^D(z^*,x_1)\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}{P^D(z^*,x_1)} + \frac{(\Lambda_1\bar{K})^{\alpha_T}}{P^D(z^*,x_1)}\right]\frac{\partial z_D}{\partial z}$$

subtracting and rearranging we get:

$$\frac{\partial C^P}{\partial z} - \frac{\partial C^D}{\partial z} = \left[\frac{1}{P(z^*,x_1)} - \frac{1}{P^D(z^*,x_1)}\frac{\partial z_D}{\partial z}\right](\Lambda_1\bar{K})^{\alpha_T} + \left[\frac{p_N(z^*,x_1)}{P(z^*,x_1)} - \frac{p_N^D(z^*,x_1)}{P^D(z^*,x_1)}\frac{\partial z_D}{\partial z}\right]\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}$$

From Remark L1.1.1 and the assumption that $\frac{\partial z_D}{\partial z} \leq \frac{z_D}{z} \leq 1$ we have that $\frac{1}{P} - \frac{1}{P^D}\frac{\partial z_D}{\partial z} \geq 0$

From Remark L1.1.2 we get that:

$$\frac{p_N}{P} - \frac{p_N^D}{P^D}\frac{\partial z_D}{\partial z} = \frac{p_N}{P}\left(1 - \left(\frac{z^*}{z_D^*}\right)^{\frac{1}{\eta}}\frac{\partial z_D}{\partial z}\right) \geq 0$$

where the inequality follows from the assumptions $\eta < 1$ and $\frac{\partial z_D}{\partial z} \leq \frac{z_D}{z}$.

From *Remark L1.1* and *Remark L1.2* we get that **Lemma 1** follows.

C Proof of Proposition 1

Proposition 1: (*Misallocation*) For any given level of debt issuance B_1, households in the decentralized equilibrium overinvest in the non-traded sector and underinvest in the traded one.

Proof: For $x = (\Lambda, B)$, recall that aggregate consumption in $t = 0$ is:

$$C_0(x) = Y\left(y_N\left(z_0,(1-\Lambda_0)\bar{K}\right), y_T\left(z_0,\Lambda_0\bar{K}\right) + y_C + q(x)B - B_0\right)$$

Define the continuation value of x as

$$V1(x) = \beta\int_{\underline{z}}^{z^*(x)} u\left(C^D(z,x)\right)dF(z) + \beta\int_{z^*(x)}^{\bar{z}} u\left(C^P(z,x)\right)dF(z)$$

and the value in $t=0$ for the social planner of choosing x_1 as:

$$V0\left(x_{1}\right)=u\left(C_{0}\left(x_{1}\right)\right)+V1\left(x_{1}\right) \tag{22}$$

The problem of the social planner is to choose $\hat{x}_1$ to maximize 22.

Given this, we can express the first-order condition of the planner's problem with respect to Λ is

$$0=\frac{\partial V1\left(\hat{x}_{1}\right)}{\partial\Lambda}+u'\left(\hat{C}_{0}\right)\frac{\partial\hat{Y}}{\partial c_{T}}\frac{\hat{\partial q}}{\partial\Lambda_{1}}\hat{B}_{1} \tag{23}$$

In equilibrium, all households choose the same capital allocation $\tilde{\lambda}_1^*=\tilde{\Lambda}_1$. Given some debt issuance B, the equilibrium allocation of capital $\tilde{\Lambda}_1\left(B\right)$is pinned down by the household's first-order condition, which can also be written in terms of the derivative of $V1$:

$$0=\frac{\partial V1\left(\tilde{\Lambda}_{1}\left(B\right),B\right)}{\partial\Lambda} \tag{24}$$

We know from Lemma 1 that

$$\frac{\partial V1\left(\hat{x}_{1}\right)}{\partial\Lambda}\leq\frac{\partial V1\left(\tilde{\Lambda}_{1}\left(\hat{B}\right),\hat{B}\right)}{\partial\Lambda}$$

so since the objective is concave then its derivative is decreasing and we get that $\tilde{\Lambda}_1\left(\hat{B}\right)\leq\hat{\Lambda}_1$.