

Sovereign Risk and Dutch Disease*

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Abstract

This paper studies the impact of natural resource discoveries on default risk. I use data of giant oil discoveries to estimate their effect on the spreads of 37 emerging economies. I find that spreads increase by up to 530 basis points following a discovery of average size. I develop a quantitative sovereign default model with production in three intermediate sectors and oil discoveries. Following a discovery investment and foreign borrowing increase, with opposite effects on spreads. The discovery also generates a reallocation of production factors away from manufactures into oil extraction and to the non-traded sector. This reallocation increases the volatility of tradable income used to finance debt payments, which undermines the effect of investment on spreads. Under the benchmark calibration the model accounts for at least one third of the increase in spreads observed in the data.

JEL Codes: E30, F34, F41

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1 Introduction

Between 1970 and 2012 sixty-four countries discovered at least one giant oil field, twelve of them had a default episode in the following five years.¹ Taking into account all the countries in the world, the unconditional probability of default in any given five year period was 6%, conditional on discovering a giant oil field this probability was 11%.² This observation appears counter-intuitive, why would a country that just became richer also become more likely to default on its debt?

This paper studies the impact that the discovery and exploitation of natural resources have on default risk. Following the sovereign default literature, I focus on emerging economies as they are more prone to default episodes. I use data of giant oil field discoveries to document the effect of a sudden increase in available natural resources on sovereign spreads. I build on the work by [Arezki, Ramey, and Sheng \(2017\)](#), who construct a measure of the net present value of a giant oil field discovery relative to the GDP of the country where the discovery happens. I use these data to estimate the effect of a discovery on the sovereign spreads of 37 emerging economies. I find that giant oil discoveries have a large and positive effect on sovereign spreads—spreads increase by up to 530 basis points following a discovery of average size.

I also estimate the effect on the current account, investment, GDP, and consumption and find evidence in favor of the permanent income hypothesis, as [Arezki, Ramey, and Sheng \(2017\)](#) did for a wider set of countries. Following a discovery, the emerging countries considered in this paper run a current account deficit and GDP, investment, and consumption increase. Additionally, I estimate the effects of giant oil discoveries on sectoral investment and the real exchange rate and find evidence of the Dutch disease: the share of investment in the manufacturing sector decreases in favor of investment in the commodities and non-traded sectors. This reallocation of investment is accompanied by an appreciation of the real exchange rate.

I develop a small open economy model of sovereign default with long-term debt, capital accumulation, and production in three intermediate sectors: a non-traded sector, a traded “manufacturing” sector, and a traded “oil” sector. All three sectors use capital and labor for production and

¹A giant oil or gas field contains at least 500 million barrels of ultimately recoverable oil equivalent. “Ultimately recoverable reserves” is an estimate of the total amount of oil or gas that could be recovered from a field.

²The data of default episodes are from [Tomz and Wright \(2007\)](#). The default probability conditional on discovery is the probability that a country experiences a default episode in any of the five years following a discovery. These calculations only consider default episodes and oil discoveries between 1970 and 2004.

the oil sector additionally requires an oil field of a certain capacity. In the model, an economy starts with a small oil field and every period has a positive probability of discovering a larger one. After an oil discovery foreign borrowing and investment increase. Also, the real exchange rate appreciates and there is a reallocation of production factors away from the manufacturing sector toward the oil and non-traded sectors, the so-called Dutch disease.

In the model, as in the data, the international price of oil is relatively more volatile than the international price of the other tradable goods.³ Higher investment decreases spreads and higher foreign borrowing increases them, however, the latter effect dominates because the reallocation of capital implied by the Dutch disease makes total tradable income more volatile. It is worth noting that, in the model, there is nothing inefficient about the Dutch disease (i.e. it is not really a disease). The reallocation of production factors is the optimal response to the increase in the availability of natural resources, which takes into account the implied increase in the cost of borrowing.

I calibrate the model to the Mexican economy, which is a typical small-open economy widely studied in the sovereign debt and emerging markets literature, and I use the oil discoveries data from [Arezki, Ramey, and Sheng \(2017\)](#) to discipline the probability and size of discoveries. It is important to note that Mexico did not have any giant oil field discoveries between 1993 and 2012, which is the period analyzed in this paper.⁴ This lack of discoveries is desirable because it allows me to discipline the parameters of the model with business cycle data that does not have any variation in endogenous variables that could be driven by oil discoveries. This allows me to then validate the theory by contrasting the responses of model variables to oil discoveries with those estimated from the data.

Under the benchmark calibration, the model can explain at least one third of the increase in spreads following an oil discovery. There are other complementary forces that could also make spreads increase and are not considered in the model. Growth externalities in the manufacturing sector could make the Dutch disease inefficient if they are not internalized, this could hamper future growth and increase spreads in the present.⁵ Also, deterioration of institutions following

³Commodities have always shown a higher price volatility than manufactures. This is a stylized fact documented by [Jacks, O'Rourke, and Williamson \(2011\)](#) with data that goes back to the 18th century.

⁴An interesting case of study for this paper would be the episode of the Mexican default in 1982, which was preceded by two giant oil field discoveries: one in 1977 and another in 1979, each with an estimated net present value of potential revenues of 50 percent of Mexico's GDP at the time. The main inconvenience with this episode is the lack of data on sovereign spreads, which are crucial to discipline the parameters in the model that control default incentives.

⁵See [Hevia, Neumeyer, and Nicolini \(2013b\)](#) for an example of such an externality.

giant oil discoveries could also cause spreads to increase. [Lei and Michaels \(2014\)](#) find evidence that giant oilfield discoveries increase the incidence of internal armed conflicts.

I use the model to calculate the welfare costs of the volatility of the price of oil for small open economies. This calculation is indicative of the value for the economy of financial instruments that allow it to hedge against fluctuations in the price of oil. I do this calculation for an economy that never discovers a large oil field and for an economy that does.⁶ I find that an economy that never discovers a large oil field is willing to forego 0.4% of lifetime consumption to eliminate the volatility of the price of oil. This number is 0.6% for an economy that does discover a giant oil field. These calculations suggest that the gains for the economy of having access to a hedge to fluctuations of the price of oil are 1.5 larger if it discovers a giant field.

Related literature.—This paper builds on the quantitative sovereign default literature following [Aguilar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#).⁷ They introduce sovereign default models that feature countercyclicality of net exports and interest rates, both consistent with what is observed for emerging markets. [Hatchondo and Martinez \(2009\)](#) and later [Chatterjee and Eyigungor \(2012\)](#) extend the baseline framework of quantitative models of sovereign default to include long-term debt. They show how this extension allows models to better account for the debt level as well as the level and volatility of spreads while also improving upon the model’s ability to account for other cyclical factors.

[Gordon and Guerron-Quintana \(2018\)](#) analyze the quantitative properties of sovereign default models with production, capital accumulation and long-term debt. They show that the model is able to fit cyclical properties of investment and GDP while also remaining consistent with other business cycle properties of emerging economies. They also find that capital has non-trivial effects on sovereign risk but that increased capital almost always reduces risk premia in equilibrium. The model presented in Section 3 extends their framework to have production in different sectors, with at least one also using natural resources for production. [Arellano, Bai, and Mihalache \(2018\)](#) document how sovereign debt crises have disproportionately negative effects on non-traded sectors. They develop a model with production in two sectors and one period debt.

[Hamann, Mendoza, and Restrepo-Echavarria \(2018\)](#) study the relation between oil exports, oil

⁶Even if the economy was not an oil producer it would still be affected by the volatility of the price of oil as long as oil is used to produce other goods, as is the case in the model presented in this paper.

⁷These papers extend the approach developed by [Eaton and Gersovitz \(1981\)](#).

reserves, and sovereign risk. They find that an oil exporting country is perceived as less risky if its oil production is high. In contrast, they also find that country risk increases as unexploited oil reserves increase. They develop a model that highlights the strategic decision to exploit oil reserves and its effects on sovereign risk. In this paper I abstract from this strategic framework and instead focus on the effects of oil discoveries on the sectoral allocation of capital and their consequences on sovereign risk.

This paper also relates to the literature that studies the macroeconomic effects of commodity-related shocks and the Dutch disease. [Pieschacon \(2012\)](#) studies the role of fiscal policy as a transmission mechanism of oil price shocks to key macroeconomic variables. She documents evidence that indicates that the predictions of the Dutch disease hold for the case of Mexico but not for that of Norway, and argues that fiscal policy is a key determinant of this difference. [Hevia, Neumeyer, and Nicolini \(2013a\)](#) analyze optimal policy in a New Keynesian model of a small open economy with shocks to terms of trade that generate Dutch disease periods. Their model features complete markets and an externality in the manufacturing sector that makes the Dutch disease inefficient. The model in Section 3 has incomplete markets and does not feature any externality.

[Arezki and Ismail \(2013\)](#) study the implications that changes in expenditure policy have on real effective exchange rate movements. They find that the real exchange rate appreciates when the oil export unit value increases, but, asymmetrically, that the real exchange rate does not change much when this unit value decreases. [Hevia and Nicolini \(2015\)](#) analyze optimal monetary policy in a small open economy with price and wage nominal frictions that specializes in the production of commodities. They find that, under both frictions, the Dutch disease implies inefficiencies and full price stability is not optimal.⁸ [Ayres, Hevia, and Nicolini \(2019\)](#) argue that shocks that move primary commodity prices account for a large fraction of the volatility of real exchange rates between developed economies and the US dollar. They suggest that considering trade in primary commodities could help models generate real exchange rate volatilities that are more in line with the data.

Layout.—Section 2 describes the data, documents the effect of giant oil discoveries on sovereign spreads and other macroeconomic aggregates, and discusses the empirical evidence that motivates

⁸In another paper ([Hevia and Nicolini \(2013\)](#)) the authors find that in an economy with only price frictions domestic price stability is optimal, even if fiscal policy cannot respond to shocks. In other words, as they explain it, the Dutch disease is not a disease.

the theoretical framework. Section 3 presents the model and discusses the theoretical mechanism. Section 4 describes the calibration. Section 5 presents the quantitative results and Section 6 concludes.

2 Giant oil discoveries in emerging economies

This section documents the effects of giant oil discoveries on thirty-seven emerging economies considered in JP Morgan's Emerging Markets Bonds Index (EMBI).⁹ Due to data availability all analyses are restricted to these thirty-seven economies and the years between 1993 and 2012. I use a measure of the net present value (NPV) of oil discoveries as a percentage of GDP constructed by [Arezki, Ramey, and Sheng \(2017\)](#). I follow their empirical strategy to estimate the effects of oil discoveries on investment, the current account, GDP, and consumption and, as they do, find evidence for the intertemporal approach to the current account (as developed by [Obstfeld and Rogoff \(1995\)](#)). Next, I estimate the effect of giant oil discoveries on the sovereign spreads of these economies and find spreads increase by up to 530 basis points following a discovery. Finally, I estimate the effect of giant oil discoveries on the real exchange rate and investment by sectors and find evidence of the Dutch disease.

2.1 Data and empirical strategy

Giant oil discoveries are a measure of changes in the future availability and potential exploitation of natural resources. As [Arezki, Ramey, and Sheng \(2017\)](#) argue, giant oil discoveries have three unique features that allow the use of a quasi-natural experiment approach: they indicate significant increases in future production possibilities, the timing of discoveries is exogenous due to uncertainty around oil and gas exploration, and there is a time delay of 5.4 years, on average, between discovery and production.¹⁰

⁹The thirty-seven countries are: Argentina, Belize, Brazil, Bulgaria, Chile, China, Colombia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Ghana, Hungary, Indonesia, Iraq, Jamaica, Kazakhstan, Republic of Korea, Lebanon, Malaysia, Mexico, Pakistan, Panama, Peru, Philippines, Poland, Russian Federation, Serbia, South Africa, Sri Lanka, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, and Vietnam.

¹⁰[Arezki, Ramey, and Sheng \(2017\)](#) mention that experts' empirical estimates suggest that it takes between four and six years for a giant oil discovery to go from drilling to production. They also made their own calculation and found that the average delay between discovery and production is 5.4 years.

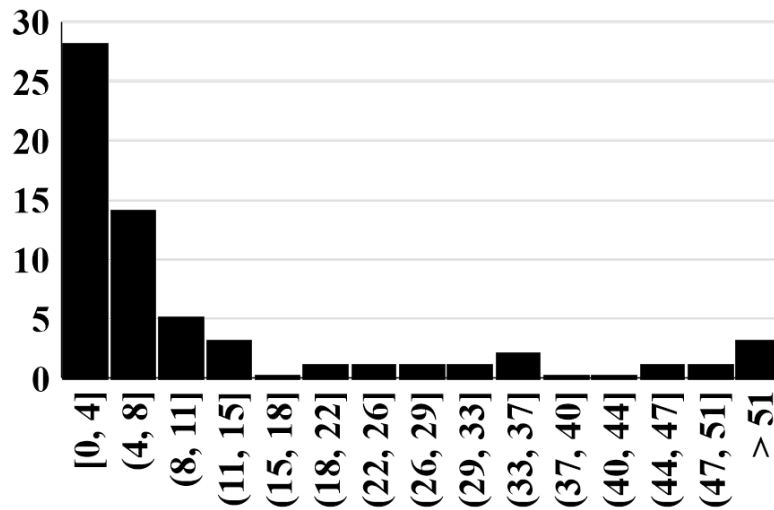
[Arezki, Ramey, and Sheng \(2017\)](#) construct a measure of the net present value (NPV) of giant oil discoveries as a percentage of GDP at the time of discovery as follows:¹¹

$$NPV_{i,t} = \frac{\sum_{j=5}^J \frac{q_{i,t+j}^{*oil price_t}}{(1+r_i)^j}}{GDP_{i,t}} \times 100$$

where $NPV_{i,t}$ is the discounted sum of gross revenue for country i at the year of discovery t . The gross revenue $q_{i,t+j}$ is derived from an approximated production profile starting five years after the field discovery up to an exhaustion year J , which is higher than 50 years for a typical field of 500 million barrels of ultimately recoverable reserves.¹²

Considering the thirty-seven economies in the EMBI and the years 1993–2012 the data show 61 giant oil discoveries in 15 of the 37 countries. The average NPV of a discovery was 18 percent of GDP and the largest was 467. Figure 1 depicts the distribution of the NPV of these discoveries.

Figure 1: Distribution of NPV of giant oil discoveries



Percent of GDP, EMBI countries, 1993 –2012.

¹¹They use the data on giant oil discoveries in the world collected by [Horn \(2014\)](#). For more details of the construction of the NPV see section IV.B. in [Arezki, Ramey, and Sheng \(2017\)](#).

¹²It is important to mention that the gross revenue $q_{i,t+j}$ considers the same price of oil for subsequent years. Since the price of oil closely resembles a random walk, the current price is the best forecast of future prices. See Appendix B of [Arezki, Ramey, and Sheng \(2017\)](#) for a detailed explanation of the approximation of the production profile of giant oil discoveries.

Investment, current account, GDP, and consumption data are from the IMF (2013) and the World Bank (2013). GDP and consumption are measured in constant prices in local currency units. Investment and the current account are measured as a percentage of GDP. Spreads data are from JP Morgan's Emerging Markets Bonds Index (EMBI) Global. The index tracks liquid, US dollar emerging market fixed and floating-rate debt instruments issued by sovereign and quasi-sovereign entities. Spreads are measured against comparable U.S. government bonds.¹³ The real exchange rate is calculated as $RER_{i,t} = \frac{e_{i,t}P_t^{US}}{P_t^i}$ where P_t^{US} and P_t^i are the US and country i 's GDP deflators, respectively, and $e_{i,t}$ is the nominal exchange rate between country i 's currency and the US dollar, these data are also from the IMF (2013). Finally, the data of investment by sector is in terms of share of total investment and is from the United Nations Statistics Division (2017).

To estimate the effect of giant oil discoveries on different macroeconomic variables I use a dynamic panel model with a distributed lag of giant oil discoveries:

$$y_{i,t} = \rho y_{i,t-1} + \sum_{s=0}^{10} \psi_s NPV_{i,t-s} + \alpha_i + \mu_t + \sum_{s=0}^{10} \xi_s p_{oil,t} \mathbb{I}_{disc,i,t-s} + \varepsilon_{i,t} \quad (1)$$

where $y_{i,t}$ is the dependent variable, including investment, the current account, log of real GDP, log of real consumption, sovereign spreads, log of the real exchange rate, and the share of investment by sector; $NPV_{i,t}$ is the NPV of a giant oil discovery in country i in year t ; α_i controls for country fixed effects; μ_t are year fixed effects; $p_{oil,t}$ is the international price of oil at time t ; $\mathbb{I}_{disc,i,t-s}$ is an indicator function of whether country i had an oil discovery in period $t-s$; and $\varepsilon_{i,t}$ is the error term. Country fixed effects control for any unobservable and time-invariant characteristics, while year fixed effects control for common shocks like world business cycles and the international price of oil.¹⁴ The interaction of the international price of oil with the indicator function controls for the fact that the reaction of the dependent variable to this common shock may differ conditional

¹³To be included, an instrument has to have at least 2.5 years until maturity, have a current face amount outstanding of at least 1 billion US dollars, and have a sovereign credit rating of BB+ or lower. The issuing country's GNI per capita must be below an Index Income Ceiling (IIC) for three consecutive years. Currently this IIC is 19,708 US dollars.

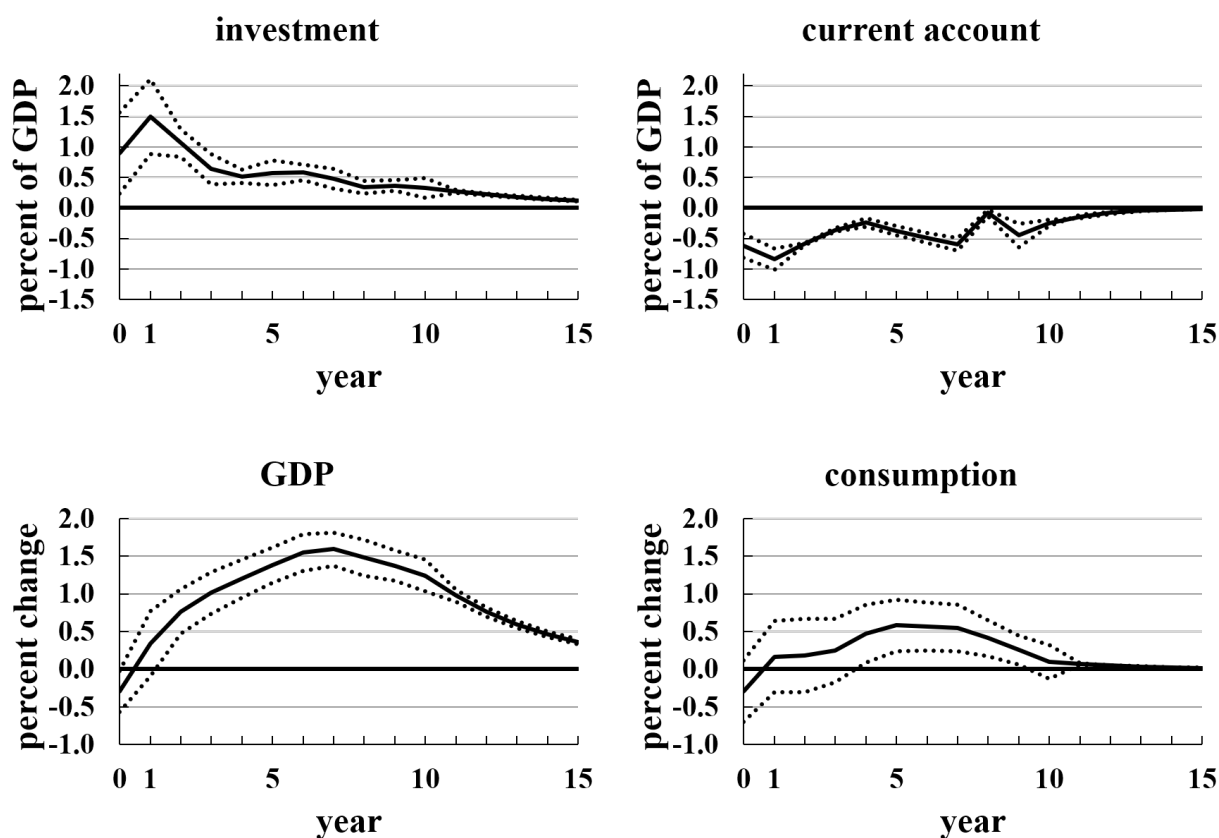
¹⁴As noted by Nickell (1981), estimates of the dynamic panel with fixed effects are inconsistent when the time span is small. He shows that this asymptotic bias is of the order $1/T$, which, in the case of the sample considered in this paper, is 0.05. Arellano and Bond (1991) developed an efficient GMM estimator for dynamic panel data models with small time span and large number of individuals. The results in this section are virtually unchanged using the Arellano-Bond estimator instead. Given the order of the Nickell bias and to keep the results comparable with those of Arezki, Ramey, and Sheng (2017) I use the above approach as the benchmark.

on having a recent discovery. Following [Arezki, Ramey, and Sheng \(2017\)](#), I exploit the dynamic feature of the panel regression and use impulse response functions to capture the dynamic effect of giant oil discoveries given by $\Delta y_{i,t} = \rho \Delta y_{i,t-1} + \sum_{s=0}^{10} \psi_s NPV_{i,t-s}$.

2.2 Permanent income hypothesis

Figure 2 shows the dynamic response of investment, the current account, GDP, and consumption to an oil discovery of average size, based on the estimated coefficients of equation (1). The dotted lines are 90% confidence intervals based on [Driscoll and Kraay \(1998\)](#) standard errors.

Figure 2: Impact of giant oil discoveries on macroeconomic aggregates



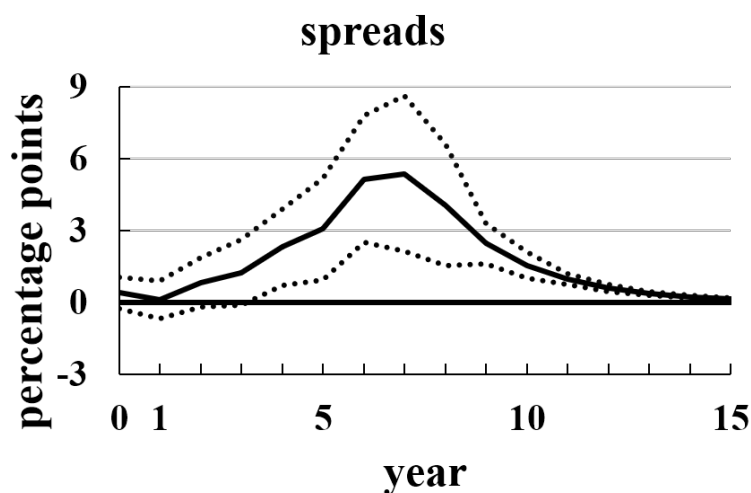
Impulse response to an oil discovery with net present value equal to 18 percent of GDP. The dotted lines indicate 90 percent confidence intervals.

The top left panel shows that the investment-to-GDP ratio increases immediately after an oil discovery and continues to be higher in the subsequent years. The top right panel shows that oil discoveries have a negative effect on the current account-to-GDP ratio, which supports the hypothesis that these countries issue debt to finance higher consumption and investment. The bottom panels show that both GDP and consumption increase after an oil discovery. A giant oil discovery indicates that income will be higher in the future, when the oil field becomes productive. The fact that the country issues debt to finance higher investment and consumption is in line with the permanent income hypothesis.

2.3 Effect on sovereign spreads

Figure 3 shows the dynamic response of the spreads. On the year of the discovery this effect is small and not significantly different from zero. However, by the sixth year after the discovery is announced spreads have increased by 530 basis points on average. This result is striking since income increases during the years following the discovery, which indicates that the country has a higher ability to service its debt.

Figure 3: Impact of giant oil discoveries on spreads



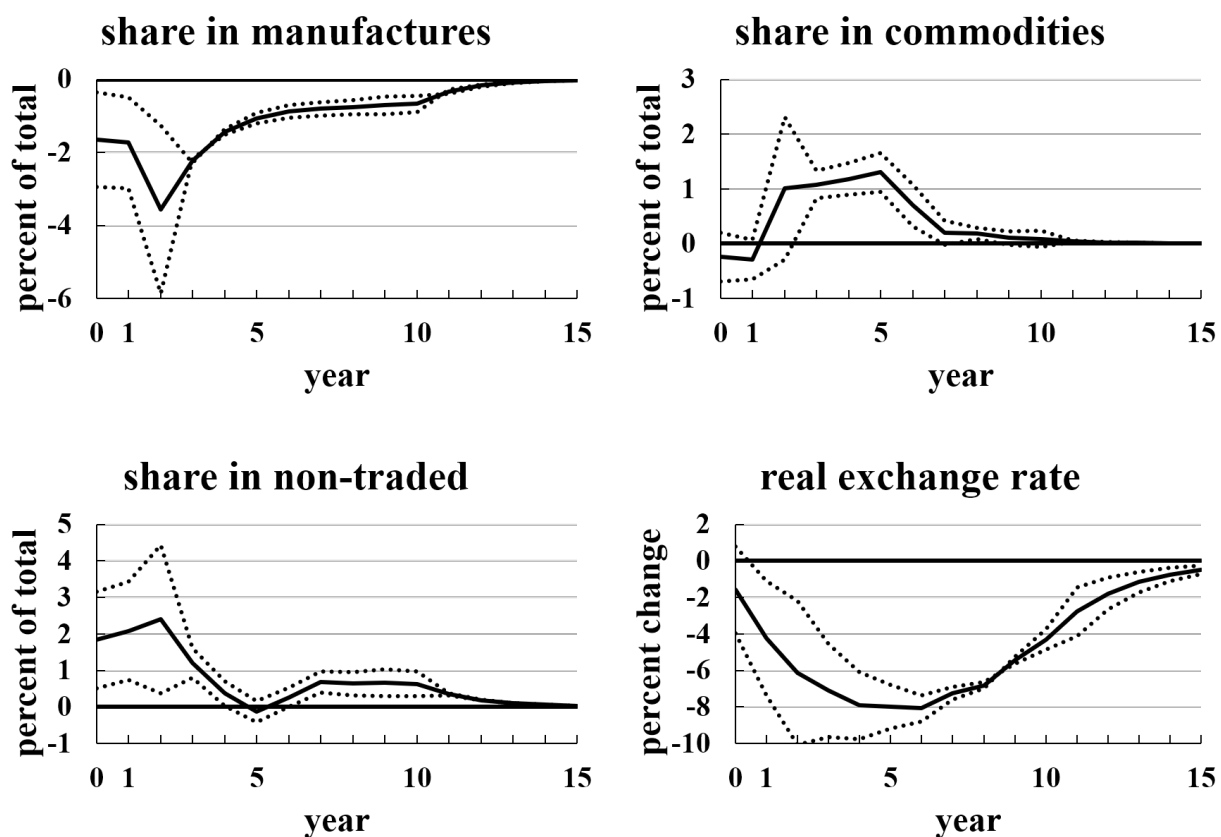
Impulse response to an oil discovery with net present value equal to 18 percent of GDP. The dotted lines indicate 90 percent confidence intervals.

The theoretical model in Section 3 shows how the effects of the Dutch disease can reconcile these empirical observations. Subsection 2.4 shows some evidence of the Dutch disease following oil discoveries.

2.4 The Dutch disease

Figure 4 shows the dynamic response of the real exchange rate, as well as the share of total investment in manufactures, commodities, and non-traded sectors. Commodities comprises agricultural and fishing activities as well as mining and quarrying. The non-traded sector includes construction and wholesale, retail, and logistics services.

Figure 4: Impact of giant oil discoveries on sectoral investment and the RER



Impulse response to an oil discovery with net present value equal to 18 percent of GDP. The dotted lines indicate 90 percent confidence intervals.

Following a discovery the share of investment in the manufacturing sector decreases and the shares in both commodities and non-traded sectors increase. The real exchange rate appreciates, which is in line with the theoretical predictions of the Dutch disease: higher income from the commodity sector increases the consumption of non-traded goods, this in turn increases the price of non-traded goods and production factors crowd out of the manufacturing into non-traded sectors and resource extraction.

3 Model

This section presents a dynamic small open economy model with long-term debt, production in different sectors, capital accumulation, and discovery of natural resources. There is a benevolent government that makes borrowing, investment, and production decisions and cannot commit to repay its debt.¹⁵

3.1 Environment

Goods and technology.—There is a final non-traded good used for consumption and capital accumulation. This good is produced with a constant elasticity of substitution (CES) technology using a bundle of an intermediate non-traded good $c_{N,t}$ and two intermediate traded goods: manufactures $c_{M,t}$ and oil, $c_{oil,t}$:

$$Y_t = \left[\omega_N^{\frac{1}{\eta}} (c_{N,t})^{\frac{\eta-1}{\eta}} + \omega_M^{\frac{1}{\eta}} (c_{M,t})^{\frac{\eta-1}{\eta}} + \omega_{oil}^{\frac{1}{\eta}} (c_{oil,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (2)$$

where η is the elasticity of substitution and ω_i are the weights of each intermediate good i in the production of the final good. The intermediate non-traded good and manufactures are produced

¹⁵In Appendix A I show how all the allocations and default decisions in this economy can be supported in a decentralized economy where households choose labor and capital investment and firms choose capital and labor allocations in different sectors.

using capital k and labor l with constant returns to scale technologies:

$$y_{N,t} = z_t k_{N,t}^{\alpha_N} l_{N,t}^{1-\alpha_N} \quad (3)$$

$$y_{M,t} = z_t k_{M,t}^{\alpha_M} l_{M,t}^{1-\alpha_M} \quad (4)$$

where z_t is aggregate productivity in the economy. Each period the economy is endowed with an oil extraction capacity n_t and oil is produced using n_t , capital and labor:

$$y_{oil,t} = z_t \left(k_{oil,t}^{\alpha_{oil}} l_{oil,t}^{1-\alpha_{oil}} \right)^{1-\zeta} n_t^\zeta \quad (5)$$

where $\zeta \in (0, 1)$ is the share of oil revenue that corresponds to the oil rent.

The resource constraint of the final non-traded good is:

$$c_t + i_{k,t} = Y_t + m_t, \quad (6)$$

where c_t is private consumption, $i_{k,t}$ is investment in capital, Y_t is production of the final non-traded good, and m_t is a transitory income shock described below. At the beginning of each period the economy starts with an aggregate stock of capital k_t that can be freely allocated to each of the three intermediate sectors so that $k_t = k_{N,t} + k_{M,t} + k_{oil,t}$. The law of motion of the aggregate stock of capital is:

$$k_{t+1} = (1 - \delta) k_t + i_{k,t} - \Psi(k_{t+1}, k_t) \quad (7)$$

where $i_{k,t}$ is investment, δ is the capital depreciation rate, and $\Psi(k_{t+1}, k_t) = \chi \left(\frac{k_{t+1} + k_t}{k_t} \right)^2 k_t$ is a capital adjustment cost function.¹⁶ Similarly, given an aggregate labor supply l_t , labor is freely allocated within sectors so that $l_t = l_{N,t} + l_{M,t} + l_{oil,t}$.

Shocks and oil discoveries.—In each period, the economy receives a transitory income shock $m_t \in [-\bar{m}, \bar{m}]$ drawn independently each period from a mean zero probability distribution with continuous CDF.¹⁷ Additionally, in each period the economy experiences one of finitely many

¹⁶Having capital adjustment costs is important in business cycle models to avoid investment being overly volatile, see [Mendoza \(1991\)](#) for a discussion of the case of small open economies. Additionally, as [Gordon and Guerron-Quintana \(2018\)](#) argue, sovereign default models with capital accumulation require capital adjustment costs to sustain positive levels of debt in equilibrium. The reason is that without adjustment costs the cheapest way for the sovereign to hedge against fluctuations is to reduce investment rather than borrowing from the rest of the world.

¹⁷This i.i.d. income shock is included to make robust computation of the model possible. In the calibration, the

events s_t that follow a Markov chain governed by the transition matrix $\pi(s_{t+1}|s_t)$. This shock s_t determines four variables: the aggregate productivity in the economy z_t , the international price of oil $p_{oil,t}$, the oil extraction capacity n_t , and the number of periods since an oil discovery τ_t .¹⁸ For simplicity, I assume that the oil extraction capacity can take one of two values, that is $n_t \in \{n_L, n_H\}$ with $0 \leq n_L < n_H$. The economy starts with oil extraction capacity n_L and with probability ϕ_1 discovers a giant oil field, which becomes available after five periods. That is, if the economy discovers a giant oil field in period j then $n_{j+\tau_t} = n_L$ for $\tau_t = 0 \dots 5$ and then $n_{j+6} = n_H$. This transition is known by all agents and happens with certainty. For $t \geq j + 7$ oil extraction capacity is $n_t = n_H$ and with probability ϕ_0 the giant oil field is exhausted, in which case $n_{t+1} = n_L$.¹⁹

Preferences.—The government has preferences over private consumption c_t and labor supply l_t represented by:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$

where $u(c, l) = \frac{(c - \chi l^\nu)^{1-\sigma} - 1}{1-\sigma}$ and β is the discount factor. Following [Greenwood, Hercowitz, and Huffman \(1988\)](#), I assume preferences such that there is no wealth effect on labor supply. This is a convenient formulation that prevents labor supply in the model from displaying countercyclical fluctuations and that has been used in sovereign default models.²⁰ Also, as discussed by [Jaimovich and Rebelo \(2008\)](#) and [Jaimovich and Rebelo \(2009\)](#), this type of preferences—along with adjustment costs to capital—are essential for business cycle models to generate comovement in response to news shocks about fundamentals. In this environment oil discoveries are akin to news about future higher total factor productivity in the oil sector.

Debt structure.—As in [Chatterjee and Eyigungor \(2012\)](#) the government issues long-term

parameter \bar{m} is chosen so that this shock is relatively small (i.e. the right-hand side of equation (6) is always positive). See [Chatterjee and Eyigungor \(2012\)](#) for a detailed theoretical discussion in an exchange economy, for a discussion of the extension to production economies see [Gordon and Guerron-Quintana \(2018\)](#).

¹⁸I assume that the international price of manufactures is fixed. As it will be discussed in Subsection 3.4, what is key is that the price of oil is relatively more volatile than the price of other traded goods, thus this assumption can be understood as a normalization.

¹⁹The average duration of a giant oil field is 50 years, longer than the time-span in the data. Moreover, as [Arezki, Ramey, and Sheng \(2017\)](#) document, the production rate is highest for the first five years after the field becomes productive and then decreases at a slow rate. A richer model of oil production would include details on the depletion of the reserves on the field through its exploitation. However, the focus of this paper is on the effect of oil discoveries and the transition between discovery and production, not so of the long life-cycle of oil fields. As shown in Section 4, this parameter is set to $\phi_0 = 0.0$ to further simplify the model as if the oil field were perpetual. I did robustness checks for small values of ϕ_0 and the main results remain unchanged.

²⁰See for example [Mendoza and Yue \(2012\)](#) and [Gordon and Guerron-Quintana \(2018\)](#).

bonds that mature probabilistically at a rate γ . Each period, the fraction $1 - \gamma$ of bonds that did not mature pay a coupon κ . The law of motion of bonds is:

$$b_{t+1} = (1 - \gamma)b_t + i_{b,t} \quad (8)$$

where b_t is the number of bonds due at the beginning of period t and $i_{b,t}$ is the amount of bonds issued in period t .²¹ The bonds are denominated in terms of the final good in the rest of the world, which is also the numeraire, and are purchased by risk-neutral, deep-pocketed foreign lenders who have access to a risk free bond that pays an interest rate r^* .

Default and the balance of payments.—At the beginning of every period the government has the option to default. If the government defaults it gets excluded from international financial markets—although it can still trade in goods—for a stochastic number of periods; the government gets re-admitted to financial markets with probability θ and zero debt. While in default productivity is $\tilde{z}_t \leq z_t$ and the transitory income shock is $-\bar{m}$.²² In default, the balance of payments is:

$$0 = p_M x_{M,t} + p_{oil,t} x_{oil,t} \quad (9)$$

where $x_{oil,t} = y_{oil,t} - c_{oil,t}$ and $x_{M,t} = y_{M,t} - c_{M,t}$ are net exports of oil and manufactures, respectively, $p_{oil,t}$ is the international price of oil in period t , and p_M is the international price of manufactures, which is assumed to be constant. Equation (9) implies that in default trade in goods has to be balanced, imports to increase consumption of manufactures (oil) have to be financed by exports of oil (manufactures).

If the government decides to pay its debt obligations then it has access to international financial markets and can issue new debt. In this case, the balance of payments is:

$$[\gamma + (1 - \gamma)\kappa]b_t = p_M x_{M,t} + p_{oil,t} x_{oil,t} + q_t i_{b,t} \quad (10)$$

²¹Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012) have an alternative formulation with no coupon payments ($\kappa = 0$). As Chatterjee and Eyigungor (2012) argue, including the parameter κ is advantageous because it allows to target data on maturity length and debt service separately.

²²Specifically, I follow Gordon and Guerron-Quintana (2018) and set . For a discussion of the advantages of this formulation see Chatterjee and Eyigungor (2012). The income shock is set to its minimal possible value to ease the computation of the equilibrium.

where q_t is the price of newly issued debt. Equation (10) shows how payments of debt obligations (left-hand side) are supported by exports of goods and issuance of new debt.

3.2 Recursive formulation

The state of the economy is the underlying stochastic variable s , the i.i.d. income shock m , the stock of capital k , the outstanding government debt b , and an indicator of whether or not the government is in default.

Let $V(s, m, k, b)$ be the value of the government that starts the period not in default. Then the government chooses to repay its debt obligations, $d = 0$, or to default, $d = 1$:

$$V(s, m, k, b) = \max_{d \in \{0, 1\}} \{ [1 - d] V^P(s, m, k, b) + d V^D(s, k) \}$$

where $V^P(s, m, k, b)$ is the value of repaying and $V^D(s, k)$ is the value of default.

If the government decides to default then its debt obligations are erased and it gets excluded from financial markets. Then, the government chooses the stock of capital next period k' , the labor supply l , static allocations of capital and labor in the current period $\vec{K} = \{k_N, k_M, k_{oil}\}$ and $\vec{L} = \{l_N, l_M, l_{oil}\}$, net exports of manufactures and oil $\vec{X} = \{x_M, x_{oil}\}$, and consumption of final and intermediate goods $\vec{C} = \{c, c_N, c_M, c_{oil}\}$ to solve:

$$V^D(s, k) = \max_{\{k', l, \vec{C}, \vec{L}, \vec{K}, \vec{X}\}} \{ u(c, l) + \beta \mathbb{E} [\theta V(s', m', k', 0) + (1 - \theta) V^D(s', k')] \}$$

subject to the resource constraint of the final good (6), the resource constraint of capital $k_t = k_N + k_M + k_{oil}$, the resource constraint of labor $l_t = l_{N,t} + l_{M,t} + l_{oil,t}$, the law of motion of capital (7), the resource constraints of intermediate goods $c_N = y_N$, $c_{oil} + x_{oil} = y_{oil}$ and $c_M + x_M = y_M$, and the balance of payments under default (9). Note that the government can trade in goods, but trade has to be balanced since it cannot issue debt.

If the government decides to repay then it chooses the stocks of capital k' and debt b' in the next period, the labor supply l , static allocations of capital and labor in the current period $\vec{K} = \{k_N, k_M, k_{oil}\}$ and $\vec{L} = \{l_N, l_M, l_{oil}\}$, net exports of manufactures and oil $\vec{X} = \{x_M, x_{oil}\}$, and

consumption of final and intermediate goods $\vec{C} = \{c, c_N, c_M, c_{oil}\}$ to solve:

$$V^P(s, m, k, b) = \max_{\{k', b', l, \vec{C}, \vec{L}, \vec{K}, \vec{X}\}} \{u(c, l) + \beta \mathbb{E}[V(s', m', k', b')]\}$$

subject to the resource constraint of the final good (6), the resource constraint of capital $k_t = k_N + k_M + k_{oil}$, the resource constraint of labor $l_t = l_{N,t} + l_{M,t} + l_{oil,t}$, the law of motion of capital (7), the law of motion of bonds (8), the resource constraints of intermediate goods $c_N = y_N$, $c_{oil} + x_{oil} = y_{oil}$ and $c_M + x_M = y_M$, and the balance of payments under repayment (10).

3.3 Equilibrium

A Markov equilibrium is value functions $V(s, k, b)$, $V^D(s, k)$, and $V^P(s, k, b)$, a policy function for capital in default $k'^D(s, k)$, a labor supply in default $l^D(s, k)$, policy functions for capital $k'(s, k, b)$ and debt issuance $b'(s, k, b)$ in repayment, a labor supply in repayment $l(s, k, b)$, a default policy function $d(s, k, b)$, policy functions for static allocations in repayment and in default, and a price schedule of bonds $q(s, k', b')$ such that: (i) given the price schedule q , the value and policy functions solve the government's problem, and (ii) given the default policy function, the price schedule is such that lenders break even in expectation:

$$q(s, k', b') = \frac{\mathbb{E}_{m, s' | s} \{[1 - d(s', m', k', b')][\gamma + (1 - \gamma)(\kappa + q(s', k'', b''))]\}}{1 + r^*} \quad (11)$$

where $k'' = k'(s', k', b')$ and $b'' = b'(s', k', b')$ are the government's decisions of future capital and debt in next period's state. Note that, given the i.i.d. nature of the transitory income shock, the price schedule q does not depend on the current realization of m .

3.4 Higher spreads and the Dutch disease

An oil discovery in period t is news that, with certainty, the economy will have a higher level of oil for extraction n in period $t + 6$. With a higher n the economy can achieve a higher level of income. However, as it will be illustrated in Section 5, spreads increase between periods t and $t + 5$, that is, between the announcement of a discovery and when higher oil production starts. In order to

understand why this is the case, I will discuss first what happens in period $t + 6$ and then iterate backwards.

In period $t + 6$, the higher level of n increases the allocation of capital and labor in the oil sector. In order to increase consumption, the allocations of capital and labor in the non-traded sector also increase, thus, the shares of capital and labor in the manufacturing sector decrease. This reallocation of production factors is the Dutch disease in the model. Additionally, the aggregate labor supply increases to take advantage of the higher n

Both higher n and higher labor supply increase the return to capital in period $t + 6$. This higher return drives an increase in investment in period $t + 5$. In order to smooth consumption this higher investment is financed with foreign debt. Investment and debt issuance have opposite effects on spreads: while higher indebtedness raises spreads higher investment reduces them.²³ However, in this environment the effect of higher investment is undermined by the reallocation of production factors away from the manufacturing sector in period $t + 6$. This is because the price of oil is more volatile than the price of manufactures, the reallocation increases the volatility of total traded income used to service the debt, as can be seen in the balance of payments equation:

$$\underbrace{[\gamma + (1 - \gamma) \kappa] b - q(s, k', b') [b' - (1 - \gamma) b]}_{\text{net debt payments}} = \underbrace{p_M [f^M(k_M, l_M) - c_M] + p_{oil} [f^{oil}(k_{oil}, l_{oil}, n) - c_{oil}]}_{\text{income to support debt payments}}$$

where the share of oil revenue on the right-hand side is higher the higher n is. This higher volatility increases the probability of defaulting in period $t + 6$ from the perspective of period $t + 5$. Since debt is long-term, a higher spread in $t + 5$ increases the spreads in $t + 4$, as can be seen in equation (11). This in turn increases spreads in the previous periods up until period t , when the news about the oil discovery is realized.

²³For a detailed discussion of the effect of investment on spreads see [Gordon and Guerron-Quintana \(2018\)](#). They show that investment has non-trivial effects on the equilibrium level of the price of new bonds. On one hand, more capital gives the sovereign the ability to avoid default in bad times by disinvesting to repay debt, which makes spreads decrease with investment; on the other, higher levels of capital increase the value of default in the future, which in turn increases the default set and spreads in the current period. They show that, given a high enough level of indebtedness, the former effect dominates the latter and, everything else constant, sovereign spreads decrease with investment.

4 Calibration

I calibrate the model to the Mexican economy for the period 1993–2012. There are two reasons that make Mexico an ideal example for the purposes of this paper: it is a typical small open emerging economy that has been widely studied in the sovereign debt literature and it did not have any giant oil field discoveries during the period of study. This lack of giant oil discovery allows me to discipline the parameters of the model with business cycle data that does not include variation in endogenous variables induced by giant oil discoveries. I then validate the model by comparing the reaction of model variables to an oil field discovery with the estimates from section 2.

A period in the model is one year. There are two sets of parameters: the first (summarized in table 1) is calibrated directly from data and the second (summarized in table 2) is chosen so that moments generated by the model match their data counterparts. I set the capital shares to $\alpha_N = 0.32$ and $\alpha_M = 0.37$ following [Valentinyi and Herrendorf \(2008\)](#), who calculate labor shares for the US for different sectors and aggregate them into tradable and non-tradable. I set the share of oil rent to $\zeta = 0.38$ and the capital share in the oil sector to $\alpha_{oil} = 0.49$ as in [Arezki, Ramey, and Sheng \(2017\)](#). I use data on sectoral GDP for Mexico between 1993 and 2012 to estimate the elasticity of substitution $\eta = 0.73$.²⁴ I set the weights $\omega_N = 0.79$, $\omega_M = 0.15$, and $\omega_{oil} = 0.06$ using aggregate consumption shares. I set the relative risk aversion parameter to $\sigma = 2$ and the risk free interest rate to $r^* = 0.04$, which are standard values in the literature. I set the curvature of the disutility of labor to $\nu = 1.45$ so that the Frisch elasticity of labor supply is $\frac{1}{\nu-1} = 2.2$.²⁵ I set the scaling parameter of the disutility of labor to $\chi = 0.8$ so that the labor supply in the steady state of the economy without default is normalized to 1.

²⁴To estimate the elasticity of substitution I follow the methodology used by [Stockman and Tesar \(1995\)](#). As discussed by [Mendoza \(2005\)](#) and [Bianchi \(2011\)](#), the range of estimates for the elasticity of substitution between tradables and non-tradables is between 0.40 and 0.83.

²⁵This value is consistent with estimates for the United States and has been used in similar business cycle models, see for example [Neumeyer and Perri \(2005\)](#) and [Mendoza and Yue \(2012\)](#).

Table 1: Parameters calibrated directly from the data

Parameter		Value	Source
Capital shares	α_N	0.32	Valentinyi and Herrendorf (2008)
	α_M	0.37	
	α_{oil}	0.49	Arezki, Ramey, and Sheng (2017)
Oil rent	ζ	0.38	
Elasticity of substitution	η	0.73	estimated for Mexico
Intermediate output shares	ω_N	0.79	shares in aggregate consumption
	ω_M	0.15	
	ω_{oil}	0.06	
Risk aversion	σ	2	standard values
Risk free rate	r^*	0.04	
Curvature parameter of labor	ν	1.45	Frisch elasticity of 2.2
Scaling parameter of labor	χ	0.80	Steady state labor supply equal 1
Bonds maturity rate	γ	0.14	7 year average duration
Probability of reentry	θ	0.40	2.5 years exclusion
Probability of giant oil discovery	ϕ_1	0.13	average for all countries perpetual oil well
Probability of exhaustion	ϕ_0	0.00	
Support of i.i.d. shock	\bar{m}	0.018	Chatterjee and Eyigungor (2012) bound is +/- 2 standard deviations
Standard deviation of i.i.d. shock	σ_m	0.009	
Persistence of price of oil	ρ_{oil}	0.48	AR(1) estimation of innovations in the real price of oil
Volatility of price of oil	v_p^2	0.23	
Mean price of oil	\bar{p}_{oil}	1.0	normalization
Price of manufactures	p_M	1.0	

I normalize the international price of manufactures to $p_M = 1$ and set the average price of oil also to $\bar{p}_{oil} = 1$. As discussed in Subsection 3.4, what is key is that the price of oil is relatively more volatile than the price of manufactures. For simplicity I assume one is volatile and one is not, assuming that the mean of the price of oil is the same as the price of manufactures highlights the role of higher volatility. I model changes in the price of oil to follow an AR(1) process:

$$\Delta \log p_{oil,t} = \rho_{oil} \Delta \log p_{oil,t-1} + v_p \varepsilon_t \quad (12)$$

where $\log p_{oil,t}$ is the natural logarithm of the price of oil and ε_t are iid with standard normal distribution. I use data on the West Texas Intermediate price of oil divided by the US GDP deflator to calculate a real price and estimate the persistence parameter in (12) $\rho_{oil} = 0.48$ and the variance

of the iid shock is $v_p^2 = 0.23$. Then I use these estimates and the normalized average price of oil \bar{p}_{oil} to approximate the process with a finite state Markov-chain using the Rouwenhorst method.²⁶

I set the probability of discovering a giant oil field to $\phi_1 = 0.13$, which is the unconditional probability of discovery in a given year in the data for all countries. The probability of exhaustion is $\phi_0 = 0.0$. The probability of re-entry to financial markets is $\theta = 0.40$, so that the average duration of exclusion is 2.5 years, as documented for recent years by [Gelos, Sahay, and Sandleirs \(2011\)](#).

For the transitory income shock m I follow [Chatterjee and Eyigungor \(2012\)](#) and assume $m \sim \text{trunc } N(0, \sigma_m^2)$ with points of truncation $-\bar{m}$ and \bar{m} . I set $\bar{m} = 0.018$ and $\sigma_m = 0.009$. For [Chatterjee and Eyigungor \(2012\)](#) these values are 0.006 and 0.003, respectively. I re-scale these values so that the relative size between the persistent and the maximum transitory component of income remains the same. The productivity shock is assumed to follow an AR(1) process:

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z v_t$$

where v_t are iid with standard normal distribution. The persistence ρ_z and variance σ_z are calibrated to match the persistence and volatility of the business cycle of the Mexican GDP (more details below).

Table 2 summarizes the parameters calibrated by simulating the model. This calibration is made in two steps: first, all parameters except n_H are chosen to match some business cycle moments for the Mexican economy in the period 1993–2012. This first step only considers simulated economies that do not have an oil discovery, as was the case for Mexico in those years. The second step considers all simulated economies and calculates oil production as a fraction of GDP in economies with a giant oil discovery to discipline n_H .

²⁶Method first proposed by [Rouwenhorst \(1995\)](#), for a discussion on its properties see [Kopecky and Suen \(2010\)](#).

Table 2: Parameters calibrated simulating the model

Parameter	Value	Parameter	Value
Discount factor	β 0.925	Persistence of productivity	ρ_z 0.93
Productivity default	d_0 -0.25	Volatility of productivity	σ_z 0.025
cost	d_1 0.312	Coupon rate	κ 0.045
Capital depreciation rate	δ 0.10	Low oil for extraction	n_L 0.05
Capital adjustment cost	ψ 3.5	High oil for extraction	n_H 0.50
Moment		Data	Model
Average spread		266	220
St. dev. of spreads		134	127
Debt-to-GDP ratio		0.13	0.11
Debt service / GDP		0.05	0.03
Capital-to-GDP ratio		1.9	2.0
$\sigma_{inv}/\sigma_{GDP}$		2.3	2.1
ρ_{GDP}		0.30	0.34
σ_{GDP}		2.22	2.22
100 * Oil GDP / GDP		0.07	0.08
$\frac{NPV}{GDP}$		19	22

There is a total of ten parameters chosen to match ten moments from the data: the average and standard deviation of spreads, the debt-to-GDP ratio, the debt service-to-GDP ratio, the capital-to-GDP ratio, the relative volatility of investment to GDP, the persistence and variance of GDP, and the oil GDP as a fraction of total GDP with n_L and n_H .

The value of the discount factor β mainly determines the debt-to-GDP ratio. The average and standard deviation of spreads are mainly pinned down by the default cost parameters d_0 and d_1 . The debt service-to-GDP ratio is mostly affected by the value of the coupon payment parameter κ . The capital-to-GDP ratio and the relative volatility of investment are mostly determined by the capital depreciation rate δ and the capital adjustment cost parameter ψ , respectively. The values of ρ_{GDP} and σ_{GDP} are estimated using data of the cyclical component of Mexican GDP and GDP series generated by the model. Both are HP Filtered with a smoothing parameter of 100. These values for the data simulated by the model are pinned down by the persistence ρ_z and variance σ_z of the productivity shock. Finally, the parameters n_L and n_H determine the fraction of oil production to GDP. I choose n_L to match the oil production as a percentage of GDP in Mexico between 1993 and 2012, which is 7 percent on average. The value of n_H is set to match the average net present

value of oil discoveries as a fraction of GDP. These net present values are calculated as:

$$NPV_t = \sum_{s=6}^{\infty} \left(\frac{1}{1+r_t} \right)^s p_{oil,t} [y_{oil}(s_t, k_t, b_t, n_H) - y_{oil}(s_t, k_t, b_t, n_L)] \quad (13)$$

where r_t is the implied yield of the government bonds at time t and $y_{oil}(s_t, k_t, b_t, n_H)$ and $y_{oil}(s_t, k_t, b_t, n_L)$ are what oil production in period t would be with n_H and n_L , respectively, given the current state.

Table 3 shows the performance of the model with non-targeted moments. The model does well with the over-volatility of consumption relative to output. The model also does a good job producing countercyclical spreads and trade balance, as well as predicting a lower correlation between investment and output relative to that of output and consumption. However, the volatility of the trade balance and the real exchange rate are not well reproduced by the model.²⁷

Table 3: Not targeted moments

Moment	Data	Model
σ_c / σ_{GDP}	1.11	1.14
σ_{TB}	4.07	1.06
σ_{RER}	6.60	2.90
$Corr(c, GDP)$	0.85	0.92
$Corr(inv, GDP)$	0.81	0.49
$Corr(TB, GDP)$	-0.03	-0.29
$Corr(spreads, GDP)$	-0.37	-0.47
$Corr(RER, GDP)$	-0.12	-0.02

The following section shows the model's predictions after an oil discovery, with special focus on the model's ability to reproduce the increase in spreads.

5 Quantitative Results

This section presents the main quantitative results of the model. First, Subsection 5.1 shows the predictions of the model following an oil discovery and how it explains one third of the increase in

²⁷Appendix A shows how the equilibrium allocations can be supported as the equilibrium of a decentralized economy. In that case, the real exchange rate is calculated as $\frac{1}{P}$ where P is the price of the final non-traded good.

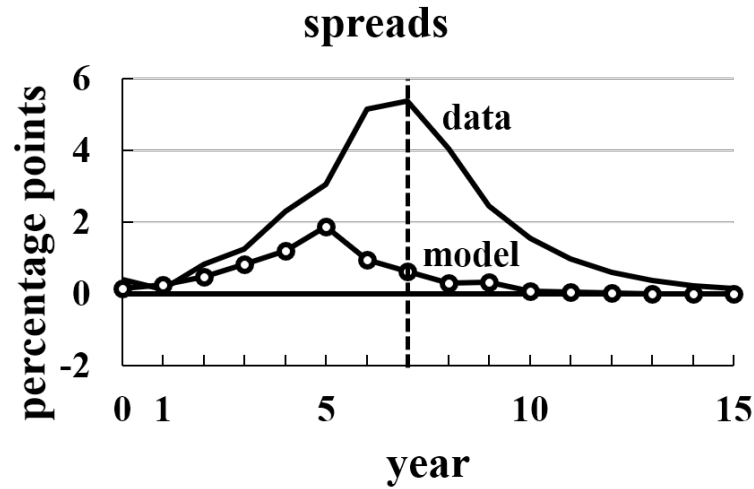
spreads. Then, Subsection 5.2 explains the procedure to calculate the welfare gains of eliminating the volatility of the price of oil and presents the results.

5.1 Response to an oil discovery

First, I generate a panel of model generated time series as follows: (i) simulate 300 economies for 5010 periods without allowing for oil discoveries, (ii) drop the first 5000 to eliminate any effect of initial conditions, (iii) allow for random oil discoveries after period 5011 and simulate 50 more periods, (iv) for each economy that experienced an oil discovery calculate the net present value of said discovery as in (13).

Then, I estimate the same regression (1) for spread using these model generated data. Figure 3 compares the impulse-response of spreads estimated with model data to that estimated from the actual data in Figure 3.

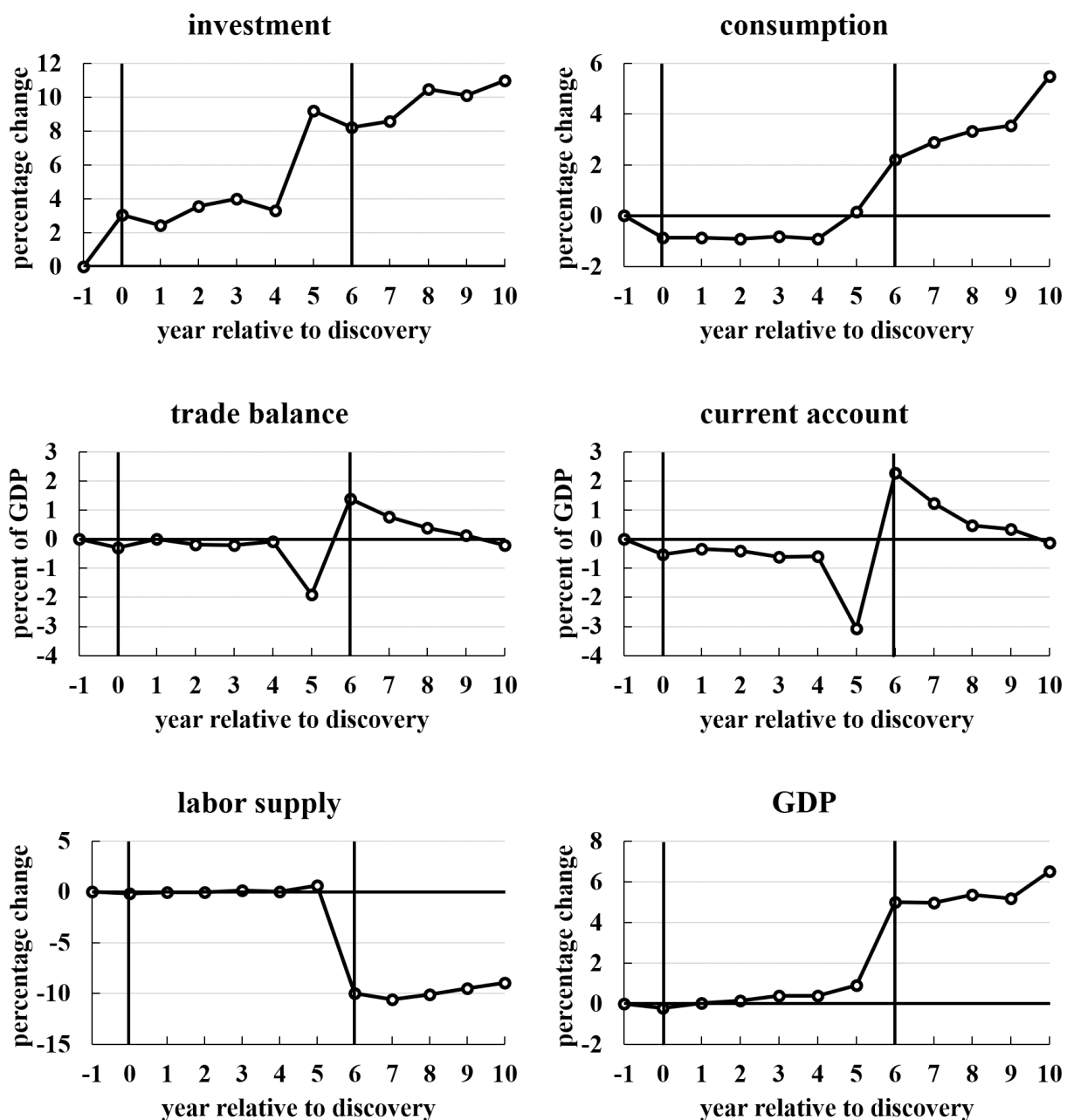
Figure 5: Impulse-response of spreads to a discovery of average size



In the data, spreads start increasing when the news of the discovery is realized and continue to increase until they peak in year 7, where they reach a maximum increase of 5.3 percentage points. In the model under the benchmark calibration spreads also increase when the news is realized and

continue to do so until period 5, exactly one period before the larger oil field is available. The model explains around one third of the increase in spreads following an oil discovery.²⁸

Figure 6: Impulse-response of other macroeconomic variables after discovery in time $t = 0$



²⁸That is, the integral under the curve labeled “model” in Figure 5 up until the seventh period is approximately one third of the integral under the solid line labeled “data”.

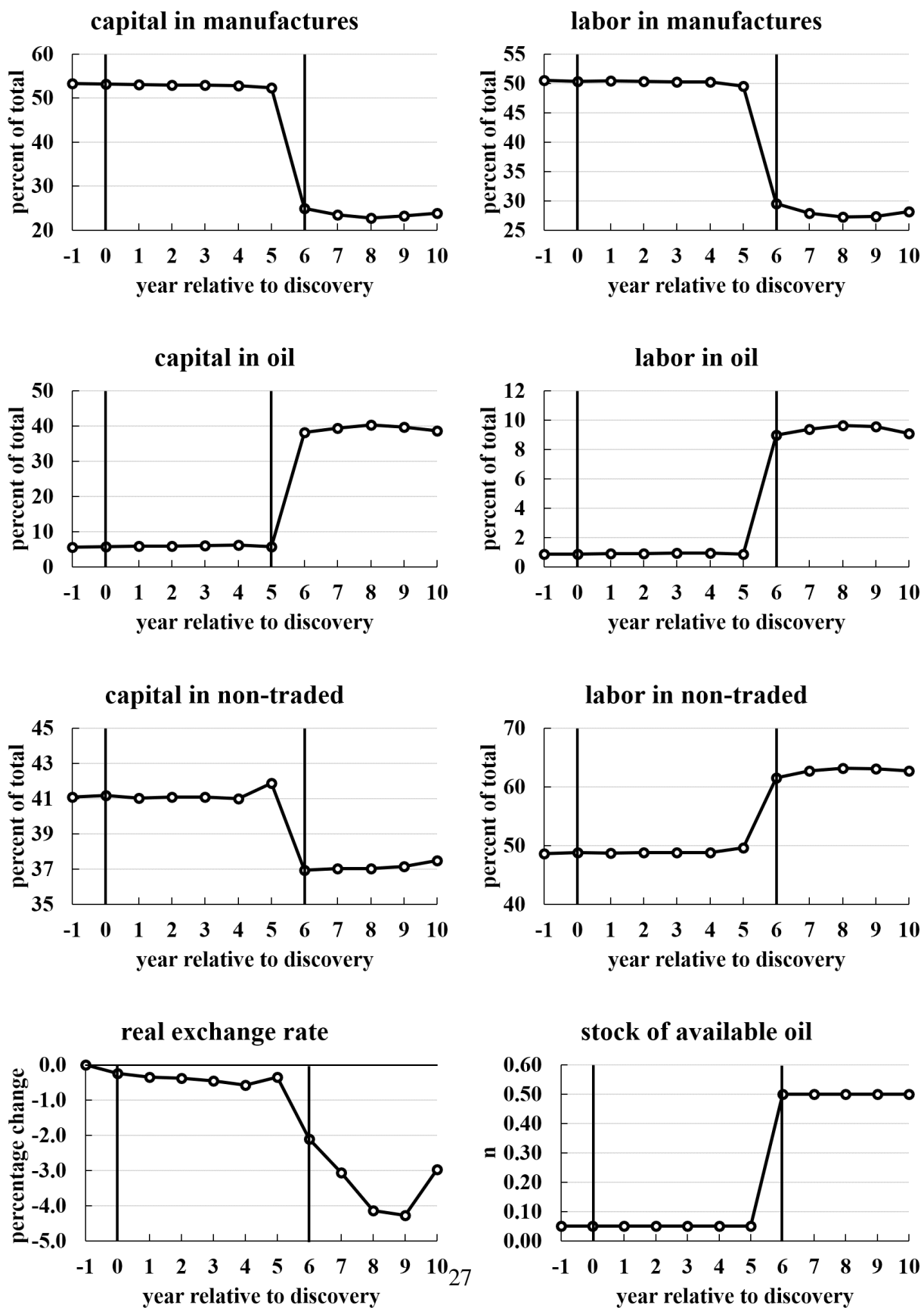
To illustrate the co-movement of endogenous variables that generate this increase in spreads Figure 6 shows the average impulse-response functions of investment, consumption, the trade balance, the current account, labor supply, and GDP. To calculate these impulse-response functions I do the following: (i) focus on the model generated economies that had an oil discovery, (ii) center the time-series so that the discovery happens at $t = 0$, (iii) average each period across all of these economies to get the average path, (iv) compute the impulse-response functions of the average paths relative to period $t = -1$.

As discussed in Subsection 3.4, investment increases in anticipation of the higher available oil for extraction n in period 6. The increase is supported by imports financed with debt issuance, both reflected on the current account and trade balance deficits. GDP and consumption only start increasing until period 5 and continue to do so in the subsequent periods.

The increase in investment and the current account deficit have opposite effects on spreads, however, the effect of the latter dominates due to the higher volatility of tradable income starting in period 6. This higher volatility is explained by a reallocation of production factors away from the manufacturing sector, which has a constant international price, into the oil sector, which has a volatile price. This factor reallocation is illustrated in Figure 7, which shows the impulse-response functions of the shares of capital and labor in each sector, as well as the impulse-response of the real exchange rate and available oil for extraction n .

In period 6, once a higher n is available, the model features a reallocation of production factors away from the manufacturing sector. The share of labor allocated in the oil and non-traded sectors increase, as well as the share of capital in the oil sector. The share of capital in the non-traded sector actually decreases, this is a result of this sector being less intensive in the use of capital.

Figure 7: Impulse-response of real exchange rate and path of capital shares after discovery in time $t = 0$



5.2 Welfare analysis

The purpose of this subsection is to calculate the welfare cost of the volatility of the price of oil. I interpret this calculation as an indication of how valuable it is for a country to have access to financial instruments that allow it to hedge against oil price risk.

I calculate the welfare cost as the fraction of consumption, in all possible histories, that the economy is willing to forego in order to eliminate the volatility of the price of oil. Denote \mathcal{E}^{Bench} as all the equilibrium objects under the benchmark calibration from Section 4. Similarly, denote \mathcal{E}^{NoRisk} as all the equilibrium objects under the same calibration but with no volatility for the price of oil.

Let $V^{bench}(s, k, b)$ and $V^{NoRisk}(s, k, b)$ be the values of the government in good financial standing from \mathcal{E}^{Bench} and \mathcal{E}^{NoRisk} , respectively. When the state is (s, k, b) , let $\chi(s, k, b)$ be the fraction of consumption—net of labor disutility—that the government is willing to sacrifice to eliminate the volatility of the price of oil if it is in good financial standing. Since the utility function is homothetic in consumption net of disutility of labor, $\chi(s, k, b)$ is pinned down by:

$$(1 - \chi(s, k, b))^{1-\sigma} [V^{NoRisk}(s, k, b) + \mathcal{C}] = V^{bench}(s, k, b) + \mathcal{C} \quad (14)$$

where $\mathcal{C} = \frac{1}{1-\sigma} \frac{1}{1-\beta}$ is a constant. Similarly, denote $V^{D,bench}(s, k)$ and $V^{D,NoRisk}(s, k)$ as the default values of the government from \mathcal{E}^{Bench} and \mathcal{E}^{NoRisk} , respectively. Define $\chi^D(s, k)$ in a similar fashion:

$$(1 - \chi^D(s, k))^{1-\sigma} [V^{D,NoRisk}(s, k) + \mathcal{C}] = V^{D,bench}(s, k) + \mathcal{C} \quad (15)$$

where $\mathcal{C} = \frac{1}{1-\sigma} \frac{1}{1-\beta}$.

To calculate a welfare cost χ_j I simulate a time series of $2500 + T$ periods for productivity, price of oil, and oil extraction capacity $\{(z_t, p_{oil,t}, n_t)\}_{t=1}^{2500+T}$. I use $\{(z_t, p_{oil,t}, n_t)\}_{t=1}^{2500+T}$ and the policy functions from \mathcal{E}^{Bench} to generate paths of values for the benchmark economy $\{V_t^{bench}\}_{t=1}^{2500+T}$, taking into account when the economy enters and exits default in the equilibrium path. Similarly, I set $p_{oil,t} = \bar{p}_{oil}$ for all t and use $\{(z_t, n_t)\}_{t=1}^{2500+T}$ and the policy functions from \mathcal{E}^{NoRisk} to generate

paths of values for the economy with no oil risk $\{V_t^{NoRisk}\}_{t=1}^{2500+T}$, also taking into account when the economy enters and exits default in the equilibrium path. I use these paths to calculate a path of welfare costs $\{\chi_t\}_{t=1}^{2500+T}$. I drop the first 2500 to eliminate the effect of initial conditions and average the rest to get the welfare cost χ_j .

I make two welfare exercises. For each of the three I calculate 300 welfare costs $\{\chi_j\}_{j=1}^{300}$ and average them. The first exercise considers an economy that has a low oil extraction capacity. Note that, even if n_L were equal to 0 the economy would still be exposed to oil price risk since oil is consumed. I follow the procedure described above but setting $n_t = n_L$ in all periods. The second exercise allows for oil discoveries. Table 4 shows the results.

Table 4: Welfare analysis

Exercise	Welfare cost
No discoveries	0.4%
With discoveries	0.6%

An economy that never experiences a discovery is willing to forego 0.4% of consumption permanently in order to eliminate the volatility of the price of oil. Conditional on discovery this figure is higher, 0.6%. Given that preferences are concave, eliminating any source of consumption volatility is always desirable, however, eliminating this particular source of volatility is 1.5 times more valuable for an economy that has an oil discovery than for an economy that does not. Note that even without any endowment of oil, the economy is still exposed to swings in its price since it is a consumed commodity. When the economy is highly indebted, the factor reallocation implied by the Dutch disease increases the probability of states in which the economy would default. This higher probability of experiencing the dead-weight loss implied by default explains the higher value of eliminating the oil price risk.

6 Conclusion

In this paper I documented the effect of giant oil field discoveries on sovereign spreads, the sectoral allocation of capital and macroeconomic aggregates. Following a giant oil discovery sovereign

spreads increase by up to 530 basis points and the share of investment in manufactures decreases in favor of investment in mining and non-traded sectors, which is accompanied by an appreciation of the real exchange rate. Countries run a current account deficit to finance imports. GDP, investment, and consumption increase.

I developed a sovereign default model with production in three sectors, investment, and discovery of oil fields. The model accounts for all the effects documented from the data. In the model, the share of capital in the oil and non-traded sectors increase and the share of capital in the manufacturing sector decreases. This shift—referred to in the literature as the Dutch disease—increases the volatility of exports that support debt payments since the price of oil is more volatile than the price of manufactures.

It is worth noting that, in the model, there is nothing inefficient about the Dutch disease (i.e. it is not really a disease). The reallocation of production factors is the optimal response to the increase in the availability of natural resources, which takes into account the implied increase in the cost of borrowing.

I interpret the model's ability to jointly reproduce these effects as a validation of the hypothesis that the future reallocation of production factors is a key determinant of the change in sovereign spreads following an oil discovery.

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A Decentralized economy

This Appendix shows how the allocations from the economy in Section 3 can be decentralized by an economy with a representative household, a government, and competitive firms. First I layout the environment and then I prove an equivalence result that is akin to a first welfare theorem.

A.1 Environment

Final Good.—There is a competitive firm that assembles the final non-traded good Y_t from the intermediate non-traded good $c_{N,t}$, manufactures $c_{M,t}$, and oil $c_{oil,t}$ and sells it to the representative household at price P_t . The firm has access to the technology:

$$Y_t = f^Y(c_{N,t}, c_{M,t}, c_{oil,t}) = \left[\omega_N^{\frac{1}{\eta}} (c_{N,t})^{\frac{\eta-1}{\eta}} + \omega_M^{\frac{1}{\eta}} (c_{M,t})^{\frac{\eta-1}{\eta}} + \omega_{oil}^{\frac{1}{\eta}} (c_{oil,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where η is the elasticity of substitution and ω_i are the weights of each intermediate good i in the production of the final good. The firm purchases manufactures and oil in international markets at prices p_M and $p_{oil,t}$ and purchases the intermediate non-traded good at price $p_{N,t}$ from a domestic producer. Cost minimization implies the demands for intermediate goods are:

$$\begin{aligned} c_{N,t} &= \left(\frac{P_t}{p_{N,t}} \right)^{\eta} Y_t \omega_N \\ c_{M,t} &= \left(\frac{P_t}{p_{M,t}} \right)^{\eta} Y_t \omega_M \\ c_{oil,t} &= \left(\frac{P_t}{p_{oil,t}} \right)^{\eta} Y_t \omega_{oil} \end{aligned}$$

and since the firm is competitive the price of the final good equals its marginal cost:

$$P = \left[\omega_N (p_{N,t})^{1-\eta} + \omega_M (p_{M,t})^{1-\eta} + \omega_{oil} (p_{oil,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

Intermediate Goods.—Manufactures $y_{M,t}$, oil $y_{oil,t}$, and the intermediate non-traded good $y_{N,t}$

are produced by competitive firms with access to technologies:

$$\begin{aligned} y_{N,t} &= f^N(z_t, k_{N,t}, l_{N,t}) \\ y_{M,t} &= f^M(z_t, k_{M,t}, l_{M,t}) \\ y_{oil,t} &= f^{oil}(z_t, k_{oil,t}, l_{oil,t}, n_t) \end{aligned}$$

where f^N and f^M are homogeneous of degree 1 in k and l and f^{oil} is homogeneous of degree 1 in k , l and n . Each period, these firms rent capital $k_{i,t}$ and labor $l_{i,t}$ from the household in exchange for a rental rate of capital r_t and a wage w_t . The manufacturing and oil firms sell their product in international markets at prices $p_{M,t}$ and $p_{oil,t}$ and the non-traded firm sells its product to the domestic final good firm at price $p_{N,t}$. Note that f^{oil} has decreasing returns to scale with respect to k and l , thus it will make positive profits. The representative household owns all the firms and get these profits which can be interpreted as the oil rent—that is, the payments for the natural resources factor n , which are assumed to be owned by the household as well.

Households.—There is a representative household with preferences over consumption c_t and labor supply l_t represented by:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$

where $u(c, l) = \frac{(c - \chi l^\nu)^{1-\sigma} - 1}{1-\sigma}$ and β is the discount factor. The household owns all the firms and makes consumption, investment and labor supply decisions subject to its budget constraint:

$$c_t + (1 + \tau_{k,t}) k_{t+1} + \Psi(k_{t+1}, k_t) \leq (1 - \tau_{l,t}) \frac{w_t}{P_t} l_t + \left(1 + \frac{r_t}{P_t} - \delta \right) k_t + \frac{\pi_t^N + \pi_t^M + \pi_t^{oil}}{P_t} + m_t + T_t$$

where $\tau_{k,t}$ and $\tau_{l,t}$ are distortionary taxes, T_t are transfers from the government, m_t is a small income shock, and π_t^N , π_t^M and π_t^{oil} are profits from the intermediate goods firms.²⁹

Government.—There is a benevolent government that can issue long term debt in international financial markets and lacks commitment to repay. The law of motion for debt is:

$$b_{t+1} = (1 - \gamma) b_t + i_{b,t}$$

²⁹Note that in equilibrium only π_t^{oil} will be positive if $n_t > 0$.

where γ is the fraction of debt that matures each period, b_t is the stock of debt in period t and $i_{b,t}$ is new debt issuances. At the beginning of each period the government chooses whether to default or not. If the government defaults then productivity in the economy is $\tilde{z}_t = z_t - \max \{d_0 z_t + d_1 z_t^2\}$. After default, the government is excluded from financial markets and is readmitted with probability θ . If the government repays then it can issue new debt. Regardless of default or repayment, the government has access to distortionary taxes $\tau_{k,t}$, and $\tau_{l,t}$ and lump-sum transfers T_t to influence the decisions of the households. The government maximizes the representative household's utility subject to its budget constraint. If the government is in good financial standing then its budget constraint is:

$$P_t \tau_{k,t} k_{t+1} + \tau_{l,t} w_t l_t + q_t (s_t, k_{t+1}, b_{t+1}) [b_{t+1} - (1 - \gamma) b_t] = P_t T_t + [\gamma + (1 - \gamma) \kappa] b_t$$

where $(1 - \gamma) \kappa b_t$ are the coupon payments for the outstanding debt. If the government decides to default then its budget constraint is:

$$P_t \tau_{k,t} k_{t+1} + \tau_{l,t} w_t l_t = P_t T_t$$

and gets readmitted to financial markets with probability θ and zero debt.

A.2 Equivalence result

In this subsection I prove that the allocations that characterize the equilibrium of the economy in Section 3 can be decentralized by the market economy described above. I do this in two steps: first, I show that, given the state and the dynamic decisions, the static allocations in each period are the same. Then I show that the dynamic problems are the same.

The recursive formulation of the problem of the government in Section 3 is:

$$V(s, m, k, b) = \max_{d \in \{0,1\}} \{ [1 - d] V^P(s, m, k, b) + d V^D(s, k) \}$$

where the value in repayment is:

$$\begin{aligned}
V^P(s, m, k, b) &= \max_{\{k', b', l, \vec{C}, \vec{L}, \vec{K}, \vec{X}\}} \{u(c, l) + \beta \mathbb{E}[V(s', m', k', b')]\} \\
s.t. \quad c + k' + \phi \left(\frac{k' - k}{k} \right)^2 k &\leq Y + (1 - \delta)k + m \\
Y &= f^Y(c_N, c_M, c_{oil}) \\
c_N &= f^N(z, k_N, l_N) \\
c_M &= f^M(z, k_M, l_M) - x_M \\
c_{oil} &= f^{oil}(z, k_{oil}, l_{oil}, n) - x_{oil} \\
q(s, k', b') [b' - (1 - \gamma)b] &= p_M x_M + p_{oil} x_{oil} + [\gamma + \kappa(1 - \gamma)]b \\
k &= k_N + k_M + k_{oil} \\
l &= l_N + l_M + l_{oil}
\end{aligned}$$

and the value in default is:

$$\begin{aligned}
V^D(s, k) &= \max_{\{k', l, \vec{C}, \vec{L}, \vec{K}, \vec{X}\}} \{u(c, l) + \beta \mathbb{E}[\theta V(s', m', k', 0) + (1 - \theta)V^D(s', k')]\} \\
s.t. \quad c + k' + \phi \left(\frac{k' - k}{k} \right)^2 k &\leq Y + (1 - \delta)k - \bar{m} \\
Y &= f^Y(c_N, c_M, c_{oil}) \\
c_N &= f^N(\tilde{z}, k_N, l_N) \\
c_M &= f^M(\tilde{z}, k_M, l_M) - x_M \\
c_{oil} &= f^{oil}(\tilde{z}, k_{oil}, l_{oil}, n) - x_{oil} \\
0 &= p_M x_M + p_{oil} x_{oil} \\
k &= k_N + k_M + k_{oil} \\
l &= l_N + l_M + l_{oil}
\end{aligned}$$

Note that these problems can be rewritten as:

$$\begin{aligned}
V^P(s, m, k, b) &= \max_{\{k', b', l, \vec{C}, \vec{L}, \vec{K}, \vec{X}\}} \{u(c, l) + \beta \mathbb{E}[V(s', m', k', b')]\} \\
s.t. \quad c + k' + \phi \left(\frac{k' - k}{k} \right)^2 k &\leq F(s, l, k, X) + (1 - \delta)k + m \\
X &= q(s, k', b') [b' - (1 - \gamma)b] - [\gamma + \kappa(1 - \gamma)]b
\end{aligned}$$

and:

$$V^D(s, k) = \max_{\{k', l, \vec{c}, \vec{l}, \vec{k}, \vec{x}\}} \{u(c, l) + \beta \mathbb{E} [\theta V(s', m', k', 0) + (1 - \theta) V^D(s', k')]\}$$

$$s.t. \quad c + k' + \phi \left(\frac{k' - k}{k} \right)^2 k \leq F^D(s, l, k) + (1 - \delta)k - \bar{m}$$

where $F(s, k, b, X)$ and $F^D(s, k)$ summarize all the static allocations given the state and the choices of (l, k', b') . In repayment F is defined as:

$$F(s, l, k, X) = \max_{c_N, c_M, c_{oil}, k_N, l_N, k_M, l_M, k_{oil}, l_{oil}, x_{oil}, x_M} f^Y(c_N, c_M, c_{oil})$$

$$s.t. \quad c_N = f^N(z, k_N, l_N)$$

$$c_M = f^M(z, k_M, l_M) - x_M$$

$$c_{oil} = f^{oil}(z, k_{oil}, l_{oil}, n) - x_{oil}$$

$$X = p_M x_M + p_{oil} x_{oil}$$

$$k = k_N + k_M + k_{oil}$$

$$l = l_N + l_M + l_{oil}$$

and in default F^D is defined as:

$$F^D(s, l, k) = \max_{c_N, c_M, c_{oil}, k_N, l_N, k_M, l_M, k_{oil}, l_{oil}, x_{oil}, x_M} f^Y(c_N, c_M, c_{oil})$$

$$s.t. \quad c_N = f^N(\tilde{z}, k_N, l_N)$$

$$c_M = f^M(\tilde{z}, k_M, l_M) - x_M$$

$$c_{oil} = f^{oil}(\tilde{z}, k_{oil}, l_{oil}, n) - x_{oil}$$

$$0 = p_M x_M + p_{oil} x_{oil}$$

$$k = k_N + k_M + k_{oil}$$

$$l = l_N + l_M + l_{oil}$$

In repayment, the first-order conditions that characterize the static allocations are:

$$f_{c_N}^Y(c_N, c_M, c_{oil}) = \lambda_{c_N} \quad (16)$$

$$f_{c_M}^Y(c_N, c_M, c_{oil}) = \lambda_{c_M} \quad (17)$$

$$f_{c_{oil}}^Y(c_N, c_M, c_{oil}) = \lambda_{c_{oil}} \quad (18)$$

$$f_k^N(z, k_N, l_N) = \frac{\lambda_k}{\lambda_{c_N}} \quad (19)$$

$$f_k^M(z, k_M, l_M) = \frac{\lambda_k}{\lambda_{c_M}} \quad (20)$$

$$f_k^{oil}(z, k_{oil}, l_{oil}, n) = \frac{\lambda_k}{\lambda_{c_{oil}}} \quad (21)$$

$$f_l^N(z, k_N, l_N) = \frac{\lambda_l}{\lambda_{c_N}} \quad (22)$$

$$f_l^M(z, k_M, l_M) = \frac{\lambda_l}{\lambda_{c_M}} \quad (23)$$

$$f_l^{oil}(z, k_{oil}, l_{oil}, n) = \frac{\lambda_l}{\lambda_{c_{oil}}} \quad (24)$$

$$\lambda_{c_{oil}} = \frac{p_{oil}}{p_M} \lambda_{c_M} \quad (25)$$

$$\lambda_{BoP} = \frac{\lambda_{c_M}}{p_M} \quad (26)$$

where λ_{c_N} , λ_{c_M} , $\lambda_{c_{oil}}$, λ_k , λ_l , and λ_{BoP} are the multipliers of the market clearing constraints for intermediate goods, for capital, for labor, and for the balance of payments, respectively. Note that equations (25) and (26) already pin down $\lambda_{c_{oil}}$ and λ_{BoP} in terms of λ_{c_M} and the international prices p_M and p_{oil} . Thus, we are left with a system of 9 first-order conditions plus 6 constraints to solve for eleven static allocations c_N , c_M , c_{oil} , k_N , l_N , k_M , l_M , k_{oil} , l_{oil} , x_{oil} , and x_M and 4 multipliers λ_{c_N} , λ_{c_M} , λ_k , and λ_l .

Now, in the market economy the final good firm solves:

$$\begin{aligned} \min_{c_N, c_M, c_{oil}} \quad & p_N c_N + p_M c_M + p_{oil} c_{oil} \\ \text{s.t.} \quad & Y \leq f^Y(c_N, c_M, c_{oil}) \end{aligned}$$

and the intermediate goods firms solve:

$$\max_{k_N, l_N} f^N(z, k_N, l_N) - rk_N - wl_N$$

$$\max_{k_M, l_M} f^M(z, k_M, l_M) - rk_M - wl_M$$

$$\max_{k_{oil}, l_{oil}} f^{oil}(z, k_{oil}, l_{oil}, n) - rk_{oil} - wl_{oil}$$

The 11 static allocations $c_N, c_M, c_{oil}, k_N, l_N, k_M, l_M, k_{oil}, l_{oil}, x_{oil}$ and x_M , 3 endogenous prices p_N, w , and r , and the multiplier μ^Y of the constraint in the minimization problem of the final good firm are pinned down by the 10 F.O.C.s of these problems:

$$f_{c_N}^Y(c_N, c_M, c_{oil}) = \frac{p_N}{\mu^Y} \quad (27)$$

$$f_{c_M}^Y(c_N, c_M, c_{oil}) = \frac{p_M}{\mu^Y} \quad (28)$$

$$f_{c_{oil}}^Y(c_N, c_M, c_{oil}) = \frac{p_{oil}}{\mu^Y} \quad (29)$$

$$f_k^N(z, k_N, l_N) = r \quad (30)$$

$$f_l^N(z, k_N, l_N) = w \quad (31)$$

$$f_k^M(z, k_M, l_M) = r \quad (32)$$

$$f_l^M(z, k_M, l_M) = w \quad (33)$$

$$f_k^{oil}(z, k_{oil}, l_{oil}) = r \quad (34)$$

$$f_l^{oil}(z, k_{oil}, l_{oil}) = w \quad (35)$$

the balance of payments, and market clearing conditions:

$$\begin{aligned}
c_N &= f^N(z, k_N, l_N) \\
c_M &= f^M(z, k_M, l_M) - x_M \\
c_{oil} &= f^{oil}(z, k_{oil}, l_{oil}, n) - x_{oil} \\
X &= p_M x_M + p_{oil} x_{oil} \\
k_N + k_M + k_{oil} &= k \\
l_N + l_M + l_{oil} &= l
\end{aligned}$$

where, recall, $X = q(s, k', b') [b' - (1 - \gamma)b] - [\gamma + \kappa(1 - \gamma)]b$, k' , b' , and l are given.

Note that if $\mu^Y = \frac{p_M}{\lambda_{C_M}}$, $p_N = \mu^Y \frac{\lambda_{C_N}}{p_M}$, $r = \mu^Y \frac{\lambda_k}{p_M}$, and $w = \mu^Y \frac{\lambda_l}{p_M}$ then the two systems of equations are the same and, thus, the allocations that satisfy them are the same.

Finally, for the dynamic allocations note that the government in the market economy has three instruments τ_k , τ_l , and T to pin down the households decisions for labor, capital in the next period, and consumption. Thus, with the correct choices of capital and transfers the two problems are equivalent.