Default Depressions*

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Abstract

Prolonged sovereign defaults are associated with deep economic depressions that persist well after their resolution. Short defaults do not have this scarring effect. I develop a novel theory that accommodates both types of default along with the standard cyclical behavior of sovereign risk. I incorporate elements from the "Big Push" literature that generate demand complementarities and increasing returns to scale in a unique-equilibrium setting. Default disrupts technology adoption and long defaults act as a "Big Pull" force that takes the economy from a "rich" to a "poor" ergodic state. These differentiated effects of long and short defaults generate a bimodal ergodic distribution, and the economy fluctuates around to regimes: "normal times" and a "default trap". In the default trap the capital stock and technology adoption are lower, defaults are more frequent, and spreads are higher and more volatile than in normal times.

Keywords: Sovereign default, Economic depressions.

JEL Codes: F34, F41, H63

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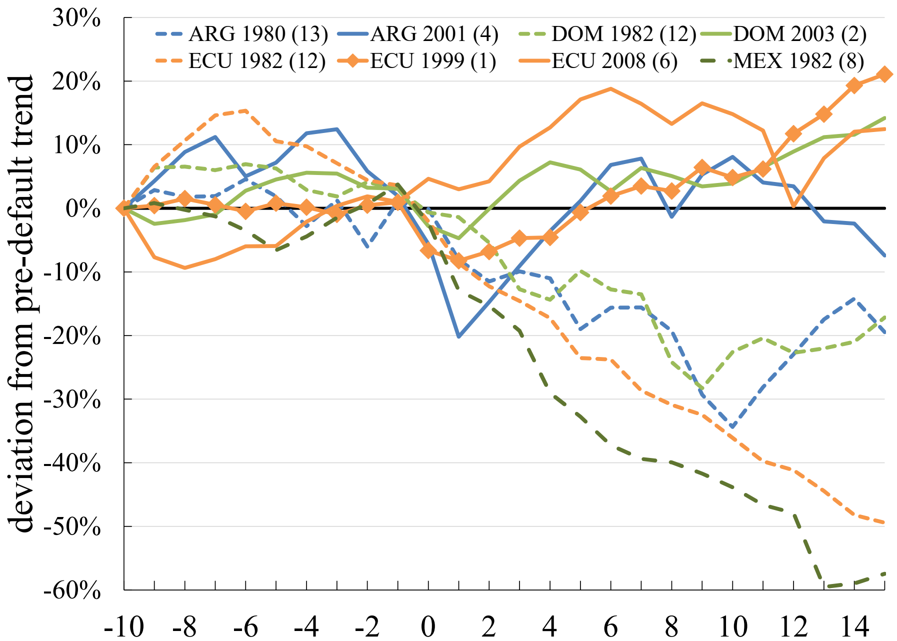
1 Introduction

Sovereign defaults are large and disruptive events, but some are more disruptive than others. Long-lasting defaults have scarring effects on GDP that are not proportionally observed for shorter ones. Figure 1 illustrates this observation with eight default events of different duration: two in Argentina (in 1980 and 2001), two in the Dominican Republic (in 1982 and 2003), three in Ecuador (1982, 1999, and 2008), and one in Mexico (1982). The labels include the year of default and in parenthesis its duration in years. Each line corresponds to log-deviations of real GDP per capita from its pre-default trend and all series are centered so that the year of default is period 0.

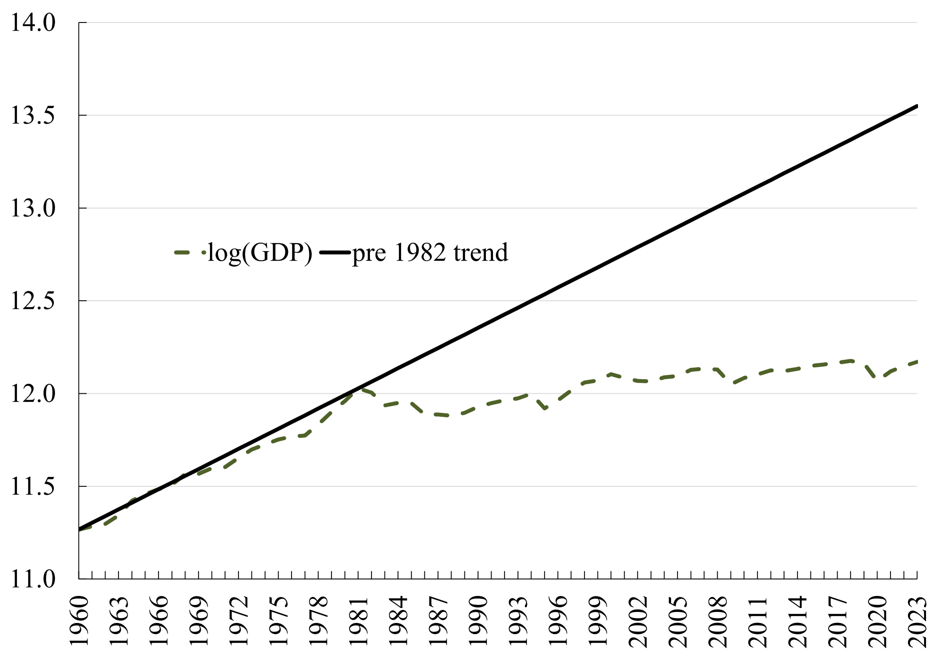
Figure 1: GDP per capita, local currency units



Each line corresponds to log-deviations of real GDP per capita from its pre-default trend. The pre-default trend is computed as a linear trend using the ten years prior to each event. The country labels correspond to Argentina (ARG), the Dominican Republic (DOM), Ecuador (ECU), and Mexico (MEX). The year of the default is stated after the country's code and the duration of each default is in parenthesis.

These events can be sorted into two groups: long defaults that pushed their economies into a depression (the dashed lines) and shorter defaults that did not. The examples from Argentina and the Dominican Republic are very stark because, for the same country, different default durations are associated with very different long-lasting effects on income. The case of Ecuador is similar, with the addition of a very short event of only one year (the orange line with square marks), after which GDP rapidly grew. The case of the 1982 Mexican default is the most dramatic in this group. The gap between GDP per capita and its pre-default trend continued to widen years after the default was resolved in period 8 in the graph. In fact, the Mexican economy never recovered from this shock, as can be seen in Figure 2. The following Section analyzes 64 default events and presents additional evidence regarding these motivating observations: long defaults have large long-lasting negative effects on GDP that remain well beyond their resolution.

Figure 2: Mexico GDP per capita and pre-1982 trend



The data are the natural logarithm of Mexican real GDP per capita and its pre-default trend. The pre-default trend is computed as a linear trend between 1960 and 1981.

Standard models of sovereign default (Arellano (2008), Aguiar and Gopinath (2006), and the literature that spanned thereafter) are not suited to study these deep and long-lasting scarring effects because they study default risk over the business cycle, with output fluctuating around a stable long-run mean. This is true even for richer models with endogenous production (see Mendoza and Yue (2012)) and capital accumulation (see Park (2017), Gordon and Guerron-Quintana (2018), Arellano, Bai, and Mihalache (2018)). Default in these models may have large negative effects on output and capital accumulation may make these effects persistent, but once a default is resolved the economy eventually converges back to its long-run mean. The main contribution of this paper is to develop a novel theory of sovereign default that can accommodate both the cyclical behavior that is studied in the literature and large scarring effects of long defaults after which the economy does not necessarily converge back to its long-run mean.

The model builds on the sovereign default literature with production and incorporates some elements from the literature that studies economic development in the context of industrialization traps (see Murphy, Shleifer, and Vishny (1989)). This literature analyzes the idea of how, in the context of imperfect competition and aggregate demand spillovers, a "Big Push" into industrialization can move an economy from a "bad" (poor) to a "good" (rich) equilibrium (or from a low- to a high-income steady state in a model of unique equilibrium as in this paper and in Schaal and Taschereau-Dumouchel (2024)). In the model that I develop, instead, long-lasting sovereign defaults act as a "Big Pull" force that pulls the economy down from a rich into a poor ergodic state. The model does not feature a corresponding "Big Push" force that is symmetric to endogenous default, which makes recovery from the default-induced poverty trap less likely. The model also has the feature that short defaults are less likely to have this scarring effect, as in the data.

There is a small-open economy in which a sovereign makes optimal borrowing, investment, and default decisions without commitment. This is the standard sovereign-default block of the model. The novelty is on the production side, which features a continuum of monopolistically competitive firms whose productivity is heterogeneous in two dimensions. One dimension is permanent and firm-specific. The other is the result of firms' choices to operate a traditional technology or to pay a fixed cost to access a modern—more productive—one. These assumptions generate increasing returns to scale for certain levels of capital and demand complementarities in the sense that, for some firms, the decision to adopt the modern technology depends on whether other firms are doing so. When capital is abundant, production cost are lower and more firms find it profitable to adopt the modern technology.

As in the model developed by Buera, Hopenhayn, Shin, and Trachter (2021), ex ante firm heterogeneity allows the model to feature a "Big Push" region, where a small increase in the capital stock has a large effect on technology adoption (in their model the main focus is on firm-level distortions and the effects of reducing them through structural reforms). The model also features other more stable regions, with significantly lower or higher levels of capital, in which changes to the aggregate stock only have moderate effects on adoption. This duality is crucial for the model to generate the bimodal ergodic distribution described below. Another advantage from the ex-ante heterogeneity of firms is that it guarantees a unique production equilibrium, which rules out multiplicity due to coordination failures simplifying the analysis.

The only source of risk in the model is an aggregate productivity shock that is exogenously penalized during periods of default, which is a standard assumption in the literature. While in default, the sovereign is in financial autarky and regains access with a constant probability. This exogenous default resolution is standard in the literature, and, more importantly, it allows me to perform experiments of different default durations without them interacting with other endogenous variables. Why some defaults last longer than others, as well as what are the sources of the default penalty on productivity, are interesting open questions in the literature that are beyond the scope of this paper (for a more detailed treatment of these questions see Benjamin and Wright (2009), Bocola (2016), Dvorkin, Sanchez, Sapriza, and Yurdagul (2021), and Arellano, Bai, and Bocola (2017)).

Default events in the model feature significantly different long-lasting effects on GDP and production decisions. After short defaults, GDP and technology adoption quickly bounce back and GDP remains close to its pre-default level in the long-run. After a long enough default, however, GDP also bounces back up, but to a level that is ten percentage points lower than before the event. GDP remains significantly below its pre-default level even 20 years after the event was resolved. These differentiated effects of default events generate a bimodal ergodic distribution, which shows that the economy fluctuates around to regimes: "normal times" and a "default trap". In normal times the capital stock is relatively high, the fraction of firms that operate the modern technology is close to one, default risk is low, and sovereign spreads are low and feature low variance. When the economy is in the default trap the capital stock is significantly lower, as well as technology adoption. In addition, defaults are more frequent and spreads are higher and more volatile. The median of the fraction of firms adopting the modern technology in the ergodic distribution is 0.07 and there is full adoption only 30 percent of the times. This indicates that the "default trap" is more common, showing the absence of a "Big Push" force to counter the "Big Pull" of lengthy defaults.

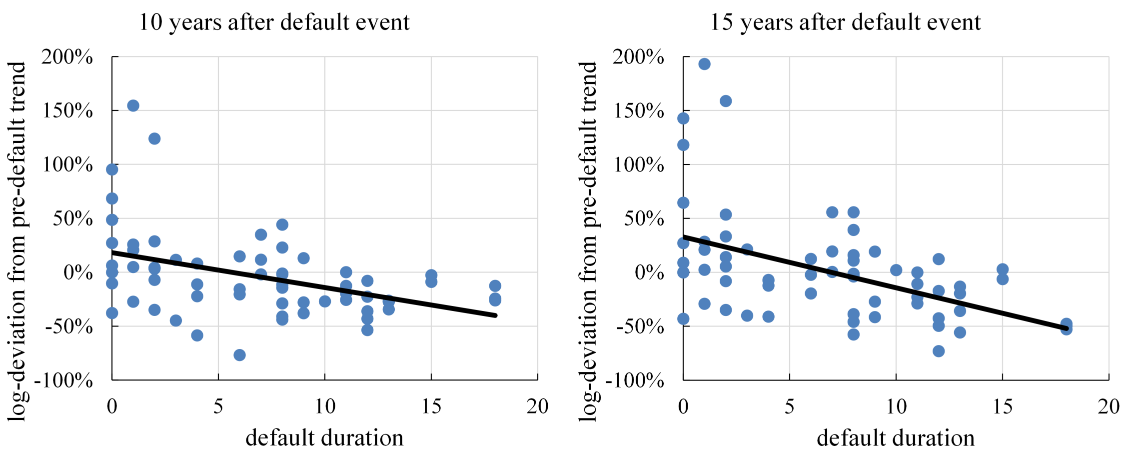
Layout.—Section 2 presents additional motivating evidence on the scarring effect of long defaults, Section 3 presents the model and discusses the main novel mechanisms, Section 4 describes the calibration exercise and presents the main results, Section 5 concludes.

2 Default depressions

This Section presents evidence on the relationship between the length of default events and long-run GDP per capita (all the results in this section are robust to using aggregate GDP data instead). I analyze 64 default events in 54 different countries reported in Laeven and Valencia (2018). The data report the year of sovereign defaults and the year of the default resolution, which is what I use as the duration variable (Appendix A lists all 64 default events and the relevant dates).

The main purpose of this exercise is to see whether longer defaults are associated with GDP being significantly below trend years later. I take real GDP expressed in constant local currency units and compute linear trends using the ten years that precede a default. Then, I compute the log-deviation from this trend in each of the years after default. Figure 3 shows two scatter plots relating the duration of each default event to the log-deviation of GDP from the pre-default trend 10 and 15 years after the event.

Figure 3: Default duration and GDP deviation from long-run trend



Each observation corresponds to a default event. The horizontal axis is the number of years it took for the default to be resolved. The vertical axes are log-deviations of GDP, measured in local currency units, from its trend in the 10 years prior to default. The left panel shows the log-deviation of GDP 10 years after the default event, and the right panel shows it 15 years after the event.

There is a clear negative correlation between default duration and GDP relative to its pre-default trend several years after the default event. I then estimate the following Probit model

$$\Pr\left(\Delta y_{ijt} < 0\right) = \Phi\left(\psi_0 + \psi_1 d_i + \overrightarrow{\psi} X\right), \tag{1}$$

where Δy_{it} is the log-deviation of GDP in country j from its pre-default trend t years after default event i, d_i is the duration (in years) of default event i, X is a vector of additional controls, and Φ is the cumulative normal distribution. The controls include a dummy variable for default events that last less than a year. This ensures that the results are not driven by the large positive observations of Δy_{ijt} in Figure 3. I also include a dummy variable for "sudden" defaults, which takes a value of 1 if the economy was growing in the two years prior to default. I include a dummy variable for defaults in the 1980s, which constitute 34 of the 64 observations. Defaults in the 1980s were much longer than defaults in more recent years (see Gelos, Sahay, and Sandleirs (2011)) and extended all over the world. As pointed out by Almeida, Esquivel, Kehoe, and Nicolini (2024), the 1980s featured the most widespread sovereign debt crisis in history and was accompanied by abnormally large real interest rates due to monetary tightening in the United States. This control is important because these events constitute roughly half of the sample and because there may have been other aggregate shocks during that decade that could have significantly affected the path of GDP. Finally, I include an interaction between the 1980s dummy variable and the duration of default events. Table 1 reports the estimated coefficients for $t = 10$ and $t = 15$.

Table 1: Probit estimation

	(1)	(2)	(3)	(4)
variable	10 years	10 years	15 years	15 years
duration	0.168***	0.256**	0.120***	0.106*
	(0.0435)	(0.129)	(0.0362)	(0.0583)
duration=0		0.683		-0.0858
		(0.641)		(0.602)
sudden default		0.112		0.154
		(0.386)		(0.357)
1980s		0.769		0.0502
		(0.773)		(0.735)
1980s*duration		-0.119		0.0122
		(0.142)		(0.0889)
Constant	-0.626**	-1.173**	-0.617**	-0.700
	(0.291)	(0.593)	(0.281)	(0.480)
Observations	64	64	64	64

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The parameter of interest ψ_1 is positive and statistically different from zero, with and without additional controls. In fact, none of the controls are significant—either together or included one at a time—which suggests that the negative relationship illustrated in Figure 3 is quite robust.

The coefficient $\psi_1 = 0.256$ in column (2) implies that each additional year in default increases the probability that GDP is below its pre-default long-run trend by 0.069. This duration effect on the probability of GDP being depressed 15 years after the event is 0.035 with the estimated coefficient $\psi_1 = 0.106$ in column (4).

The data suggest that long defaults are more likely to be associated with an economic depression, with GDP being significantly below its pre-default trend well after the default was resolved. Standard models of sovereign default are not suited to study such deep and long-lasting effects because they model the cyclical effects of default risk only. The following section develops a novel theory of sovereign default that can accommodate both the cyclical behavior and these large scarring effects of long defaults.

3 Model

There is a small-open economy populated by a sovereign and a continuum of firms. The sovereign cannot commit to future policies and makes optimal borrowing, investment, and default decisions. As is standard in the literature, the economy suffers an exogenous productivity cost while in default and is excluded from financial markets for a random number of periods. The key innovation of the model is in the production side of the economy, which is laid out in detail below.

3.1 Environment

Technology.—There is a competitive firm that produces a final tradable good using technology

$$Y_t = \left(\int_0^1 (y_{it})^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where y_{it} are differentiated non-tradable varieties of intermediate goods, and $\epsilon > 1$ is the elasticity of substitution across varieties. There is a measure one of monopolistically competitive firms $i \in [0, 1]$ that produce the intermediate goods using capital k_{it} and labor l_{it}. Each intermediate good producer is characterized by an individual productivity level $z_i \geq 1$, which is fixed across time. These productivity levels follow a Pareto distribution with CDF $G(z_i) = 1 - z_i^{-\xi}$, where ξ is the tail parameter. Each period, the productivity of all firms is scaled by an aggregate productivity shock z_t, which follows a standard Markov process. This is the only source of risk in the model. At the beginning of each period firms choose whether to operate a "traditional" technology

$$y_{it}^{T} = z_t z_i z_T k_{it}^{\alpha} l_{it}^{1-\alpha}, \tag{3}$$

where $\alpha \in (0,1)$, or pay a fixed cost f to operate a "modern" technology

$$y_{it}^{M} = z_t z_i z_M k_{it}^{\alpha} l_{it}^{1-\alpha}, \tag{4}$$

where $z_T < z_M = 1$ indicates that the modern technology is more productive (the assumption $z_M = 1$ is made without loss of generality). The fixed cost is expressed in terms of the final good and has to be paid each period that the firm chooses to operate the modern technology.

Preferences.—The sovereign has preferences for consumption of the final good and labor represented by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - \frac{L_t^{1+\nu}}{1+\nu} \right)^{1-\sigma}}{1-\sigma} \right], \tag{5}$$

where $\beta \in (0,1)$ is the discount factor, $\sigma > 0$ is the coefficient of constant relative risk aversion, and $1/\nu$ is the Frisch elasticity of labor supply. This formulation removes the wealth effect on the labor supply preventing it from sharply rising when TFP is low or when consumption drops, which are responses that are at odds with the data (see Greenwood, Hercowitz, and Huffman (1988)). This is particularly important during default events, which are characterized by low TFP and consumption in the model. The sovereign owns all firms and capital in the economy and supplies labor and capital to the intermediate firms for a wage w_t and a rental rate r_t, respectively. The law of motion of capital is

$$K_{t+1} = I_t + (1-\delta) K_t, \tag{6}$$

where $\delta \in (0,1)$ is the depreciation rate and I_t is investment. The final good can be used for investment, but there is a capital adjustment cost $\Psi(K_t, K_{t+1}) = \frac{\phi}{2} \frac{(K_{t+1}-K_t)^2}{K_t}$. This assumption, which is standard in open-economy business cycle models, prevents investment from being significantly more volatile in the model than in the data and prevents the sovereign from sharply depleting the capital stock to pay foreign debt in periods of distress, as discussed by Gordon and Guerron-

Quintana (2018).

Borrowing and default.—The sovereign issues long-term debt in international financial markets and cannot commit to repay it. Following the literature (see Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012)), I assume that debt matures at a constant rate $\gamma \in (0,1)$. The law of motion of debt is

$$B_{t+1} = i_{bt} + (1-\gamma) B_t, \tag{7}$$

where i_{bt} is current debt issuance that is sold to competitive risk-neutral international investors for a price q_t. If the sovereign defaults then the government is excluded from financial markets and gets readmitted with probability θ and all debt forgiven. While in default, aggregate productivity is $z_D(z_t) = z_t - \max\left\{0, d_0 z_t + d_1 z_t^2\right\}$, with $d_0 < 0 < d_1$. This formulation was introduced by Chatterjee and Eyigungor (2012) in a pure exchange economy and has been widely implemented in production economies as a cost to TFP (see Gordon and Guerron-Quintana (2018), Arellano, Bai, and Mihalache (2018), and Esquivel (2024)). What is important is that it makes the cost of defaulting large in "good times" when productivity is high and small or zero in "bad times", which is essential for default events and default risk to be countercyclical. Mendoza and Yue (2012) show how such an asymmetric cost of defaulting can emerge in a production economy similar to the one laid out here as a result of firms losing access to working-capital to finance purchases of certain intermediate inputs. While the above environment can be extended to feature a similarly endogenous cost, this extension would come at the expense of clarity of the novel mechanism that generates a depression after a lengthy default, which is discussed in detail below.

3.2 Static industry equilibrium

This section characterizes the choices of individual firms and shows how increasing returns to scale arise as a result of the technology assumptions. Moreover, since all firm choices are static, this section defines a static industry equilibrium that characterizes aggregate output as a function of only the productivity shock z_t and the aggregate capital stock K_t. Crucially, the combination of (z_t, K_t) determines the mass of firms that choose to adopt the modern technology, which will be a key driver of depressions following a lengthy default episode.

Operating profits.—The final demand for variety i is

$$y_{it} = \left(\frac{P_t}{p_{it}}\right)^{\epsilon} Y_t, \tag{8}$$

where Y_t is output of the final good and P_t is its corresponding price index

$$P_t = \left(\int_0^1 (p_{it})^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}. \tag{9}$$

Given a production plan y, cost minimization for firm i yields the total cost function

$$C_{it}^{j}(y) = \frac{\mu_t}{z_{it}^{j}} y, \tag{10}$$

where $\mu_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^{\alpha}$ and z_{it}^{j} is firm i's productivity in period t, which depends on its individual level z_i, on the current realization of the aggregate shock z_t, and on whether the firm chose to operate the traditional technology $z_{it}^{T} = z_t z_i z_T$ or the modern (M) one $z_{it}^{M} = z_t z_i$. Given a technology choice $j \in \{T, M\}$, operating profits of firm i in period t are

$$\pi_{it}^{Oj} = \max_{y} \left\{ p_{it}(y)\, y - C_{it}^{j}(y) \right\}, \tag{11}$$

where $p_{it}(y)$ is the inverse demand function implied by (8). The solution to the problem in (11) implies the standard choice of monopoly pricing

$$p_{it}^{j} = \frac{\epsilon}{\epsilon - 1} \frac{\mu_t}{z_{it}^{j}}, \tag{12}$$

charging a markup $\frac{\epsilon}{\epsilon-1}$ over the firm's marginal cost μ_t / z_{it}^{j}. Plugging (12) into (8) and using the assumption that firms satisfy their demand we get that operating profits are

$$\pi_{it}^{Oj} = \frac{1}{\epsilon} \left(\frac{\epsilon - 1}{\epsilon} \frac{z_{it}^{j}}{\mu_t} \right)^{\epsilon - 1} (P_t)^{\epsilon} Y_t. \tag{13}$$

Technology adoption.—A firm i chooses to adopt the modern technology if

$$\pi_{it}^{OM} - P_t f \geq \pi_{it}^{OT}, \tag{14}$$

that is, if the additional profits of operating the modern technology more than compensate for the fixed cost of adoption. Note that, all else equal, firms with high z_i are more profitable, so the assumption that z_i are distributed Pareto implies that there is always a positive mass of firms that adopts the modern technology. In addition, note that firms with high z_{it}^j produce more and charge lower prices. This implies that total output Y_t increases as the mass of modern firms increases. This in turn increases demand for all varieties (equation (8)) making it more attractive for low z_i firms to adopt the modern technology.

This virtuous cycle of adoption in which firms adopt the modern technology as long as other firms do so is what the development literature highlights as "the big push". Some of the work in this literature introduces such a mechanism in environments with identical firms, which gives rise to multiplicity of equilibra and generates a scope for government intervention as a coordinating force. In this model, however, the assumptions regarding ex ante heterogeneity of firms guarantee uniqueness with respect of the adoption decision, making the coordination failure motive for government intervention mute. This simplifies the analysis on the interactions between increasing returns and default events, which is the main focus of the paper. Appendix (B) shows that if $(\epsilon - 1) > \frac{1+\nu}{\alpha+\nu}$ then the adoption decision is uniquely characterized by a cutoff $z_i = z_t^*$ such that equation (14) holds with equality. The Appendix also shows that under this assumption the mass of firms adopting the modern technology $m_t = \left(\frac{1}{z_t^*}\right)^{\xi}$ is increasing in the aggregate shock z_t and in the aggregate stock of capital K_t.

TFP and final output.—Define total factor productivity as

$$A_t = \left(\int_0^1 \left(z_{it}^{j(i,t)} \right)^{\epsilon - 1} di \right)^{\frac{1}{\epsilon - 1}}, \tag{15}$$

where $j(i,t) \in \{T, M\}$ is firm i's adoption choice in period t. The assumption that z_i follow a

Pareto distribution with tail parameter $\xi > \epsilon - 1$ implies that

$$A_t = A\left(z_t, z_t^*\right) = z_t \left(\frac{\xi}{1+\xi-\epsilon}\left[1+\left(\hat{z}^{\epsilon-1}-1\right)(m_t)^{\frac{1+\xi-\epsilon}{\xi}}\right]\right)^{\frac{1}{\epsilon-1}}, \tag{16}$$

where z_t is the aggregate shock and m_t is the mass of firms adopting the modern technology. The Pareto-distribution assumption allows for a closed-form characterization of A_t, and significantly simplifies the analysis and computation of equilibrium. This assumption, however, is not necessary to generate the endogenous increasing returns to scale in aggregate output. Assuming that z_i are distributed log-normal, for instance, would work similarly at the expense of having to compute A_t numerically.

From the market clearing conditions for production factors and using individual firm's optimality conditions we can derive total output as a function of aggregates and TFP:

$$Y_t = A_t (K_t)^{\alpha} (L_t)^{1-\alpha}, \tag{17}$$

where L_t is the aggregate labor supply. Finally, with GHH preferences the labor supply dos not depend on the current consumption level, which makes it a static decision that can be written as a function of A_t and K_t:

$$L_t = \left[(1-\alpha)\frac{\epsilon-1}{\epsilon}A_t (K_t)^{\alpha}\right]^{\frac{1}{\alpha+\nu}}. \tag{18}$$

This implies that final output is also a function of z_t and K_t only:

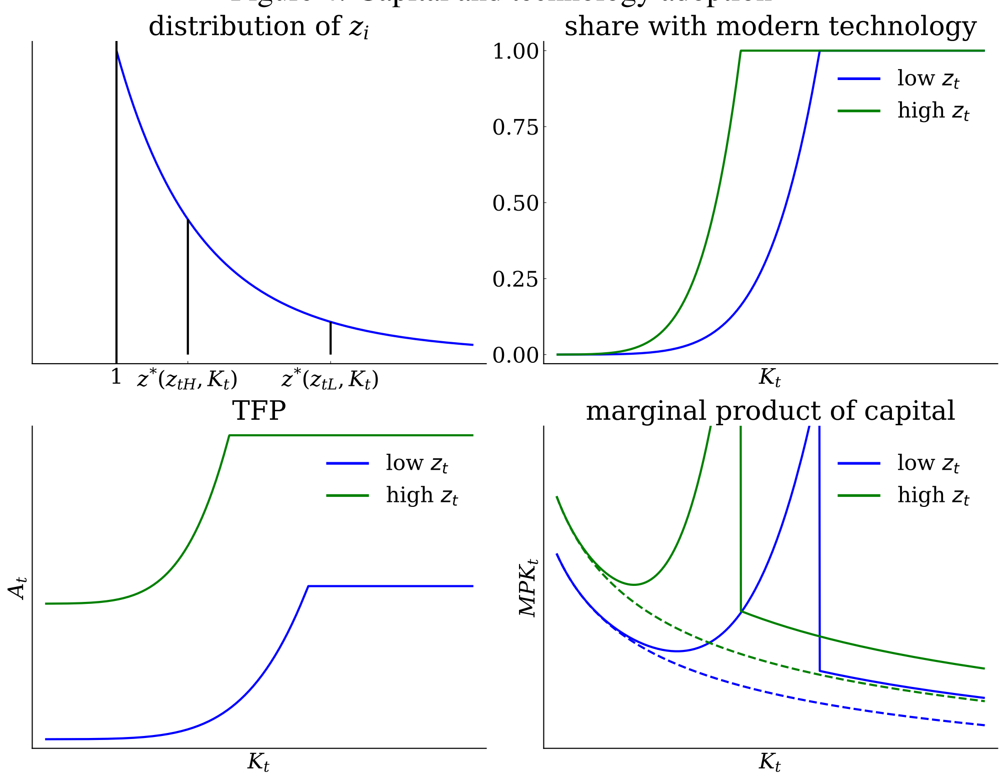
$$Y_t = F(z_t, K_t) = \left[(1-\alpha)\frac{\epsilon-1}{\epsilon}\right]^{\frac{1-\alpha}{\alpha+\nu}} (A_t)^{\frac{1+\nu}{\alpha+\nu}} (K_t)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)}, \tag{19}$$

(see Appendix (B) for the full derivation of these equations). These characterizations imply that the productivity cutoff implicitly defined as the solution to equation 14 is only a function of z_t and K_t

$$z_t^* = z^*(z_t, K_t) \tag{20}$$

Increasing returns to scale.—Figure 4 illustrates how capital may have increasing returns to scale in some regions of the state space. The top-left panel shows the distribution of individual firm productivities z_i. There is a large mass of firms with levels of z_i close to 1, which is the lowest value, and a small and decreasing mass of firms with high z_i. The figure also shows two productivity cutoff values corresponding to a high and low realization of the shock $z^*(z_{tH}, K_t)$ and $z^*(z_{tL}, K_t)$ for a given level of capital K_t. For the given state, all firms with productivity z_i to the right of the cutoff adopt the modern technology. As mentioned before, since the support of the Pareto distribution is $[1, \infty)$ there is always a positive mass of adopting firms. This mass, however, is lower for low realizations of z_t and close to zero for low enough levels of capital K_t, as illustrated by the top-right panel of Figure 4.

Figure 4: Capital and technology adoption



The top-right panel also shows how, for a large enough level of capital K_t all firms adopt the modern technology $m_t = 1$. The bottom-left panel shows the corresponding values of TFP A_t for high and low z_t and different levels of capital. As suggested by the behavior of m_t, TFP is low and close to being flat for low levels of capital, rapidly increasing for medium levels, and high and flat for high levels. These dynamis have important implications for the marginal product of capital MPK_t, which is shown on the bottom-right panel. For low enough levels of capital MPK_t is decreasing in K_t, as is standard with a Cobb-Douglas production function. Once capital reaches a level at which adoption starts to quickly increase then the marginal product of capital becomes increasing as the increase in A_t more than compensates for the decreasing returns to scale in the production function. Once capital reaches the level at which all firms adopt the modern technology $m_t = 1$, the MPK_t function features a discontinuity and becomes decreasing, but at higher levels relative to the case in which A_t were constant (illustrated by the continuing dashed lines).

3.3 Recursive formulation of dynamic problem

The state of the economy is (B, K, z), where B is the debt level, K is the aggregate capital stock, and z is the aggregate productivity shock. At the beginning of a in good financial standing, the value of the sovereign is:

$$V(B,K,z) = \max_{D\in\{0,1\}} \left\{ DV^D(K,z) + (1-D)V^P(B,K,z) \right\}, \tag{21}$$

where V^D and V^P are the values of defaulting and repaying, respectively, and D is the default choice. If the sovereign decides to repay $D = 0$, the value is

$$V^P(B,K,z) = \max_{\{B',K',c\}} \{u(c, L(z,K)) + \beta\mathbb{E}[V(B',K',z')]\} \tag{22}$$
$$s.t. \qquad c + K' + \Psi(K,K') + \gamma B \leq F(z,K) + (1-\delta)K + q(B',K',z)[B' - (1-\gamma)B]$$

where q is the price schedule for government debt (defined below), and L and F are the functions defined in the previous subsection that result from the static industry equilibrium. If the sovereign defaults $D = 1$, the value is

$$V^D(K,z) = \max_{\{K',c\}} \left\{ u(c, L(z_D(z),K)) + \beta\theta\mathbb{E}[V(0,K',z')] + \beta(1-\theta)\mathbb{E}\left[V^D(K',z')\right] \right\} \tag{23}$$
$$s.t. \qquad c + K' + \Psi(K,K') \leq F(z_D(z),K) + (1-\delta)K,$$

where $z_D(z) = z - \max\left\{0, d_0 z + d_1 z^2\right\} \leq z$ is aggregate productivity in default.

Equilibrium.—An equilibrium is value and policy functions in default V^D, K^D, c^D; value and policy functions in repayment V, V^P, D, B^P, K^P, c^P; and a price schedule q such that: (i) given the price schedule, the value and policy functions solve the system of Bellman equations (21), (22), and (23); and (ii) the price schedule q satisfies

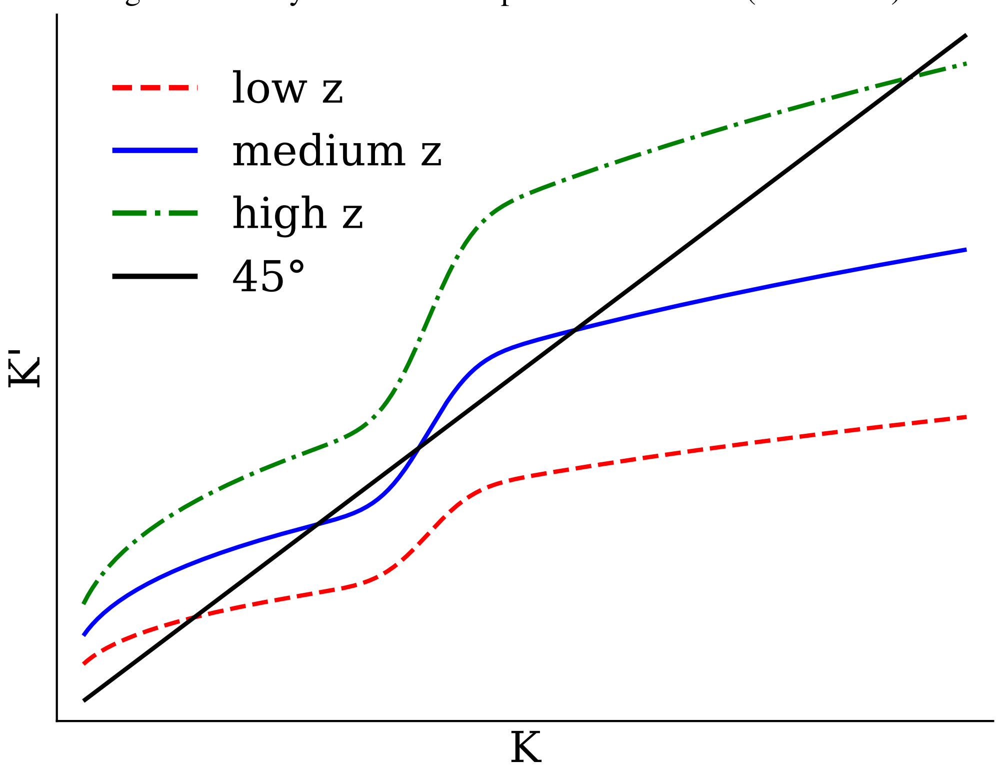
$$q\left(B', K', z\right) = \frac{\mathbb{E}\left[\left(1 - D\left(B', K', z'\right)\right)\left(\gamma + (1-\gamma)\, q\left(B'', K'', z'\right)\right)\right]}{1 + r}, \tag{24}$$

where $B'' = B^P\left(B', K', z'\right)$, $K'' = K^P\left(B', K', z'\right)$, and r is the international risk-free interest rate.

3.4 The "Big Pull" effect of default

To analyze the dynamic implications of the production structure, consider first the deterministic closed-economy version of the environment laid out above. Figure 5 illustrates the shape that the policy function for capital accumulation takes for different fixed values for the productivity level z. For a high enough z (the green-dashed-dotted line) there is a unique steady state with a high level of capital. Similarly, for a low-enough z (the red-dashed line) there is a unique steady state with low capital.

Figure 5: Policy function for capital accumulation (illustrative)



The most interesting case is illustrated by the solid-blue line, which features three steady-state levels of capital. Two of these, the lowest and the highest, are stable and the middle one is unstable in the sense that an epsilon-deviation from it would put the economy's capital level on a dynamic path toward one of the other steady states.

In an economy with productivity shocks, different realizations of z shift similar-looking policy functions up and down. Starting at a high enough level, capital fluctuates around the high-capital steady state for small enough shocks, but a large (and persistent) negative shock to z may push it into a trayectory toward the low-capital steady state. The converse is true for a low starting point: capital fluctuates around the low steady state and a large persistent shock may push it toward the high steady state.

This symmetric transition between the high and low steady states breaks in the economy with endogenous default. This is because default, which is associated with low realizations of z, triggers an additional productivity reduction through the exogenous default penalty. A lengthy default makes low values of z more persistent because high subsequent realizations are hindered by it. When default happens in the region of the state space with high levels of capital it has a "big pull" effect on technology adoption. When the economy is in the low-capital region there is no corresponding positive event that would have a "big push" effect similar to the pull-down that default has when capital is high.

4 Quantitative analysis

I use value function iteration to solve for the equilibrium using the formulas derived in Subsection 3.2 to compute aggregate output and the labor supply as functions of the state. Following Hatchondo, Martinez, and Sapriza (2010), I compute the limit of the finite-horizon version of the economy. I jointly solve for investment and borrowing decisions using a non-linear optimization routine, and approximate value functions and the price schedule using linear interpolation. To compute expectations over the productivity shock I use a Gauss-Legendre quadrature.

4.1 Calibration

A period in the model is one quarter. Most of the parameters are taken from the literature and summarized on Table 2 below. The relative risk aversion parameter is $\sigma = 2$ and the risk-free rate $r^* = 0.01$, which are standard values in the business cycle literature. I set the discount factor of the sovereign to $\beta = 0.95$, which is close to values that have been calibrated to the Argentinean economy in similar models (Arellano (2008) chooses 0.953, Chatterjee and Eyigungor (2012) choose 0.954, and Gordon and Guerron-Quintana (2018) choose 0.946). Following Mendoza and Yue (2012), I set $\nu = 0.455$ so that the Frisch elasticity is equal to 2.2.

Table 2: Calibrated parameters

Parameter		Value	Source
Relative risk aversion	σ	2	Standard value
Risk-free rate	r^*	0.01	Standard value
Discount factor	β	0.95	Standard value
Labor disutility parameter	ν	0.455	Mendoza and Yue (2012)
Debt duration	γ	0.05	Chatterjee and Eyigungor (2012)
Probability of reentry	θ	0.0625	Gelos, Sahay, and Sandleirs (2011)
Capital share	α	0.33	Standard value
Elasticity of substitution	ϵ	2.92	Schaal and Taschereau-Dumouchel (2024)
Depreciation rate	δ	0.05	Standard value
Adjustment cost	ϕ	21.2	Gordon and Guerron-Quintana (2018)
Mean of productivity shock	μ_z	0.768	High-capital steady state $y_{ss}^H = 1$
Persistence of productivity	ρ	0.95	Gordon and Guerron-Quintana (2018)
Variance of productivity	σ^2	0.017	Gordon and Guerron-Quintana (2018)
Pareto curvature	ξ	20.5	high-capital steady state with $m_t = 1$
Fixed cost	f	0.04	low-capital steady state with $m_t < 0.1$
Productivity of traditional technology	z_T	0.898	$y_{ss}^L / y_{ss}^H = 0.60$

I set the debt duration parameter to $\gamma = 0.05$ to match the maturity information for Argentina reported in Broner, Lorenzoni, and Schmukler (2013), as done by Chatterjee and Eyigungor (2012). I set the probability of reentry to financial markets to $\theta = 0.0625$ for an average duration in autarky of 16 quarters, which is the median duration of default events documented by Gelos, Sahay, and Sandleirs (2011).

On the production side, I set the elasticity of substitution to $\epsilon = 2.92$, following Schaal and Taschereau-Dumouchel (2024) who calibrate a similar model of heterogeneous firms. This is close to the value of 3, which is standard in the literature (see the discussion in Hsieh and Klenow (2009)). The capital share $\alpha = 0.33$ and the capital depreciation rate $\delta = 0.05$ are standard values. The capital adjustment cost parameter is $\phi = 21.2$, which is the same as the calibrated parameter in Gordon and Guerron-Quintana (2018). The aggregate productivity shock follows an AR(1) process

$$\log z_t = (1 - \rho) \log \mu_z + \rho \log z_{t-1} + \sigma_z \epsilon_t, \tag{25}$$

where the persistence $\rho = 0.95$ and variance $\sigma^2 = 0.017$ are taken from Gordon and Guerron-

Quintana (2018), who calibrate them for Argentina using a Cobb-Douglas production technology. I set the mean value of the shock $\mu_z = 0.768$ so that output $y^H_{ss} = 1$ in the deterministic steady-state in which all firms adopt the modern technology. Let y^L_{ss} be the level of output in the hypothetical steady state in which no firms adopt the modern technology, I set the productivity level of the traditional techology $z_T = 0.898$ so that $y^L_{ss} = 0.6$, which is the average drop of output from trend observed in depressions that follow lengthy defaults. Finally, I jointly choose the pareto tail parameter $\xi = 20.5$ and the fixed cost of adoption $f = 0.04$ so that $m_t = 1$ at the steady-state level of capital with full adoption k^H_{ss} and so that $m_t < 0.1$ at the hypothetical steady-state level of capital with no adoption k^L_{ss}. These choices will allow the ergodic distribution to feature long periods of high and low technology adoption.

Table 3 reports some business cycle statistics from the model and compares them to their data counterparts for Argentina, which is a widely studied example in the sovereign default literature. The main objective of the quantitative analysis in this paper is to study how long defaults can generate an economic depression. To that end, I interpret as “normal times” the periods in which there is full adoption of the modern technology $m_t = 1$, which corresponds to fluctuations around the high-adoption high-capital steady state. Column (2) in Table 3 reports long-run statistics from the model moments conditional on $m_t = 1$.

Table 3: Business cycle statistics

		model	
	(1)	(2)	(3)
moment	data	full adoption	low adoption
Default frequency	0.03	0.01	0.03
Av. spreads	8.15	8.31	19.34
Std. spreads	4.43	6.52	25.49
debt-to-GDP ratio	0.25	0.28	16.19
Capital stock (total)	n.a.	3.61	1.73
σ_c/σ_y	1.23	1.45	1.14
σ_i/σ_y	2.65	2.06	2.10
$\sigma_{TB/y}$	2.34	2.16	2.71
$Cor\,(\text{spreads}, y)$	-0.79	-0.52	-0.20
$Cor\,(TB/y, y)$	-0.68	-0.34	0.04

To compute the model moments I draw 100 samples of 11,000 periods and drop the first 1,000. Column (2) drops all periods in which the mass of adopters is not 1. Column (3) drops all periods in which the mass of adopters is larger than the median. Except for spreads and the debt-to-GDP ratio, all variables are HP-Filtered with a smoothing parameter of 1,600. In the data, debt corresponds to External debt stocks, public and publicly guaranteed from the World Development Indicators database. All model moments are conditional on the economy being in good financial standing.

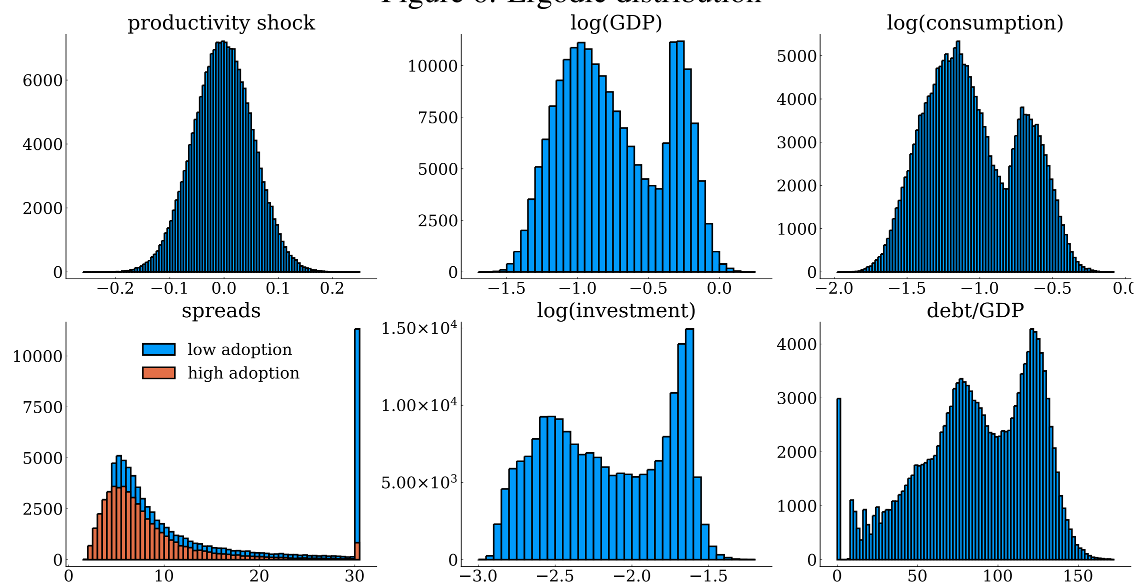
The model does a good job in replicating long-run business cycle statistics of Argentina during model "normal times", despite none of these moments being directly targeted by the calibration exercise. This, however, is not that surprising given that the parameters that govern default dynamics and the business cycle were taken from similar models calibrated to Argentina. What is reassuring is that the introduction of the endogenous risk of a depression from a lengthy default does not significantly alter the overall model performance during normal times.

Column (3) reports model statistics conditional on adoption below its median level in the ergodic distribution $m_t \leq 0.07$ (while the median adoption level is well below 1, there is full adoption 30 percent of the time). It is in this part of the ergodic distribution that spreads are significantly higher and more volatile, the aggregate capital stock is less than half of what it is in normal times, and the economy is not able to sustain as much debt. Once the economy gets pulled down to this part of the ergodic distribution, more frequent defaults (even if they are short) make transitioning back to normal times less likely. This point is explored further in the following subsections.

4.2 Bimodal ergodic distribution and the lack of "Big Push"

As suggested by Table 3 and consistent with the previous analysis regarding two stable steady states, Figure 6 shows that the ergodic distribution of the model is bimodal. It presents histograms for the natural logarithm of the productivity shock (relative to its mean), the natural logarighm of GDP, consumption, and investment, the debt-to-GDP ratio and spreads from 200,000 periods simulated from the model.

Figure 6: Ergodic distribution

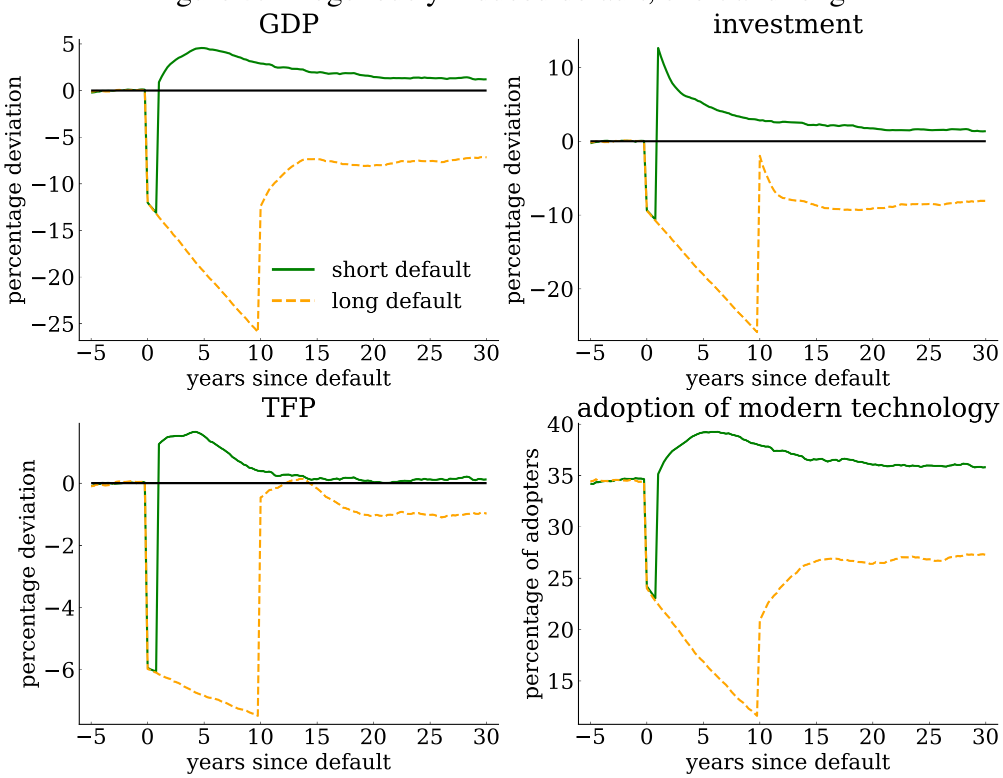


First, note that while the shock distribution appears normal (as expected, given the Gaussian assumption for its stochastic process) the other distributions feature two modes. Also, note that the histograms for GDP, consumption, debt, and investment accumulate more mass on the lower end of the distribution, illustrating how the model features an endogenous force that pulls the economy down it to the low steady state (default events) while a corresponding force to push it back to the high steady state is absent. Thus, states with high capital and low default risk are less frequent. Finally, the panel for spreads shows two conditional histograms: one with high adoption ($m_t = 1$) and one with adoption below the median. The distribution of the low-adoption periods is to the right of that of the high adoption and has a fatter tail. In addition, it has an overwhelmingly larger number of very high realizations (spreads above 30 percentage points, note that these are large values considering that spreads reported here are conditional on not defaulting).

4.3 Default depressions

To analyze the differentiated long-lasting effects that short and long defaults have in the model I proceed in two steps. First, I consider a random draw from the ergodic distribution and induce a default, regardless of whether the sovereign would have chosen to default or not. Then I simulate the economy thereafter imposing two durations for the default state: one and ten years. Then I let the economy be readmitted to financial markets (after one and ten years) and continue the simulation for up to 30 years after the induced default event. Figure 7 shows the average of 10,000 of such paths on a window of five years before and 30 years after the default.

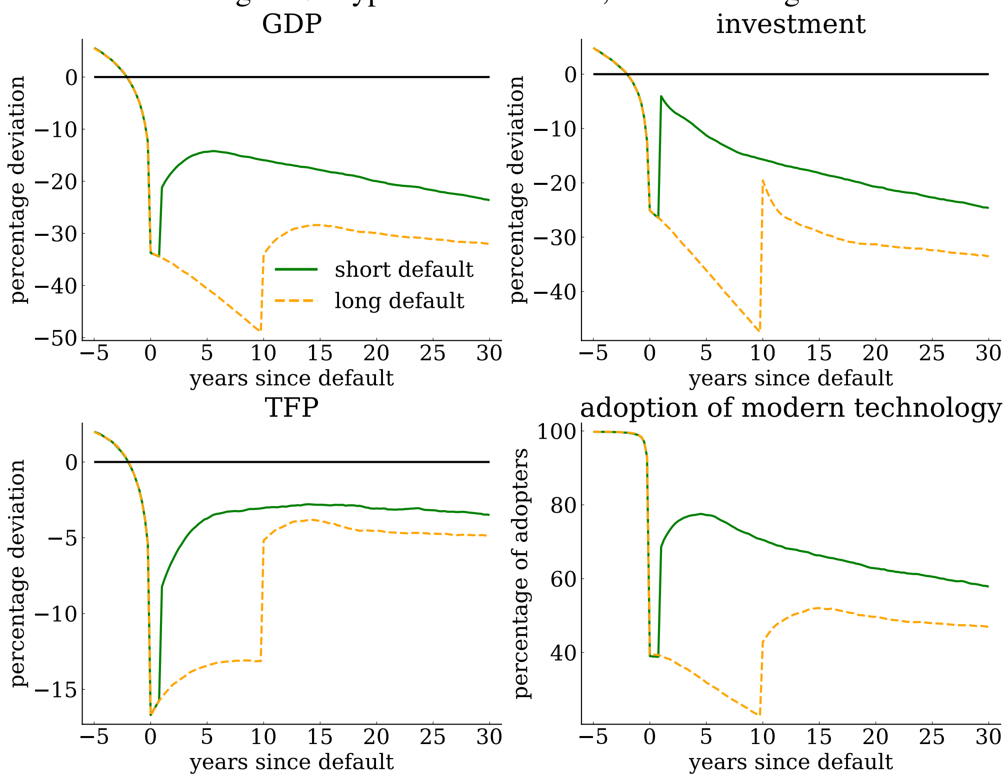
Figure 7: Exogenously induced default, short and long



The differences depicted on Figure 7 can be interpreted as the direct effect of the duration of a default, independent from other endognous choices that may determine the initial default and subsequent ones. The top-left panel shows that GDP not only immediately bounces back after the short default, but is actually higher during the following periods. This overshooting is indicative of the significant non-linearities in the model that drive its asymmetric ergodic distribution. The higher GDP level is supported by higher TFP due to significantly higher adoption. In contrast, GDP in the long default event bounces back to a level that is 10 percentage points lower than what GDP was on average prior to the default. Moreover, GDP remains well below its pre-default level even 30 years after the event and 20 after the event ended. Technology adoption does not recover in this case and capital and TFP remain at much lower levels.

Figure 8 does a similar exercise for the typical default in the model in normal times (i.e. defaults that endogenously happen when adoption is at its highest level). As is standard in models of sovereign default, default events are preceded by a sequence of bad shock realizations, which explain why GDP is already dropping in the years prior.

Figure 8: Typical default event, short and long



As before, the duration of the default event matters. GDP, TFP, investment, and adoption all bounce back to a higher level in the economy with the short default episode. The long default event has a differentiated and long-lasting negative effect that remains 20 years after the event was over. Average adoption of the modern technology does not fully recover and continues to slowly fall after default in both cases, increasing the vulnerability of the economy to future negative shocks.

5 Conclusion

Motivated by the effects of long defaults in the data, this paper presented a novel theory of default-induced depressions. In the model, long default episodes pull the economy down to a default trap in which the capital stock and technology adoption are lower, defaults are more frequent, and spreads are higher and more volatile than in normal times. Besides these novel dynamics, the model features during normal times all the standard business cycle properties of emerging economies that face sovereign risk, as all the workhorse models in this literature.

The model sheds light on a plausible mechanism through which an emerging (or even an advanced) economy may become trapped in a low-income and high-default risk regime. Moreover, the model results and the motivating evidence stress the importance of understanding the sources of lengthy default episodes and different policies to prevent them. It is tempting to conclude that the model makes a strong case for swift resolutions to sovereign crises through the intervention of international financial institutions. Moreover, one may conclude that this case is even stronger for crises in rich economies, who stand a lot more to lose (falling into a default trap). The effects of such interventions, however, are not innocuous because of the potential ex ante effects they may have on borrowing and default incentives. This scope for moral hazard would also have significant consequences on the regime switching dynamics introduced in this paper. Better understanding the effects of such interventions and other similar policy proposals is an exciting avenue for future research.

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A Data on default events

Table 4 summarizes the relevant data on default events from Laeven and Valencia (2018).

Table 4: Default events

Country	Default year	Default resolution	Duration
Albania	1990	1992	2
Argentina	1980	1993	13
Argentina	2001	2005	4
Argentina	2014	2016	2
Bulgaria	1990	1994	4
Belize	2007	2007	0
Bolivia	1980	1992	12
Brazil	1983	1994	11
Chile	1983	1990	7
Cote d'Ivoire	1984	1997	13
Cote d'Ivoire	2001	2009	8
Cote d'Ivoire	2010	2010	0
Cameroon	1989	1992	3
Congo, Dem. Rep.	1976	1989	13
Congo, Rep.	1986	1992	6
Costa Rica	1981	1990	9
Cyprus	2013	2013	0
Dominica	2002	2004	2
Dominican Republic	1982	1994	12
Dominican Republic	2003	2005	2
Ecuador	1982	1994	12
Ecuador	1999	2000	1
Ecuador	2008	2014	6
Egypt, Arab Rep.	1984	1992	8
Gabon	1986	1994	8
Guinea	1985	1992	7
Gambia, The	1986	1988	2
Greece	2012	2012	0
Grenada	2004	2005	1
Guyana	1982	1992	10
Honduras	1981	1992	11
Indonesia	1999	2002	3

Table 5: Default events (continued)

Country	Default year	Default resolution	Duration
Iran, Islamic Rep.	1992	1994	2
Jamaica	1978	1990	12
Jamaica	2010	2010	0
Jordan	1989	1993	4
Morocco	1983	1990	7
Moldova	2002	2002	0
Madagascar	1981	1992	11
Mexico	1982	1990	8
Malawi	1982	1988	6
Niger	1983	1991	8
Nigeria	1983	1992	9
Nicaragua	1980	1995	15
Panama	1983	1996	13
Peru	1978	1996	18
Philippines	1983	1992	9
Paraguay	1982	1994	12
Russian Federation	1998	2000	2
Sudan	1979	1985	6
Senegal	1981	1996	15
Sierra Leone	1977	1995	18
Seychelles	2008	2009	1
Togo	1979	1997	18
Trinidad and Tobago	1989	1989	0
Turkiye	1978	1982	4
Tanzania	1984	1992	8
Ukraine	1998	1999	1
Ukraine	2015	2015	0
Uruguay	1983	1991	8
Uruguay	2002	2003	1
Venezuela, RB	1982	1990	8
South Africa	1985	1993	8
Zambia	1983	1994	11

B Characterization of the adoption cutoff

B.1 Problem of a firm

- Given a production plan y_{it}, the cost minimization problem of the firm is

$$\begin{aligned} &\min_{k_{it},l_{it}} r_t k_{it} + w_t l_{it} \\ s.t. \qquad & y_{it} \leq z_{it} k_{it}^{\alpha} l_{it}^{1-\alpha} \end{aligned}$$

the F.O.C.s are

$$\begin{aligned} r_t &= \eta_{it} \alpha \frac{z_{it} k_{it}^{\alpha} l_{it}^{1-\alpha}}{k_{it}} \\ w_t &= \eta_{it} \left(1-\alpha\right) \frac{z_{it} k_{it}^{\alpha} l_{it}^{1-\alpha}}{l_{it}} \end{aligned}$$

where $\eta_{i,t}$ is the multiplier of the constraint. Assume the constraint binds so $\eta_{it} > 0$, rearranging we get

$$\frac{k_{it}}{l_{it}} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}$$

plugging into the constraint we get

$$\begin{aligned} l_{it} &= \left(\frac{1-\alpha}{\alpha} \frac{r_t}{w_t}\right)^{\alpha} \frac{y_{it}}{z_{it}} \\ k_{it} &= \left(\frac{\alpha}{1-\alpha} \frac{w_t}{r_t}\right)^{1-\alpha} \frac{y_{it}}{z_{it}} \end{aligned}$$

so the cost function is

$$\begin{aligned} C_{it}\left(y_{it}\right) &= \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^{\alpha} \frac{y_{it}}{z_{it}} \\ &= \frac{\mu_t}{z_{it}} y_{it} \end{aligned}$$

where $\mu_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^{\alpha}$

- Operating profits are

$$\pi_{it}^{O} = \max_{y_{it}} p_{it}\left(y_{it}\right) y_{it} - \frac{\mu_t}{z_{it}} y_{it}$$

using the demand curve the F.O.C. implies

$$p_{it} = \frac{\epsilon}{\epsilon - 1} \frac{\mu_t}{z_{it}}$$
$$y_{it} = \left(\frac{P_t}{p_{it}}\right)^{\epsilon} Y_t$$

price is a markup $\frac{\epsilon}{\epsilon-1}$ over marginal cost μ_t / z_{it}. Operating profits are then

$$\begin{aligned}\pi_{it}^{O} &= \frac{1}{\epsilon - 1} \frac{\mu_t}{z_{it}} y_{it} \\ &= \frac{1}{\epsilon} \left(\frac{\epsilon - 1}{\epsilon} \frac{z_{it}}{\mu_t}\right)^{1-\epsilon} \left(P_t\right)^{\epsilon} Y_t.\end{aligned}$$

B.2 Price of final good and aggregate productivity

- The price of the final good satisfies

$$\begin{aligned}P_t^{1-\epsilon} &= \int_0^1 \left(p_{it}\right)^{1-\epsilon} di \\ &= \int_0^1 \left(\frac{\epsilon}{\epsilon - 1} \frac{\mu_t}{z_{it}}\right)^{1-\epsilon} di \\ &= \left(\frac{\epsilon - 1}{\epsilon} \frac{1}{\mu_t}\right)^{\epsilon-1} \int_0^1 \left(z_{it}\right)^{\epsilon-1} di\end{aligned}$$

define a productivity aggregator as

$$A_t = \left(\int_0^1 \left(z_{it}\right)^{\epsilon-1} di\right)^{\frac{1}{\epsilon-1}}$$

then the price of the final good is

$$P_t = \frac{\epsilon}{\epsilon - 1} \frac{\mu_t}{A_t}$$

- Note that

$$\int_0^1 \left(z_{it}\right)^{\epsilon-1} di = \int_1^{\infty} \left(z_{it}\right)^{\epsilon-1} dF\left(z_i\right)$$

let the break-even cutoff productivity to operate the modern technology be z_t^*, then

$$\int_0^1 (z_{it})^{\epsilon-1}\, di = \int_1^{z_t^*} (z_{it})^{\epsilon-1}\, dF(z_i) + \int_{z_t^*}^{\infty} (z_{it})^{\epsilon-1}\, dF(z_i)$$

recall that z_i are distributed Pareto, so

$$\begin{aligned} F(z_i) &= 1 - z_i^{-\xi} \\ dF(z_i) &= \xi z_i^{-\xi-1} \end{aligned}$$

plugging in and rearranging

$$\int_0^1 (z_{it})^{\epsilon-1}\, di = \xi\,(\hat{z} z_t)^{\epsilon-1} \int_1^{z_t^*} (z_i)^{\epsilon-\xi-2}\, dz_i + \xi\,(z_t)^{\epsilon-1} \int_{z_t^*}^{\infty} (z_i)^{\epsilon-\xi-2}\, dz_i$$

which implies that the integrals are

$$\begin{aligned} \int_1^{z_t^*} (z_i)^{\epsilon-\xi-2}\, dz_i &= \frac{(z_i)^{\epsilon-\xi-1}}{\epsilon-\xi-1}\Big|_1^{z_t^*} \\ &= \frac{1}{1+\xi-\epsilon}\left[1-\left(\frac{1}{z_t^*}\right)^{1+\xi-\epsilon}\right] \\ \int_{z_t^*}^{\infty} (z_i)^{\epsilon-1}\, dF(z_i) &= \frac{(z_i)^{\epsilon-\xi-1}}{\epsilon-\xi-1}\Big|_{z_t^*}^{\infty} \\ &= \frac{1}{1+\xi-\epsilon}\left(\frac{1}{z_t^*}\right)^{1+\xi-\epsilon} \end{aligned}$$

and note that the second is well defined iff $0 < 1+\xi-\epsilon$. Plugging in and rearranging

$$\int_0^1 (z_{it})^{\epsilon-1}\, di = (z_t)^{\epsilon-1}\,\frac{\xi}{1+\xi-\epsilon}\left[z_T^{\epsilon-1} + \left(1-z_T^{\epsilon-1}\right)\left(\frac{1}{z_t^*}\right)^{1+\xi-\epsilon}\right]$$

- So the productivity aggregator is

$$A_t = A\left(z_t^*\right) = z_t\left(\frac{\xi}{1+\xi-\epsilon}\left[z_T^{\epsilon-1} + \left(1-z_T^{\epsilon-1}\right)\left(\frac{1}{z_t^*}\right)^{1+\xi-\epsilon}\right]\right)^{\frac{1}{\epsilon-1}}$$

note that in the limit where no firm adopts the modern technology we get

$$\lim_{z_t^* \to \infty} A\left(z_t^*\right) = z_t z_T \left(\frac{\xi}{1+\xi-\epsilon}\right)^{\frac{1}{\epsilon-1}}$$

and in the limit where all firms adopt the modern technology we get

$$\lim_{z_t^* \to 1} A\left(z_t^*\right) = z_t \left(\frac{\xi}{1+\xi-\epsilon}\right)^{\frac{1}{\epsilon-1}}$$

B.3 Aggregate production

- Taking stock, we know:

$$\begin{aligned}
l_{it} &= \left(\frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right)^{\alpha}\frac{y_{it}}{z_{it}} \\
k_{it} &= \left(\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{1-\alpha}\frac{y_{it}}{z_{it}} \\
\mu_t &= \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t}{\alpha}\right)^{\alpha} \\
p_{it} &= \frac{\epsilon}{\epsilon-1}\frac{\mu_t}{z_{it}} \\
y_{it} &= \left(\frac{P_t}{p_{it}}\right)^{\epsilon} Y_t \\
p_{it}y_{it} &= \left(p_{it}\right)^{1-\epsilon}\left(P_t\right)^{\epsilon} Y_t \\
A_t &= \left(\int_0^1 \left(z_{it}\right)^{\epsilon-1} di\right)^{\frac{1}{\epsilon-1}} \\
P_t &= \frac{\epsilon}{\epsilon-1}\frac{\mu_t}{A_t}
\end{aligned}$$

- Combining individual factor demands with optimal production plans and rearranging we get

$$\begin{aligned}
l_{it} &= \frac{1-\alpha}{w_t}\frac{\epsilon-1}{\epsilon}\left(p_{it}\right)^{1-\epsilon}\left(P_t\right)^{\epsilon} Y_t \\
k_{it} &= \frac{\alpha}{r_t}\frac{\epsilon-1}{\epsilon}\left(p_{it}\right)^{1-\epsilon}\left(P_t\right)^{\epsilon} Y_t
\end{aligned}$$

integrating both sides over all firms i

$$\int_0^1 l_{it} di = \int_0^1 \frac{1-\alpha}{w_t} \frac{\epsilon - 1}{\epsilon} (p_{it})^{1-\epsilon} (P_t)^{\epsilon} Y_t di$$
$$\int_0^1 k_{it} di = \int_0^1 \frac{\alpha}{r_t} \frac{\epsilon - 1}{\epsilon} (p_{it})^{1-\epsilon} (P_t)^{\epsilon} Y_t di$$

simplifying and using market clearing

$$L_t = \frac{1-\alpha}{w_t} \frac{\epsilon - 1}{\epsilon} P_t Y_t$$
$$K_t = \frac{\alpha}{r_t} \frac{\epsilon - 1}{\epsilon} P_t Y_t$$

so we get the aggregate capital-to-labor ratio is

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} = \frac{k_{it}}{l_{it}} \qquad \forall i$$

- Production of firm i is

$$y_{it} = \left(\frac{P_t}{p_{it}} \right)^{\epsilon} Y_t$$

plugging in for p_{it} and P_t we get

$$y_{it} = \left(\frac{\frac{\epsilon}{\epsilon-1} \frac{\mu_t}{A_t}}{\frac{\epsilon}{\epsilon-1} \frac{\mu_t}{z_{it}}} \right)^{\epsilon} Y_t$$
$$= \left(\frac{z_{it}}{A_t} \right)^{\epsilon} Y_t$$

plugging in for the production function and rearranging

$$\left(\frac{k_{it}}{l_{it}} \right)^{\alpha} l_{it} = (z_{it})^{\epsilon-1} \left(\frac{1}{A_t} \right)^{\epsilon} Y_t$$

plugging in for the aggregate capital-to-labor ratio

$$\left(\frac{K_t}{L_t} \right)^{\alpha} l_{it} = (z_{it})^{\epsilon-1} \left(\frac{1}{A_t} \right)^{\epsilon} Y_t$$

integrating over i and rearranging we get that aggregate production is

$$Y_t = A_t (K_t)^{\alpha} (L_t)^{1-\alpha}$$

B.4 Labor supply

- Let households have GHH preferences, then

$$\sum_{t=0}^{\infty} \beta^t \frac{\left(c - \frac{L^{1+\nu}}{1+\nu}\right)^{1-\sigma}}{1-\sigma}$$

the F.O.C. with respect to labor implies

$$L^{\nu} = w_t$$

recall the capital-to-labor ratio is

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}$$

plugging in and rearranging

$$L^{1+\nu} = \frac{1-\alpha}{\alpha} K_t r_t$$

- Now, recall from the individual capital demand we had

$$k_{it} = \frac{\alpha}{r_t} \frac{\epsilon - 1}{\epsilon} (p_{it})^{1-\epsilon} (P_t)^{\epsilon} Y_t$$

integrating over i and using market clearing and the definition of P_t we get

$$K_t = \frac{\alpha}{r_t} \frac{\epsilon - 1}{\epsilon} P_t Y_t$$

normalizing $P_t = 1$, plugging in for Y_t and rearranging we get

$$K_t r_t = \alpha \frac{\epsilon - 1}{\epsilon} A_t (K_t)^{\alpha} (L_t)^{1-\alpha}$$

- Plugging into the household's optimality condition from labor supply we get

$$L_t = \left[(1-\alpha) \frac{\epsilon - 1}{\epsilon} A_t (K_t)^{\alpha} \right]^{\frac{1}{\alpha+\nu}}$$

  the labor supply L_t as a function of the state (z_t, K_t) and of the cutoff z_t^* (note A_t depends on the cutoff)

- So output is

$$\begin{aligned} Y_t &= A_t (K_t)^{\alpha} (L_t)^{1-\alpha} \\ &= \left[(1-\alpha) \frac{\epsilon - 1}{\epsilon} \right]^{\frac{1-\alpha}{\alpha+\nu}} (A_t)^{\frac{1+\nu}{\alpha+\nu}} (K_t)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)} \end{aligned}$$

B.5 Adoption cutoff

- Operating profits of a firm i are

$$\pi_{it}^{O} = \frac{1}{\epsilon - 1} \frac{\mu_t}{z_{it}} y_{it}$$

  plugging in for $y_{it} = \left(\frac{P_t}{p_{it}}\right)^{\epsilon} Y_t$ and using the normalization $P_t = 1$ we get

$$\pi_{it}^{O} = \frac{1}{\epsilon - 1} \frac{\mu_t}{z_{it}} \left(\frac{1}{p_{it}}\right)^{\epsilon} Y_t$$

  plugging in for $p_{it} = \frac{\epsilon}{\epsilon-1} \frac{\mu_t}{z_{it}}$ we get

$$\pi_{it}^{O} = \frac{1}{\epsilon} \left(\frac{\epsilon - 1}{\epsilon} \frac{z_{it}}{\mu_t} \right)^{\epsilon-1} Y_t$$

- The productivity cutoff z_t^* satisfies

$$\pi_{it}^{OT} = \pi_{it}^{OM} - f_m (1 + r_t)$$

plugging in for π_{it}^{OT}

$$\frac{1}{\epsilon}\left(\frac{\epsilon-1}{\epsilon}\frac{z_t z_t^* z_T}{\mu_t}\right)^{\epsilon-1} Y_t = \frac{1}{\epsilon}\left(\frac{\epsilon-1}{\epsilon}\frac{z_t z_t^*}{\mu_t}\right)^{\epsilon-1} Y_t - f_m\left(1+r_t\right)$$

rearranging

$$\left(\frac{\epsilon-1}{\epsilon}\frac{z_t z_t^*}{\mu_t}\right)^{\epsilon-1}\left[1-\left(z_T\right)^{\epsilon-1}\right] = \frac{\epsilon f_m\left(1+r_t\right)}{Y_t}$$

plugging in for $Y_t = \left[(1-\alpha)\frac{\epsilon-1}{\epsilon}\right]^{\frac{1-\alpha}{\alpha+\nu}}\left(A_t\right)^{\frac{1+\nu}{\alpha+\nu}}\left(K_t\right)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)}$ and rearranging:

$$\left(z_t^*\right)^{\epsilon-1} = \frac{\epsilon f_m\left(1+r_t\right)}{\left(A_t\right)^{\frac{1+\nu}{\alpha+\nu}}\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(K_t\right)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)}\left[1-\left(z_T\right)^{\epsilon-1}\right]}\left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}+(\epsilon-1)}\left(\frac{\mu_t}{z_t}\right)^{\epsilon-1}$$

- Note that μ_t is a function of A_t:

$$\mu_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t}{\alpha}\right)^{\alpha}$$
$$w_t = \frac{1-\alpha}{\alpha}\frac{K_t}{L_t}r_t$$
$$r_t = \alpha\frac{\epsilon-1}{\epsilon}A_t\left(K_t\right)^{\alpha-1}\left(L_t\right)^{1-\alpha}$$
$$L_t = \left[(1-\alpha)\frac{\epsilon-1}{\epsilon}A_t\left(K_t\right)^{\alpha}\right]^{\frac{1}{\alpha+\nu}}$$

plugging in and rearranging

$$\mu_t = \frac{\epsilon-1}{\epsilon}A_t$$

- Plugging for μ_t and rearranging we get

$$\left(z_t^*\right)^{\epsilon-1} = \frac{\epsilon f_m\left(1+r_t\right)}{\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(K_t\right)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)}\left[1-\left(z_T\right)^{\epsilon-1}\right]}\left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(\frac{1}{z_t}\right)^{\epsilon-1}\left(A_t\right)^{(\epsilon-1)-\frac{1+\nu}{\alpha+\nu}}$$

- Note that A_t is decreasing in z_t^*. If we assume $(\epsilon-1) > \frac{1+\nu}{\alpha+\nu}$ then the RHS is decreasing in z_t^*. As $z_t^* \to \infty$ we get

$$\infty > \frac{\epsilon f_m\left(1+r_t\right)}{\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(K_t\right)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)}\left[1-\left(z_T\right)^{\epsilon-1}\right]}\left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(\frac{1}{z_t}\right)^{\frac{1+\nu}{\alpha+\nu}}\left(z_T\left(\frac{\xi}{1+\xi-\epsilon}\right)^{\frac{1}{\epsilon-1}}\right)^{(\epsilon-1)-\frac{1+\nu}{\alpha+\nu}}$$

so the LHS is higher with high enough cutoff z_t^*. This means that there's always a positive mass of firms adopting the modern technology. To have an interior cutoff we need

$$1 < \frac{\epsilon f_m (1+r_t)}{(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}} (K_t)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)} \left[1-(z_T)^{\epsilon-1}\right]} \left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}} \left(\frac{1}{z_t}\right)^{\frac{1+\nu}{\alpha+\nu}} \left(\left(\frac{\xi}{1+\xi-\epsilon}\right)^{\frac{1}{\epsilon-1}}\right)^{(\epsilon-1)-\frac{1+\nu}{\alpha+\nu}}$$

otherwise we get the corner solution of full adoption $z_t^* = 1$. This could happen with a large enough stock of capital or a large enough productivity shock z_t.

- If the above inequality does not hold then the equation

$$\left(z_t^*\right)^{\epsilon-1} = \frac{\epsilon f_m (1+r_t)}{(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}} (K_t)^{\alpha\left(\frac{1+\nu}{\alpha+\nu}\right)} \left[1-(z_T)^{\epsilon-1}\right]} \left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1-\alpha}{\alpha+\nu}} \left(\frac{1}{z_t}\right)^{\epsilon-1} \left(A\left(z_t^*\right)\right)^{(\epsilon-1)-\frac{1+\nu}{\alpha+\nu}}$$

pins down $z_t^* \in (1,\infty)$. Once we know z_t^* we know A_t and all other variables.