$$S(k) - 2S(k-1) + S(k-2) = 2$$
$$S(0) = 25, \quad S(1) = 16$$

~~Asymmetric~~ Linear Recurrence Relation

Homogeneous

$S(k) + C_1 S(k-1) + C_2 S(k-2) - - - C_n S(k-n) = 0$

Non-homogeneous

$S(k) + C_1 S(k-1) + C_2 S(k-1) + \dots + C_n S(k-n) = f(k)$

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Generating function (Numeric functions):

Let S_n be an sequence defined for $n \geq 0$ then the indinite sum denoted by $G(S, z)$ is defined as $S_0 + S_1 z + S_2 z^2 + \cdots \infty$

$$= G(S, z) = \sum_{i=0}^{\infty} S_i z^i$$

Que Suppose there is sequence of no. that is defined as 2^n. Find generating function

Sol $2^0, 2^1, 2^2, 2^3, \ldots$

Generation func^n $G(S, z) = \sum_{i=0}^{\infty} S_i z^i$

$$= \sum_{i=0}^{\infty} 2^i z^i$$

$$= 1 + 2z + (2z)^2 + \cdots$$

$$= \frac{1}{1-2z} \quad \left[\because \frac{a}{1-r}\right]$$

Que Sequence $= ba^n$. Find generating func^n $G(S, z)$

Sol $ba, ba^2, ba^3, \ldots$

$$G(S, z) = \sum_{i=0}^{\infty} ba^i z^i$$

$$= b \sum_{i=0}^{\infty} a^i z^i$$

$$= b\left[1 + az + a^2 z^2 + \cdots\right]$$

$$= \frac{b}{1-az}$$

Que obtain Partial fraction decomposition for ~~the~~ expression $G(S, z) = \dfrac{6-29z}{1-11z+30z^2}$ and identify sequence having this expression as generating function

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Sol. $G(z) = \dfrac{6-29z}{1-11z+30z^2}$

i) ~~A~~ Partial Fraction

$$1-11z+30z^2 = 30z^2-11z+1$$
$$= 30z^2-6z-5z+1$$
$$= 6z(5z-1)-(5z-1)$$
$$= (6z-1)(5z-1)$$
$$= (1-6z)(1-5z)$$

$$\cancel{\frac{A}{(5z-1)} + \frac{B}{(6z-1)} = \frac{6-29z}{1-11z+30z^2}}$$

$$\cancel{(6z-1)A + (5z-1)B = 6-29z}$$

$$\cancel{(6A+5B)z - (A+B) = 6-29z}$$

$$\cancel{6A+5B = -29 \quad \text{--(i)}} \qquad \cancel{-(A+B) = 6 \quad \text{(ii)}}$$

$$\cancel{6A+5B = -29}$$
$$\cancel{-6A-6B = 30}$$

$B =$

$$\frac{A}{1-5z} + \frac{B}{1-6z} = \frac{6-29z}{(1-5z)(1-6z)}$$

$$A = -1$$
$$\therefore B = 7$$

$$\frac{-1}{1-5z} + \frac{7}{1-6z}$$

Sequence :- $-1 \cdot 5^n + 7 \cdot 6^n$

10 marks que

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How to ~~generate~~ solve ~~retr~~ recurrence relation using generating func^n

$S(k) - 3S(k-1) - 2 = 0 \quad , \quad k \geq 1 \quad , S(0) = 1$

Order $= k(k-1) = 1$

$S(k)\, z^k - 3S(k-1)\, z^k =$

Step I — Multiply whole eq^n with z^k

Step II — Summing up for $k \geq 1$

$$\underbrace{\sum_{k=1} S(k) \cdot z^k}_{\mathrm{I}} - 3 \underbrace{\sum_{k=1}^{\infty} S(k-1)\, z^k}_{\mathrm{II}} = 2 \underbrace{\sum_{k=1}^{\infty} z^k}_{\mathrm{III}}$$

~~Input~~ I Part

From definition of $G(S,z)$

$$G(S,z) = \sum_{k=0}^{\infty} S(k) \cdot z^k$$

$$= S(0) + S(1) z + S(2) z^2 + \dots + S(n)\, z^n + \dots$$

$$G(S,z) = S(0) + \sum_{k=1}^{\infty} S(k) \cdot z^k$$

$$G(S,z) - S(0) = \sum_{k=1}^{\infty} S(k)\, z^k$$

II Part

$$\sum_{k=1}^{\infty} S(k-1)\, z^k = z \sum_{k=1}^{\infty} S(k-1)\, z^{k-1}$$

change k to $k+1$ — bcz infinite series मे एक term add या subtract करने पर कोई फरक नहीं पड़ता

$$= z \sum_{k=0}^{\infty} S(k) \cdot z^k$$

$$= z \cdot G(S,z)$$

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III Part

$$2 \sum_{k=1}^{\infty} z^k$$

$$= 2\left[z+z^2+z^3+\cdots\right]$$

$$= 2z\left[1+z+z^2+\cdots\right]$$

$$= \frac{2z}{1-z}$$

~~Final eqn~~ $$\boxed{G(s,z) - s(0) - 3z\,G(s,z) = \frac{2z}{1-z}}$$

~~Now obtain sequence from this~~

$$\cancel{G(s,z)[1 - 3z\,G(s,z) - s(0)]} =$$

$$G(s,z) - 1 - 3z\,G(s,z) = \frac{2z}{1-z}$$

$$G(s,z) - 3z\,G(s,z) = \frac{2z}{1-z} + 1 = \frac{2z+1-z}{1-z}$$

$$G(s,z) - 3z\,G(s,z) = \frac{z+1}{1-z}$$

$$G(s,z) = \frac{z+1}{(1-z)(1-3z)}$$

$$\frac{A}{1-z} + \frac{B}{1-3z} = \frac{1+z}{(1-z)(1-3z)}$$

$$A = -1$$
$$B = 2$$

$$\text{Final} = \frac{-1}{1-z} + \frac{2}{1-3z} = -1\cdot(1)^n + 2\cdot(3)^n$$

$$= -1 + 2\cdot 3^n$$

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