Linear Regression

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Basic Elements of Linear Regression

Basic Elements of Linear Regression

Linear regression, which dates to Gauss and Legendre, is perhaps the simplest, and by far the most popular approach to solving regression problems. What makes linear regression *linear* is that we assume that the output truly can be expressed as a *linear* combination of the input features.

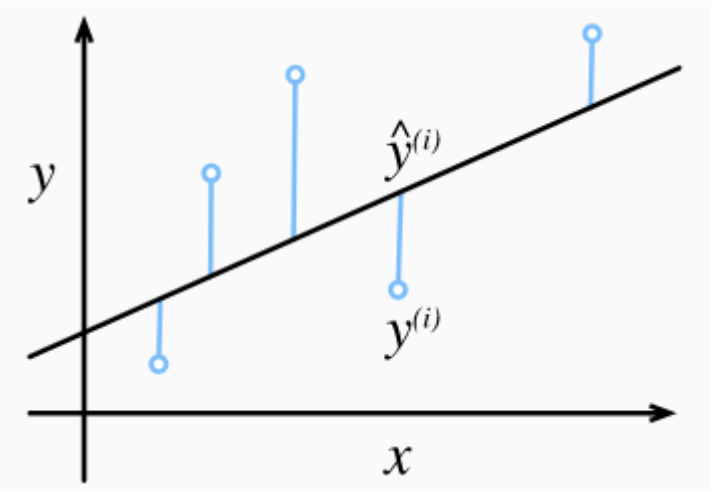
Training Data

Typically, we will use n to denote the number of samples in our dataset. We index the samples by I, denoting each input data point as $x^{(i)} = [x_1^{(i)}, x_2^{(i)}]$ and the corresponding label as $y^{(i)}$

Loss Function

$$l^{(i)}(\mathbf{w}, b) = \frac{1}{2}\left(\hat{y}^{(i)} - y^{(i)}\right)^2,$$

$$L(\mathbf{w}, b) = \frac{1}{n}\sum_{i=1}^{n} l^{(i)}(\mathbf{w}, b) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{2}\left(\mathbf{w}^\top \mathbf{x}^{(i)} + b - y^{(i)}\right)^2.$$



$$\mathbf{w}^*, b^* = \underset{\mathbf{w}, b}{\operatorname{argmin}}\ L(\mathbf{w}, b).$$

Analytic Solution

$$\mathbf{w}^* = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T y$$

Gradient descent

$$(\mathbf{w}, b) \leftarrow (\mathbf{w}, b) - \frac{\eta}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} \partial_{(\mathbf{w}, b)} l^{(i)}(\mathbf{w}, b)$$

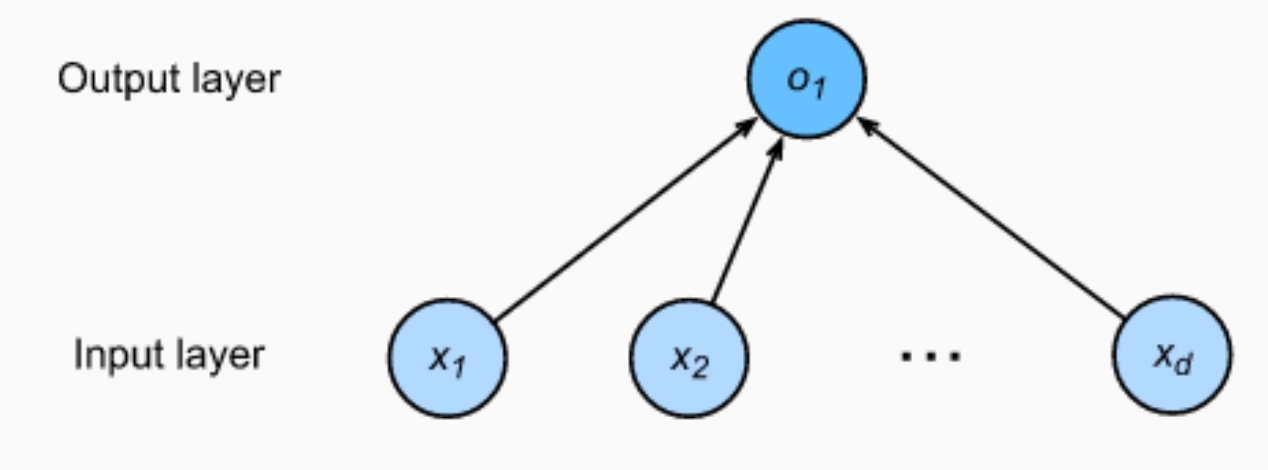
$$\begin{aligned}
\mathbf{w} &\leftarrow \mathbf{w} - \frac{\eta}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} \partial_{\mathbf{w}} l^{(i)}(\mathbf{w}, b) &&= w - \frac{\eta}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} \mathbf{x}^{(i)} \left(\mathbf{w}^\top \mathbf{x}^{(i)} + b - y^{(i)} \right), \\
b &\leftarrow b - \frac{\eta}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} \partial_b l^{(i)}(\mathbf{w}, b) &&= b - \frac{\eta}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} \left(\mathbf{w}^\top \mathbf{x}^{(i)} + b - y^{(i)} \right).
\end{aligned}$$

Model Prediction

$$\hat{\mathbf{w}}^\top x + \hat{b},$$

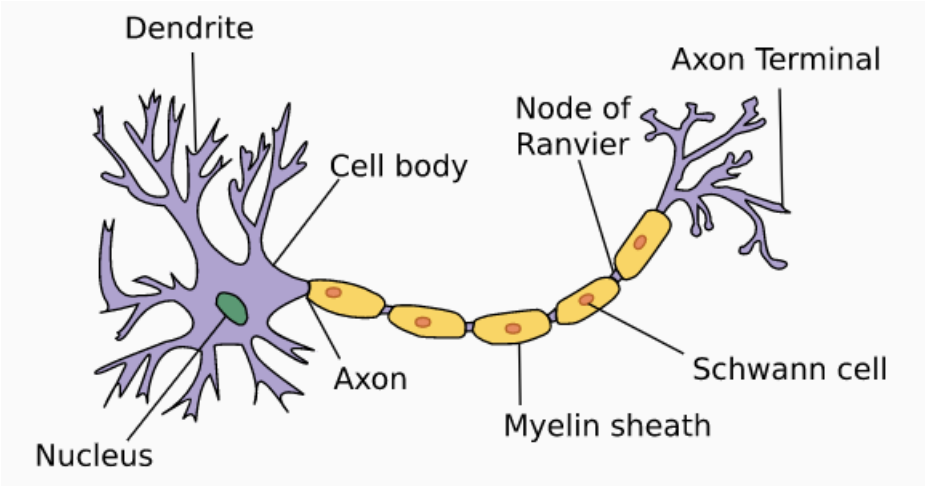
From Linear Regression to Deep Networks

Neural Network Diagram



Linear regression is a single-later neural network.

Biology



The real neuron

Thanks!

Does anyone have any questions?

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Resources

Dive into Deep Learning