```
2. Calculate
```

T(n)

and the order of complexity of each algorithm in the

big-Oh

or

Theta

notation using any of the methods we discussed in class. (a) Show your calculations and

derivations step by step in detail, then (b) state the polynomial (or recurrence relation for

the recursive algorithms) T(n) and (c) determine the complexity order of the algorithm.

Incomplete answers to this part will get a zero, not a partial grade

Algorithm 1:

There are 3 nested loops, each iterates through a dimension of the matrices a,b, and c. The outer loop goes from 1 to p, middle loop from 1 to r, and the inner loop from 1 to q. The innermost loop has constant time ops.

```
The time complexity is O(p^* r * q)
The recurrence relation is T(n) = O(p * r * q)
```

Algorithm 2:

Loops are divided into blocks the size of t. The outer loops iterate from 1 to p and from 1 to r in steps of T, and the inner loop is iterated from 1 to q in steps of T. Constant time operations inside the innermost loop.

```
Time complexity is O((p * r * q) / (T^2))
Recurrence relation is T(n) = O((p * r * q) / (T^2))
```

Algorithm 3:

```
If (p,q,r) < 8, iterative algorithm is used
```

Time complexity is
$$T(n) = O(p * r * q)$$

If matrices are split until reaching base case, time complexity would be O(n^log2(7)) where n is the dimension of the matrices.

Algorithm 4:

Uses divide and conquer strategy with block partitioning. Partitions into 4 equal sixe blocks until base cased is reached

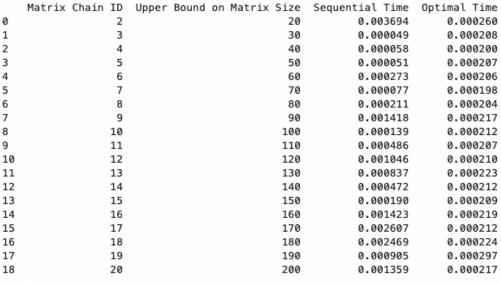
The recurrence relation is $T(n) = 8*T(n/2) + O(n^2)$ for matrix multiplication and O(1) for other operations

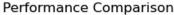
The complexity order is O(n^3) using the master theorem.

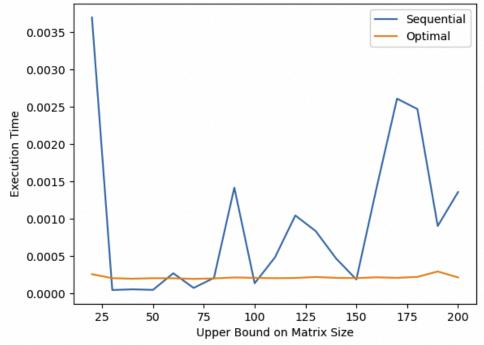
Algorithm 5:

Uses block partitioning and dividend conquer. Partitions A and B into 4 equal sized blocks recursively until base case. Performs 7 recursive matrix multiplications, then combines them

The recurrence relation is $T(n) = 7 * T(n/2) + O(n^2)$

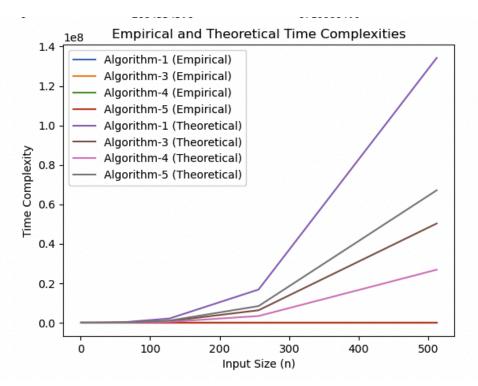






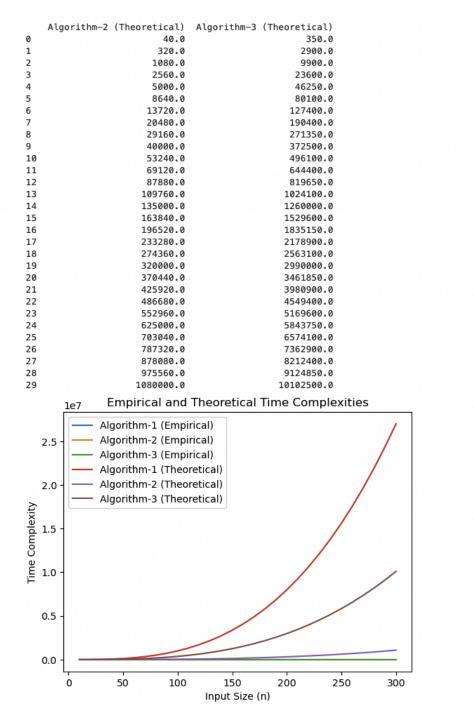
Matrix chain multiplication graph: This graph shows that the optimal solution outperforms the sequential solution significantly. Even though the sequential solution is faster at specific data points, the optimal solution is a more consistent performer that could be over twice as fast in certain scenarios.

```
Size
        Algorithm-1 (Empirical) Algorithm-3 (Empirical) \
0
                    2.384186e-07
                                               2.384186e-07
      1
1
      2
                    4.291534e-07
                                               4.291534e-07
2
      4
                     0.000000e+00
                                               0.000000e+00
3
      8
                    1.192093e-07
                                               1.192093e-07
4
                    0.000000e+00
                                               0.000000e+00
     16
5
     32
                    0.000000e+00
                                               0.000000e+00
6
    64
                    1.907349e-07
                                               1.907349e-07
7
    128
                    0.000000e+00
                                               0.000000e+00
8
    256
                     0.000000e+00
                                               0.000000e+00
9
    512
                     5.245209e-07
                                               5.245209e-07
   Algorithm-4 (Empirical) Algorithm-5 (Empirical)
0
              2.384186e-07
                                         2.384186e-07
1
              4.291534e-07
                                         4.291534e-07
2
              0.000000e+00
                                         0.000000e+00
3
              1.192093e-07
                                         1.192093e-07
4
              0.000000e+00
                                         0.000000e+00
5
              0.000000e+00
                                         0.000000e+00
6
              1.907349e-07
                                         1.907349e-07
7
              0.000000e+00
                                         0.000000e+00
8
              0.000000e+00
                                         0.000000e+00
9
              5.245209e-07
                                         5.245209e-07
   Algorithm-1 (Theoretical) Algorithm-3 (Theoretical)
                                             1.250000e-01
0
                            1
                            8
1
                                             2.000000e+00
2
                           64
                                             2.000000e+01
3
                          512
                                             1.760000e+02
4
                         4096
                                             1.472000e+03
5
                        32768
                                             1.203200e+04
6
                                             9.728000e+04
                       262144
7
                      2097152
                                             7.823360e+05
8
                                             6.275072e+06
                    16777216
9
                    134217728
                                             5.026611e+07
   Algorithm-4 (Theoretical)
                               Algorithm-5 (Theoretical)
0
                          0.2
                                                       0.5
1
                          1.6
                                                      4.0
2
                                                     32.0
                         12.8
3
                        102.4
                                                    256.0
4
                        819.2
                                                   2048.0
5
                       6553.6
                                                  16384.0
6
                                                 131072.0
                      52428.8
7
                    419430.4
                                                1048576.0
8
                   3355443.2
                                                8388608.0
9
                   26843545.6
                                               67108864.0
```



Empirical vs. Theoretical time complexities: Experiment 2: For this experiment, the empirical time does not line up with the theoretical complexity for the smaller input sizes. This could be due to the presence of hidden constants. However, as the size of the input increases, the empirical time seems to be closer to the theoretical complexity figure, indicating correct behavior.

Size Algorithm-1 (Empirical Algorithm-2 (Empirical 1.748493=e)		61		7 \	
1 20 1.986821e-07 1.986821e-0 2 30 3.973643e-07 3.973643e-0 3 40 0.00000e+00 0.00000e+0 4 50 2.384186e-07 2.384186e-0 5 60 1.351039e-07 1.351039e-0 6 70 1.271566e-07 1.27156e-0 7 80 3.337860e-07 3.337860e-0 8 90 4.371007e-07 4.371007e-0 9 100 1.986821e-07 1.986821e-0 10 110 1.033147e-07 1.43511e-0 11 120 1.436511e-07 1.43651e-0 11 120 1.436511e-07 1.43651e-0 13 140 2.384186e-07 3.814697e-0 14 150 3.019969e-07 3.019969e-0 15 160 5.563100e-08 5.563100e-0 16 170 1.589457e-07 1.589457e-0 17 180 1.033147e-07 1.781550e-0 19 200 2.463659e-07 2.463659e-0 20 210 2.145767e-07 2.145767e-0 21 220 1.827876e-07 1.827876e-0 22 230 2.861023e-07 2.861023e-0 24 250 1.986821e-07 1.986821e-0 25 260 2.622604e-07 2.861023e-0 26 270 2.861023e-07 2.861023e-0 27 288 290 2.304713e-07 2.304713e-0 28 290 2.304713e-07 2.304713e-0 29 300 2.940496e-07 2.940496e-0 Algorithm-3 (Empirical) Algorithm-1 (Theoretical) \ 0 1.748403e-07 2.940496e-07 2.940496e-0 26 270 2.861023e-07 2.861023e-07 2.861023e-0 27 280 2.70277e-0 7.702077e-0 28 29 3.3973643e-07 2.940496e-07 2.940496e-0 1 1.986821e-07 3.337860e-07 3.337800e-0 1 1.986821e-07 3.337800e-0 1 1.986821e-07 3.337800e-0 1 1.986821e-07 3.304713e-0 2 3.3973643e-07 2.940496e-07 2.940496e-0 1 1.748403e-07 3.337800e-0 1 1.986821e-07 3.337800e-0 2 2.361023e-07 3.861000e-0 1 1.936821e-07 3.337800e-0 2 3.3973643e-07 3.337800e-0 2		Size	-		-
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17 180 1.033147e-07 1.033147e-0 18 190 2.781550e-07 2.781550e-0 19 200 2.463659e-07 2.463659e-0 20 210 2.145767e-07 2.145767e-0 21 220 1.827876e-07 1.827876e-0 22 230 2.861023e-07 2.861023e-0 23 240 7.152557e-08 7.152557e-0 24 250 1.986821e-07 1.986821e-0 25 260 2.622604e-07 2.622604e-0 26 270 2.861023e-07 2.861023e-0 27 280 2.702077e-07 2.702077e-0 28 290 2.304713e-07 2.304713e-0 29 300 2.940496e-07 2.940496e-0 Algorithm-3 (Empirical) Algorithm-1 (Theoretical) \ 0 1.748403e-07 1000 1 1.986821e-07 8000 2 3.937643e-07 27000 3 0.00000e+00 64000 4 2.384186e-07 125000 5 1.351039e-07 216000 6 1.271566e-07 343000 7 3.337860e-07 125000 8 4.371007e-07 729000 9 1.986821e-07 1000000 1 0.033147e-07 1331000 11 1.430511e-07 1728000 12 3.814697e-07 1728000 13 2.384186e-07 2744000 14 3.019969e-07 3375000 15 5.563100e-08 4096000 16 1.589457e-07 1910000 17 1.033147e-07 1832000 18 2.781550e-07 8000000 20 2.145767e-07 9261000 21 1.827876e-07 19648000 22 2.861023e-07 10648000 23 7.152557e-08 13824000 24 1.986821e-07 10648000 25 2.62604e-07 17576000 26 2.861023e-07 19653000 27 2.78077e-07 12952000 28 2.304713e-07 15625000 26 2.861023e-07 15625000 27 2.770277e-07 21952000 28 2.304713e-07 17576000 26 2.861023e-07 15625000 27 2.770277e-07 21952000 28 2.304713e-07 1755000	15	160	5.56310	5.563100e-0	
18 190	16	170	1.58945	1.589457e-0	
19 200	17	180	1.03314	7e-07	1.033147e-0
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19	17		1.033147e-07		5832000
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25	23		7.152557e-08		13824000
26 2.861023e-07 19683000 27 2.702077e-07 21952000 28 2.304713e-07 24389000	24		1.986821e-07		15625000
27 2.702077e-07 21952000 28 2.304713e-07 24389000	25		2.622604e-07		17576000
28 2.304713e-07 24389000	26		2.861023e-07		19683000
28 2.304713e-07 24389000	27		2.702077e-07		21952000
29 2.940496e-07 27000000	28		2.304713e-07		24389000
	29		2.940496e-07		27000000



Empirical vs. Theoretical time complexities: Experiment 1: for this experiment, the empirical time does not consistently match each algorithm's predicted complexity as input size increases. This seems to be occurring because all the empirical time is the same. This could be a result of the same input sizes giving a consistent answer for empirical time. Ensuring different matrix pairs could help reflect the true behavior of the algorithms.

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Algorithm-1: 90,100,110,120;202,228,254,280;314,356,398,440;426,484,542,600 Algorithm-2: 90,100,110,120;202,228,254,280;314,356,398,440;426,484,542,600 Algorithm-3: 90,100,110,120;202,228,254,280;314,356,398,440;426,484,542,600 Algorithm-4: 90,100,110,120;202,228,254,280;314,356,398,440;426,484,542,600 Algorithm-5: 90,100,110,120;202,228,254,280;314,356,398,440;426,484,542,600
```

Matrix multiplication output