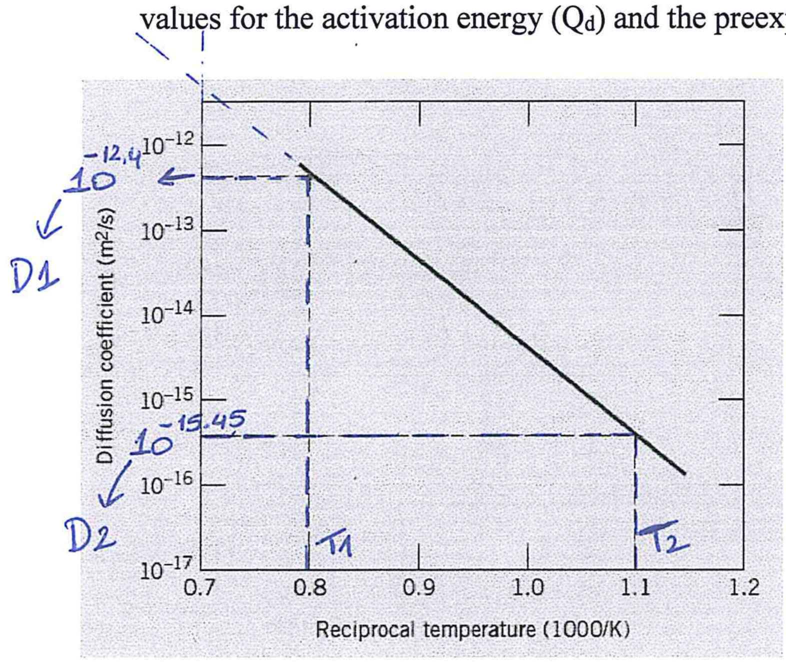
SOLUTIONS OF IN CLASS 2

1) A plot of the logarithm of the diffusion coefficient versus reciprocal of absolute temperature is shown in the below figure, for the diffusion of copper in gold. Determine values for the activation energy (Q_d) and the preexponential (D_o).



$$\log D = \log D_o - \frac{Q_d}{2{,}3R}\left(\frac{1}{T}\right)$$

This equation can be taken as a form of a straight line:

$$y = b + mx$$

So; $b = \log D_o$ → intercept

$m = -\dfrac{Q_d}{2{,}3R}$, m → slope

$$Q_d = -2{,}3R\,(\text{slope})$$

$$Q_d = -2{,}3R\left[\frac{\Delta \log D}{\Delta\left(\frac{1}{T}\right)}\right] = -2{,}3R\left[\frac{\log D_1 - \log D_2}{\frac{1}{T_1} - \frac{1}{T_2}}\right]$$

$$Q_D = -2{,}3 \cdot (8{,}31\ \text{J/mol.K})\left[\frac{\log 10^{-12{,}40} - (+\log 10^{-15.45})}{0{,}8.10^{-3}(K^{-1}) - 1{,}1.10^{-3}.(K^{-1})}\right]$$

$$Q_D = -2{,}3 \cdot 8{,}31 \cdot \left[\frac{-12{,}40 - (-15.45)}{0{,}8.10^{-3} - 1{,}1.10^{-3}}\right]$$

$$Q_D = 194.315\ \text{J/mol}$$

$$Q_D = 194{,}315\ \text{kJ/mol}$$

Rather than trying to make an extrapolation to determine D_0, you can obtain a more accurate value by using equation:

$$\log D_0 = \log D + \frac{Q_D}{2{,}3R}\left(\frac{1}{T}\right)$$

for values of D_2 and T_2;

$$\log D_0 = \log 10^{-15.45} + \frac{194.315\ J/mol\ .(1{,}1 . 10^{-3} . K^{-1})}{2{,}3\ .\ 8{,}31\ J/mol.K}$$

$$\log D_0 = -15{,}45 + \frac{194315\ .\ 1{,}1 . 10^{-3}}{2{,}3\ .\ 8{,}31}$$

$$\log D_0 = -4{,}27$$

$$D_0 = 10^{-4{,}27}\ m^2/s$$

$$D_0 \cong 5.2 \times 10^{-5}\ m^2/s$$

2) The diffusion coefficients for copper in aluminum at 500 and 600°C are 4.8×10^{-14} and 5.3×10^{-13} m^2/s, respectively. Determine the approximate time at 500°C that will produce the same diffusion result (in terms of concentration of Cu at some specific point in Al) as a 10 h heat treatment at 600°C.

For some specific concentration of solute (C_1) in an alloy;

$$\frac{C_1 - C_o}{C_s - C_o} = \text{constant}$$

$$\frac{x}{2\sqrt{Dt}} = \text{constant}$$

or $$\frac{x^2}{Dt} = \text{constant}$$

The composition in both diffusion situations will be equal at the same position; x is also constant. So,

$$Dt = \text{constant}$$

$$D_{500} \cdot t_{500} = D_{600} \cdot t_{600}$$

$$t_{500} = \frac{D_{600} \cdot t_{600}}{D_{500}} = \frac{(5.3 \cdot 10^{-13}\ m^2/s)(10 \cdot h)}{(4.8 \cdot 10^{-14}\ m^2/s)}$$

$$t_{500} = 110.4 \text{ hours}$$