1. Consider a single crystal of some hypothetical metal that has the FCC crystal structure and is oriented such that a tensile stress is applied along a $[\bar{1}02]$ direction. If slip occurs on a (111) plane and in a a $[\bar{1}01]$ direction, compute the stress at which the crystal yields if its critical resolved shear stress is 3.42 MPa.

$$\theta = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}}\right] \qquad \sigma_y = \frac{\tau_{crss}}{(\cos\phi\cos\lambda)_{max}}$$

First, it is necessary to determine the values of ϕ and λ.

λ is the angle between $[\bar{1}02]$ and $[\bar{1}01]$.

Therefore, $u_1 = -1$ $\quad v_1 = 0$ $\quad w_1 = 2$

$u_2 = -1$ $\quad v_2 = 0$ $\quad w_2 = 1$

$$\lambda = \cos^{-1}\left[\frac{(-1)(-1) + (0)(0) + (2)(1)}{\sqrt{((-1)^2 + (0)^2 + (2)^2).((-1)^2 + (0)^2 + (1)^2)}}\right]$$

$$\lambda = \cos^{-1}\left(\frac{3}{\sqrt{10}}\right) = 18.4^\circ$$

The normal to the (111) slip plane is [111] direction.

ϕ is the angle between $[\bar{1}02]$ and $[111]$.

Therefore, $u_1 = -1$ $\quad v_1 = 0$ $\quad w_1 = 2$

$u_2 = 1$ $\quad v_2 = 1$ $\quad w_2 = 1$

$$\phi = \cos^{-1}\left[\frac{(-1)(1)+(0)(1)+(2)(1)}{\sqrt{\left((-1)^2+(0)^2+(2)^2\right)\cdot\left((1)^2+(1)^2+(1)^2\right)}}\right]$$

$$\phi = \cos^{-1}\left(\frac{3}{\sqrt{15}}\right) = 39.2^\circ$$

$$\sigma_y = \frac{\tau_{crss}}{\cos\phi \cdot \cos\lambda}$$

$$\sigma_y = \frac{3.42 \text{ MPa}}{\left(\frac{3}{\sqrt{10}}\right)\cdot\left(\frac{3}{\sqrt{15}}\right)} = 4.65 \text{ MPa}$$

2. The lower yield point for an iron that has an average grain diameter of 5×10^{-2} mm is 135 MPa. At a grain diameter of 8×10^{-3} mm, the yield point increases to 260 MPa. At what grain diameter will the lower yield point be 205 MPa?

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

σ_y (MPa)	d (mm)	$d^{-1/2}$ $(mm)^{-1/2}$
135	5×10^{-2}	4.47
260	8×10^{-3}	11.18

$$135 = \sigma_0 + 4.47 \cdot k_y$$

$$260 = \sigma_0 + 11.18\, k_y \rightarrow \sigma_0 = 260 - 11.18\, k_y$$

$$135 = (260 - 11.18\, k_y) + 4.47\, k_y$$

$$6.71\, k_y = 125 \qquad k_y = 18.63 \text{ MPa} \cdot mm^{1/2}$$

$$\sigma_0 = 51.7 \text{ MPa}$$

At a yield strength of 250 MPa;

$$250 = 51.7 + 18.63 \cdot d^{-1/2}$$

$$d^{-1/2} = 8.23 \; mm^{-1/2}$$

$$d = 1.48 \times 10^{-2} \text{ mm}$$