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Cognitive load and mental rotation: structuring orthographic projection for learning and problem solving

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Abstract. Cognitive load theory was used to generate a series of three experiments to investigate the effects of various worked example formats on learning orthographic projection. Experiments 1 and 2 investigated the benefits of presenting problems, conventional worked examples incorporating the final 2-D and 3-D representations only, and modified worked examples with several intermediate stages of rotation between the 2-D and 3-D representations. Modified worked examples proved superior to conventional worked examples without intermediate stages while conventional worked examples were, in turn, superior to problems. Experiment 3 investigated the consequences of varying the number and location of intermediate stages in the rotation trajectory and found three stages to be superior to one. A single intermediate stage was superior when nearer the 2-D than the 3-D end of the trajectory. It was concluded that (a) orthographic projection is learned best using worked examples with several intermediate stages and that (b) a linear relation between angle of rotation and problem difficulty did not hold for orthographic projection material. Cognitive load theory could be used to suggest the ideal location of the intermediate stages.

In engineering, architecture and other related professions, the manner in which 3 dimension (3-D) objects are represented as 2 dimensional (2-D) drawings is standardised. Three orthogonal views are drawn directly from the top, front and the left and these views permit details of the physical structure to be communicated to others. This procedure, known as orthographic projection, requires a process of rotation. The object must be rotated so that it presents a view perpendicular to each of the three faces to be drawn. While rotation sometimes may be physical when the object is available and can be rotated, frequently mental rotation must occur. This paper is concerned with the cognitive processes associated with mental rotation and most importantly, given those cognitive processes, with effective procedures for presenting and learning orthographic projection.

Substantial work has been carried out on mental rotation (e.g. see Shepard & Metzler, 1971; Bethell-Fox & Shepard, 1988; Cooper & Shepard, 1982; Metzler & Shepard, 1982; Carpenter & Just, 1986; Olson & Bialystok, 1983; Rock, Wheeler & Tudor, 1989) and some of this work is relevant to current concerns. Metzler & Shepard (1982: 67) argued on the basis of a series of experiments involving the mental rotation of 3-D objects that 'if the angle of rotation is at all large, the subjects can only prepare for the appearance of the object in the rotated orientation by first passing through other states of preparation corresponding to an ordered sequence of intermediate orientations'. Metzler and Shepard assume that people construct mental representations of the object and that these representations are rotated. The intermediate rotations preserve the essential structure of the initial model.

Consider people who must learn to transform 3-D objects into 2-D drawings and vice-versa. What procedures might they use? Cooper (1988) isolated two strategies used by subjects required to manipulate orthographic projections. The most common strategy involved the mental construction of a 3-D object. An alternative strategy required a more direct matching and extrapolation of parts of figures. Importantly, while this alternative strategy does not require the construction of 3-D representations, evidence was obtained that such representations were constructed. Cooper concluded that 3-D representations were generated because they increased problem solving efficiency by making structural features explicit. If so, it is plausible to suggest that the most common way of transforming 2-D to 3-D representations and vice-versa is by constructing a 3-D representation and rotating it. One needs to ask what the consequences might be of the requirement to engage in this mental process.

Cognitive load theory and orthographic projection

Cognitive load theory (see Sweller, 1988) was devised initially to explain why people could solve problems but learn little of their structure. It is argued that cognitive load imposed by task-information may exhaust cognitive resources of individuals and consequently affect learning (Halford, 1993). Halford describes cognitive resources as being the individual's cognitive ability to deal with a task and cognitive load being the demand made by a task on the individual's mental effort for the successful completion of the task. When the demand made by a task is low a greater improvement is achieved from the given increase in available cognitive resource. Similarly, according to cognitive load theory, learning frequently fails not because of the intrinsic complexity of the information, but rather because it is presented in a manner that requires wasteful use of cognitive resources. Sometimes, material, because of its format, must be mentally reformulated before learning can commence. Complex cognitive activities must be engaged in, not because they are essential to learning but because the structure of the information must be re-organised in order to permit learning. A heavy, extraneous cognitive load may be imposed that interferes with the assimilation of essential information.

Problem solving, under some circumstances, provides one example of a cognitive activity that is inappropriate from a learning perspective. Assume experts use schemas to solve problems that allow them to be classified according to the moves that permit solution (see Chi, Glaser & Rees, 1982; Larkin, McDermott, Simon & Simon, 1980). Sweller (1988) suggested that the means-ends strategy used by novices to solve problems (searching for differences between each problem state encountered and the goal state and then searching for operators to reduce those differences) misdirected attention and imposed a heavy cognitive load. For learning to occur, attention needed to be directed to each problem state and its associated moves, not to a cognitively demanding search process for differences between problem states and for relevant operators. This search process may be irrelevant to schema acquisition and so prevent cognitive resources from being used for learning. A similar discordance may occur between the cognitive requirements of solving orthographic projection problems and learning to solve such problems.

In this paper it is assumed that a person faced with the problem of drawing orthogonal 2-D views of a 3-D representation must learn to mentally rotate the object on three different occasions in three different directions in order to have the front, top and left perpendicular to the viewing point. When faced with the alternative task of determining the structure of an object from three, 2-D representations, it is assumed that a person must learn to rotate each of the 2-D representations in an appropriate direction to permit all views to be integrated into a single structure. Constructing images of intermediate stages of rotation presumably requires cognitive resources. Cooper (1988: 85) states, "... the construction of an internal model corresponding to a three-dimensional object must require time, effort and processing resources ...." If a cognitive load is imposed, then the nature of the load needs to be analysed to determine its possible effects on learning.

While the search for and construction of intermediate stages by the solver may be essential to solving the problem, paradoxically, it may not be essential to *learning how* to construct the images. In order to construct a rotated image, a person may need to engage in a trial and error process involving a search for the correct set of relations between the various entities of the rotated object. In turn, this process requires discovering the correct set of relations between the original and rotated objects. These activities are an essential precursor to learning the relations between rotated images only because of the manner in which the material is presented. Learning these relations need not include problem solving search. Search and learning are different processes and learning can occur without search. There are grounds for hypothesising that cognitive resources required to engage in a trial and error search for and a construction of intermediate (and final) stages may be unavailable for learning about the relations brought into play when an object is rotated. If searching for appropriate images requires different cognitive processes to learning but draws on the same pool of cognitive resources, then the search process may impose a heavy cognitive load that interferes with learning.

Worked examples and orthographic projection

Some worked examples appropriately direct attention and reduce cognitive load. Cooper & Sweller (1987) and Sweller & Cooper (1985) found the presentation of many algebra worked examples resulted in superior learning to having students solve the equivalent algebra problems. Studying a worked example utilises a forward working strategy which eliminates the search for subgoals and operators thereby freeing cognitive resources which can be engaged in learning the problem solution. Given these results, it can be hypothesised that the presentation of many orthographic projection examples should be superior to the presentation of equivalent problems.

Worked examples include both the 2-D and the 3-D representations. Problems include only the 2-D *or* the 3-D representations with problem solvers required to find the alternative representation. Worked examples eliminate the need to either search for or construct the goal (which will be a 2-D or 3-D representation depending on the starting point). Searching for or constructing an appropriate 2-D or 3-D structure is a quite different cognitive activity to studying a particular rotation. Neither search nor the process of construction is carried out with the intention of learning. Any learning is incidental. In contrast, studying is intended to result in learning. Cognitive resources freed from the search or construction process can be used to learn how to rotate the images.

If nothing more than 2-D and 3-D representations are presented to students, resources will need to be devoted to establishing how one perspective was transformed into the other. It can be hypothesised that the presentation of examples that include intermediate rotational stages would be even more advantageous than merely presenting the start and the goal states. The intermediate stages will reduce further the need for cognitive activity extraneous to learning by indicating the relations needed to transform one perspective into another. By physically presenting intermediate stages, students would not have to generate them mentally. Cognitive resources otherwise used to generate intermediate stages can be used for learning instead.

The inclusion of intermediate rotational stages also should be beneficial because they in effect, integrate the 2-D representations and 3-D representations. Sweller, Chandler, Tierney & Cooper (1990), Tarmizi & Sweller (1988) and Ward & Sweller (1990) found that worked examples and other instructional procedures were not effective if they required students to split their attention between disparate sources of information that required integration before they could be understood (e.g. a text and associated diagram that could be understood only when mentally integrated). It was suggested that cognitive resources required for integration were unavailable for learning. By physically integrating multiple sources of information, extraneous cognitive load was reduced and the examples became effective. In the same way, by eliminating the need for students to mentally integrate 2-D and 3-D representations, extraneous cognitive load should be reduced and resources made available for learning. The intermediate stages should, in effect, physically integrate the 2-D and 3-D representations eliminating the need for mental integration.

While it has been argued on the basis of cognitive load theory that having to generate mental images imposes an extraneous cognitive load that interferes with learning, there is nothing necessary about this argument. For example, one could argue that the increase in processing resources required to search for intermediate stages over having them presented *assists* in learning. The act of searching for and constructing intermediate images may in fact help in learning the solution procedure. On this hypothesis, the cognitive resources required to generate an intermediate image are also used in *learning* to generate the solution images. If those cognitive resources are not used because the intermediate image is presented, learning will be correspondingly reduced.

This argument clearly contradicts cognitive load theory with its central assumption that resources required for problem solving are not available for learning. Experimentally, it is a simple matter to test between these two hypotheses. Both hypotheses suggest that processing time should be increased by having to generate images. Cognitive load theory suggests that this increase in time available for learning should, nevertheless, result in a decrease in learning, as indicated on subsequent tests. The alternative hypothesis suggests that increases in learning time should result in increases in learning. Performance on test items thus distinguishes between these two hypotheses.

To sum the argument from a cognitive load theory perspective:

1. When transforming a 3-D to 2-D perspective using the rules of orthographic projection, students must mentally rotate the images through a series of intermediate stages.
2. Rotating images through a series of intermediate stages requires either a search or construction process for both the intermediate stages and the goal states.

3. Searching for or constructing intermediate stages and the goal requires cognitive resources that consequently are unavailable for learning.
4. By providing the intermediate stages and the goal, cognitive resources can be devoted to learning how the transformations occur rather than searching for or constructing appropriate transformations.

The experiments reported in this paper were designed primarily to test the fourth point. The first experiment, using 3-D to 2-D transformations, compared groups given (a) conventional problems to solve, (b) conventional worked examples to study in which the 2-D representations were presented and (c) modified worked examples in which the intermediate stages as well as the 2-D representations were present. The second experiment was identical to the first except that the transformation was from 2-D to 3-D.

Differential performance can be hypothesised during both a learning/acquisition phase and the subsequent test phase. During acquisition, a conventional problem group must generate both the final goal state and many intermediate rotational stages. A conventional worked example group only has to generate the intermediate stages while a modified worked example group does not have to generate any stages because they are all provided. Tentatively, it can be hypothesised that conventional problems, because of the large number of rotational stages that must be generated, will impose a heavy cognitive load and so take a long time to process. Modified worked examples, with no stages to generate should take less time while conventional worked examples should be intermediate. These hypotheses can only be tentative because it is under the control of subjects how long they choose to study worked examples. The stronger hypotheses concern performance on test problems. If generating stages imposes a heavy cognitive load that interferes with learning, then not only should modified worked examples require less time in acquisition, more should be learned and enhanced learning should be demonstrated on subsequent test problems.

The third experiment investigated the cognitive load consequences of reducing the number of intermediate stages and the degree of rotation of the intermediate stages.

Experiment 1

Experiment 1 had two related functions. Firstly, it was designed to compare the effects of presenting conventional problems with conventional worked examples to students learning orthographic projection. Secondly, and more importantly, an appropriate structure for the worked example was investigated.

From previous work using cognitive load theory it is known that studying appropriately structured worked examples can facilitate learning more than solving the equivalent problems. From the work on mental rotation we have evidence that people construct intermediate images when identifying rotated objects. It is plausible to hypothesise that when learning to rotate a 3-D representation of an object in order to construct 2-D representations, extraneous cognitive load is reduced maximally by providing a worked example with several intermediate stages included. The modified worked example group of this experiment used this procedure. The conventional worked example group used the same procedure without the intermediate stages. The conventional problem group used problems that the subjects had to solve, rather than worked examples.

Methods

Subjects. The subjects were thirty, year 10 students from a Sydney metropolitan high school. These students had no previous exposure to technical drawing but had at least an intermediate level of mathematics for their age. Average age of subjects was 15.8 years. Subjects were selected by strict randomization with no reference to gender.

Procedure. The initial instructional material was identical for all groups. It contained, in written form, an introduction to orthographic drawing. This introduction included projection theory which involves perceiving the orthogonal views from different perspectives – top, front and left – and rotating them into a correct orientation for the 2-D representations (see Figure 1).

The acquisition material consisted of four problems in which 3-D representations of cubes stacked together in various configurations had to be depicted as 2-D representations. Presentation modes differed for the three groups. The conventional problem group was presented with four conventional problems. A 3-D representation of a solid object was given and the solver was required to construct the 2-D representations and draw them in their respective positions. The conventional worked example group was presented with the same four problems, but Problems 1 and 3 were presented as conventional worked examples, each followed by a conventional problem identical to Problems 2 and 4 in the conventional problem group. The worked examples had a 3-D representation of the solid object and the three 2-D representations of the given solid. The modified worked example group was again presented with the same four problems but this time Problems 1 and 3 were modified worked examples that included a sequence of intermediate states of the transforma-

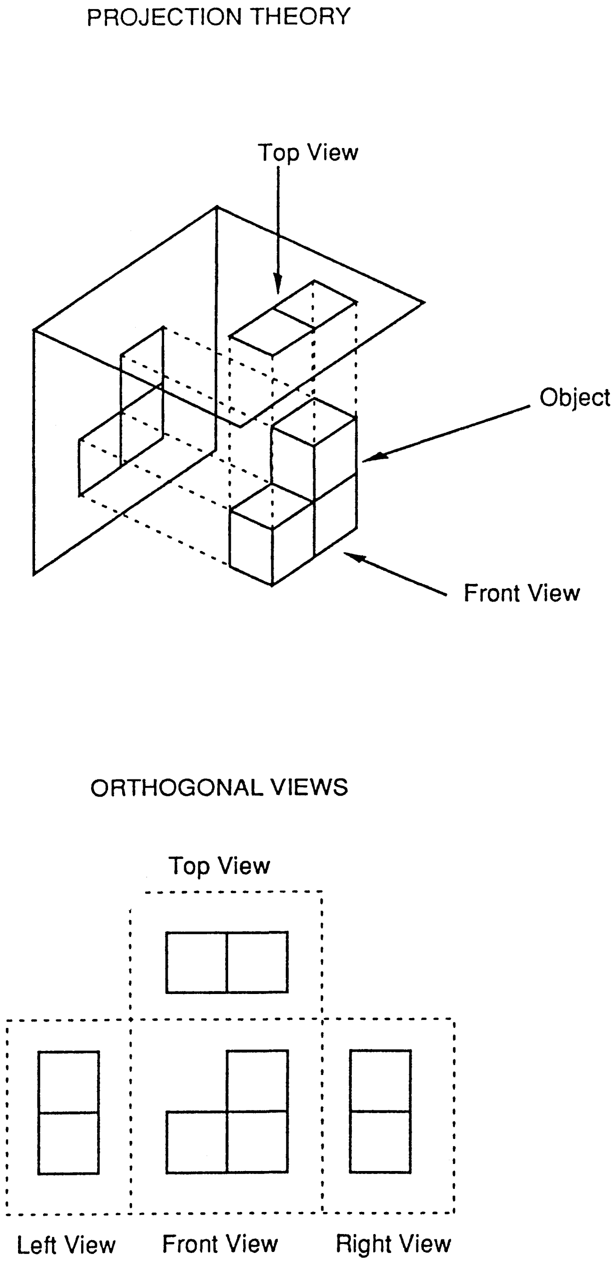

Figure 1. Diagrammatic illustration of projection theory and relevant orthogonal views.

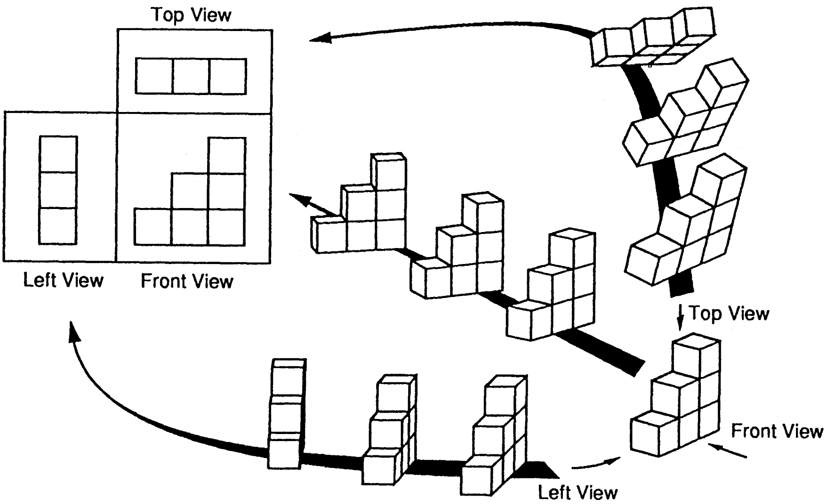

Figure 2. A modified worked example used during acquisition in Experiments 1, 2, and 3.

tion as an integrating device (see Figure 2). Problems 2 and 4 were identical to the equivalent problem presented to the conventional problem group.

The test material was common to all groups and consisted of three conventional problems all of which were similar to those used in the acquisition phase.

Subjects were randomly assigned to three groups (10 subjects per group) and tested individually. They were given up to ten minutes to read the initial instructional material. During acquisition, the subjects were given five minutes to solve each problem or study a worked example, after which they had to move to the next problem or worked example irrespective of whether they had correctly solved the problem or understood the worked example. The only feedback subjects received during the five minutes was whether a presented solution was correct or not. If incorrect, additional solution attempts continued until either the problem was solved or five minutes had elapsed. If, after five minutes, subjects were still unable to solve a problem, then the correct solution was shown.

Following acquisition, all subjects were presented with the set of test problems. They were given five minutes to solve each problem with no feedback or correction. Time taken to solve during acquisition and test was recorded as well as errors made during the two phases. An error was recorded if a line was drawn incorrectly in the 2-D representations. Every line in the orthogonal views is a potential error. If three wrong attempts were made on a line it was recorded as three errors.

Table 1. Times to solution and errors during acquisition and test of Experiment 1.

Problem	Conventional problem		Conventional worked example		Modified worked example	
ACQUISITION			Times to solution			
	Mean	S.D.	Mean	S.D.	Mean	S.D.
1	191.1	86.46	66.6	23.86	66.8	15.54
2	118.1	32.19	153.4	95.70	89.8	36.51
3	81.2	32.18	45.9	15.84	45.8	12.40
4	217.0	58.10	190.0	52.09	168.4	53.63
			Errors on Problems 2 and 4			
	3.4	1.64	2.9	1.37	1.9	1.19
TEST			Times to solution			
1	191.4	84.88	145.6	62.69	92.9	29.34
2	214.2	78.36	142.0	60.00	116.5	51.19
3	210.0	58.17	176.4	62.26	152.0	38.17
			Errors			
	4.8	1.23	3.2	0.89	2.1	0.62

[Solution times are in seconds and errors are frequency of incorrect responses.]

Results and discussions

Mean acquisition times and standard deviations are displayed in Table 1. Orthogonal contrasts indicated that when using combined times of worked examples (Problems 1 & 3), the modified worked example group spent significantly less time with worked examples than the conventional worked example group (on worked examples) and the conventional group (on conventional problems), $F(1,27) = 10.824$. (A one-tailed test with an alpha of 0.05 is used throughout this paper.) Similarly, the combined times of the following, similar problems (Problems 2 & 4) indicated more rapid solution by the modified worked example group compared to the other two groups, $F(1,27) = 7.290$.

Contrasting the conventional problem group with the conventional worked example group on Problems 1 and 3 combined, indicated that the conventional worked example group spent significantly less time studying their worked examples than the conventional problem group required to solve the equivalent problems, $F(1,27) = 32.604$. A similar comparison using Problems 2 and

4 (presented as problems to solve for both groups) indicated no significant difference between the two groups, $F(1,27) = 0.057$.

The results conclude that the modified worked example group spent substantially less time on acquisition than a combination of the other two groups, while the conventional worked example group required less time to process worked examples than the conventional problem group required to solve the equivalent problems. Inspection of Table 1 indicates that on Problems 1 and 3, while there was a large difference between the conventional problem group and the worked example groups, the means of the latter 2 groups were very close. Nevertheless, evidence for a difference between the worked example groups during acquisition may be seen on Problems 2 and 4. While the modified worked example group was significantly different from the conventional problem and conventional worked example groups, the latter two groups did not differ on these problems. (Differences between conventional and modified worked example groups are investigated in detail in Experiment 3.)

The results, in accordance with the cognitive load theory, suggest differences in acquisition times. The fact that modified worked examples generally were processed rapidly suggests that the procedure reduced cognitive load compared to conventional worked examples and problems. Whether the reduced cognitive load of modified worked examples freed the resources for learning can be determined from the test results.

The effects of acquisition on the test problems are of major interest. A comparison between the groups using orthogonal contrasts indicated that the modified worked example group was significantly more rapid than a combination of the conventional problem and conventional worked example groups on all problems: Problem 1, $F(1,27) = 9.548$; Problem 2, $F(1,27) = 6.150$ and Problem 3 $F(1,27) = 3.880$, $0.05 < p < 0.1$. The conventional problem and the conventional worked example groups did not differ on Problem 1, $F(1,27) = 2.624$ or Problem 3, $F(1,27) = 1.932$. The conventional worked example group solved Problem 2 significantly more rapidly than the conventional problem group, $F(1,27) = 6.350$. Based on these results, it is clear that the modified worked example group, despite spending less time on acquisition, solved the test problems more rapidly than the other two groups. This is the major finding of Experiment 1.

Mean errors on the acquisition and test problems are indicated in Table 1. Errors were defined as lines incorrectly drawn in the 2-D representations. Contrasting errors in acquisition, the modified worked example group made significantly fewer errors on the 2nd & 4th problems than the other two groups, $F(1,27) = 5.198$. (There was no opportunity for errors by the worked example groups on the 1st & 3rd problems.) The conventional problem group and the conventional worked example did not differ significantly, $F(1,27) = 0.624$.

An analysis of errors during the test phase showed that the modified worked example group made fewer errors (across all problems) than the conventional problem and the conventional worked example groups, $F(1,27) = 10.112$. The conventional problem and conventional worked example groups also differed on this measure, $F(1,27) = 5.382$.

The results of this experiment provide evidence that modifications of worked examples by progressive adjustments, while maintaining the structure of the representation, facilitate learning. In accordance with the theories of mental rotation, it is hypothesised that the transformation information provided through progressive adjustments from 3-D representation to 2-D representations, when not given, had to be processed mentally. From cognitive load theory, it is hypothesised that the cognitive resources required for this process are not devoted to learning. As a consequence, the conventional problem and the conventional worked example groups should learn less than the modified worked example group. This result was obtained. There was no evidence of increased acquisition times translating into increased learning.

Experiment 2

The results of Experiment 1 provided evidence that appropriately structured worked examples are useful in learning to transform 3-D representations to 2-D representations. Experiment 2 was designed to test the effectiveness of providing progressive adjustments rather than requiring students to generate the intermediate steps, using 2-D to 3-D transformations.

Method

Subjects. Thirty, year 10 students from a Sydney metropolitan high school were used as subjects. Their background was similar to the subjects used in Experiment 1. Average age of this group was 15.4 years.

Procedure. The initial instructional material was similar to Experiment 1 but subjects were advised verbally to place some emphasis on the 2-D to 3-D transformation when studying the information. Following the instructional material, a set of four acquisition problems was presented to three groups. The conventional problem group was presented the four problems, each of which had three, 2-D representations given. The problem task was to find the correct

3-D representation that corresponded to the given 2-D representations from a set of 4 possible 3-D representations. The conventional worked example group was presented identical problems to the conventional problem group except that Problems 1 and 3 were presented as worked examples where both the 2-D representations and the 3-D representations were given. Subjects were verbally informed that the direction of the transformation was 2-D to 3-D. The modified worked example group also used identical problems except that Problems 1 and 3 were worked examples which included a series of drawings illustrating intermediate stages of the transformation from the three, 2-D representations to the 3-D representations. The direction of transformation (2-D to 3-D) was indicated by an arrow.

Subjects were assigned randomly to the three groups (10 subjects per group) and tested individually. Up to ten minutes was given to read the instructional material. In acquisition, all groups were given a total of five minutes to solve the problems or study the worked examples. The conventional problem subjects were given 3 minutes to study the orthogonal views during which time they were asked to construct a mental picture of the solid. Once the 3 minutes was up, subjects were asked to select the correct 3-D representations from 4 possible solutions given. The 2-D representations were still available during this period. Subjects were allowed up to 2 minutes to identify correctly the 3-D representation. The time to identify (up to 2 minutes) was recorded. Subjects were informed if their choice was correct and if not correct were asked to choose again. If they had not chosen the correct alternative within 2 minutes, they were shown the correct solution.

The conventional worked example group was given the same problems except Problems 1 and 3 were presented as conventional worked examples, each followed by a conventional problem identical to Problems 2 and 4 of the conventional problem group. The times recorded for the worked examples excluded the first 3 minutes in order that these times could be compared to problem solution times (conventional problem subjects could not choose a 3-D representation until 3 minutes had elapsed). The modified worked example group followed a procedure identical to the conventional worked example group except that modified worked examples were used.

Following the acquisition phase, four test problems were presented to all subjects. The test problems used were similar to the acquisition problems. The procedure used for the test problems was similar to that used for the conventional problems in acquisition. Each problem consisted of the three, 2-D representation of a solid and four 3-D representations, one of which corresponded to the 2-D representations. The 3-D representations were shown only after subjects had studied the 2-D representations for 3 minutes. Subjects were given up to 2 minutes to identify the correct solution. No feedback was

Table 2. Times to solution and errors during acquisition and test of Experiment 2.

Problem	Conventional problem		Conventional worked example		Modified worked example	
ACQUISITION			Times to solution			
	Mean	S.D.	Mean	S.D.	Mean	S.D.
1	54.13	24.07	55.87	17.22	43.52	12.23
2	46.42	23.55	54.33	31.21	34.32	18.48
3	40.94	14.78	29.54	12.95	25.07	12.62
4	44.09	17.99	35.23	12.03	30.00	17.02
			Errors on Problems 2 and 4			
	1.6	0.51	1.2	0.42	0.6	0.60
TEST			Times to solution			
1	77.93	54.94	54.49	18.66	32.67	16.46
2	74.38	49.15	56.94	21.33	31.21	22.44
3	91.22	40.05	69.66	16.22	48.99	16.49
4	52.51	27.15	47.92	19.80	40.30	15.53
			Errors			
	2.0	0.66	1.0	0.84	0.7	0.67

[Solution times are in seconds and errors are frequency of incorrect responses].

given in the test phase. After attempting a problem for five minutes, subjects were asked to go on to the next problem irrespective of whether the correct choice was made. Time taken to solve and the number of errors made were recorded. Errors in Experiment 2 were the number of wrongly identified 3-D representations.

Results and discussion

The mean times to solve the acquisition problems, shown in Table 2, indicated that the modified worked example group was significantly faster than both the conventional problem and the conventional worked example groups on the combined times of Problems 1 and 3, $F(1,27) = 6.400$ and the combined times of Problems 2 and 4, $F(1,27) = 4.243$. Similar comparisons of the conventional worked example group and the conventional problem group indicated no significant difference on either Problems 1 and 3, $F(1,27) = 0.960$, or Problems 2 and 4, $F(1,27) = 0.0049$.

The results show that during the acquisition phase, the modified instructions were processed more rapidly than the conventional problem and conventional worked example materials. A significant difference was not found between the conventional problem and conventional worked example groups. As was the case in Experiment 1, the rapid processing by the modified worked example group concurs with the findings of the cognitive load theory. Again, rapid processing could have reduced learning time. Test results can be used to throw light on the issue.

The modified worked example group solved the first three test problems significantly more rapidly than a combination of the conventional problem and the conventional worked example groups: Problem 1, $F(1,27) = 6.760$; Problem 2, $F(1,27) = 5.664$; Problem 3, $F(1,27) = 4.708$. Problem 4 did not show a significant difference on this measure, $F(1,27) = 1.440$. This may be due to practice on the previous test problems. The conventional problem and conventional worked example groups did not differ significantly on any of the problems: Problem 1, $F(1,27) = 1.716$; Problem 2, $F(1,27) = 0.774$; Problem 3, $F(1,27) = 0.640$ and Problem 4, $F(1,27) = 0.2304$. However, it might be noted that total test times contrasted on the above measure did show a significant difference, $F(1,27) = 3.204$, $0.05 < p < 0.1$, in favour of the conventional worked example group.

In summary, reduced times occurred for the modified instruction group compared to the conventional problem and conventional worked example groups for both the acquisition and the test problems. The conventional problem and conventional worked example groups differed significantly in favour of worked examples on total test times only.

Analysis of errors showed that with the exception of 3 cases during test and 1 in acquisition, subjects were able to identify the correct solution of all the remaining problems within two attempts. On Problems 2 and 4 of the acquisition phase (for which all subjects were presented problems rather than worked examples), the modified worked example group made significantly fewer errors than the conventional problem and conventional worked example groups, $F(1,27) = 13.690$. A similar difference was obtained on the test problems, $F(1,27) = 14.668$. Analysis of errors made by the conventional problem and the conventional worked example groups did not show a significant difference during either acquisition (Problems 2 & 4), $F(1,27) = 2.500$ or test, $F(1,27) = 1.488$.

Experiment 2 confirmed the improved performance obtained by providing the external graphical simulation of the transformation from a 2-D to 3-D representation, which otherwise had to be processed mentally. The results of Experiments 1 and 2 showed that the external simulation of the spatial transformation is effective in both transforming from 3-D representations to 2-D or vice-versa. In both cases, reduced acquisition times have been associated with increased rather than decreased learning.

Experiment 3

Having established that external simulation of internal representations assists in processing mental rotation transformation problems, we now explore the possibility of presenting the external simulation with fewer intermediate stages. Mental rotation transformations go through a trajectory of intermediate stages which allow subjects to maintain a relation between the given and each new stage. The benefit of the external simulation may not be in the number of intermediate stages but in the ability of the intermediate stages to provide a relation between the 3-D and 2-D representations. Reduced information from the trajectory which still provides the necessary relation and obviates the need for subjects to resort to search strategies may provide further information on both the cognitive factors associated with spatial transformation and on how we learn to make transformations. For example a single intermediate stage between the 2-D and 3-D representations may be just as effective as the multiple stages used in the previous experiments. If so, this result would suggest that subjects could see readily the relation between a single intermediate stage and the 2-D and 3-D representations. Additional stages may be redundant.

From cognitive load theory, we can hypothesise also that the precise location of the intermediate stage on the trajectory should be significant. According to the theory, cognitive resources not devoted directly to acquiring knowledge of various relevant problem configurations, interfere with learning. Near the 2-D end of the trajectory, more of the object is obscured from view than at the 3-D end. For mental rotation to occur, the obscured sections must be generated by the subject from the very small amount of information provided by a single, orthogonal view. Much less need be generated by the subject at the 3-D end of the trajectory because much more information is provided. Generating information can impose a heavy extraneous cognitive load compared to being given the same information. This cognitive load may interfere with learning. It follows that providing information at the 2-D end of the trajectory may be more beneficial than at the 3-D end because of the sparseness of readily codable information obtainable from the 2-D representations. Intermediate stages placed near the 2-D representations may facilitate learning more than images placed near the 3-D end of the trajectory.

There are other grounds for hypothesising that images near the 2-D end of the trajectory may be more important than those near the 3-D end. Carpenter & Just (1986) and Just & Carpenter (1985) suggested that people have difficulty in representing and storing spatial information in rotation tasks because they must keep track of the elements of a representation that can rotate out of the mind's eye and then reappear in a new location. This problem might be exacerbated at the 2-D end of the spectrum because so much more of the object is hidden.

Experiment 3 tested the effects of reducing the three intermediate stages used in Experiments 1 and 2 to one intermediate stage. The proximity of the intermediate stage to the 2-D and the 3-D representations was tested also.

Method

Subjects. Forty year 9 students from a Sydney secondary school were used as subjects. None of the subjects had any prior knowledge of orthographic drawing. All had intermediate ability in mathematics for their age. Average age of the group was 14.7 years.

Procedure. Subjects were randomly divided into four groups (10 per group) and tested individually. The general procedure used was similar to Experiment 1. All subjects were given a common instruction phase which had written information identical to Experiment 1.

Following the instruction stage, each group was given acquisition materials consisting of four problems of which two were presented as worked examples, each followed by a similar conventional problem. The conventional worked example and modified worked example (2-D, 3-D) groups received material identical to the conventional worked example and modified worked example groups in Experiment 1. As can be seen in Figure 2, three intermediate stages were used: one close to the 2-D representation, one close to the 3-D representation, and one midway between the two representations. The modified worked example (2-D) group had material similar to the modified worked example (2-D, 3-D) group but with only one intermediate stage simulating the transformation. The intermediate stage used was the one closest to the 2-D representation. The modified worked example (3-D) group used material similar to the modified worked example (2-D) group but this time the intermediate stage was the one closest to the 3-D representation.

Subjects were given three minutes to solve or study each problem and worked example. If the subjects made mistakes during acquisition, within three minutes, they were informed that a mistake had been made and asked

Table 3. Times to solution and errors during acquisition and test of Experiment 3.

Problem	Conventional worked example		Modified worked example (2-D, 3-D)		Modified worked example (2-D)		Modified worked example (3-D)	
ACQUISITION	Times to solution							
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
1	96.5	31.72	75.8	20.60	70.9	19.73	94.0	36.44
2	106.5	40.03	70.8	17.78	78.9	19.14	102.0	36.44
3	62.1	11.64	46.6	8.70	41.4	6.30	60.5	15.60
4	95.6	28.14	61.4	14.03	75.1	21.00	93.5	21.53
	Errors during acquisition							
	1.2	0.78	0.2	0.42	0.4	0.51	1.1	1.28
TEST	Times to solution							
1	94.6	37.53	61.4	17.68	75.1	18.82	88.2	23.56
2	97.0	32.01	51.2	5.75	60.4	6.22	75.9	15.41
3	75.2	15.06	53.6	8.94	60.4	10.22	75.4	16.37
4	133.3	38.11	73.3	23.25	86.4	11.78	113.7	29.46
	Errors during test							
	2.7	2.75	0.6	0.66	1.0	1.05	2.3	1.70

[Solution times are in seconds and errors are frequency of incorrect responses.]

to try again. The correct solution was shown only when a subject had tried to solve the problem unsuccessfully for three minutes.

After attempting the acquisition problems, all subjects were given a test phase which involved solving four conventional problems. Each problem had a 3-D representation of an object given and subjects were asked to draw 2-D representations: the front, left, and top views. Three of these test problems were similar to the problems used in the instructions and acquisition. Subjects were given three minutes to solve each problem. If an incorrect solution was made within the three minutes, subjects were asked to try again and correct the error. The use of an eraser was not allowed. Subjects had to leave all incorrect lines on the diagram with an identifying mark on them. There was no other feedback given. Errors were measured by the number of incorrect attempts made on each problem.

Results and discussion

A comparison between the groups of the times to solution and errors made during acquisition and test times (see Table 3) was carried out by using orthogonal contrasts. A set of three contrasts was used. Contrasting the acquisition times of the conventional worked example group with the three modified worked example groups on Problems 1 and 3 (which were worked examples) indicated that the conventional worked example group was slower in studying its worked examples, $F(1,36) = 6.250$. A similar result was obtained on Problems 2 and 4 (which were conventional problems), $F(1,36) = 6.553$.

Differences in the acquisition times between the modified worked example (2-D, 3-D) group and the modified worked example (2-D) and modified worked example (3-D) groups combined on Problems 1 and 3 (which were worked examples) may represent a real effect favouring the modified worked example (2-D, 3-D) group, $F(1,36) = 2.992$, $0.05 < p < 0.1$. A significant difference favouring the modified worked example (2-D, 3-D) group was obtained on Problems 2 and 4 (which were conventional problems), $F(1,36) = 7.507$.

Contrasting acquisition times between the modified worked example (2-D) group and modified worked example (3-D) group on Problems 1 and 3 (worked examples) indicated a significant difference in favour of the modified worked example (2-D) group, $F(1,36) = 6.604$. A similar difference was obtained on Problems 2 and 4 (conventional problems), $F(1,36) = 3.648$, $0.05 < p < 0.1$.

The acquisition results confirm that conventional worked examples with no intermediate stages are processed more slowly than worked examples with intermediate stages. This result is in accord with the suggestion that having to generate intermediate stages imposes a heavy cognitive load. The number and location of those stages also has cognitive load implications. While additional stages facilitated performance as might be expected if cognitive load is reduced, a stage closer to the 2-D representation resulted in faster processing than a stage closer to the 3-D representation.

Comparing test times to solution, the conventional worked example group was significantly slower than the modified worked example groups combined, on all test problems: Problem 1, $F(1,36) = 4.410$; Problem 2, $F(1,36) = 26.832$; Problem 3, $F(1,36) = 6.451$, and Problem 4, $F(1,36) = 17.640$. Test times to solution of the modified worked example (2-D, 3-D) group was significantly faster than the modified worked example (2-D) group and modified worked example (3-D) group: Problem 1, $F(1,36) = 4.161$; Problem 2, $F(1,36) = 5.760$; Problem 3, $F(1,36) = 8.008$ and Problem 4, $F(1,36) = 7.562$. Comparing test solution times between the modified worked example (2-D) group and modified worked example (3-D) group indicated a significant difference in favour of the modified worked example (2-D) group on Problem 2, $F(1,36) = 3.600$, $0.05 < p < 0.1$; Problem 3, $F(1,36) = 6.604$ and Problem 4, $F(1,36) = 4.972$. Problem 1 did not yield a significant difference, $F(1,36) = 1.299$.

Errors made during the acquisition and test phase showed that the conventional worked example group made significantly more errors than the three other groups: total acquisition errors, $F(1,36) = 4.410$ and total test errors, $F(1,36) = 6.100$. Comparing the modified worked example (2-D, 3-D) group with the modified worked example (2-D) and modified worked example (3-D) groups showed significant differences in the number of errors made: total acquisition errors, $F(1,36) = 2.958$, $0.05 < p$ 0.1 and total test errors, $F(1,36) = 3.062$, $0.05 < p$ 0.1. The modified worked example (2-D) group made significantly fewer errors than the modified worked example (3-D) group: total acquisition errors, $F(1,36) = 3.610$, $0.05 < p < 0.1$ and total test errors, $F(1,36) = 3.496$, $0.05 < p$ 0.1.

The pattern of results among the three groups that varied the number and the location of the intermediate stages was of primary interest in this experiment. Firstly, three intermediate stages proved superior to one stage. It may be reasonable to propose that, for maximum learning, the distance between stages must be sufficiently small to permit subjects to follow the transformation without having to generate stages themselves. The modified worked example (2-D, 3-D) group may have been closest to this ideal.

The second finding of interest in this experiment concerned the superiority of the modified worked example (2-D) group over the modified worked example (3-D) group. It was more important for intermediate stages to provide a bridge to the 2-D than the 3-D representation. It may be reasonable to suggest that because the 2-D representation is more abstract, mental rotation is more difficult at this point than elsewhere. If so, this finding indicates that, when dealing with orthographic projection, difficulty of rotation may depend not just on degree of rotation but on position on the trajectory. From an experimental point of view, additional intermediate stages close to the 2-D representation may be important.

General discussion

The experiments described in this article were designed to throw light on presentation procedures and the cognitive demand imposed when transforming 3-D representations into 2-D orthogonal views, and vice-versa. Traditional orthographic projection was used for this purpose. Based on theories of mental rotation, we assumed that learning orthographic projection involved learning to picture objects along a trajectory of intermediate orientations. Further, based on cognitive load theory, we hypothesised that the generation of these intermediate orientations would require cognitive resources that consequently would not be available for learning. By presenting the intermediate stages of rotations rather than having subjects generate them, extraneous cognitive load should be reduced and learning enhanced.

These hypotheses were supported. The first two experiments demonstrated that the inclusion of both the final and intermediate stages during practice exercises involving both 3-D to 2-D (Experiment 1) and 2-D to 3-D (Experiment 2) transformations, enhanced learning. Worked examples with several intermediate orientations proved superior to conventional worked examples with no intermediate orientations, while conventional worked examples were, in turn, superior to problems in which the initial 3-D or 2-D representations only was presented with subjects required to generate the correct solution themselves. The superiority manifested itself both during the initial acquisition and the subsequent test phase.

The third experiment demonstrated that a reduction in the number of intermediate orientations presented reduced the effectiveness of the worked examples. The major result was that effectiveness was reduced far more by eliminating orientations close to the 2-D representations than the 3-D representations. This result suggested that the orientation close to the 2-D representations were more important than those close to 3-D representation. Orientations close to the 2-D representation may be harder to generate than those close to the 3-D representation and so more useful if provided.

Cognitive load theory was used to generate the experiments described in this paper. The theory assumes that many cognitive activities, while essential precursors to learning, are unnecessary if the materials are appropriately structured. Depending on the structure of the learning materials, cognitive resources can be devoted either to appropriately restructuring the information so that it is in a form suitable for learning, or resources can be directed immediately to learning. Many search processes fall into the category of cognitive activity essential prior to learning but not essential to the learning process itself. The current experiments provided evidence that searching for an appropriate way to rotate an object may not be an effective learning device when learning 3-D to 2-D rotation or vice-versa. Presenting the necessary intermediate stages proved superior to students generating the images themselves.

Because the theory was used to generate the experiments, it may be difficult to find alternative, post-hoc explanations for the data of all three experiments. For example, it may be plausible to argue that the modified worked example groups of Experiments 1 and 2 were superior, not because they reduced extraneous cognitive load but rather, because they demonstrated an effective rotational strategy to subjects without any cognitive load consequences. The difficulty with this argument is that firstly, a rotational strategy is likely to be effective precisely because it reduces cognitive load and secondly, the results of Experiments 3 become uninterpretable. The three modified worked example groups of Experiment 3 all demonstrated a rotational strategy and yet differed amongst themselves according to cognitive load theory. Presumably a strategy hypothesis should not distinguish between number and location of intermediate stages. Of course, it does not follow from this argument that strategies are irrelevant to all processes of mental rotation. It does follow that in the present experiments, results are consistent with findings of cognitive load theory.

Lastly, the results of these experiments have clear practical implications. Students in many technical areas must learn the process of orthographic projection. We can conclude that current procedures should be substantially altered. Firstly, a heavier than normal emphasis on worked examples should be of benefit. Secondly, those worked examples should include intermediate images. Thirdly, more emphasis needs to be placed on intermediate images closer to the 2-D representations than 3-D representations. Our results suggest that with these alterations, learning should be enhanced substantially.

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