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Using Worked Examples as an Instructional Support in the Algebra Classroom

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In the 2 experiments reported here, high school students studied worked examples while learning how to translate English expressions into algebraic equations. In Experiment 1, worked examples were used as part of the regular classroom instruction and as a support for homework. In Experiment 2, students in a remedial mathematics class received individual instruction. Students using worked examples outperformed the control group on posttests after completing fewer practice problems; they also made fewer errors per problem and fewer types of errors during acquisition time, completed the work more rapidly, and required less assistance from the teacher.

Learning algebraic concepts and procedures is important for success in various educational and occupational pursuits. The student who fails to obtain competency either in writing and solving equations or in understanding the meanings and the uses of variables has limited future options in classes and careers. However, according to Nielsen (1990), less than half of U.S. students go beyond introductory algebra, and large numbers who do complete high school algebra show only a fragmented sense of these ideas and procedures. International tests and studies have consistently placed U.S. students near the bottom in mathematics performance (McKnight et al., 1989; Stigler, Lee, & Stevenson, 1990). Students from urban areas and of low socioeconomic status are especially at risk for low achievement in mathematics and hence for avoiding elective mathematics classes (Jones, Burton, & Davenport, 1984; Mullis, Dossey, Owen, & Phillips, 1993). Although the reasons are complex, one major factor is that the elementary school often does not prepare students for the breadth and the depth of the mathematics that is expected of them in high school (Flanders, 1987; McKnight et al., 1989), nor does it provide them a role in regulating their own learning (Stodolsky, 1988).

One line of research that has shown promise for improving performance in the secondary school mathematics classroom is that of John Sweller and his colleagues (Cooper & Sweller, 1987; Sweller, 1989; Sweller & Cooper, 1985). Sweller's studies of worked examples in teaching mathematics are grounded in research on mental schemata, domain-specific knowledge, automaticity, and expert–novice comparisons of cognitive psychology. Expertise in a domain, Sweller and his colleagues suggest, is not due to general problem-solving abilities but to a highly automatized and interconnected knowledge base or schemata, which are cognitive structures that help the problem solver to categorize a situation according to relevant features and that then indicate the appropriate solutions for that problem type (Brown & Borko, 1992; Chi, Glaser, & Rees, 1982; Simon & Simon, 1978). Sweller hypothesizes that expertise in algebra is characterized by this domain-specific schema-based knowledge as well as by rule automation (Cooper & Sweller, 1987; Owen & Sweller, 1985; Sweller, 1989).

Several studies provide evidence for the importance of mental schemata in mathematical domains: in categorizing, representing, and solving mathematical word problems (Carpenter & Moser, 1983; Fuson, 1992; Hinsley, Hayes, & Simon, 1977; Reed, 1984; Riley & Greeno, 1988); in constructing word problems (Lewis & Mayer, 1987); in transferring knowledge across problem types (Novick, 1992); in recalling relevant information from word problems (Mayer, 1982); and in recalling algebraic equations versus meaningless strings (Sweller & Cooper, 1985). These researchers question the dichotomy between conceptual understanding and use of rote procedures. Under good learning conditions and in application of knowledge, understanding and skill are highly interrelated and enhance one another.

If the acquisition of schemata specific to the domain of algebra is necessary to gain expertise, then the following is an important question: What methods of instruction enhance schema acquisition? The research of Sweller and his colleagues (Cooper & Sweller, 1987; Sweller, 1989; Sweller & Cooper, 1985) suggests that an extensive use of worked examples may be conducive to developing such schemata. In his cognitive load theory, Sweller hypothesizes that the conventional practice format used in many mathematics classrooms, in which lecture and a few examples are followed by massed practice, interferes with learning because of the heavy reliance on means–ends problem solving (Chandler & Sweller, 1991; Owen & Sweller, 1985; Sweller, 1988). During the solving of practice problems, novices focus on goal attainment (i.e., solving the problem); attention is focused on the current problem state and the goal state, thus leaving little cognitive capacity for learning. In contrast, the use of various worked examples frees cognitive capacity for more rapid knowledge acquisition because this range of examples presents categories of problems in their initial state and illustrates correct steps for that problem type; the very information that should be encoded in a schema.

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In laboratory studies on the effect of worked examples on learning algebra and other high school mathematics, Sweller and his colleagues found that students who were presented with a heavy dose of worked examples learned as well or better than did students who were presented with a few examples followed by conventional practice (Cooper & Sweller, 1987; Sweller & Cooper, 1985). Zhu and Simon (1987) reported on an instructional format wherein worked examples were used to teach algebra topics under whole class conditions in the People's Republic of China. Although they had no control group, Zhu and Simon's findings were similar to those of Sweller and his colleagues: Students mastered topics in less time than in a typical classroom situation. Furthermore, Zhu and Simon reported that the students had not merely memorized rules but had acquired an understanding of considerable depth.

Apart from cognitive load theory, there are several reasons why an increased use of worked examples could be an effective teaching tool in high school. First, worked examples encourage active mental participation on the part of the students by shifting more responsibility for instruction to them. Despite reform recommendations (National Council of Teachers of Mathematics, 1989; National Research Council, 1989), mathematics in many classrooms can still be characterized by a teacher-lecture–student-practice pattern where students typically see the teacher as the sole source of instruction and assistance (Stodolsky, 1988; Stodolsky, Salk, & Glaessner, 1991). If students could successfully use worked examples for self-instruction and support, then this classroom pattern, as well as students' beliefs about their ability to learn mathematics, could be changed. Second, during a typical high school mathematics lesson, a limited number of examples are presented, thus allowing many students to make faulty inductions (e.g., Matz, 1980; VanLehn, 1986). A greater range of examples should help to correct abstraction of the relevant features and solutions, especially among students with low achievement. Finally, many students report that they have no one to assist them with algebra problems outside of the classroom, especially students whose parents may have never studied algebra. A homework format that includes worked examples may provide the scaffolding needed as students study at home.

The purpose of these experiments is to extend the research on worked examples to an urban high school in which students begin to study algebra after 8 or more years of arithmetic. Although it seems that these students may have much to gain from the support and instruction of worked examples, there are also reasons to be cautious. In their study of how students generate explanations in physics, Chi and her colleagues found that although students who achieved well used worked examples productively, students who were low achievers were often unsuccessful (Chi, Bassok, Lewis, Reimann, & Glaser, 1989). Furthermore, Sweller and his colleagues (Cooper & Sweller, 1987; Sweller, 1989; Sweller & Cooper, 1985) generally chose students with average or above-average ability, and Zhu and Simon's (1987) research with students in mainland China cannot be expected to generalize to an urban U.S. high school. In this paper the focus is on two experiments that were part of a longer research project carried out in a general urban high school.

Experiment 1

Learning to map between words describing situations and correct equations is important for success in a number of domains. In solving problems involving complex situations and numbers, success is often dependent on the student's ability to model the problem in terms of an equation or some other mathematical representation that can be easily manipulated and solved. However, despite the importance of writing equations, the translation process is not mastered by many students (e.g., Rosnick & Clement, 1980), thus they are left unable to apply the power of algebra. Because of their lack of familiarity both with formal notation involving variables and with mathematical meanings for words, many students have difficulty even with direct translations between English expressions like *five less than a number* and the corresponding expression. In this experiment, students received instruction on this translation process.

Method

Participants. The participants were students (19 female, 21 male, 15–17 years of age) enrolled in a general high school located in a large midwestern city in the United States. As in many urban schools, the dropout rate approaches 50%, and standardized test scores are far below the national norms. On the Test of Achievement and Proficiency (Riverside Publishing Company, 1989), 77% of the students at the school performed below the median in reading (47% were in the bottom quartile); in mathematics, 82% were below the median (53% were in the bottom quartile; Chicago Panel on Public School Policy and Finance, 1992). About 1 month before the experiment, all 71 students in three levels (honors, regular, basic) of first-year algebra classes taught by the same instructor were administered a 20-item test, which I constructed, that assessed the students' knowledge of arithmetic and basic algebra. Within each of the three classes, students were paired on the basis of their scores and randomly assigned to either a worked example or a conventional practice group. Because of attrition, only 20 pairs participated in this experiment.

The score on the placement test was used to define a second variable, achievement. Students who scored above the median were defined as higher achievers, and students who scored below the median were defined as lower achievers. However, the term higher achiever needs to be considered in light of the school population and the standardized test scores. Most of the higher achieving students would be considered average achievers at many other high schools. By class, none of the honors, 56% of the regular, and 71% of the basic algebra students were defined as lower achievers. By sex, 13 of the 20 lower achievers were male (7 in the worked example and 6 in the conventional practice group). Eight of the 20 higher achievers were male, with 4 in each learning condition.

Procedure. Immediately before this experiment, students in the three classes had been solving linear equations by both informal (e.g., inspection, trial and error with calculators) and formal methods (standard transformations). The instruction period began with all students studying a worksheet on writing equations, which included three examples and three practice problems. After a discussion of the worksheet and the sample problems, the students were asked to complete the three practice problems. These practice problems were subsequently checked as a class. I solicited additional questions of the students before proceeding. The few questions asked were all of a procedural nature concerning the three practice problems.

When these questions had been answered and the instructional examples and problems collected, the students received a practice worksheet with 24 problems. For the worked examples group, the worksheet contained 12 worked examples, each followed by one similar practice problem. On these worked examples, key phrases (*five less than a number*) were explicitly linked to mathematical terms ($x - 5$) by lines. These worked examples were similar in structure to the examples in the usual classroom text, but they differed in the number of examples given and in the placement of the examples, which was alternated with practice problems. The conventional practice group received a worksheet with the same 24 problems in the same order, but none of the problems was worked. Thus, the worked example students had additional instruction in the form of the worked examples but had fewer practice problems than the conventional practice students. Only the target problems that were to be solved by both groups were used in my subsequent analyses of errors.

During the 20-min acquisition time, the instructor was available to all students. Students were assisted in the order in which they requested help. Worked example students were first reminded to study the example and were then given additional support, if needed. The conventional practice students received more typical classroom support. Conventional practice students were first asked to think about the initial problems or about a similar problem they had completed. Additional guidance about the meaning of words (e.g., "What does twice a number means mean?") or the structure of equations was provided as needed, and when necessary, the instructor helped the student correctly complete the equation. By group, teacher support differed only in that the worked example students were reminded to use the examples whereas the conventional practice students were given more direct guidance from the instructor.

At the end of this 20-min acquisition time, the worksheets were collected, and an 8-item test was given to the students. After the test, a 20-item worksheet was assigned for homework. As on the classroom worksheets, 10 of the problems were worked, and each problem was followed by one similar practice problem for the worked example group; the CP group had the same 20 problems, but none of them were worked. When the students returned to class the following day, their homework was collected, and they were administered a 12-item test. Ten of the items involved direct translation as on the worksheets. The final two transfer problems were stories involving changes in temperature and in money. I included these story problems so that I could determine whether students could transfer their knowledge of writing equations to less direct situations. No worked examples were available to students during either test.

Results and Discussion

A 20 (block) × 2 (achievement level) × 2 (group) randomized block analysis of variance (ANOVA; with the 20 matched pairs composing the blocks) was carried out on the 8-item test that was administered at the end of class (posttest) and on the first 10 items on the 12-item test (because the other 2 were in the transfer analysis) that was administered at the beginning of the second day (delayed posttest), with number of errors as the dependent variable in each analysis. ANOVAs were also carried out on the number of errors on the in-class worksheet, on the homework, and on the two transfer problems on the delayed posttest. For the worksheet and homework measures, only the target problems (i.e., the nonexamples) that were to be solved by both groups were analyzed. Homework was not returned by 6 of the worked example and 6 of the conventional practice students. Because 2 of these conventional practice students and 2 of these worked example students who did not return the homework were paired, this removed only 10 pairs from the homework analysis. Three pairs were deleted from the delayed posttest analysis because of absences.

In these ANOVAs, group was significant on the worksheet, $F(1, 18) = 19.40$, $p < .001$, $MS_e = 5.11$, on the posttest, $F(1, 18) = 5.83$, $p < .05$, $MS_e = 3.13$, on the homework, $F(1, 8) = 5.45$, $p < .05$, $MS_e = 4.85$, and on the delayed posttest, $F(1, 15) = 5.02$, $p < .05$, $MS_e = 4.27$. On all five measures, the worked example group outperformed the conventional practice group, although the difference was not significant on the two delayed posttest transfer problems, $F(1, 15) = 2.22$, $p > .15$, (see Table 1 for means). Achievement was significant only on the worksheet, $F(1, 18) = 5.50$, $p < .05$, $MS_e = 6.23$ with high achievers outperforming low achievers. No Group × Achievement Level interactions were significant, all Fs < 3.57, ps $> .05$.

Observational notes taken immediately after class indicated that students' behaviors also differed by group during the practice period. Students in the conventional practice group required more assistance, with their questions centering around the meanings of words (*decreased, one-half of*) and the structure of more complex equations. Even with the instructor's assistance, the conventional practice students skipped 9% of the target problems. In contrast, the worked example students generally completed all items on the worksheets long before their counterparts and required almost no assistance from the instructor; only 2% of the problems were unattempted.

Because the difference in the scores on the worksheet might only be due to the worked example group completing more of the target problems (because all of their problems were target problems evaluated by the analysis), an analysis was done only on the target problems that were actually attempted by students in both groups. In the conventional practice group, 66 of the 218 attempted target problems (30%) resulted in errors, whereas in the worked example

Table 1
Experiment 1: Mean Errors by Group

Measure	WE group	CP group	No. pairs
In-class worksheet	1.25	4.40	20*
Posttest	0.90	2.25	20*
Homework	1.10	3.40	10*
Delayed posttest	1.29	2.94	17*
Two transfer problems	0.77	1.24	17*

Note. Only problems solved by both groups were counted for errors on the in-class worksheet and homework. WE = worked example; CP = conventional practice.
* Mean difference is significant at $p < .05$.

group, only 23 of the 238 problems (10%) attempted resulted in errors. Furthermore, 94% of the errors made by the worked example group were valid algebraic equations compared with only 71% in the conventional practice group. An analysis of target problems on the homework showed a similar pattern. In addition to attempting fewer of the target problems, the conventional practice group was spending more time practicing errors.

A further analysis of these in-class worksheets showed that the worked examples were helpful not only in reducing errors but also in constraining types of errors made by students. One error of interest was reversals made on subtraction problems (i.e., writing *five less than a number* as $5 - x$). On the in-class worksheets on which there were three target problems of this form, both groups made nine reversals. However, the conventional practice group had 10 additional errors and three non-attempts on these problems, whereas the worked example group made only three other errors. These results were reflected on the posttest, which contained two similar problems: Both groups made six reversal errors, but the conventional practice group made an additional eight errors, whereas the worked example group made only one other error. Thus, although the worked examples did not initially seem to affect the number of reversals, other types of errors were constrained. However, the delayed posttest, which also contained two subtraction equations, showed quite different results by group. After additional practice in the form of homework, the conventional practice group made 13 reversals and five other errors on the two problems of interest, whereas the worked example group made only three reversal errors and two additional errors.

Another problem of interest was the one target problem on the in-class worksheet that did not match the worked example that preceded it. Results on this problem suggest further how the examples might have been useful in constraining errors. On the worked example, *three times a number was decreased by seven,* whereas on the target problem *twice a number was increased by two.* Students in the worked example group made a total of four errors, all copying the operation from the worked example; one worked example student did not attempt the problem. One of the four copy errors also involved transporting a numeral from the worked examples into the target problem. In contrast, conventional practice students made seven types of errors, and an additional four students skipped the problem. Overall, 75% of the worked example students answered this problem correctly compared with 45% of the conventional practice students, despite the difference between the operation in the example and the practice problem. If students were not using the worked examples, their results should have been more like the conventional practice group, and if they were simply copying between the example and the target problem, more errors of operation should have occurred.

Most important, these group differences (both in number of errors and in types of errors) carried over onto posttest measures where examples were not available. This indicates that the worked example students were not just mindlessly copying the examples; rather, they were forming schemata that linked English words to algebraic representations.

The worked examples appear to have been especially helpful for students who were in the process of learning English or who were deficient in mathematical language. For example, one student in the worked example group was a Vietnamese girl who had good skills in mathematics, but whose lack of understanding of English interfered when she had to interpret problems involving vocabulary. In this experiment, she navigated her way independently through the problems using the examples and went from 9 out of 10 problems incorrect on the pretest to 1 out of 8 incorrect and 2 out of 10 incorrect on the posttest and the delayed posttest, respectively. Another Vietnamese student, with whom she was paired, had 8 out of 10 incorrect on the pretest, 7 out of 10 on the posttest, and 6 out of 10 on the delayed posttest. A similar pattern was found for four basic algebra students identified as learning disabled (two were in each learning condition).

Although there were no Group × Achievement Level interactions, results by class suggest that the worked examples were a very useful tool for students identified as low-achieving. In the basic algebra class, the worked example students had a mean error rate of .86 (SD = .38) on the posttest, whereas those in the conventional practice group had a mean error rate of 3.43 (SD = 1.99); in the regular class, the worked example group had a mean error of 1.00 (SD = 1.12), whereas those in the conventional practice group had a mean error rate of 2.11 (SD = 3.10) on this test. In the honors class, the mean error rates for the worked example and conventional practice groups were .75 (SD = .96) and .50 (SD = .58), respectively. Similar results were found for the delayed posttest.

Experiment 2

Although both algebra students who were identified as high- and low-achieving in the worked example group made significant gains in relation to their counterparts in the conventional practice group in Experiment 1, it was not clear how they interpreted the role of the worked examples or in what manner they actually used the examples. Error patterns on worksheets and homework papers suggested that some students were using the examples as they wrote the practice equation but that perhaps they were not studying the examples first. The purpose of this experiment was to examine more closely the ways in which students identified as low achievers, in this case students in a remedial mathematics course, used worked examples to complete an unfamiliar algebra task. Although students who are low achievers may not use worked examples as richly or effectively as students who are high achievers (Chi et al., 1989), the examples may provide enough motivation and support during practice to help these students construct and use correct schemata despite their deficits in background knowledge (e.g., Novick, 1992). So that students' performance could be observed, each student was individually instructed, observed during the acquisition period and the posttest, and interviewed after the posttest. After the posttest, students were asked to write equations for three additional problems and to explain their thinking as they wrote the equations.

Method

Participants. The participants were 24 students (12 female, 12 male, ages 14–16 years) enrolled in prealgebra in the same high school as in Experiment 1. All of these students were classified as low achievers; they were placed in the class because of extremely low mathematics scores on standardized tests. One week before participating in the experiment, these students were administered a 12-item pretest on writing equations from written expressions. This pretest was identical to the delayed posttest in Experiment 1. On the basis of these scores, students were paired and assigned to either the worked example or the conventional practice learning condition.

Procedure. The 40-min instruction period began with an explanation of how to translate word problems into equations. In addition to illustrating why equations are helpful in solving difficult problems, the instructional worksheet and accompanying instruction included three worked examples showing how words mapped to mathematical symbols. After instruction, explanation, and discussion, each student was given three practice problems with English expressions to translate into equations. During this time, the three worked examples were available, and the experimenter answered any questions and assisted the student as necessary. Problems were checked, and any errors made by students on the three practice problems were corrected and explained.

The initial instruction was followed by a 10-min acquisition period during which students received worksheets. Both groups completed the identical problems in the same order. Three sheets with 12 problems each were available, and as students completed one worksheet, they were administered the next one until they had completed all three or until the 10 min had elapsed. For the conventional practice group, each worksheet contained 12 practice problems. As in Experiment 1, students in the worked example group had half of the problems worked, and each worked example was followed by one similar practice problem. Before beginning, worked example students were instructed to study each example before doing the similar practice problem that followed and to refer to the examples when necessary. The conventional practice students were told that the problems they had were similar to the ones they had just studied and practiced. The investigator gave the student no assistance during this time, and the instructional sheet was not available. After this acquisition time, the worksheets were removed, and the students were administered a posttest identical to the pretest. A three-problem, think-aloud interview followed the posttest.

Paired t tests were carried out on four posttest measures: errors during acquisition time (on target problems), practice problems undone, acquisition time (the time needed to study the worked examples and complete the practice problems) and posttest. Only the target problems, those that were to be solved by both groups, were analyzed.

Results and Discussion

The worked example group outperformed the conventional practice group on all four of the postinstruction measures (see Table 2). Paired t tests showed that the learning condition was significant on errors during acquisition time, $t(11) = 12.09, p < .001$, practice problems undone, $t(11) = 5.58, p < .001$, acquisition time, $t(11) = 2.65, p < .05$, and posttest, $t(11) = 2.40$, $p < .05$. Paired t tests by group showed a significant improvement between the pretest and the posttest for the worked example group, $t(11) = 3.76$, $p < .005$, but not for the conventional practice group, $t(11) = 2.13$, $p > .05$.

Table 2
Experiment 2: Mean Errors and Acquisition Time by Group

Measure	WE group *M*	WE group *SD*	CP group *M*	CP group *SD*
Errors during acquisition	2.25	1.36	13.50*	3.00
Practice problems undone	0.33	0.65	4.92*	2.64
Total acquisition time (in minutes)	8.11	1.65	9.36*	0.82
Posttest errors	5.08	2.47	6.42*	2.11

Note. Sample size is 24: Twelve students in each instructional condition. Only target problems to be done by both groups are counted on the practice problems undone and errors during practice. WE = worked example; CP = conventional practice.
* Mean difference is significant at $p < .05$.

How actively did the students use the examples during acquisition time? Students in the worked example condition spent little time examining the worked example before attempting the accompanying practice problem in spite of the directions to study and to try to understand each worked example before continuing. Instead they proceeded quickly to the practice problem and then referred back to the example as they wrote or completed the equation. This was evident by the finding that 37% of the relatively few worksheet errors for the worked example group involved copying a symbol from the example into the practice problem. It appears that these students used the examples as an on-line solution guide rather than first abstracting the solution pattern and then applying this knowledge.

Although the worked examples were generally not used by the students in the manner in which they were intended (study them for understanding, and then attempt the accompanying problem), the examples were nevertheless quite successful in constraining errors and in motivating the students to stay on task. Considering only the target problems (the nonworked examples) on the first two practice worksheets, which all students had time to complete, all of these problems were attempted by the worked example group, whereas there were 10 nonattempts by conventional practice students (they just skipped over these problems). Furthermore, of these same 12 problems that were attempted by the conventional practice group, 33 (24%) of the answers were nonequations compared with 2 (8%) for the worked example group. As in Experiment 1, these gains by the worked example group were evident on the posttest, when the worked examples were no longer available, indicating that more than rote copying had taken place.

No differences between the groups were noted on the think-aloud problems, which followed the posttest. All students found it extremely difficult to explain what they were doing as they solved the problem and required assistance in interpreting the statements, even on problems they wrote correctly during the posttest. Although it seems unrealistic to expect students, especially students with such low achievement, to show evidence of schema development after a 10-min practice period, it is somewhat puzzling that none of the posttest differences were apparent during the interviews.

General Discussion

The two experiments reported in this article support previous research on worked examples in teaching mathematics (e.g., Sweller, 1989; Zhu & Simon, 1987). Students who were given worked examples required less acquisition time, needed less direct instruction, made fewer errors, and made fewer types of errors during practice. Although the worked example group had fewer practice problems, the children in this group continued to outperform those in the conventional practice group on posttests on which the worked examples were no longer available.

Importantly, the worked examples were helpful to students defined as lower achievers, including students with a history of failure in mathematics and students identified as learning disabled. This is an important consideration if we take seriously the belief that all students should have the opportunity to succeed at high school mathematics. In Experiment 1, the results indicated that students in all three algebra classes benefited from worked examples more than they did from conventional practice. In Experiment 2, prealgebra students who were given worked examples to study during the 10-min acquisition period significantly outperformed students who were only given instruction and practice problems.

Observations in Experiment 2 seemed in agreement with the analysis of Chi and her colleagues about how students with low academic achievement use worked examples (Chi et al., 1989). Rather than studying the examples first, these students were more likely to use the examples on-line as they solved the problems. However, in both experiments students identified as lower achieving profited by having a large number of examples to study. It may be that learning by analogy during problem solving is a more meaningful and effective learning strategy for many students, especially those who have gaps in prior knowledge and do not make successful elaborations on their own. In using a worked example to solve an accompanying problem, a student may be learning to look beyond the surface features of the problem to the underlying structural similarities, a process that would facilitate construction of a base schema adequate for transfer (Pierce, Duncan, Gholson, Ray, & Kamhi, 1993). The fact that the problems were paired with worked examples may have helped cue students to look for the underlying similarities and enabled them to recognize these similarities.

Furthermore, the worked examples seem to have helped to prevent the practicing of incorrect solutions and the learning of incorrect associations (e.g., Siegler, 1988). Because they provide a scaffolding for learning, illustrating the correct form of equations, the proper use of symbols, and the meaning of words (e.g., increased), students in the worked example condition were less likely than students in the conventional practice condition to be practicing errors in class and at home, and this carried over to posttests when the examples were no longer available. Although worked examples initially seemed to be less effective in correcting reversal errors on directional statements involving subtraction in Experiment 1, after additional homework practice the worked example group made one-fourth as many reversals as the conventional practice group did on the delayed posttest.

The two experiments suggest that a more extensive use of worked examples in the classroom will facilitate learning of certain topics. The students' success in using examples seems to have motivated them to stay on task longer during practice at school and at home, as indicated by the fewer number of homework errors made by the worked example group (Table 1). Students also reported that having the examples helped them to remember how to do the problems at home. During discussions with the students in Experiment 2 following the posttest, all of them said that they found the worked examples helpful, especially as the problems became more difficult. In a related study with different students, three teachers in the same school were asked to use a worked example format in teaching addition and subtraction of polynomials over three days. These teachers reported that students needed less initial teacher explanation and less teacher support during practice than usual in their classrooms. Furthermore, their students actively used the worked examples to self-correct mistakes and to monitor their understanding. Thoughtful use of the worked examples may help to change the character of the secondary mathematics classroom from one in which the teacher lectures and demonstrates and the students practice a given procedure to one in which students have higher stakes in constructing meaningful mathematical ideas and solutions. Outside of the classroom, well-planned examples may be especially helpful as a pedagogical tool in individualized learning programs or for students receiving support services in school resource rooms, especially when teachers lack expertise in mathematics.

A number of research questions are raised by these experiments. What is the depth of mathematical knowledge that students gain as they study and apply worked examples? How does this compare with traditional instruction and with other innovative means of instruction? The purpose of this study is not to suggest that worked examples be the only, or even the major, means of instruction in the high school mathematics classroom. Nor is it to suggest that rote learning is more appropriate for at-risk students. Learning how to take part in mathematical discussions, to propose, justify and test solutions, or to solve novel problems should be important features of all mathematics classrooms. However, certain topics and ideas can be presented effectively by worked examples with minimal teacher input, thus engaging the students as active learners. How well these different modes of good instruction can interact and support each other is worth considering.

In discussions with students, some reported that they were unsuccessful when attempting to use the worked examples in their algebra books. Because they are novices, even slight differences in the surface features of examples and practice problems may make transfer difficult. However, is this a skill that develops or can be developed (i.e., do students identified as lower achievers learn how to use fewer examples more effectively as they develop expertise in a domain)? Or is this a difference that persists between students who perform well and those who do not?

Finally, what is a good worked example in algebra? Sweller and his colleagues have addressed this issue in other domains and show that some examples interfere with learning, rather than enhance it (Ward & Sweller, 1990). However, text formats vary widely in how they present worked examples. The format used in these experiments was to alternate examples with similar practice problems. What is the most effective balance between written text, instruction, worked examples, and practice problems for students? As the results suggest, studying examples and applying a few practice problems may be more effective than the massed practice typical in many textbooks. Addressing these questions will help to improve mathematical instruction and learning for all students.

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