ASEN 5519 Small UAS Guidance and Control

Homework 3 Assignment

Assigned: Thursday, February 9, 2023

Due: 11:59 PM, Thursday, February 16, 2023

Background

Solving for Trim Conditions

In this assignment students will derive functions to calculate the equilibrium condition for an aircraft in straight, wings-level flight with possibly non-zero air relative flight path angle (i.e. the aircraft could be ascending or descending). Students will also derive functions to calculate the trim condition to be in a cordinated turn. Although a single function could be created to handle both cases, this assignment treats them separately.

Straight, Wings-Level Flight

In order to determine the full state and control input vector at equilibrium, we need to define the desired equilibrium condition. For a straight, level equilibrium condition we need to specify three terms which then allow us to determine the remaining variables. We will refer to the *trim definition* as specification of the air speed V_a, air-relative flight path angle γ_0, and height h_0 (which is related to density) of the aircraft equilibrium condition

$$\text{trim\_definition} = \mathbf{x}_{td} = [V_a,\ \gamma_0,\ h_0]^T. \tag{1}$$

Given the trim definition we can determine there are only three remaining independent *trim variables*: the angle of attack α_0, elevator deflection δ_{e0}, and throttle δ_{t0}

$$\text{trim\_variables} = \mathbf{x}_{tv} = [\alpha_0,\ \delta_{e0},\ \delta_{t0}]^T. \tag{2}$$

All state variables for the trim condition are zero (ϕ, v^E, p, q, r), can be derived from the trim definition and variables (z_E, θ, u^E, w^E), or do not matter (x_E, y_E, ψ).

In the case of straight, wings-level flight an equilibrium condition is one in which the total force and moment acting on the aircraft is zero. Since we need both vectors to equal zero, it is sufficient to say that we need the sum of the magnitudes of the total force vector and the total moment vector to be zero. Since the magnitude is always positive, the only way for the sum to be zero is if each term is zero. Define the objective function $J\left(\mathbf{x}_{tv}|\mathbf{x}_{td}, ap\right)$ which determines this sum as a function of the trim variables given the trim definition and aircraft parameter structure ap

$$J\left(\mathbf{x}_{tv}|\mathbf{x}_{td}, ap\right) = \|\mathbf{f}_B\| + \|\mathbf{G}_B\|. \tag{3}$$

Given the trim definition and aircraft parameter structure, the actual trim variables for that condition is the vector that minimizes the function (since the function is always greater than or equal to zero, its minimum is zero)

$$\mathbf{x}_{tv}^{*} = \arg\min_{\mathbf{x}_{tv}} J\left(\mathbf{x}_{tv}|\mathbf{x}_{td}, ap\right). \tag{4}$$

Solving this function analytically is often impossible. However, as written it can be seen that solving for the trim variable $\mathbf{x}_{tv}^{*}$ is equivalent to solving a numerical minimization problem. It turns out that Matlab provides a variety of functions (e.g. `fmincon.m`) for solving these types of problems, which in turn lets us solve for the trim condition of the aircraft.

Note, this is the only time in the course where we use the expression trim to include an equilibrium condition that could be ascending or descending, i.e. that is not flying horizontally. Because the forces and moments acting on an aircraft can be balanced in this case the aeronautics community often refers to it as a trim condition. We will see shortly that this condition is not stable since ascending (or descending) causes the height to change, which causes the density to change, which changes the force and moment balance.

Coorindated Turn

A coordinated turn differs from straight-level flight in several ways. First, the trim definition now has an additional turn, the radius R_0 of the turn

$$\text{trim\_definition} = \mathbf{x}_{td} = [V_a,\ \gamma_0,\ h_0,\ R_0]^T. \tag{5}$$

Likewise, there are now seven independent *trim variables* because we include: the roll angle ϕ_0, sideslip β_0, aileron deflection δ_{a0}, and rudder deflection δ_{r0}

$$\text{trim\_variables} = \mathbf{x}_{tv} = [\alpha_0,\ \delta_{e0},\ \delta_{t0},\ \phi_0,\ \beta_0,\ \delta_{a0},\ \delta_{r0}]^T. \tag{6}$$

All state variables for the trim condition can be derived from the trim definition and variables $(z_E, \phi, \theta, u^E, v^E, w^E, p, q, r)$, or do not matter (x_E, y_E, ψ).

The objective function is more complex in this case since the forces do not sum to zero. Instead, the forces must provided the desired acceleration vector $\mathbf{a}^{des}$ to make the turn. This is best expressed in inertial coordinates but can easily be transformed into body coordinates based on Euler angles. Further, the cordinated turn occurs with zero side force Y. Thus, the objective function becomes

$$J\left(\mathbf{x}_{tv} | \mathbf{x}_{td}, ap\right) = \|\mathbf{f}_B - m\mathbf{a}_B^{des}\| + \|\mathbf{G}_B\| + Y^2. \tag{7}$$

Problems

Submit all code through the course web site as well as a pdf or txt document answering the questions below. Only submit code for running one of the parts of Problem 3. Unlike past assignments, this assignment does not specify the exact form or decomposition of the functions you must create.

Problem 1

Create the following for the case of straight, wings-level flight:

1. A function that takes as input the trim definition and trim variable and returns the aircaft trim state vector and control surface vector.

2. A function that takes as input the trim variable, trim definition, and aircraft parameter structure, and returns the cost $J\left(\mathbf{x}_{tv}|\mathbf{x}_{td}, ap\right)$. This function will use the function from Part 1, the provided `stdatmo.m`, and other functions created in Homework 1 and Homework 2.

3. A function that takes as input the trim definition and aircraft paramater structure and returns the aircraft state and control vector to be in the defined trim condition. This function should use `fmincon.m` to minimize the function from Part 2.

Problem 2

Create the following for the case of a coordinated turn:

1. A function that takes as input the trim definition and trim variable and returns the aircaft trim state vector and control surface vector.

2. A function that takes as input the trim variable, trim definition, and aircraft parameter structure, and returns the cost $J\left(\mathbf{x}_{tv}|\mathbf{x}_{td}, ap\right)$. This function will use the function from Part 1 the provided `stdatmo.m`, and other functions created in Homework 1 and Homework 2.

3. A function that takes as input the trim definition and aircraft paramater structure and returns the aircraft state and control vector to be in the defined trim condition. This function should use `fmincon.m` to minimize the function from Part 2.

Problem 3

Use the functions created in Homework2 to simulate the Ttwistor aircraft in the following conditions. Plot and describe the results.

1. Determine the trim state and control surfaces to achieve straight, wings-level flight with height $h = 1655$ m (Boulder's elevation), airspeed $V_a = 18$ m/s, and flight path angle $\gamma_0 = 0$. Simulate the aircraft with these initial conditions (and zero background wind) and verify that the aircraft flies straight and level. Note the axis labels and don't be concerned if you see some very very small oscillations.

2. Consider a background wind $\mathbf{w}_E^E = [10,\ 10,\ 0]^T$ m/s. Determine the aircraft state such that the same trim definition of Problem 3.1 applies, i.e. equilibrium with the stated airspeed. Simulate to verify the result.

3. Determine the trim state and control surfaces to achieve straight, wings-level flight with height $h = 1655$ m (Boulder's elevation), airspeed $V_a = 18$ m/s, and flight path angle $\gamma_0 = 10$ deg. Simulate the aircraft with these initial conditions (and zero background wind). Describe the resulting motion. Be sure to let the simulation run long enough.

4. Determine the trim state and control surfaces to achieve a coordinated turn at height $h = 200$ m, airspeed $V_a = 20$ m/s, flight path angle $\gamma_0 = 0$ deg, and radius $R_0 = 500$ m. Simulate the aircraft with these initial conditions (and zero background wind). Run the simulation for at least 300 seconds. Describe the resulting motion. **Hint**: One way to verify your code for a coordinated turn is to run it with a very large radius and compare to the straight, wings-level case.