Problem 4

Part1:

For Short Period Mode :
Damping Ratio: 0.3517726583144173
Natural Frequency: 3.7753929083414826

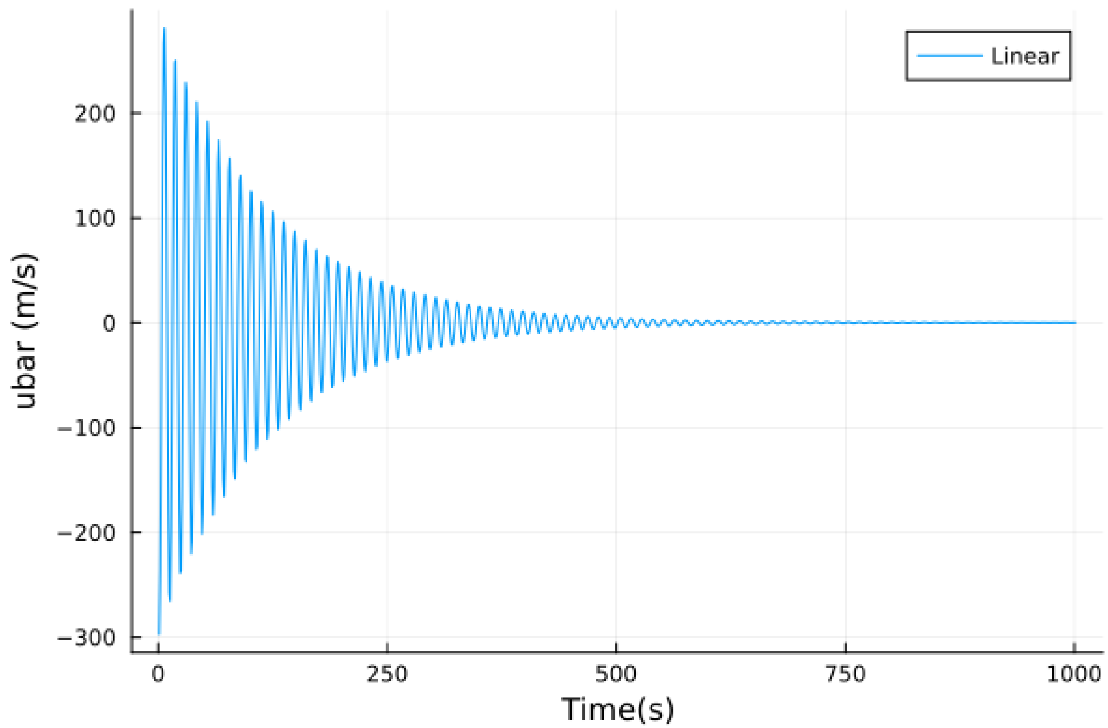
For Phugoid Mode :
Damping Ratio: 0.015662714197962942
Natural Frequency: 0.5302783415889679

Part2:

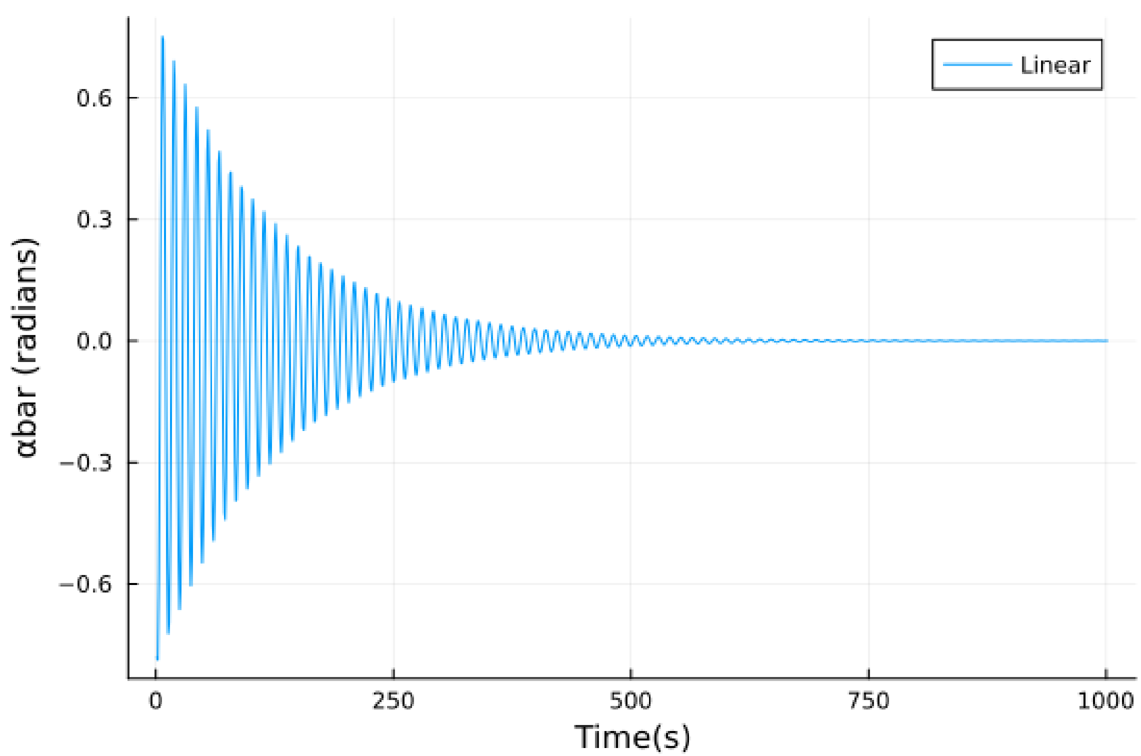
For Short Period Mode :
Eigenvalue : -1.3280799995486825 - 3.5340904243035336im
Eigenvector :
[
 0.05200873086912191 - 0.22616200256732813im
 -0.13427077743526278 + 0.11942307681502852im
 0.3053623657388001 + 0.5694776206843573im
 -0.16965064823186435 + 0.022651580265484635im
 0.6833812670502114 - 0.0im
]

For Phugoid Mode :
Eigenvalue : -0.008305598109677771 - 0.530213293494596im
Eigenvector :
[
 -0.3612050346417226 + 0.04138233464217913im
 -0.0009502074203060539 - 0.00028856315070276535im
 -0.010704528067482972 + 0.00013391596586432154im
 6.366925143638189e-5 - 0.02018810050144037im
 0.9312868896617936 - 0.0im
]

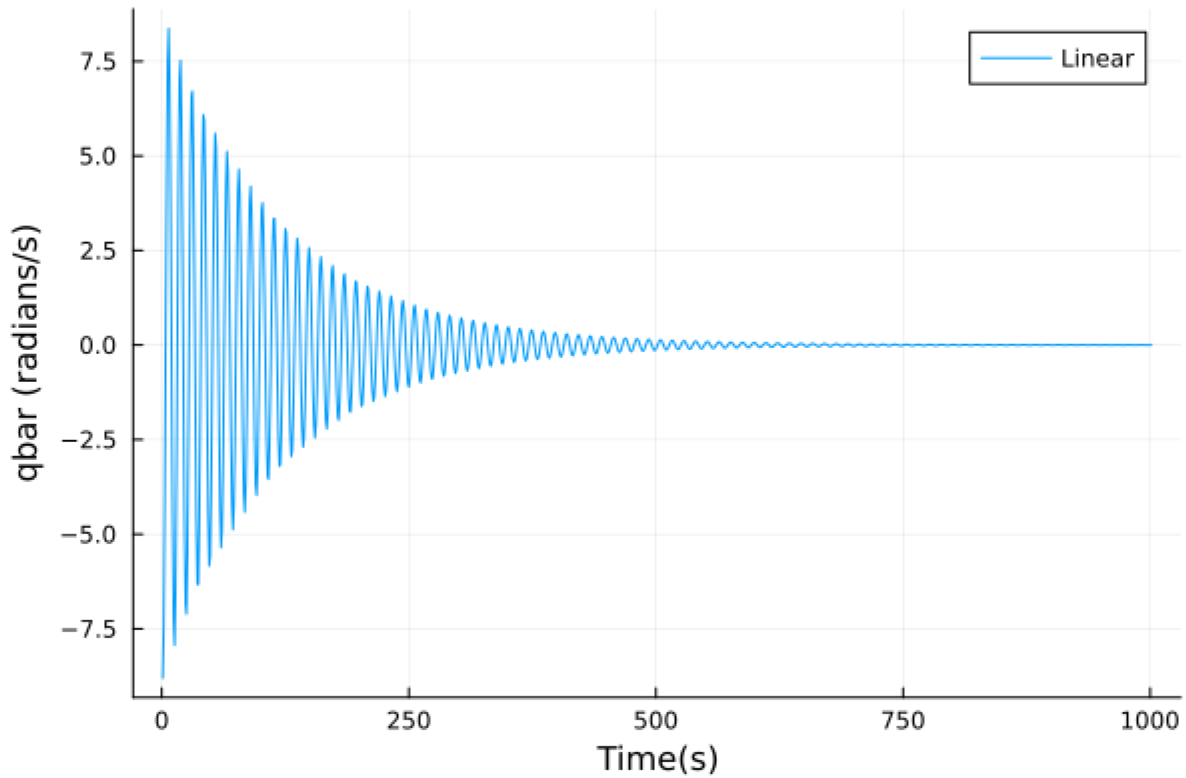
Part3:



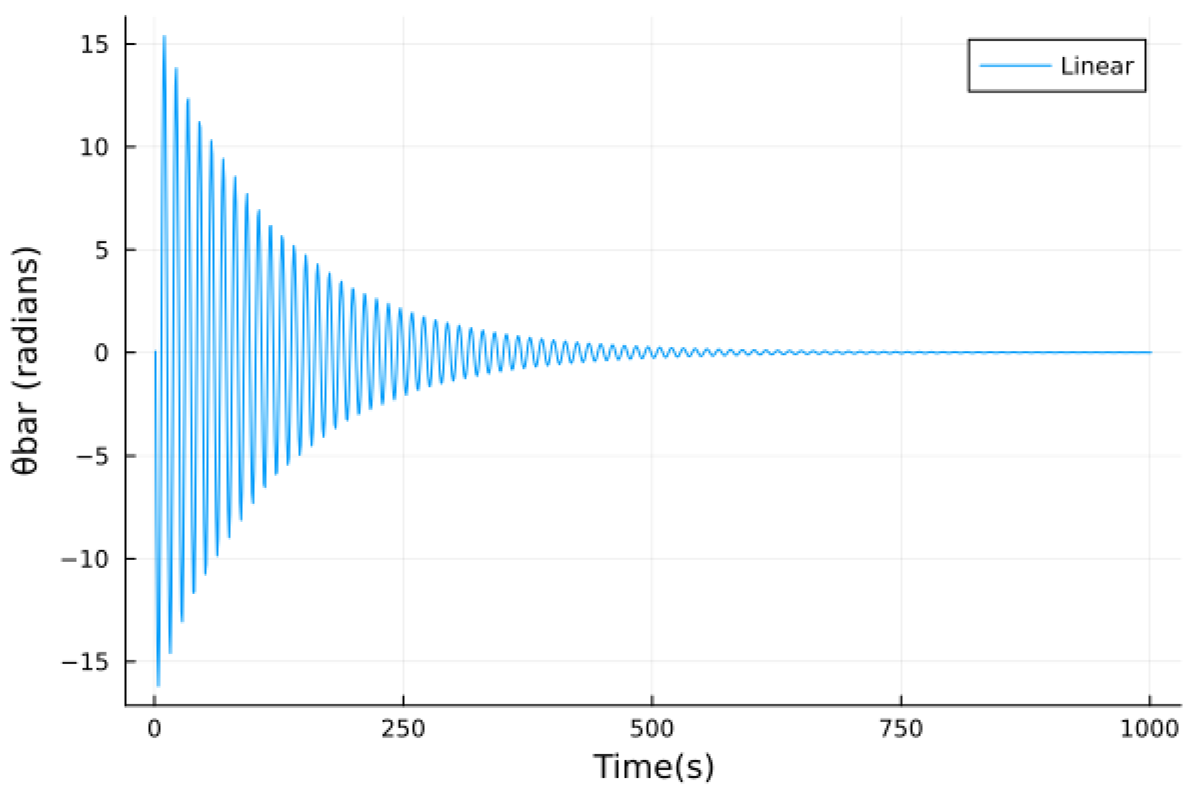
Linear
ubar (m/s)
200
100
0
−100
−200
−300
0
250
500
750
1000
Time(s)



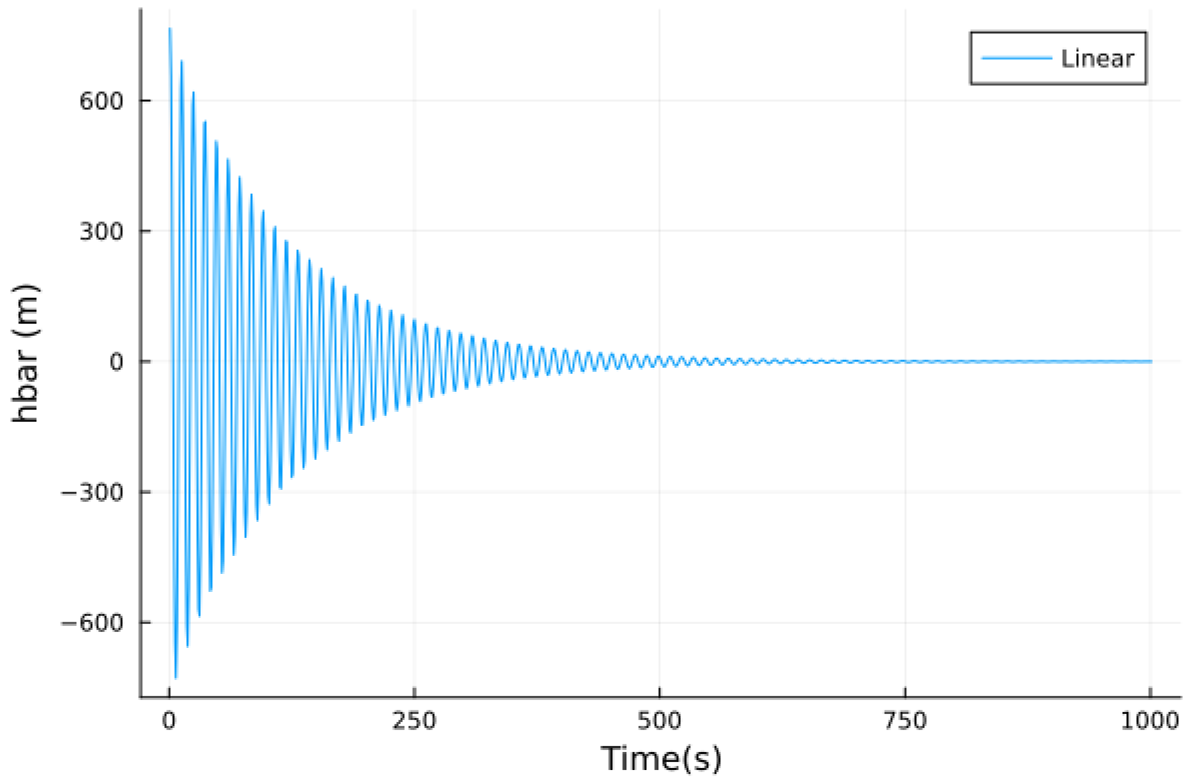
Linear
αbar (radians)
0.6
0.3
0.0
−0.3
−0.6
0
250
500
750
1000
Time(s)



qbar (radians/s)
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
0
250
500
750
1000
Time(s)
Linear



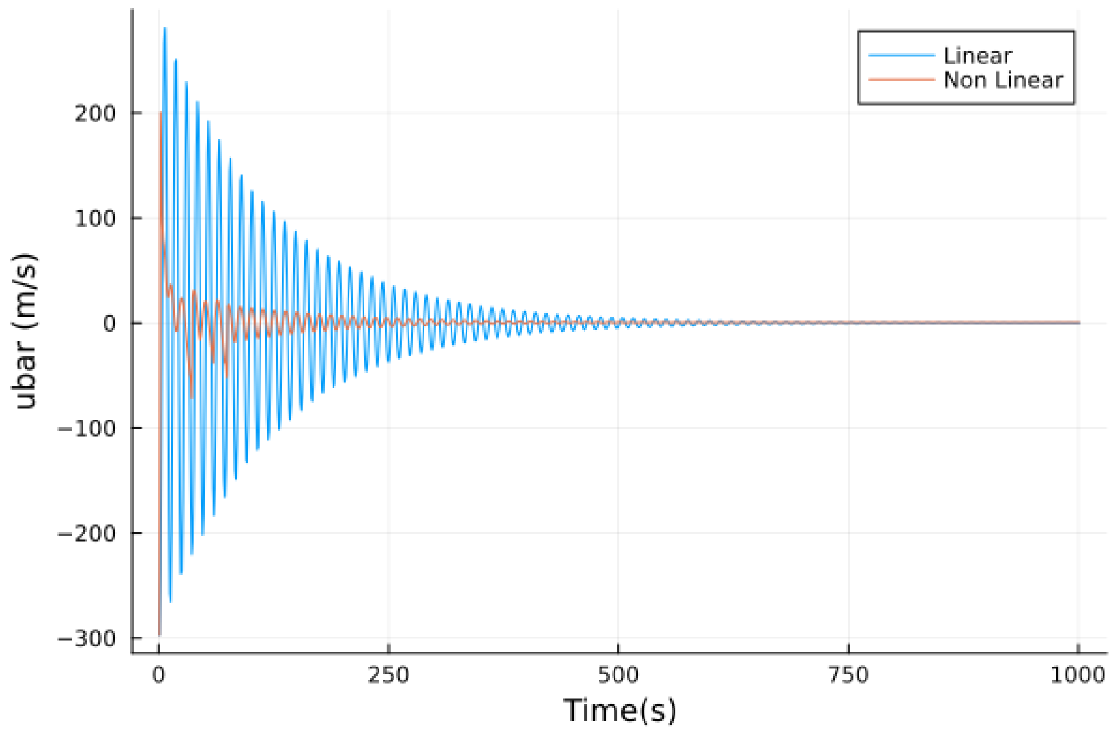
θbar (radians)
15
10
5
0
−5
−10
−15
0
250
500
750
1000
Time(s)
Linear

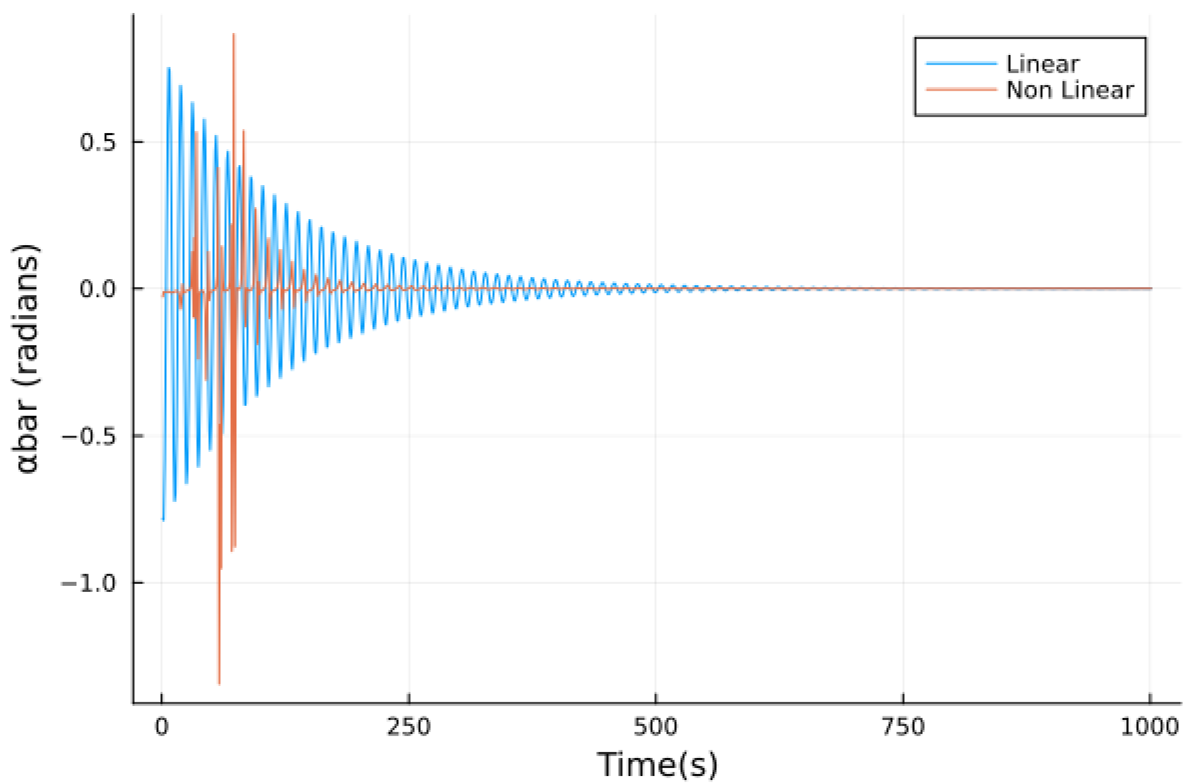


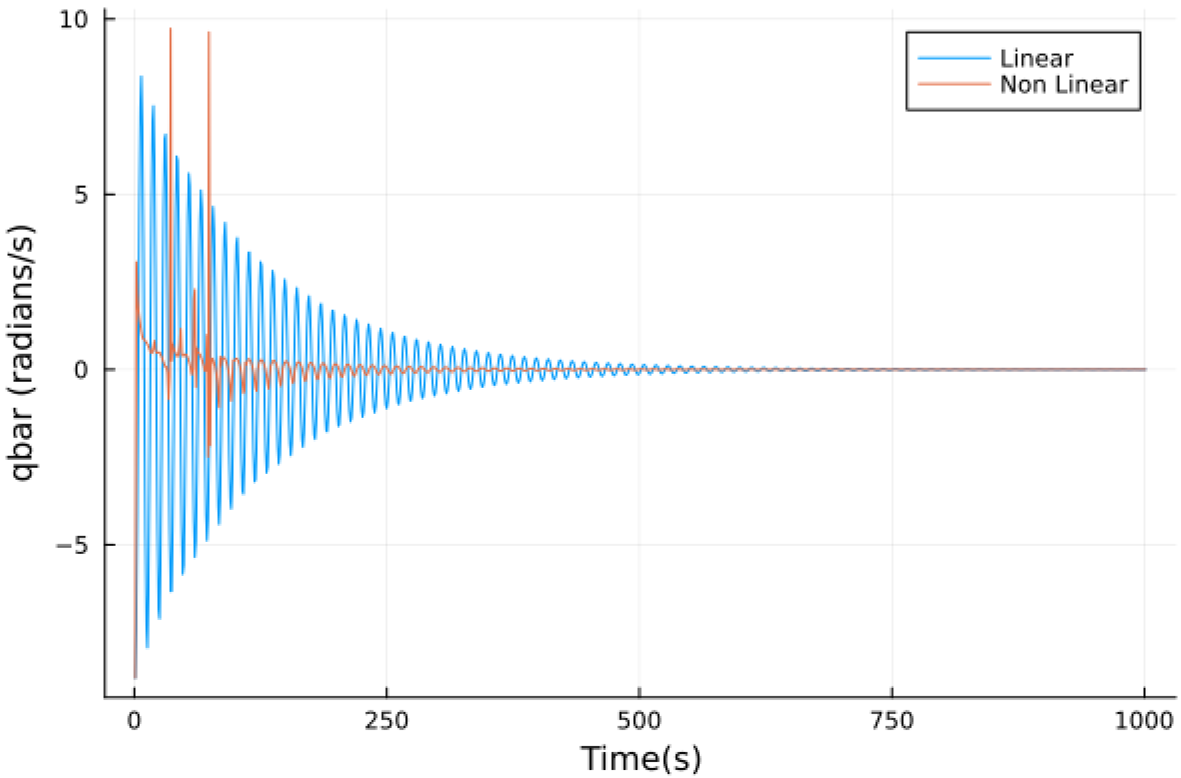
These results are for the linearized model. Since phugoid mode is a STABLE mode, the observed behavior of all the deviations converging to 0 was expected.

Part4:

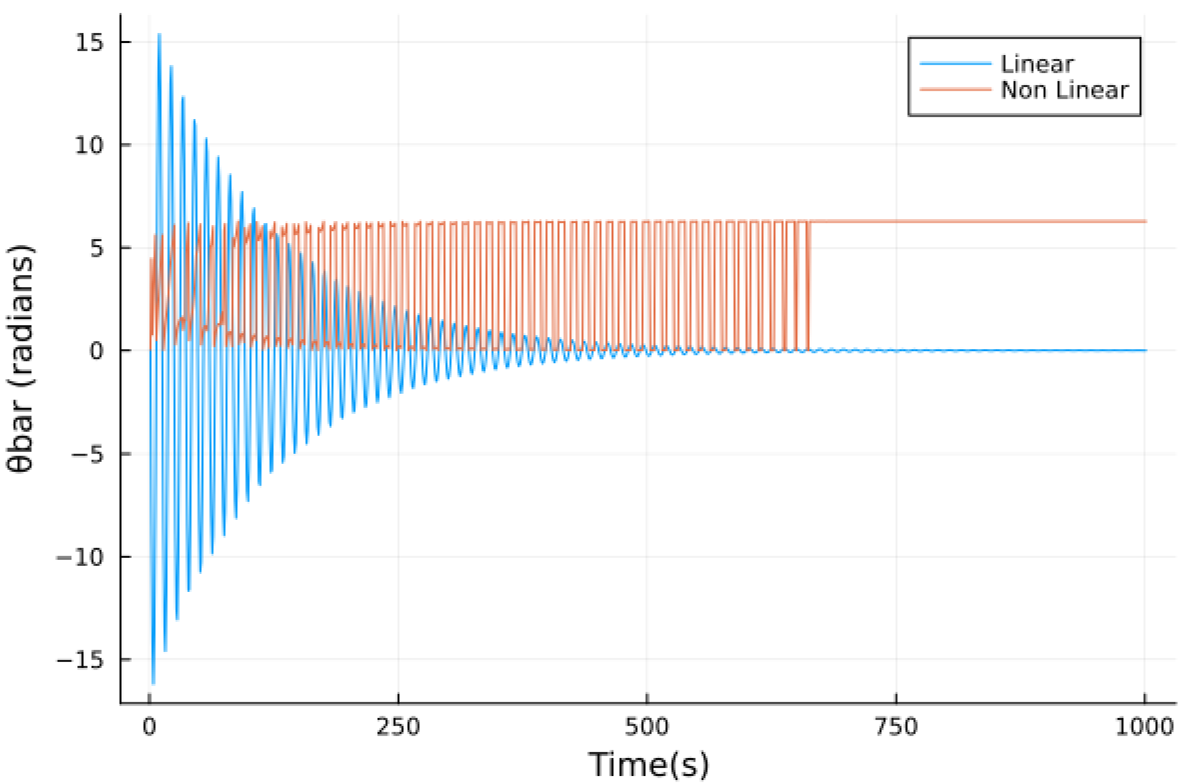
The following plots are for the time interval 0 to 1000 seconds.



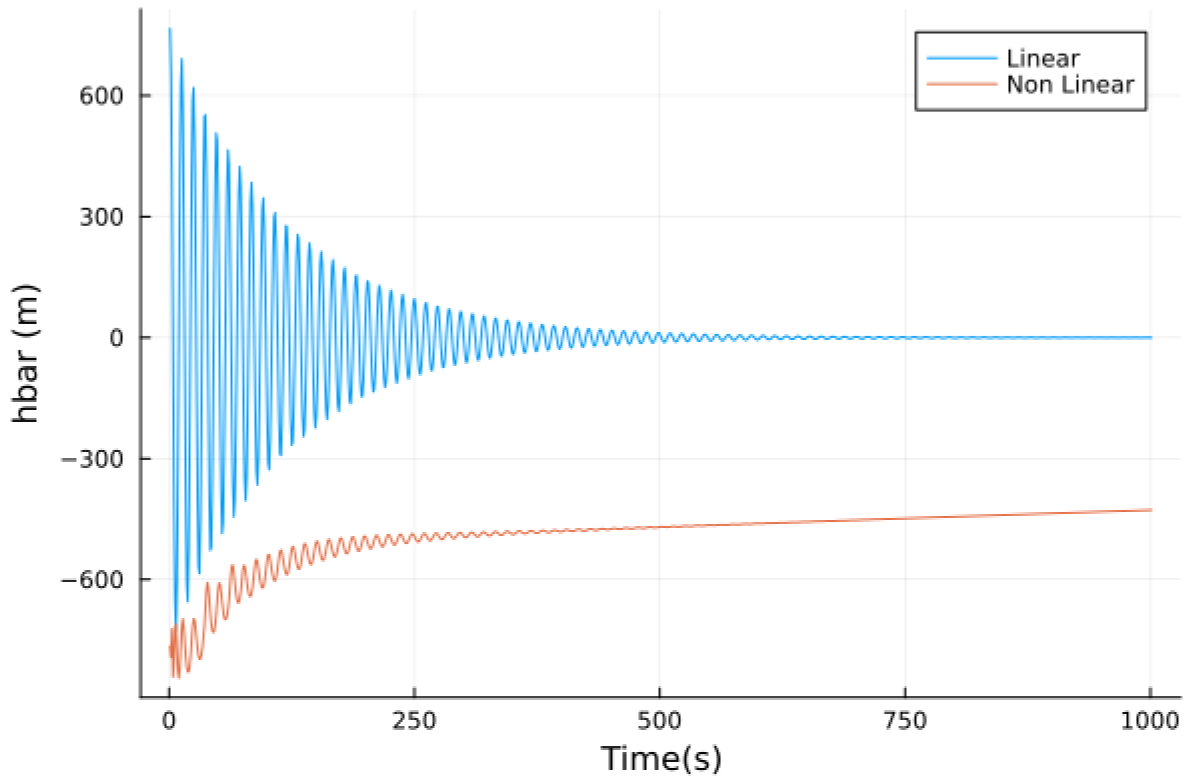




Linear
Non Linear
qbar (radians/s)
Time(s)

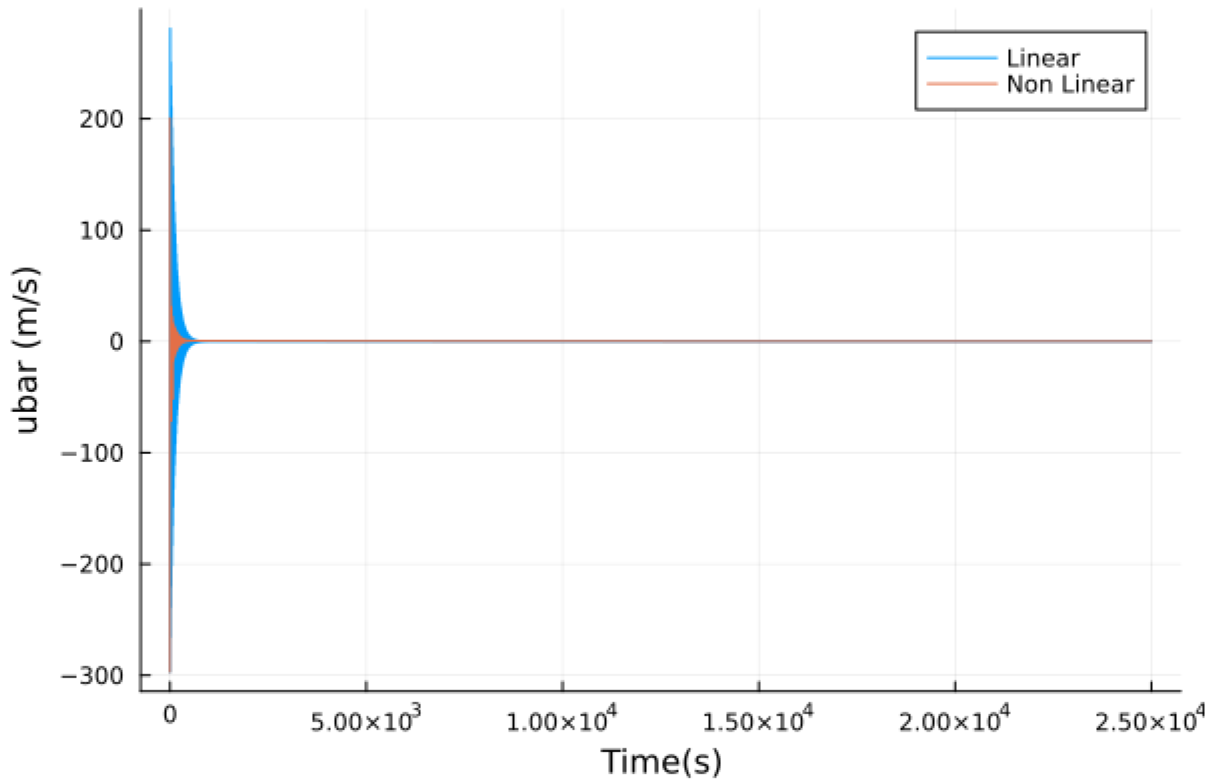


Linear
Non Linear
θbar (radians)
Time(s)

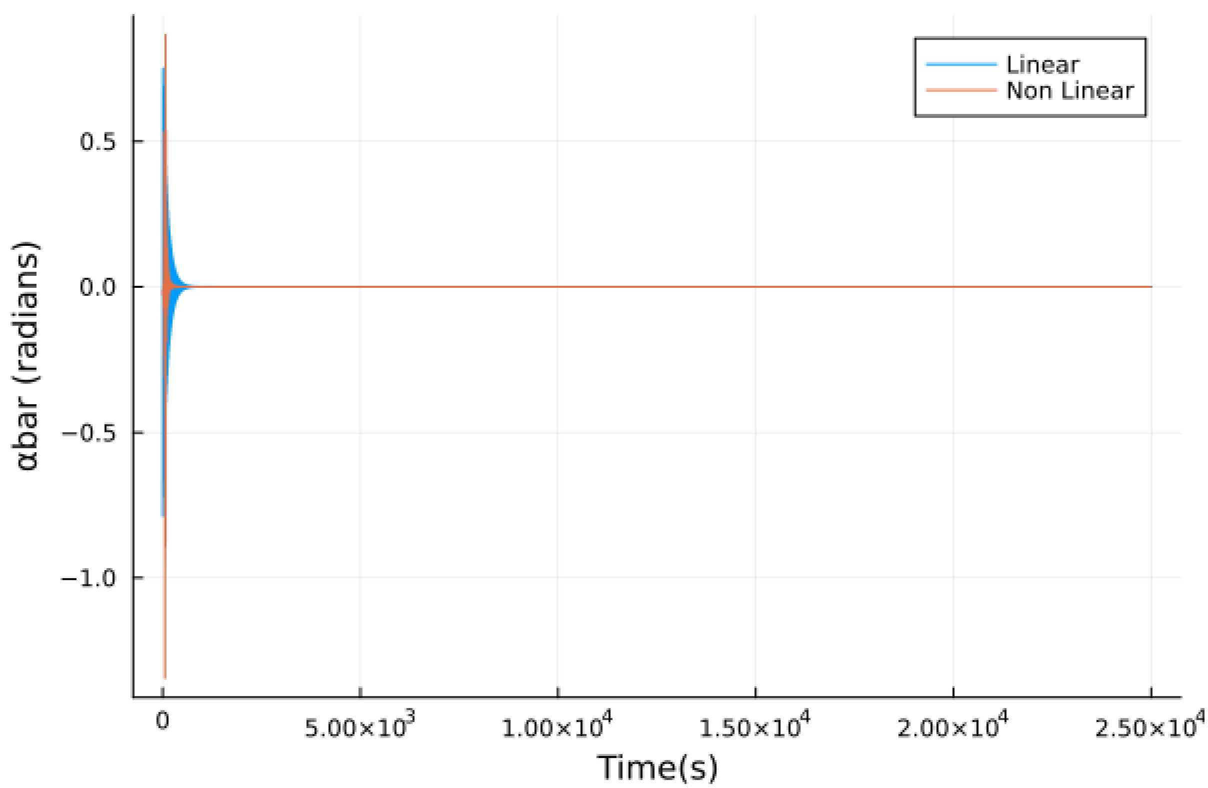


The plots show that the perturbations in u, alpha, and q go to zero for both the linearized and nonlinear models. It is a bit misleading from the plot, but the same happens for perturbation in theta. The converged perturbation value for pitch shown in the plot is close to 2pi (which, when wrapped, is close to 0). The scaling required to get the initial perturbation of 3 degrees in pitch resulted in a very high perturbation value for height. As a result, it takes a lot of time for the height perturbation to go to 0, but it eventually does go to 0, as seen in the plots below. So, yes, the linear model is a good enough approximation of the nonlinear model.

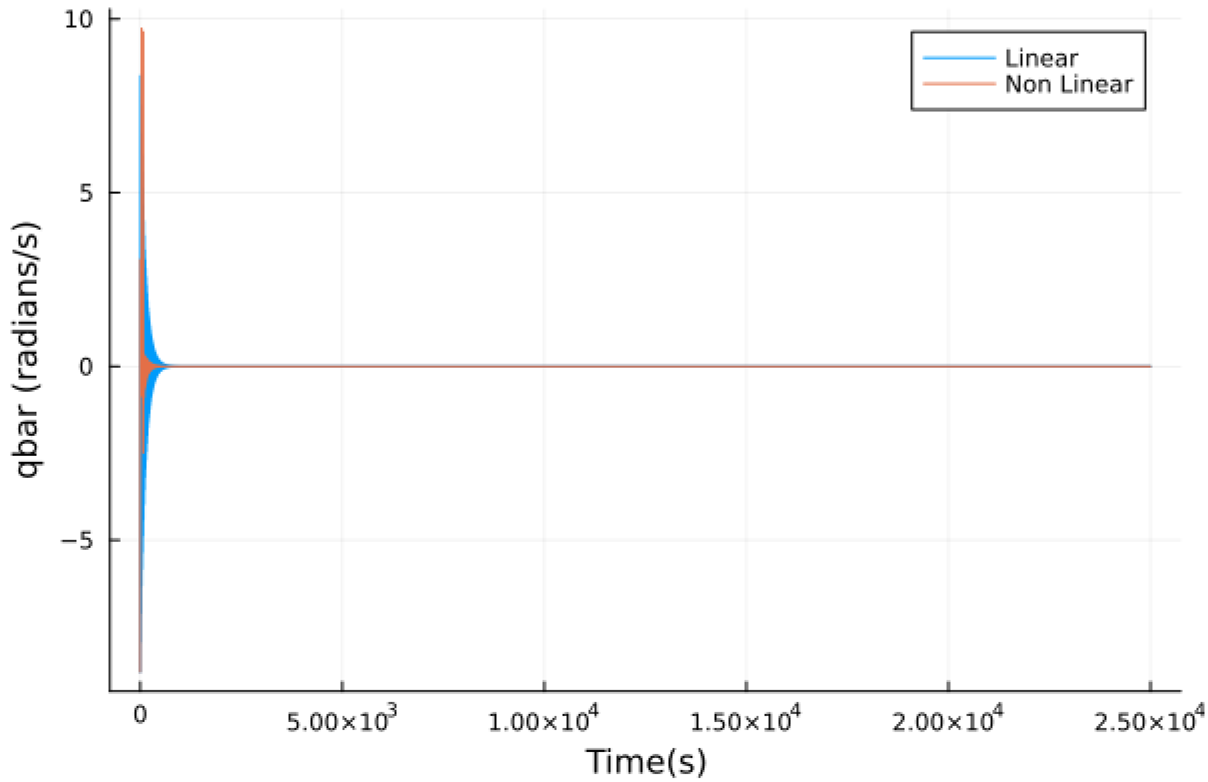
The following plots are for the time interval 0 to 25000 seconds.



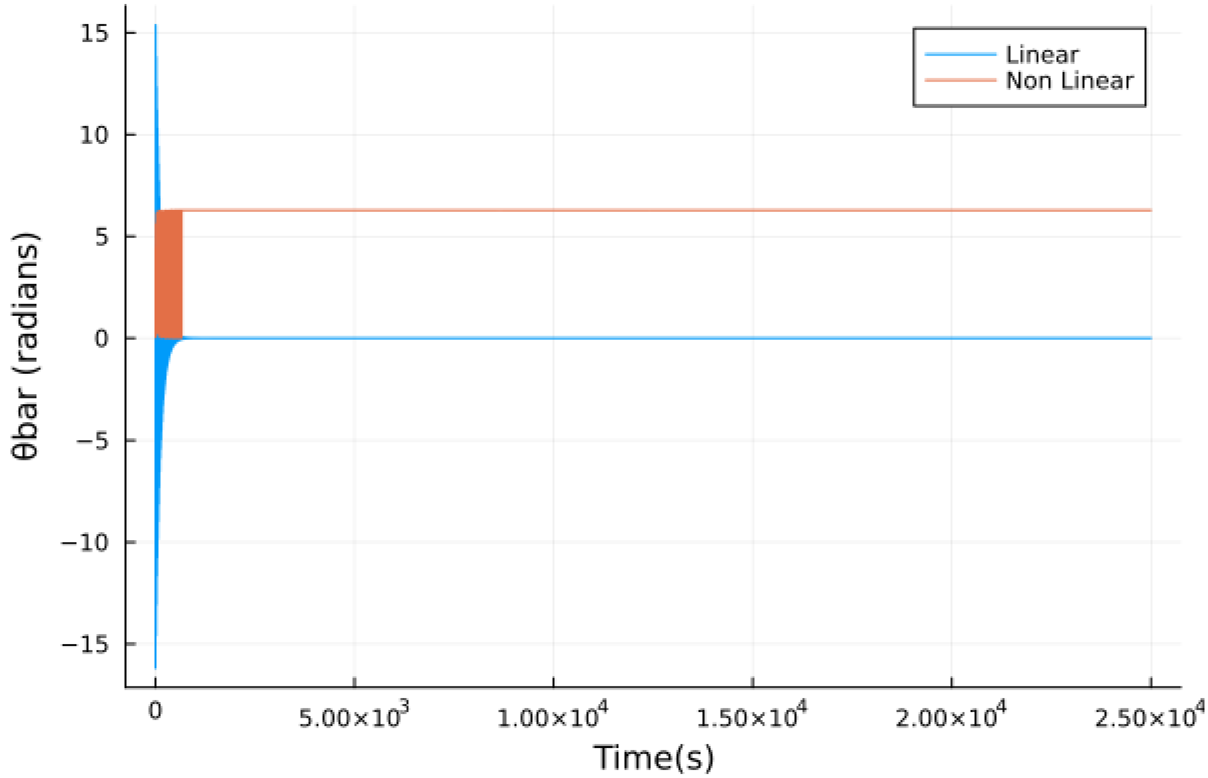
Linear
Non Linear
200
100
0
−100
−200
−300
ubar (m/s)
0
5.00×10^3
1.00×10^4
1.50×10^4
2.00×10^4
2.50×10^4
Time(s)



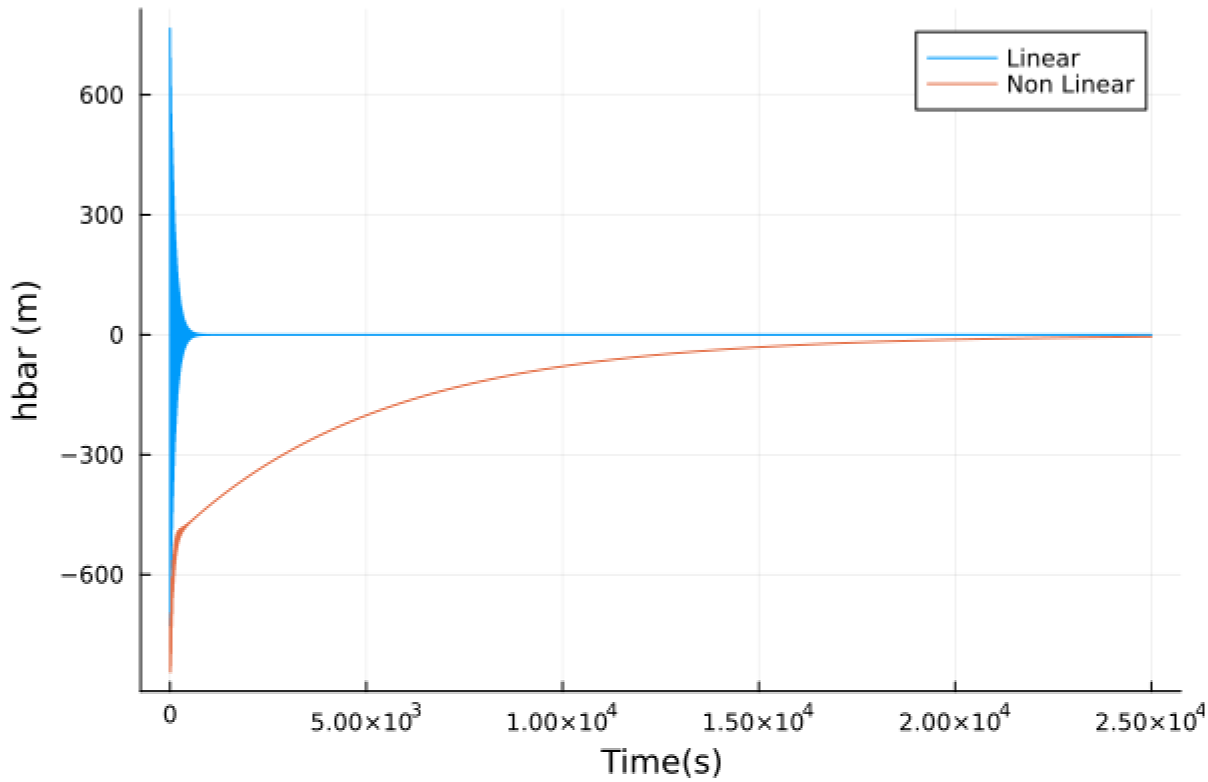
Linear
Non Linear
0.5
0.0
−0.5
−1.0
αbar (radians)
0
5.00×10^3
1.00×10^4
1.50×10^4
2.00×10^4
2.50×10^4
Time(s)



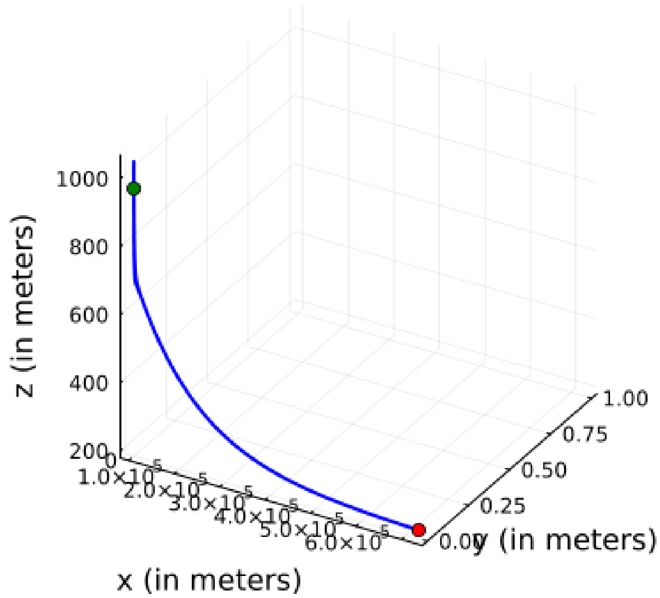
Linear
Non Linear
qbar (radians/s)
10
5
0
−5
0
5.00×10^3
1.00×10^4
1.50×10^4
2.00×10^4
2.50×10^4
Time(s)



Linear
Non Linear
θbar (radians)
15
10
5
0
−5
−10
−15
0
5.00×10^3
1.00×10^4
1.50×10^4
2.00×10^4
2.50×10^4
Time(s)

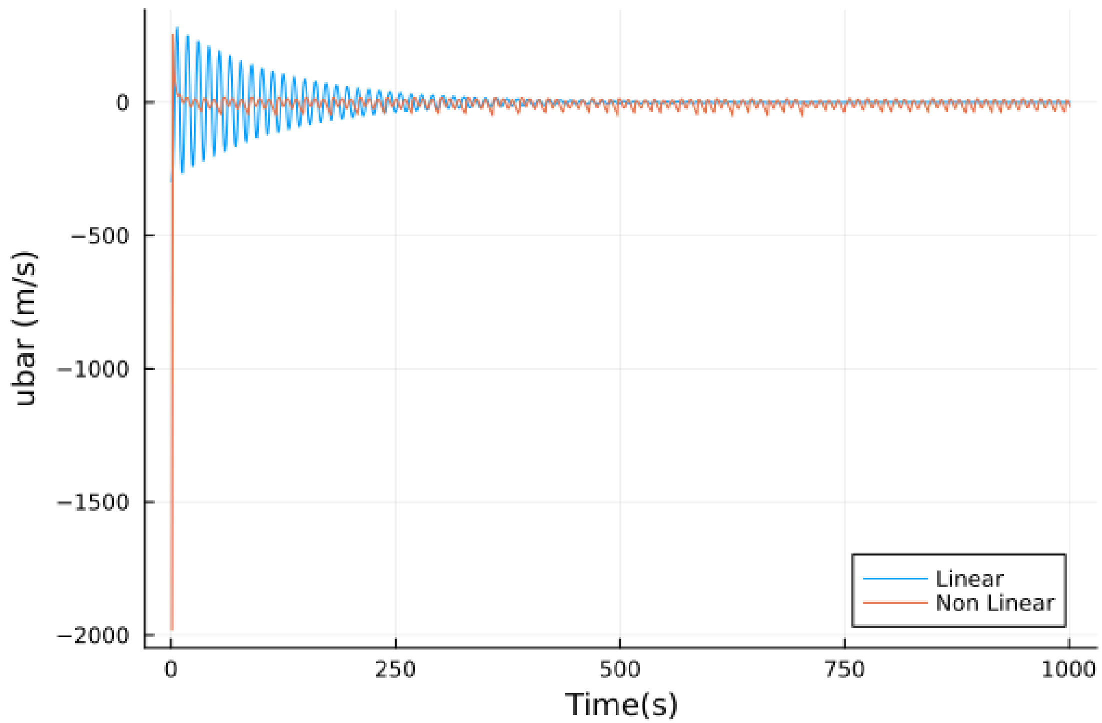


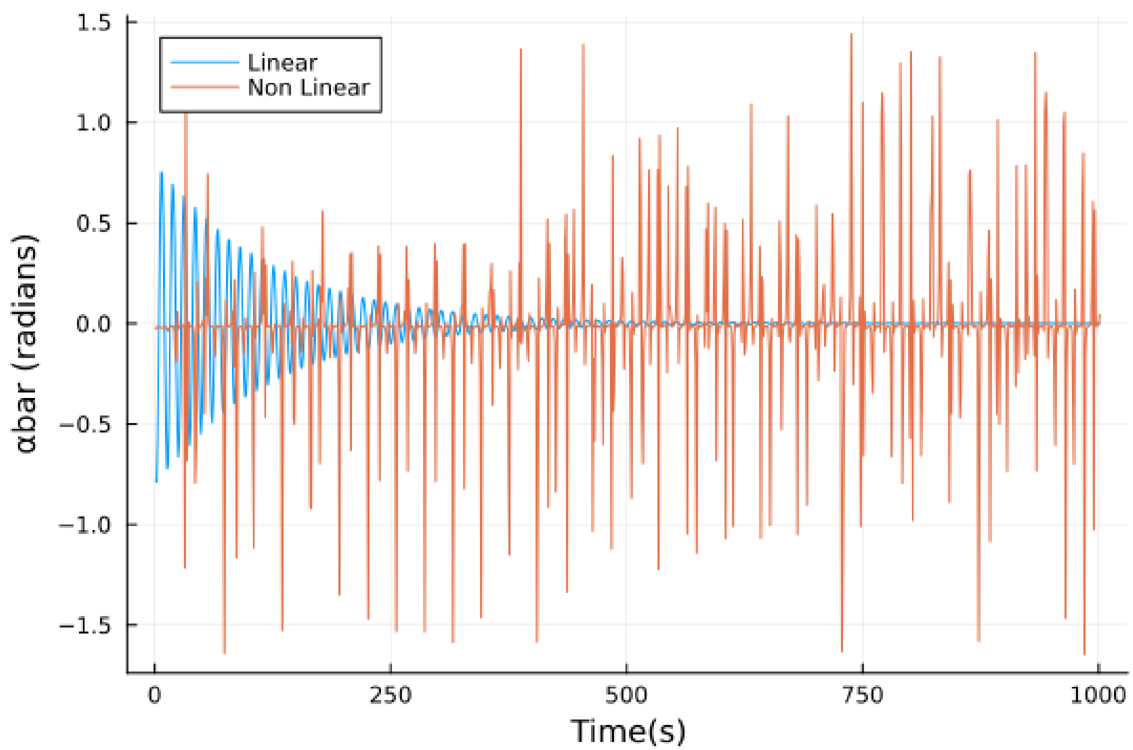
As visible, the perturbation in h eventually goes to 0.

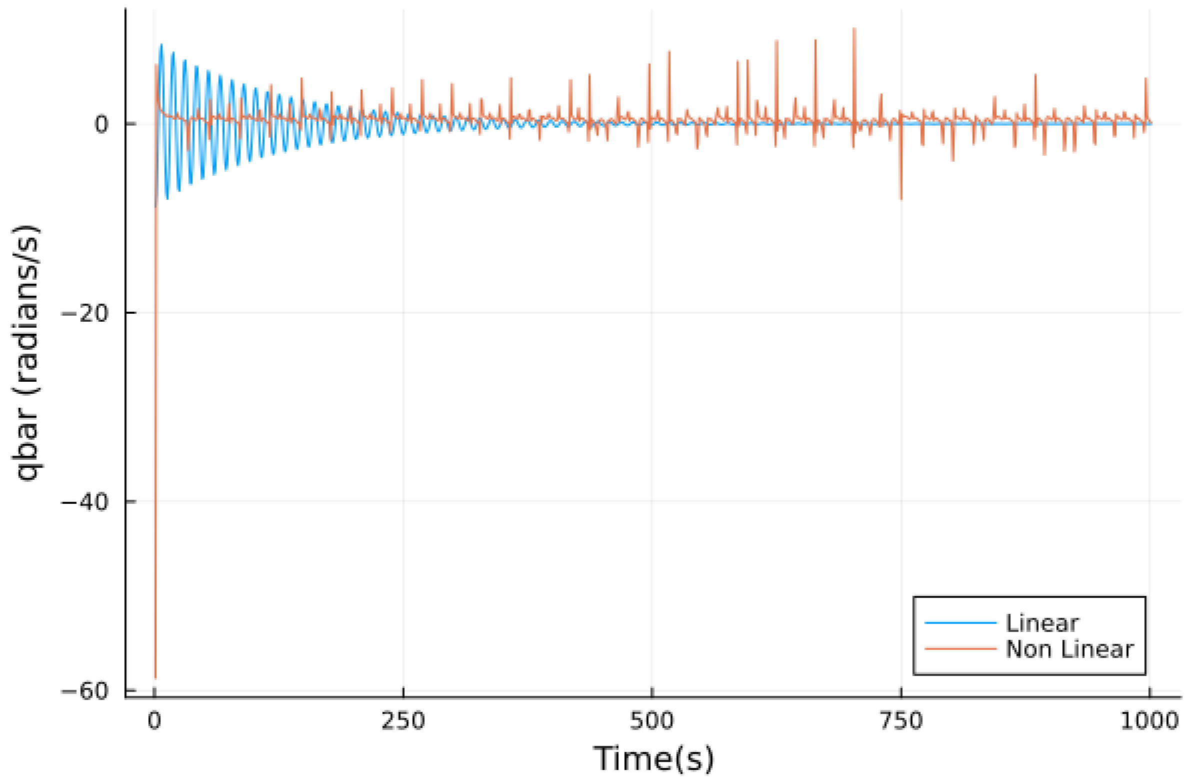


Part5:

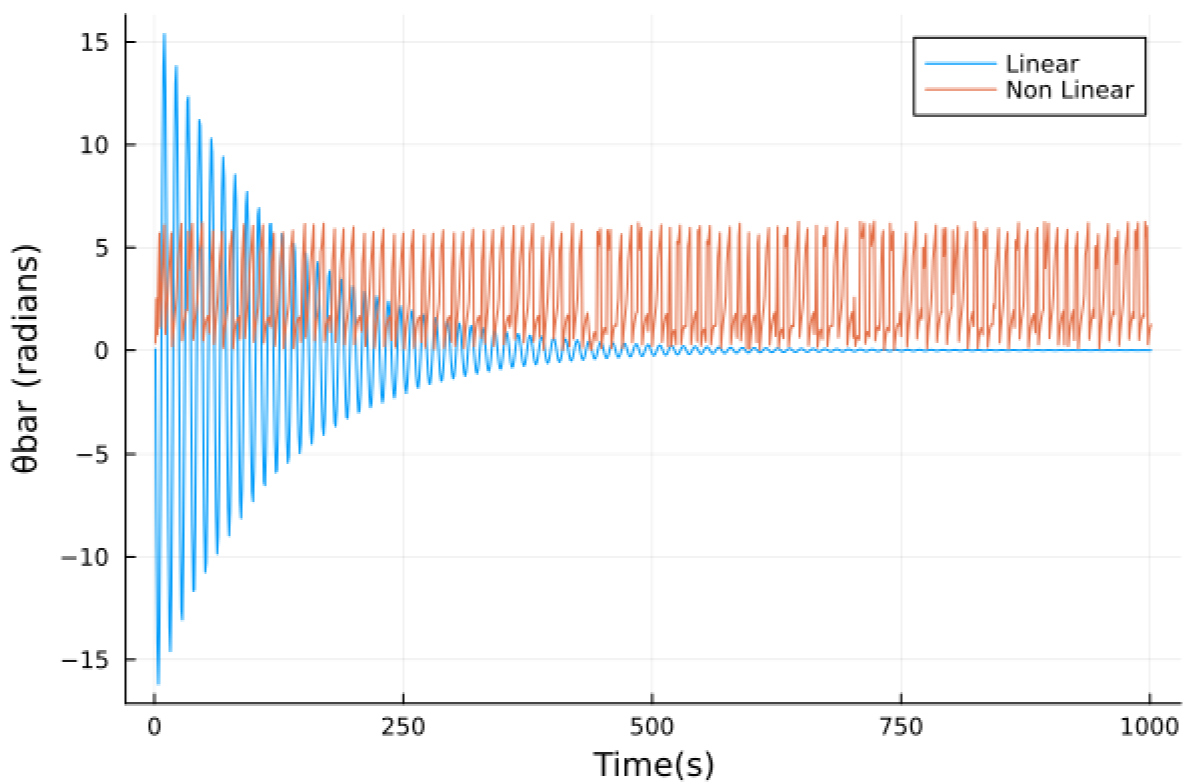
The following plots are for the time interval 0 to 1000 seconds.



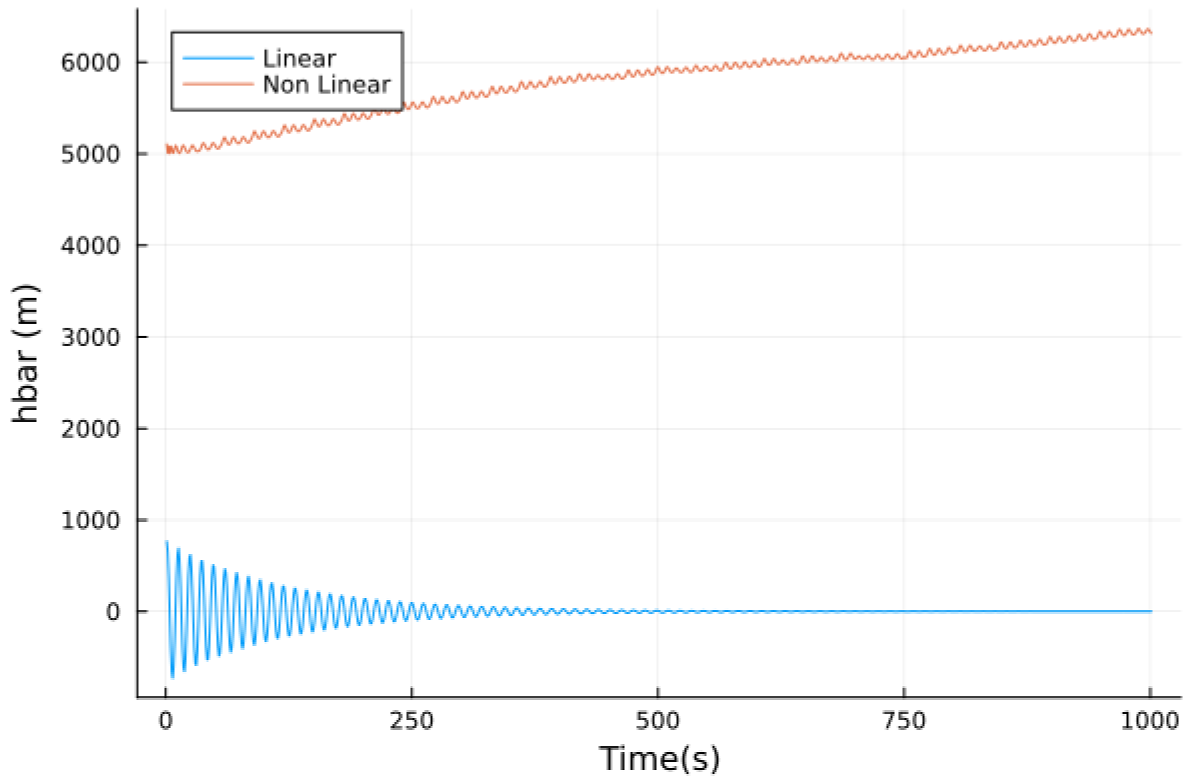




0
−20
−40
−60
qbar (radians/s)
0
250
500
750
1000
Time(s)
Linear
Non Linear

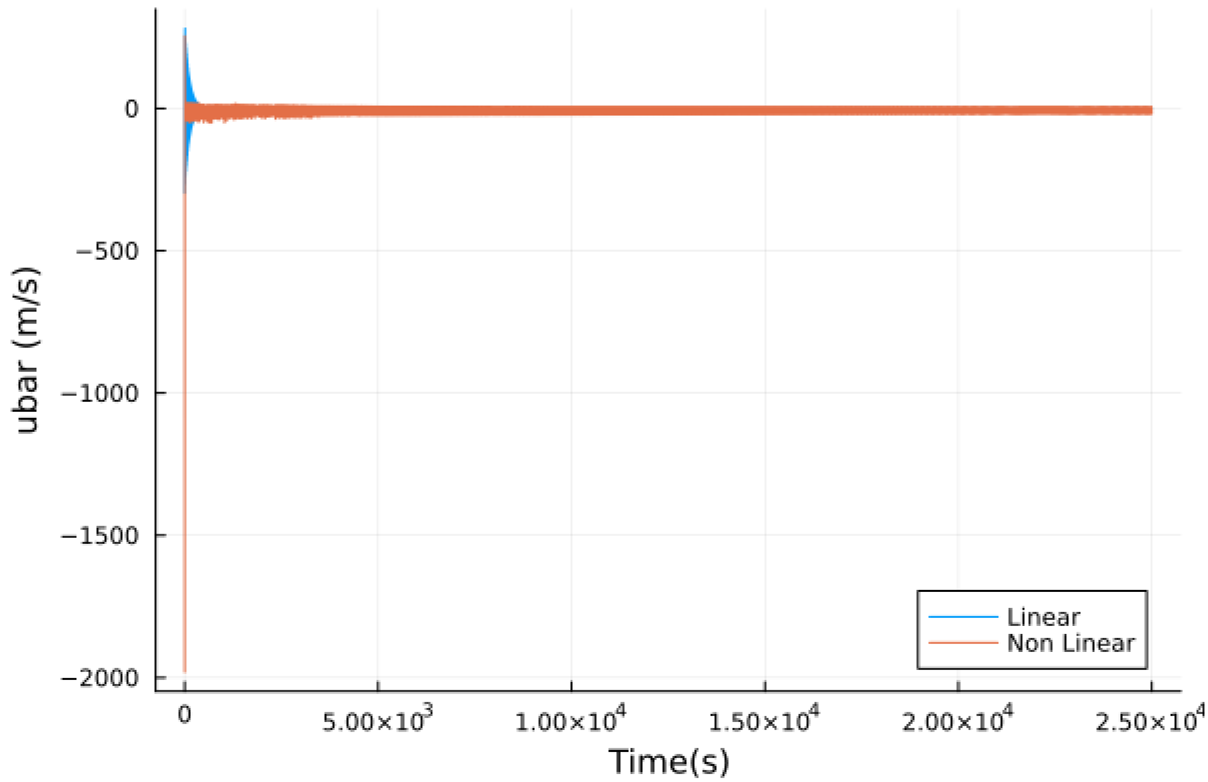


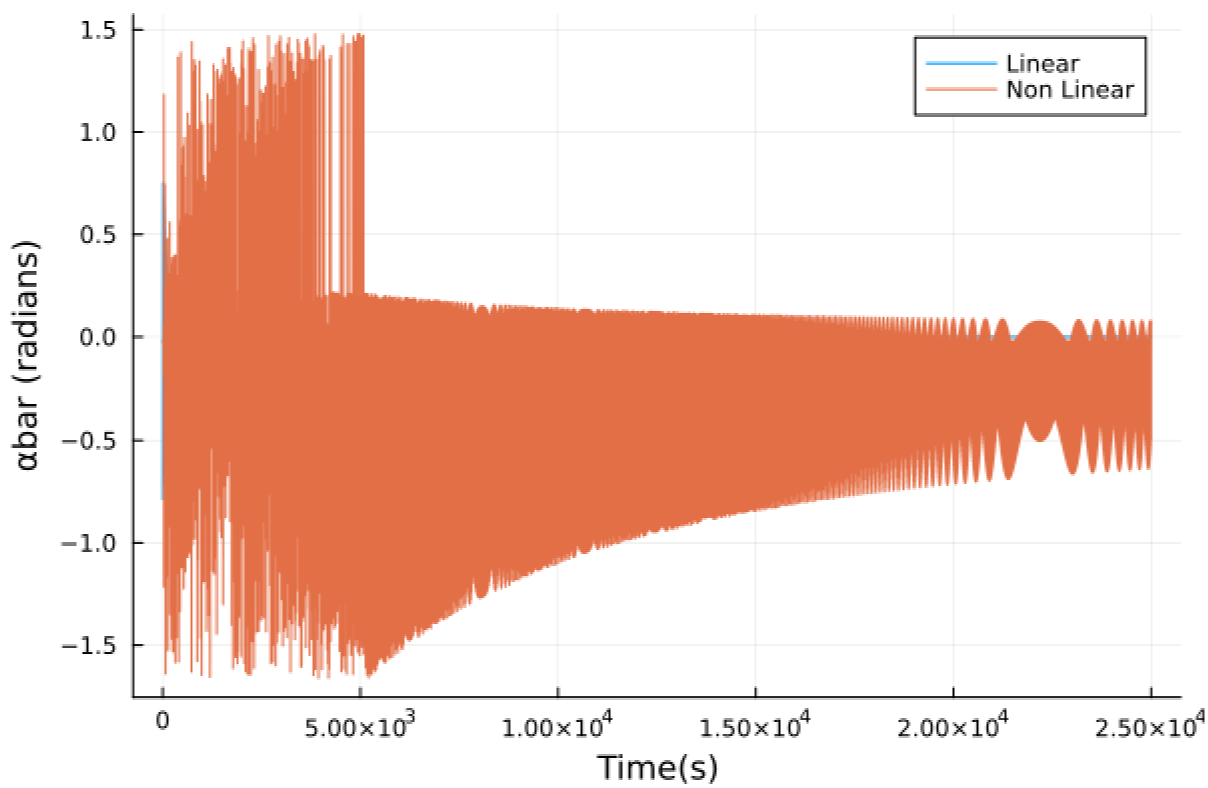
15
10
5
0
−5
−10
−15
θbar (radians)
0
250
500
750
1000
Time(s)
Linear
Non Linear

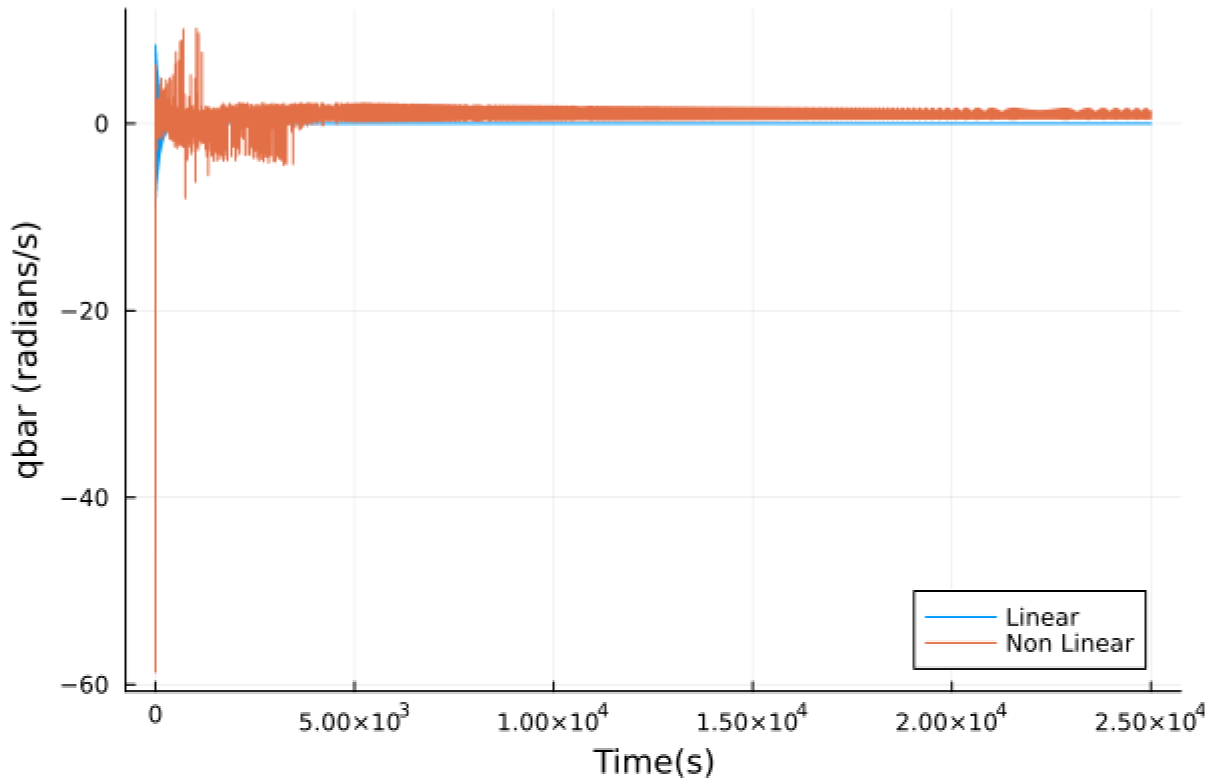


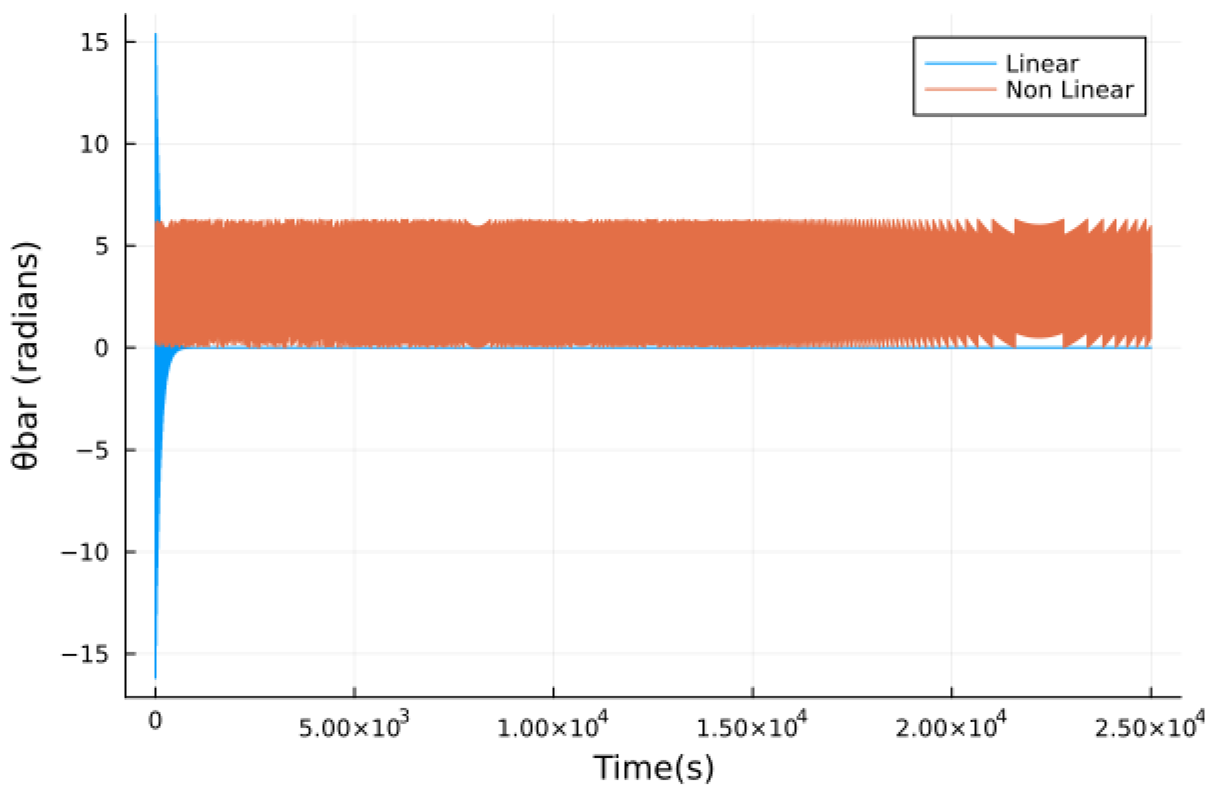
The plots show that the perturbations in u, and q go to zero for both the linearized and nonlinear models. However, there is a lot of noise. The converged perturbation value for pitch shown in the plot is close to 2pi (which, when wrapped, is close to 0). There is a lot of noise in the perturbation of the angle of attack as well. The scaling required to get the initial perturbation of 20 degrees in pitch resulted in a very high perturbation value for all the variables. Since the perturbation is no longer small, our approximated linear model doesn't behave like the more accurate nonlinear model. So, the linear model is no longer a good approximation. This is also visible from the plots when the simulation is run for a really long period of time.

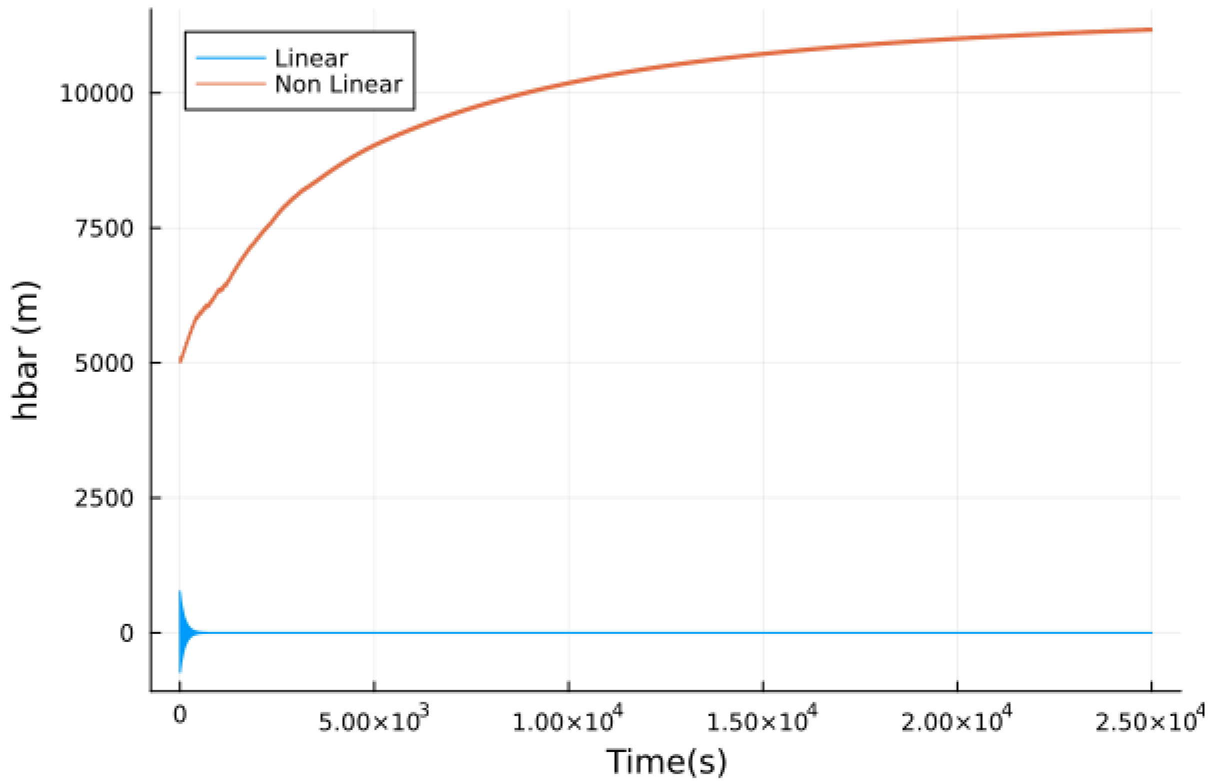
The following plots are for the time interval 0 to 25000 seconds.


ubar (m/s)
0
−500
−1000
−1500
−2000
0
5.00×10^3
1.00×10^4
1.50×10^4
2.00×10^4
2.50×10^4
Time(s)
Linear
Non Linear


αbar (radians)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
5.00×10^3
1.00×10^4
1.50×10^4
2.00×10^4
2.50×10^4
Time(s)
Linear
Non Linear


qbar (radians/s)
0
−20
−40
−60
0
5.00×10^3
1.00×10^4
1.50×10^4
2.00×10^4
2.50×10^4
Time(s)
Linear
Non Linear


θbar (radians)
15
10
5
0
−5
−10
−15
0
5.00×10^3
1.00×10^4
1.50×10^4
2.00×10^4
2.50×10^4
Time(s)
Linear
Non Linear



As visible, the perturbation in h is not going to 0 and is increasing exponentially.