

#Problem 1

```
function GetLateralEOMCoefficients( $\Gamma$ ::Array{Float64}, ap::AircraftParameters)
    #For roll rate p
    Cp0 =  $\Gamma$ [3]*ap.CL0 +  $\Gamma$ [4]*ap.Cn0
    Cp $\beta$  =  $\Gamma$ [3]*ap.Clbeta +  $\Gamma$ [4]*ap.Cnbeta
    Cpp =  $\Gamma$ [3]*ap.Clp +  $\Gamma$ [4]*ap.Cnp
    Cpr =  $\Gamma$ [3]*ap.Clr +  $\Gamma$ [4]*ap.Cnr
    Cp $\delta_a$  =  $\Gamma$ [3]*ap.Clda +  $\Gamma$ [4]*ap.Cnda
    Cp $\delta_r$  =  $\Gamma$ [3]*ap.Cldr +  $\Gamma$ [4]*ap.Cndr

    #For yaw rate r
    Cr0 =  $\Gamma$ [4]*ap.CL0 +  $\Gamma$ [8]*ap.Cn0
    Cr $\beta$  =  $\Gamma$ [4]*ap.Clbeta +  $\Gamma$ [8]*ap.Cnbeta
    Crp =  $\Gamma$ [4]*ap.Clp +  $\Gamma$ [8]*ap.Cnp
    Crr =  $\Gamma$ [4]*ap.Clr +  $\Gamma$ [8]*ap.Cnr
    Cr $\delta_a$  =  $\Gamma$ [4]*ap.Clda +  $\Gamma$ [8]*ap.Cnda
    Cr $\delta_r$  =  $\Gamma$ [4]*ap.Cldr +  $\Gamma$ [8]*ap.Cndr

    CoefficientsDict = Dict(
        "Cp0" => Cp0,
        "Cp $\beta$ " => Cp $\beta$ ,
        "Cpp" => Cpp,
        "Cpr" => Cpr,
        "Cp $\delta_a$ " => Cp $\delta_a$ ,
        "Cp $\delta_r$ " => Cp $\delta_r$ ,
        "Cr0" => Cr0,
        "Cr $\beta$ " => Cr $\beta$ ,
        "Crp" => Crp,
        "Crr" => Crr,
        "Cr $\delta_a$ " => Cr $\delta_a$ ,
        "Cr $\delta_r$ " => Cr $\delta_r$ 
    )

    return CoefficientsDict
end

function GetLongitudinalEOMCoefficients(tc::AircraftControl,  $\alpha$ ::Float64, ap::AircraftParameters)

    CLtrim = ap.CL0 + (ap.CLalpha* $\alpha$ ) + (ap.CLde*tc. $\delta_e$ )
    CDtrim = ap.CDmin + (ap.K * (CLtrim - ap.CLmin)^2)
    CXtrim = ( CLtrim*sin( $\alpha$ ) ) - ( CDtrim*cos( $\alpha$ ) )
    CZtrim = ( -CLtrim*cos( $\alpha$ ) ) - ( CDtrim*sin( $\alpha$ ) )

     $\delta_{CD\delta CL}$  = 2*ap.K*(CLtrim-ap.CLmin)
    CD $\alpha$  = ap.CLalpha* $\delta_{CD\delta CL}$ 
    CDq = ap.CLq* $\delta_{CD\delta CL}$ 
    CD $\delta_e$  = ap.CLde* $\delta_{CD\delta CL}$ 

    CX $\alpha$  = -CD $\alpha$ *cos( $\alpha$ ) + CDtrim*sin( $\alpha$ ) + ap.CLalpha*sin( $\alpha$ ) + CLtrim*cos( $\alpha$ )
    CZ $\alpha$  = -CD $\alpha$ *sin( $\alpha$ ) - CDtrim*cos( $\alpha$ ) - ap.CLalpha*cos( $\alpha$ ) + CLtrim*sin( $\alpha$ )

    CXq = -CDq*cos( $\alpha$ ) + ap.CLq*sin( $\alpha$ )
    CZq = -CDq*sin( $\alpha$ ) - ap.CLq*cos( $\alpha$ )

    CX $\delta_e$  = -CD $\delta_e$ *cos( $\alpha$ ) + ap.CLde*sin( $\alpha$ )
    CZ $\delta_e$  = -CD $\delta_e$ *sin( $\alpha$ ) - ap.CLde*cos( $\alpha$ )

    CoefficientsDict = Dict(
        "CLtrim" => CLtrim,
        "CDtrim" => CDtrim,
        "CXtrim" => CXtrim,
        "CZtrim" => CZtrim,
        "CX $\alpha$ " => CX $\alpha$ ,
        "CZ $\alpha$ " => CZ $\alpha$ ,
        "CXq" => CXq,
        "CZq" => CZq,
        "CX $\delta_e$ " => CX $\delta_e$ ,
        "CZ $\delta_e$ " => CZ $\delta_e$ 
    )

    return CoefficientsDict
end

function GetLinearizedModel(::Type{LateralAircraftState}, ts::AircraftState, tc::AircraftControl, ap::AircraftParameters)

    p = stdatmo(-ts.z)
    S = ap.S
    b = ap.b
    m = ap.m
    AirSpeedVector = [ts.u, ts.v, ts.w] #because it has been given that wind speed is zero.
    wind_angles = AirRelativeVelocityVectorToWindAngles(AirSpeedVector)
    Va = wind_angles.Va
     $\beta$  = wind_angles. $\beta$ 
    af_term = Va*cos( $\beta$ )
    Q = 0.25*p*Va*S*b/m
     $\Gamma$  = GetGammaValues(ap)
    lc = GetLateralEOMCoefficients( $\Gamma$ , ap)

    A11 = 0.25*p*S*b*ts.v*( (ap.CYp*ts.p) + (ap.CYr*ts.r) )/(m*Va)
    A11 += p*S*ts.v*(ap.CY0 + ap.CYbeta* $\beta$  + ap.CYda*tc. $\delta_a$  + ap.CYdr*tc. $\delta_r$ )/(m)
    A11 += 0.5*p*S*ap.CYbeta*sqrt(ts.u^2 + ts.w^2)/m
    A12 = ts.w + Q*ap.CYp
    A12 = A12/af_term
    A13 = -ts.u + Q*ap.CYr
    A13 = A13/af_term
    A14 = ap.g*cos(ts. $\theta$ )*cos(ts. $\phi$ )
```

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A14 = A14/af_term
A15 = 0.0

A21 = 0.25*ρ*S*b*ts.v*( (lc["Cpp"]*ts.p) + (lc["Cpr"]*ts.r) )/(Va)
A21 += ρ*S*b*ts.v*(lc["Cp0"] + lc["Cpβ"]*β + lc["Cpδa"]*tc.δa + lc["Cpδr"]*tc.δr)
A21 += 0.5*ρ*S*b*lc["Cpβ"]*sqrt(ts.u^2 + ts.w^2)
A21 = A21*af_term
A22 = (Γ[1]*ts.q) + (Q*b*m*lc["Cpp"])
A23 = (-Γ[2]*ts.q) + (Q*b*m*lc["Cpr"])
A24 = 0.0
A25 = 0.0

A31 = 0.25*ρ*S*b*ts.v*( (lc["Crp"]*ts.p) + (lc["Crr"]*ts.r) )/(Va)
A31 += ρ*S*b*ts.v*(lc["Cr0"] + lc["Crβ"]*β + lc["Crδa"]*tc.δa + lc["Crδr"]*tc.δr)
A31 += 0.5*ρ*S*b*lc["Crβ"]*sqrt(ts.u^2 + ts.w^2)
A31 = A31*af_term
A32 = (Γ[7]*ts.q) + (Q*b*m*lc["Crp"])
A33 = (-Γ[1]*ts.q) + (Q*b*m*lc["Crr"])
A34 = 0.0
A35 = 0.0

A41 = 0.0
A42 = 1.0
A43 = cos(ts.φ)*tan(ts.θ)
A44 = (ts.q*cos(ts.φ)*tan(ts.θ)) - (ts.r*sin(ts.φ)*tan(ts.θ))
A45 = 0.0

A51 = 0.0
A52 = 0.0
A53 = cos(ts.φ)*sec(ts.θ)
A54 = (ts.p*cos(ts.φ)*sec(ts.θ)) - (ts.r*sin(ts.φ)*sec(ts.θ))
A55 = 0.0

A = [A11 A12 A13 A14 A15
      A21 A22 A23 A24 A25
      A31 A32 A33 A34 A35
      A41 A42 A43 A44 A45
      A51 A52 A53 A54 A55
      ]

B11 = (1/b)*2*Va*Q*ap.CYda
B11 = B11/af_term
B12 = (1/b)*2*Va*Q*ap.CYdr
B12 = B12/af_term

B21 = 2*Va*m*Q*lc["Cpδa"]
B22 = 2*Va*m*Q*lc["Cpδr"]

B31 = 2*Va*m*Q*lc["Crδa"]
B32 = 2*Va*m*Q*lc["Crδr"]

B41 = 0.0
B42 = 0.0

B51 = 0.0
B52 = 0.0

B = [B11 B12
      B21 B22
      B31 B32
      B41 B42
      B51 B52
      ]

return A,B
end

function GetLinearizedModel(::Type{LongitudinalAircraftState},ts::AircraftState,tc::AircraftControl,ap::AircraftParameters)

ρ = stdatmo(-ts.z)
S = ap.S
c = ap.c
m = ap.m
km = ap.kmotor
AirSpeedVector = [ts.u, ts.v, ts.w] #because it has been given that wind speed is zero.
wind_angles = AirRelativeVelocityVectorToWindAngles(AirSpeedVector)
Va = wind_angles.Va
α = wind_angles.α
af_term = Va*cos(α)
lc = GetLongitudinalEOMCoefficients(tc,α,ap)

A11 = ts.u*ρ*S*lc["CXtrim"]/m
A11 -= 0.5*ρ*S*ts.w*lc["CXα"]/m
A11 += 0.25*ρ*S*c*lc["CXq"]*ts.u*ts.q/(m*Va)
A11 += ρ*ap.Sprop*ap.Cprop*tc.δt*( km*ts.u*(1-2*tc.δt)/Va + 2*ts.u*(tc.δt-1) )/m
A12 = -ts.q + ts.w*ρ*S*lc["CXtrim"]/m
A12 += 0.25*ρ*S*c*lc["CXq"]*ts.w*ts.q/(m*Va)
A12 += 0.5*ρ*S*ts.u*lc["CXα"]/m
A12 += ρ*ap.Sprop*ap.Cprop*tc.δt*( km*ts.w*(1-2*tc.δt)/Va + 2*ts.w*(tc.δt-1) )/m
A12 = A12*af_term
A13 = -ts.w + (0.25*ρ*Va*S*lc["CXq"]*c)/m
A14 = -ap.g*cos(ts.θ)
A15 = 0.0

A21 = ts.q + ts.u*ρ*S*lc["CZtrim"]/m
A21 -= 0.5*ρ*S*lc["CZα"]*ts.w/m
A21 += 0.25*ts.u*ρ*S*lc["CZq"]*c*ts.q/(m*Va)

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A21 = A21/af_term
A22 = ts.w*p*S*lc["CZtrim"]/m
A22 += 0.5*p*S*lc["CZα"]*ts.u/m
A22 += 0.25*ts.w*p*S*lc["CZq"]*c*ts.q/(m*Va)
A23 = ts.u + (0.25*p*Va*S*lc["CZq"]*c)/m
A23 = A23/af_term
A24 = -ap.g*sin(ts.θ)
A24 = A24/af_term
A25 = 0.0

A31 = ts.u*p*S*c*(ap.Cm0 + ap.Cmalpha*α + ap.Cmde*tc.δe)
A31 -= 0.5*p*S*c*ap.Cmalpha*ts.w/ap.Iy
A31 += 0.25*p*S*c*c*ap.Cmq*ts.q*ts.u/(ap.Iy*Va)
A32 = ts.w*p*S*c*(ap.Cm0 + ap.Cmalpha*α + ap.Cmde*tc.δe)
A32 += 0.5*p*S*c*ap.Cmalpha*ts.u/ap.Iy
A32 += 0.25*p*S*c*c*ap.Cmq*ts.q*ts.w/(ap.Iy*Va)
A32 = A32*af_term
A33 = (0.25*p*Va*S*c*c*ap.Cmq)/ap.Iy
A34 = 0.0
A35 = 0.0

A41 = 0.0
A42 = 0.0
A43 = 1.0
A44 = 0.0
A45 = 0.0

A51 = sin(ts.θ)
A52 = -cos(ts.θ)
A52 = A52*af_term
A53 = 0.0
A54 = (ts.u*cos(ts.θ)) + (ts.w*sin(ts.θ))
A55 = 0.0

A = [A11 A12 A13 A14 A15
     A21 A22 A23 A24 A25
     A31 A32 A33 A34 A35
     A41 A42 A43 A44 A45
     A51 A52 A53 A54 A55
     ]

B11 = 0.5*p*Va*Va*S*lc["CXδe"]/m
B12 = ρ*ap.Sprop*ap.Cprop*( Va*(km-Va) + 2*tc.δt*(km-Va)*(km-Va) )/m

B21 = 0.5*p*Va*Va*S*lc["CZδe"]/m
B21 = B21/af_term
B22 = 0.0

B31 = 0.5*p*Va*Va*S*c*ap.Cmde/ap.Iy
B32 = 0.0

B41 = 0.0
B42 = 0.0

B51 = 0.0
B52 = 0.0

B = [B11 B12
     B21 B22
     B31 B32
     B41 B42
     B51 B52
     ]

return A,B
end

trim_definition = TrimDefinitionSL(18.0,0.0,1800.0)
state, control, results = GetTrimConditions(trim_definition, aircraft_parameters)
wind_inertial = [0.0,0.0,0.0]
Alat,Blat = GetLinearizedModel(LateralAircraftState,state,control,aircraft_parameters)
# Alat,Blat = @enter GetLinearizedModel(LateralAircraftState,state,control,aircraft_parameters)
Along,Blong = GetLinearizedModel(LongitudinalAircraftState,state,control,aircraft_parameters)
# Alat,Blat = @enter GetLinearizedModel(LateralAircraftState,state,control,aircraft_parameters)

println("A_lat")
display(Alat)
println("B_lat")
display(Blat)
println("A_long")
display(Along)
println("B_long")
display(Blong)

#=
Problem 1 - Part 2)
=#

reduced_Along = Along[1:4,1:4]
long_eigenvals, long_eigenvectors = eigen(reduced_Along)

reduced_Alat = Alat[1:4,1:4]
lat_eigenvals, lat_eigenvectors = eigen(reduced_Alat)

#Short Period Mode

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short_natural_frequency = get_natural_frequency(long_eigenvals[1])
short_damping_ratio = get_damping_ratio(long_eigenvals[1])
#Phugoid Mode
phugoid_natural_frequency = get_natural_frequency(long_eigenvals[3])
phugoid_damping_ratio = get_damping_ratio(long_eigenvals[3])
#Roll Mode
roll_natural_frequency = get_natural_frequency(lat_eigenvals[1])
roll_damping_ratio = get_damping_ratio(lat_eigenvals[1])
#Dutch Roll Mode
dutch_roll_natural_frequency = get_natural_frequency(lat_eigenvals[2])
dutch_roll_damping_ratio = get_damping_ratio(lat_eigenvals[2])
#Spiral Mode
spiral_natural_frequency = get_natural_frequency(lat_eigenvals[4])
spiral_damping_ratio = get_damping_ratio(lat_eigenvals[4])

println("")
println("For Short Period Mode : ")
println("Damping Ratio : ", short_damping_ratio)
println("Natural Frequency : ", short_natural_frequency)
println("")
println("For Phugoid Mode : ")
println("Damping Ratio : ", phugoid_damping_ratio)
println("Natural Frequency : ", phugoid_natural_frequency)
println("")
println("For Dutch Roll Mode : ")
println("Damping Ratio : ", dutch_roll_damping_ratio)
println("Natural Frequency : ", dutch_roll_natural_frequency)

#=#
#Problem 1 - Part 2)
=#

println("")
println("For Roll Mode : ")
println("Damping Ratio : ", roll_damping_ratio)
println("Natural Frequency : ", roll_natural_frequency)
println("Time Constant : ", 1/roll_natural_frequency)
println("The roll mode is stable.")
println("")
println("For Spiral Mode : ")
println("Damping Ratio : ", spiral_damping_ratio)
println("Natural Frequency : ", spiral_natural_frequency)
println("Time Constant : ", 1/spiral_natural_frequency)
println("The spiral mode is unstable.")

#Problem 2

function PulseControl(t::Float64,true_control::Union{AircraftControl,Array{Float64,1}},pulse_control::Array{Float64,1},pulse_time::Array{Float64,1})

    if (t==0.0)
        return true_control
    end
    if( length(pulse_time) == 1 )
        if(t<=pulse_time[1])
            control = true_control + pulse_control
        else
            control = true_control
        end
    elseif( length(pulse_time) == 2)
        if(t<=pulse_time[1])
            control = true_control + pulse_control
        elseif(t>pulse_time[1] && t<=pulse_time[2])
            control = true_control - pulse_control
        else
            control = true_control
        end
    end
    return control
end

function AircraftEOMPulsed(t::Float64,aircraft_state::AircraftState,controls::AircraftControl,control_function::Function,
    pulse_control::Array{Float64,1},pulse_time::Array{Float64,1},wind_inertial::Array{Float64,1},aircraft_parameters::AircraftParameters)

    aircraft_surfaces = control_function(t,controls,pulse_control,pulse_time)
    return AircraftEOM(t,aircraft_state,aircraft_surfaces,wind_inertial,aircraft_parameters)
end

function aircraft_dynamics_pulsed!(du,u,p,t)
    aircraft_state = AircraftState(u...)
    control_function = p[1]
    control_inputs = AircraftControl(p[2]...)
    pulse_control = p[3]
    pulse_time = p[4]
    wind_inertial = p[5]
    aircraft_parameters = p[6]
    x_dot = AircraftEOMPulsed(t,aircraft_state,control_inputs,control_function,pulse_control,pulse_time,wind_inertial,aircraft_parameters)
    for i in 1:length(u)
        du[i] = x_dot[i]
    end
end

filename = "tttwistor.mat"
aircraft_parameters = AircraftParameters(filename)

```

```

location = "/HW5/plots"
save_plots = true
case_num = 3

trim_definition = TrimDefinitionSL(18.0,0.0,1800.0)
state, control, results = GetTrimConditions(trim_definition, aircraft_parameters)
initial_state = collect(values(state))
control_input = collect(values(control))
wind_inertial = [0.0,0.0,0.0]

if(case_num == 1)
    pulse_control = [pi/10,0.0,0.0,0.0]
    pulse_time = [1.0]
end

if(case_num == 2)
    pulse_control = [0.0,pi/10,0.0,0.0]
    pulse_time = [1.0,2.0]
end

if(case_num == 3)
    pulse_control = [0.0,0.0,pi/10,0.0]
    pulse_time = [1.0,2.0]
end

time_interval = [0.0,200.0]
extra_parameters = [PulseControl,control_input,pulse_control,pulse_time,wind_inertial,aircraft_parameters]
trajectory_states = simulate(aircraft_dynamics_pulsed!,initial_state,time_interval,extra_parameters,1.0)
time_values = [i for i in 0:length(trajectory_states)-1]
control_array = [AircraftControl(PulseControl(1.0*(i-1)),control_input,pulse_control,pulse_time...) for i in 1:length(trajectory_states)]
PlotSimulation(time_values, trajectory_states, control_array, location, save_plots)

```

Problem 1: Part 1)

A_{lat}

```
[ -0.371441  0.0458582 -0.980285  0.544278  0.0  
-20.6405   -15.7093    2.67741  0.0    0.0  
11.5196    -1.23555   -0.566909  0.0    0.0  
0.0        1.0        0.0515269  0.0    0.0  
0.0        0.0        1.00133   0.0    0.0  
]
```

B_{lat}

```
[ -0.000762643  0.00309103  
-5.35799      0.0315897  
-0.256564     -0.130882  
0.0           0.0  
0.0           0.0  
]
```

A_{long}

```
[ -0.0609492  11.4379 -0.910211 -9.797   0.0  
-0.0425166  -6.32127  0.940645 -0.0280821 0.0  
0.108966   -38.0151 -3.28532  0.0    0.0  
0.0         0.0    1.0    0.0    0.0  
0.0514586  -17.9523  0.0    18.0    0.0  
]
```

B_{long}

```
[ 0.00185606  3.38461  
-0.00686655  0.0  
-1.3996      0.0  
0.0          0.0  
0.0          0.0  
]
```

Problem 1: Part 2)

For Short Period Mode :

Damping Ratio: 0.6380975226632497

Natural Frequency: 7.54563457743263

For Phugoid Mode :

Damping Ratio: 0.030058661111159526

Natural Frequency: 0.6293676188992627

For Dutch Roll Mode :

Damping Ratio: 0.1510641420677276

Natural Frequency: 3.7508625777027906

Problem 1: Part 3)

For Roll Mode :

Damping Ratio: 1.0

Natural Frequency: 15.5882856117332

Time Constant: 0.06415073632262079

The roll mode is stable.

For Spiral Mode :

Damping Ratio: 1.0

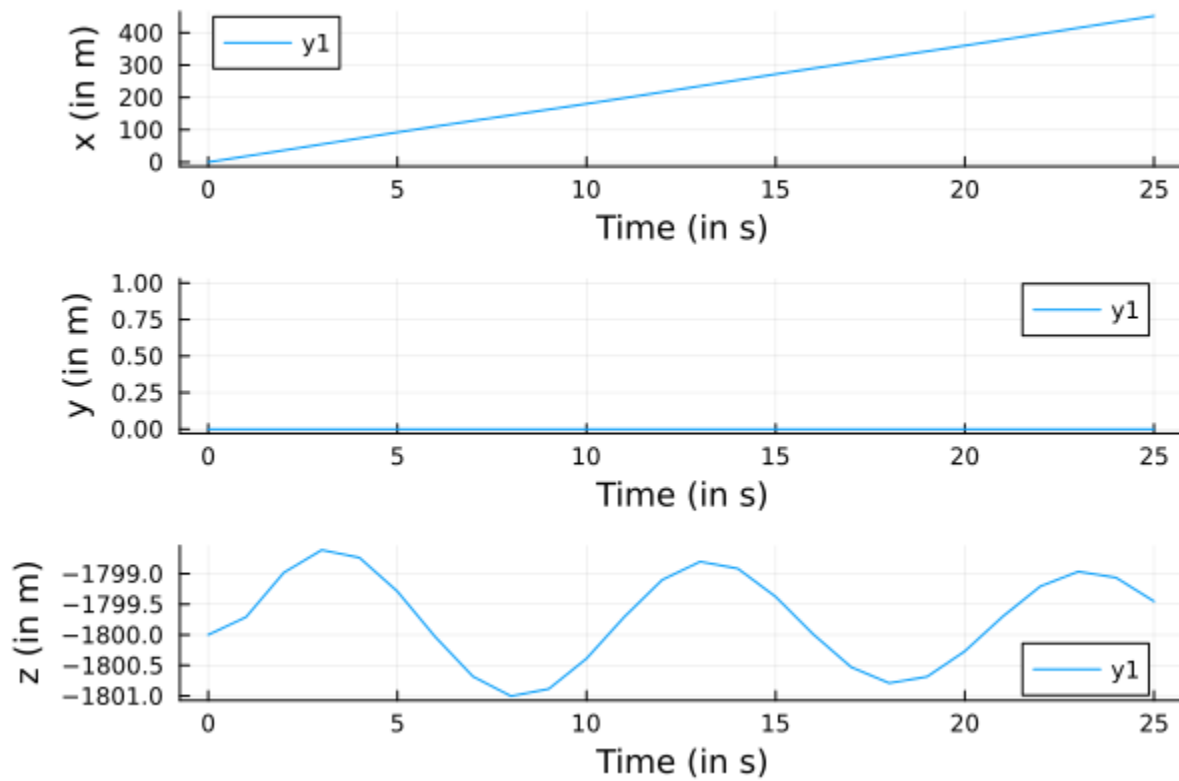
Natural Frequency: 0.07390663902876486

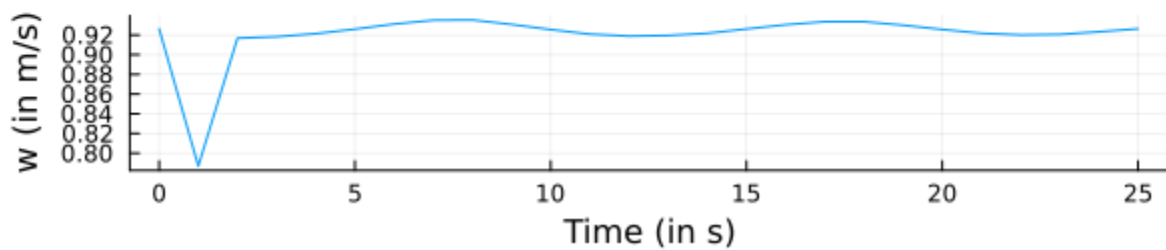
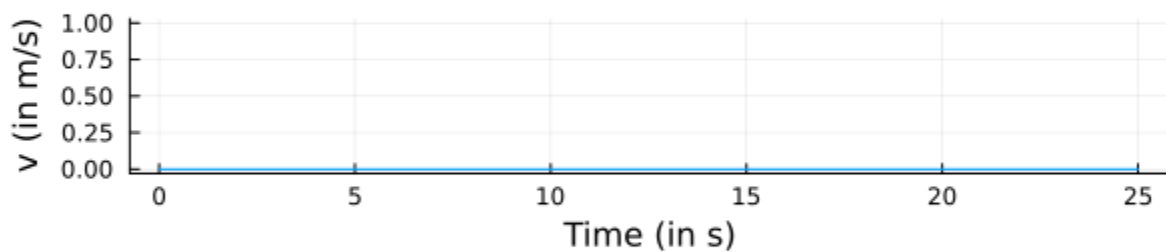
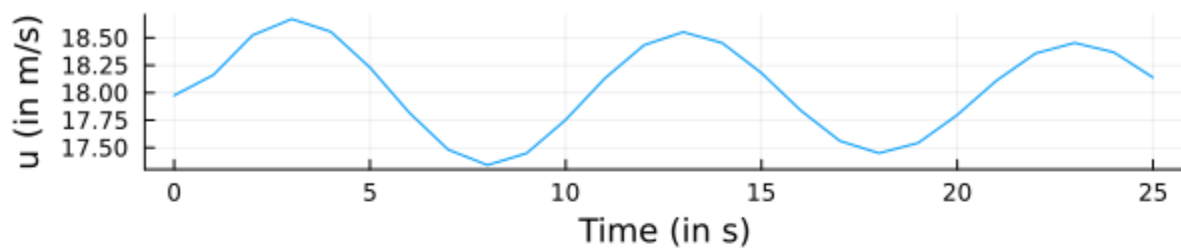
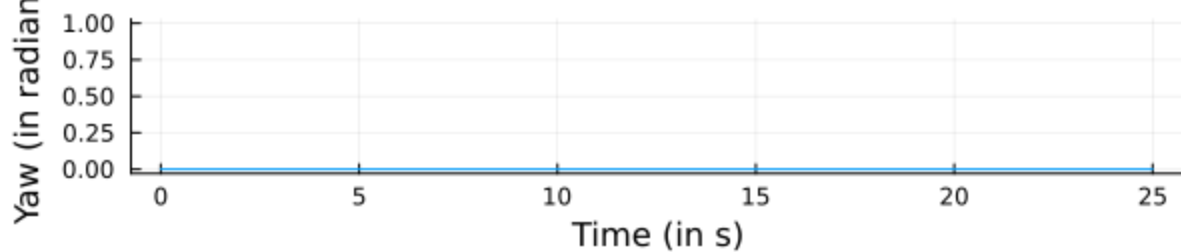
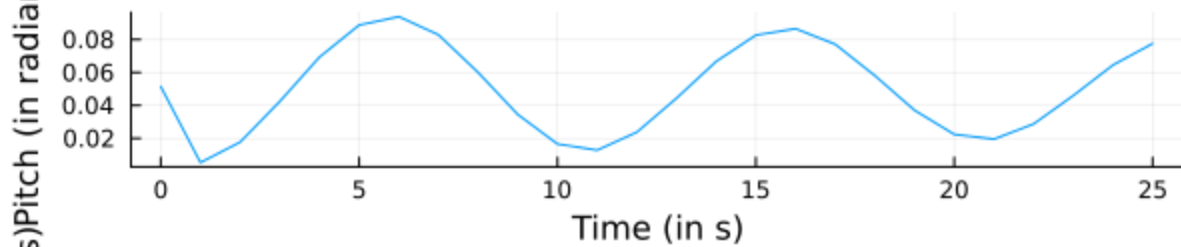
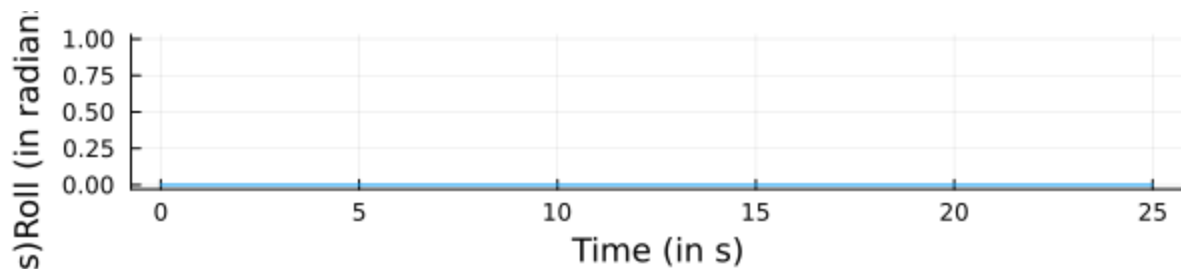
Time Constant: 13.530584168640042

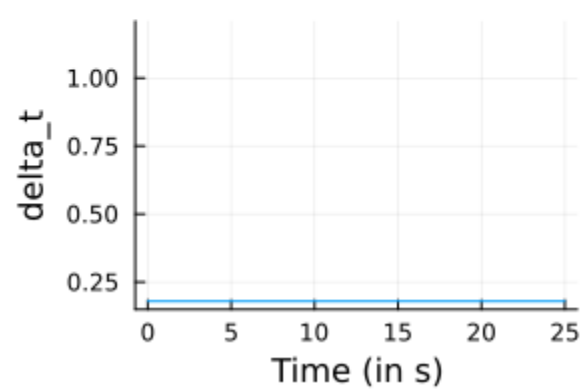
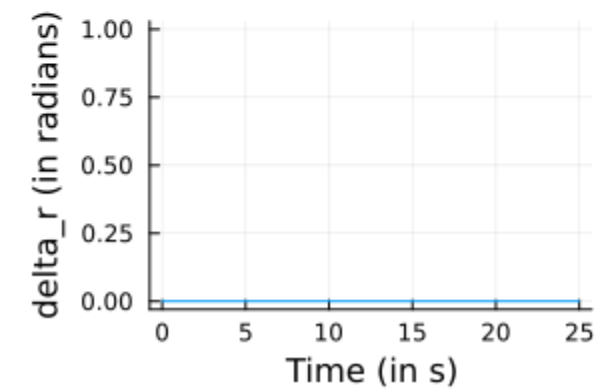
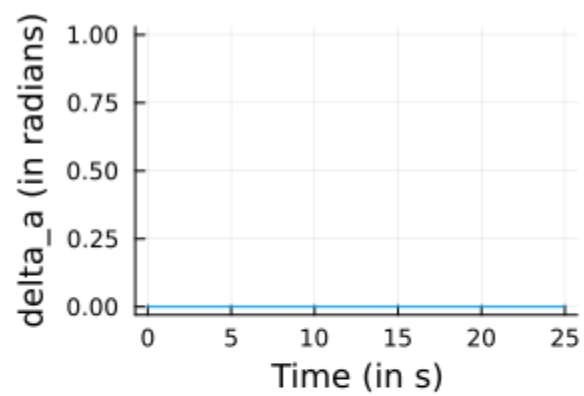
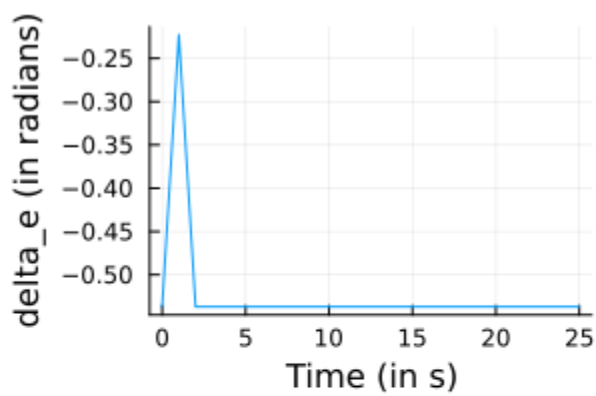
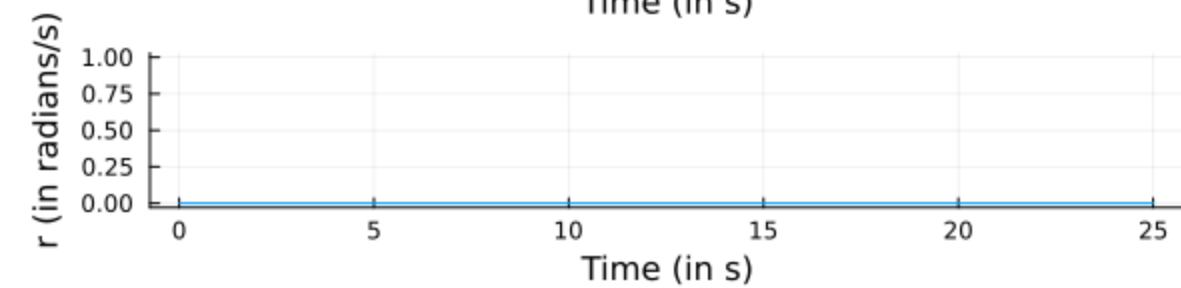
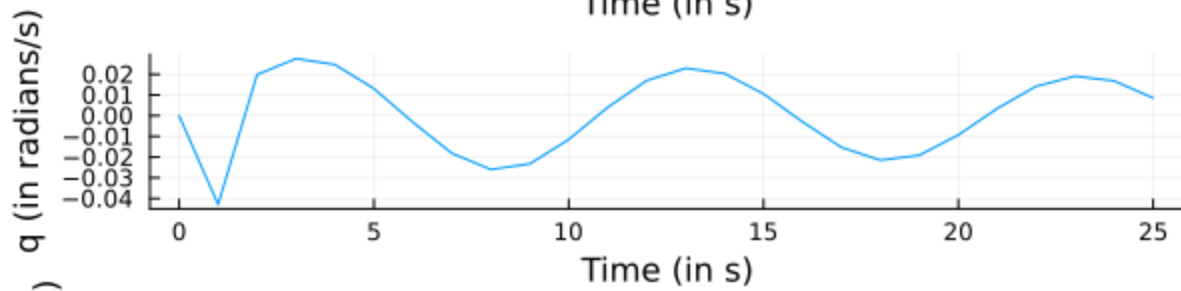
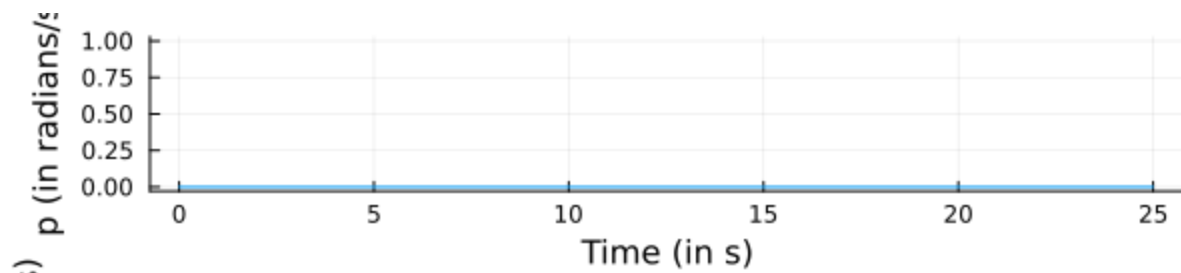
The spiral mode is unstable.

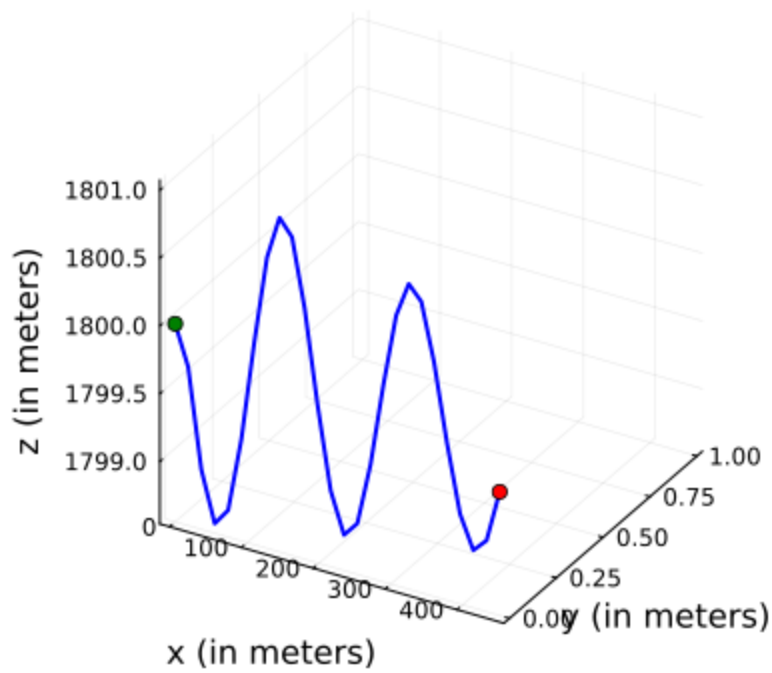
Problem 2: Part 1)

First, plots are generated for just 25 seconds to show the initial short-period behavior.





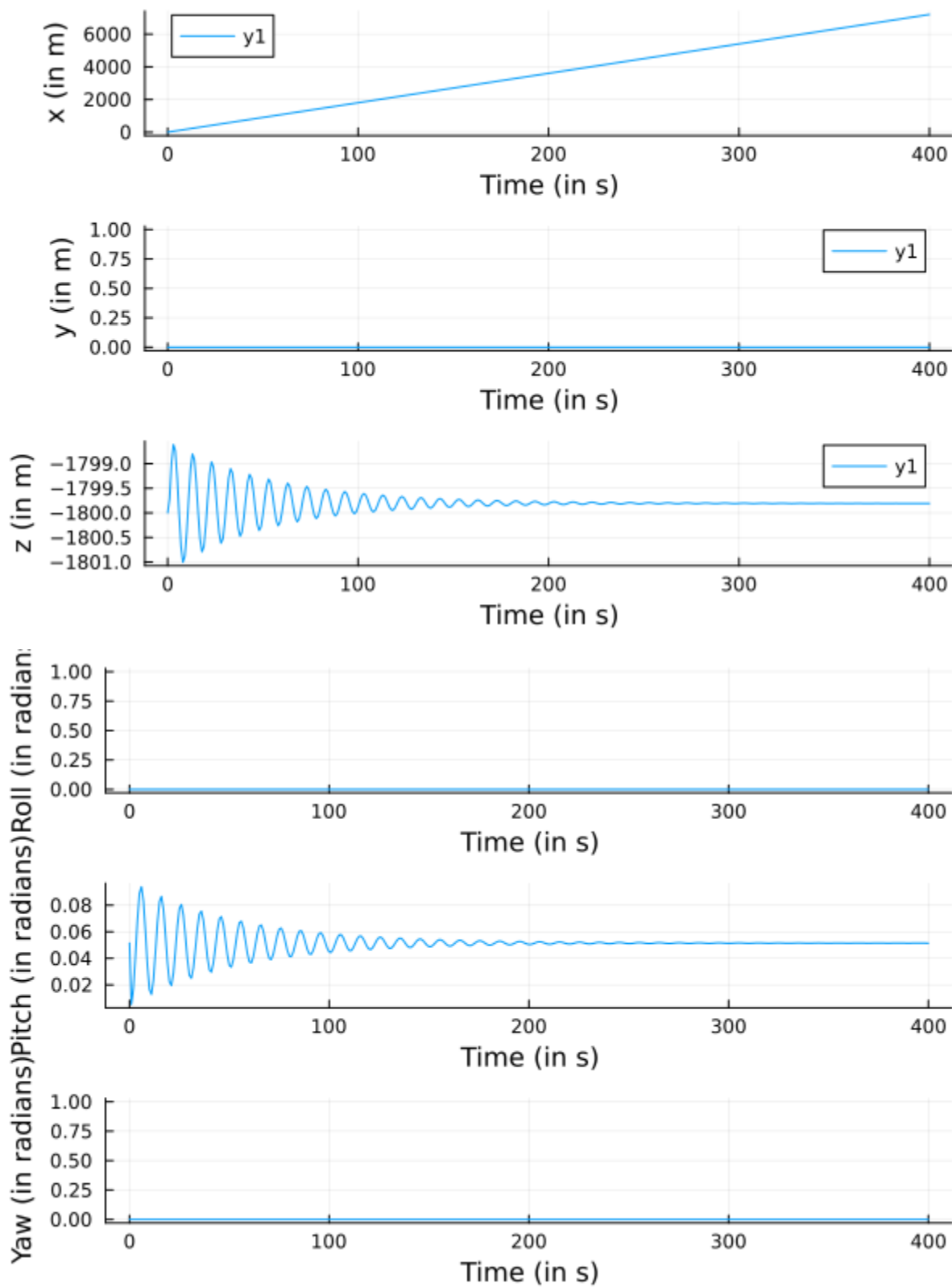


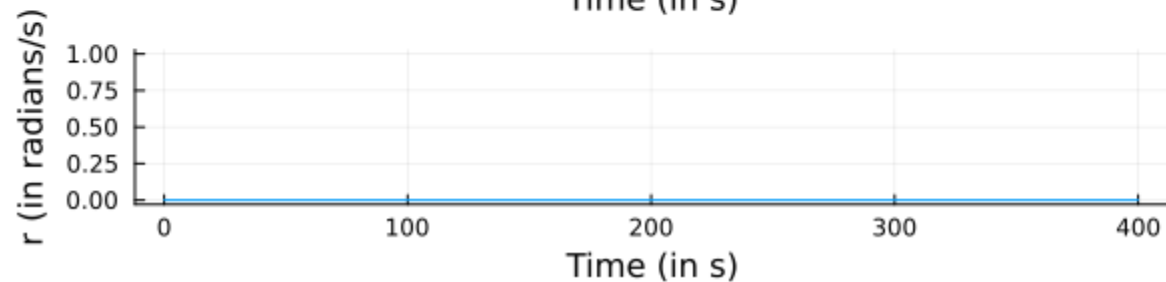
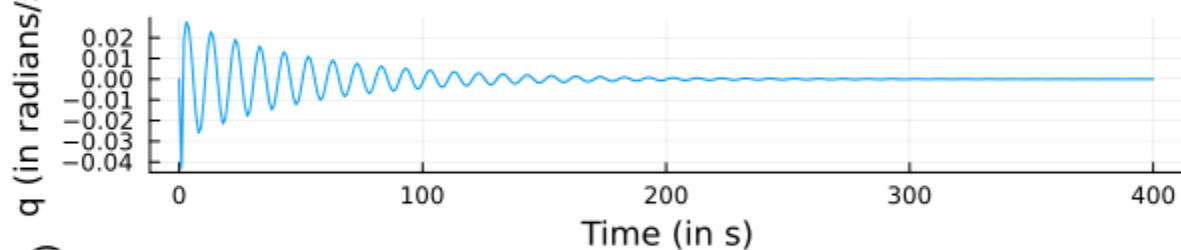
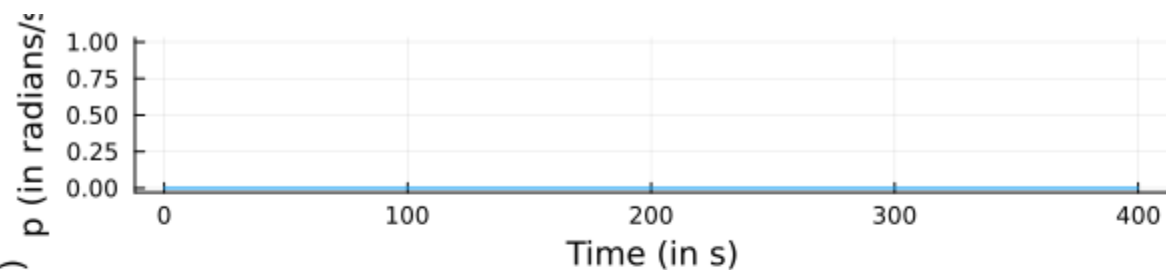
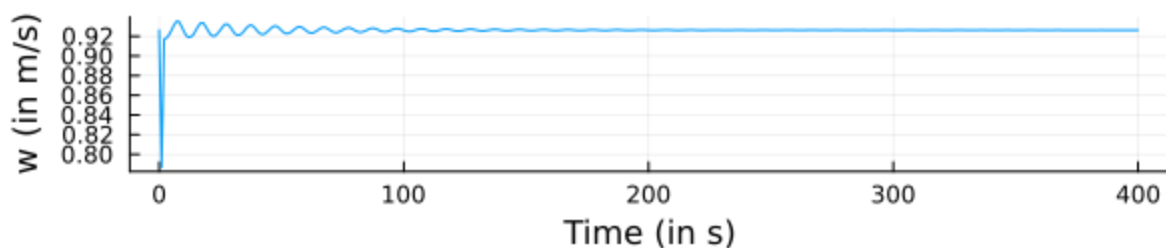
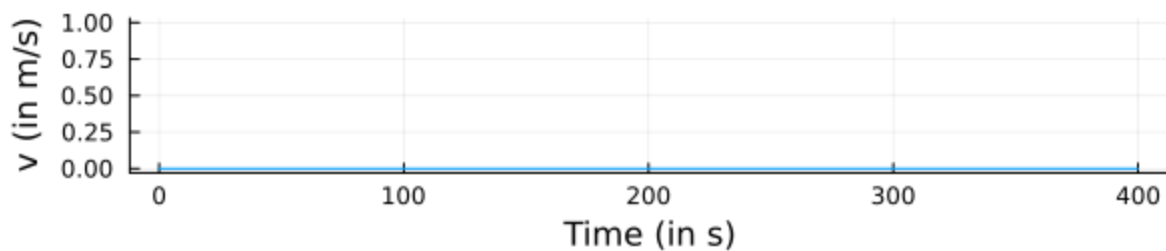
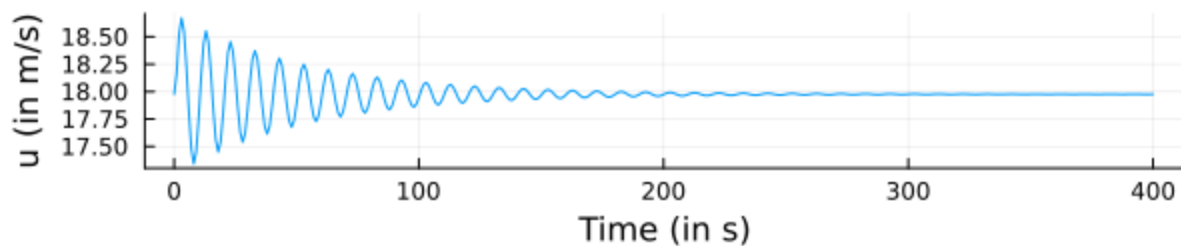


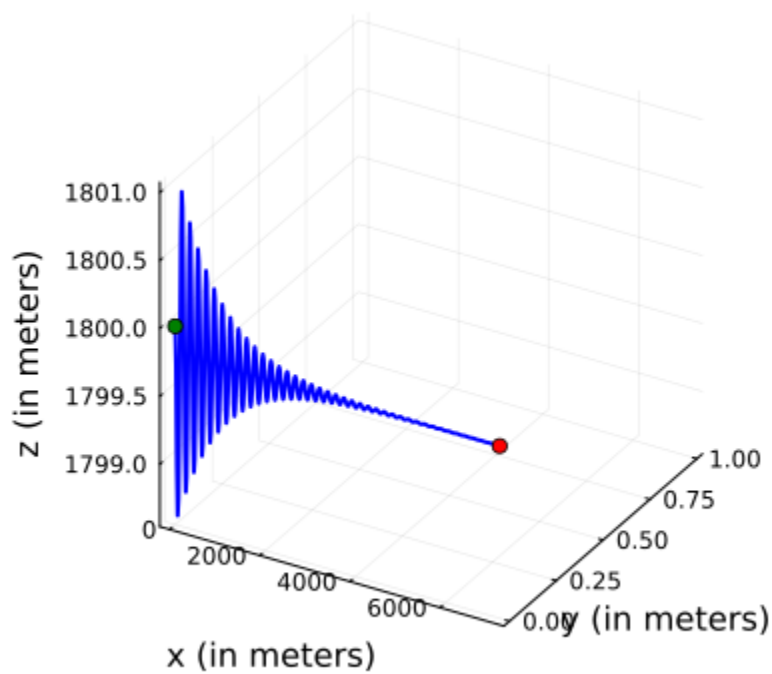
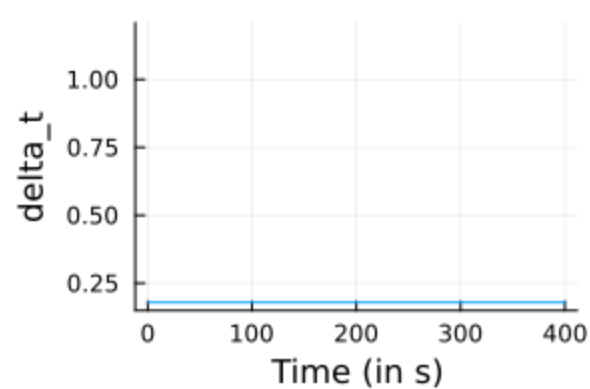
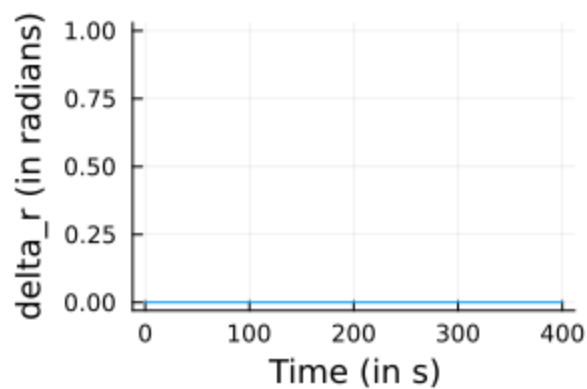
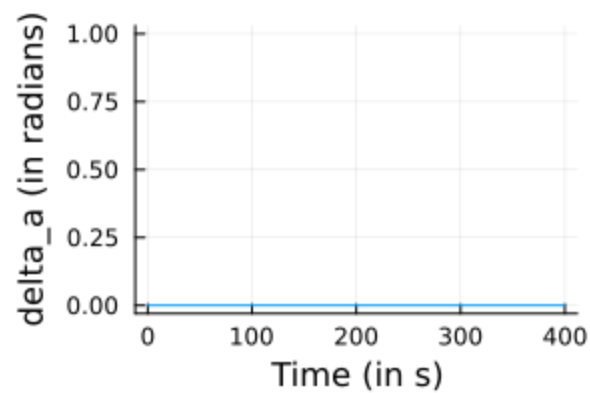
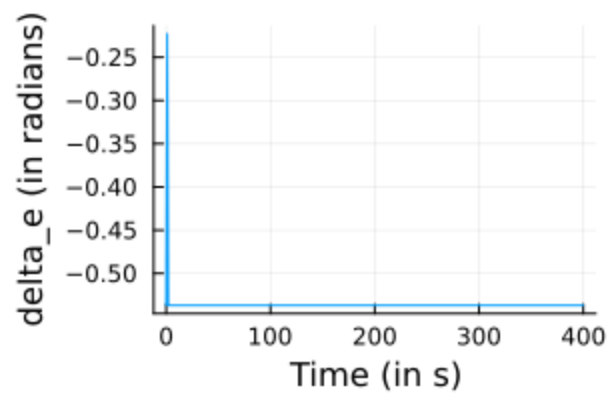
Applying an impulse to the elevator when in trim excites the short-period mode and phugoid mode, which can be seen from the aircraft's trajectory, and the plot of pitch angle with respect to time.

Since both the modes are stable modes, the disturbances soon go to zero after a sufficient amount of time, as can be seen from the plots below.

Now, plots are generated for 400 seconds to show the long-period behavior.

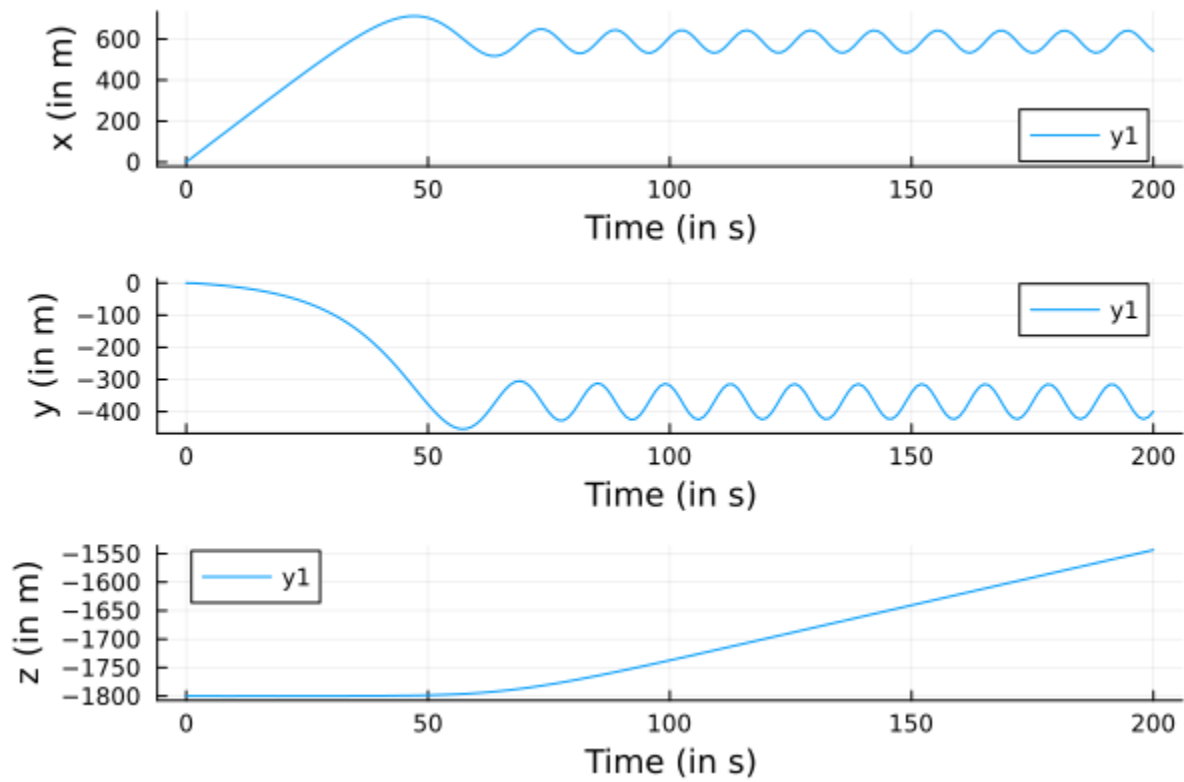


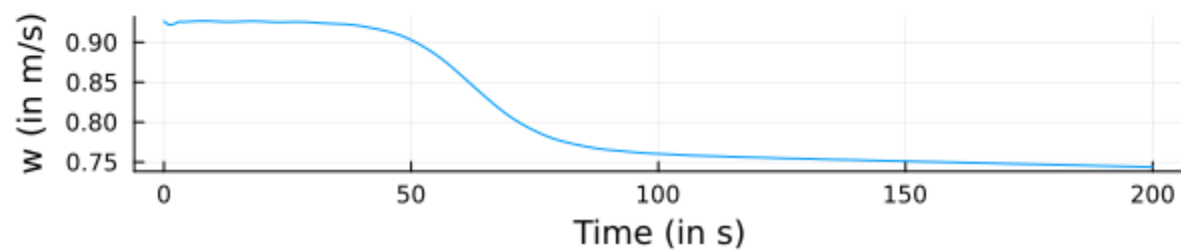
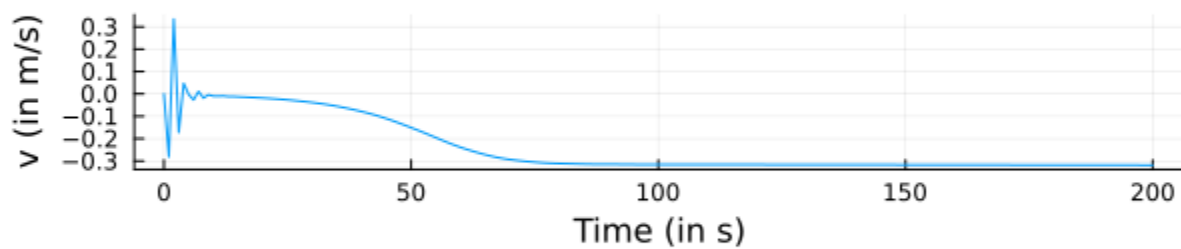
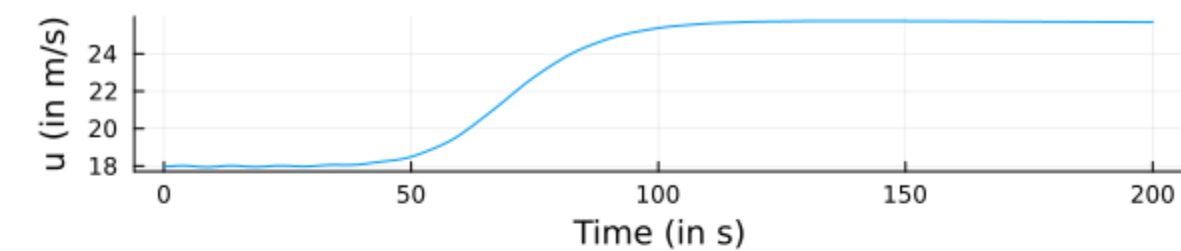
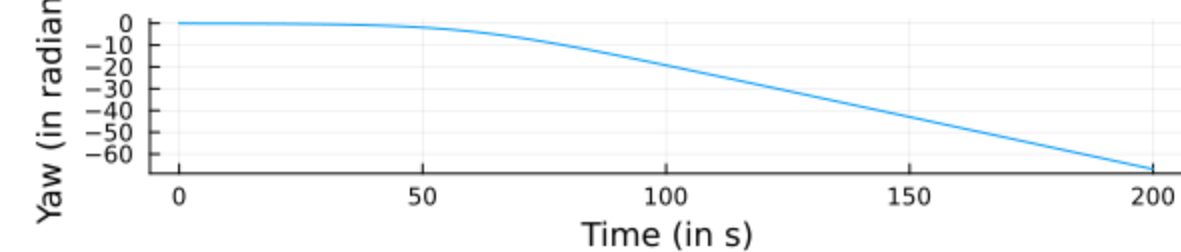
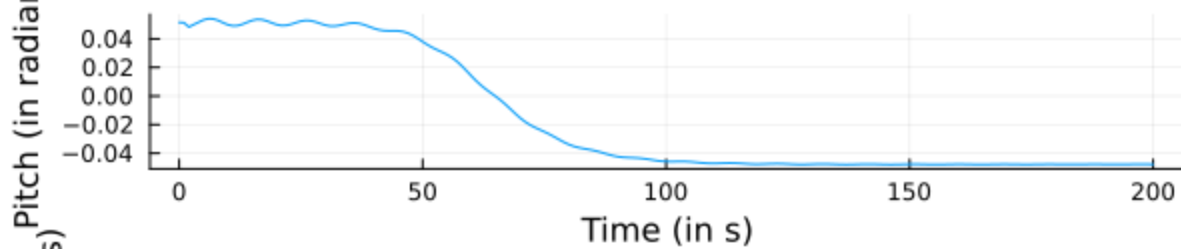
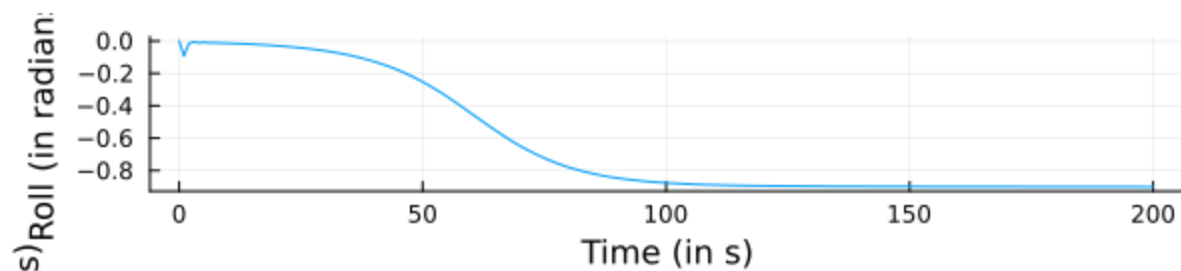


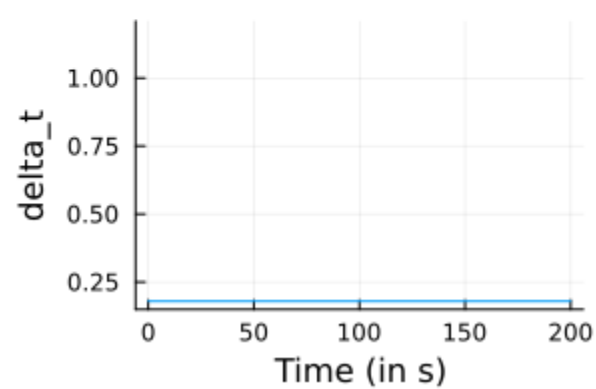
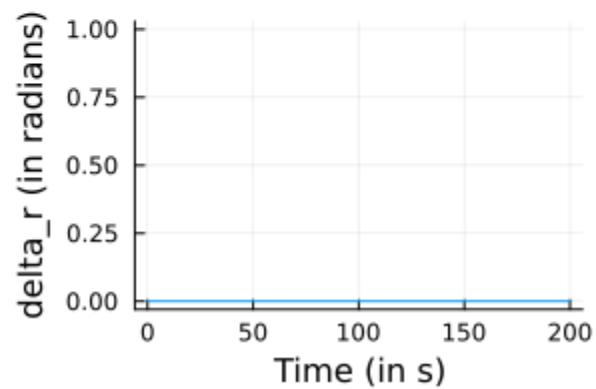
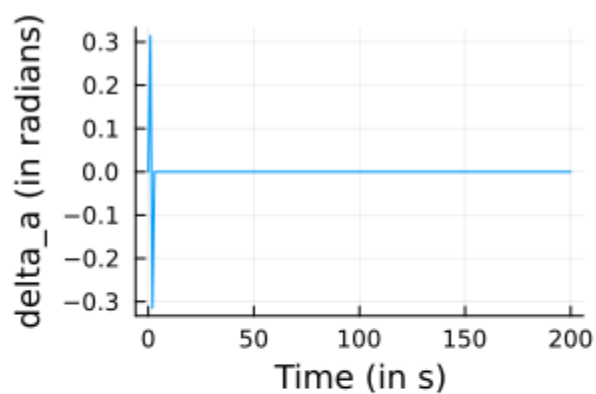
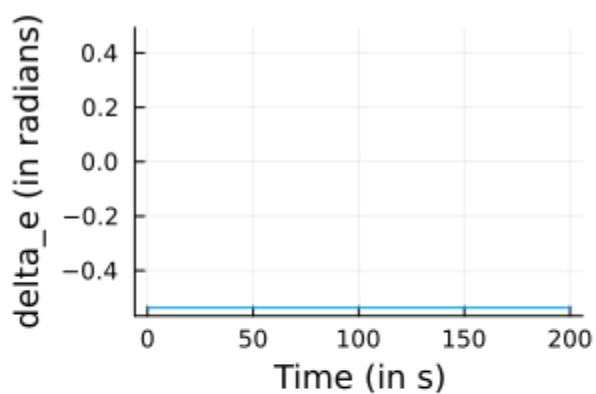
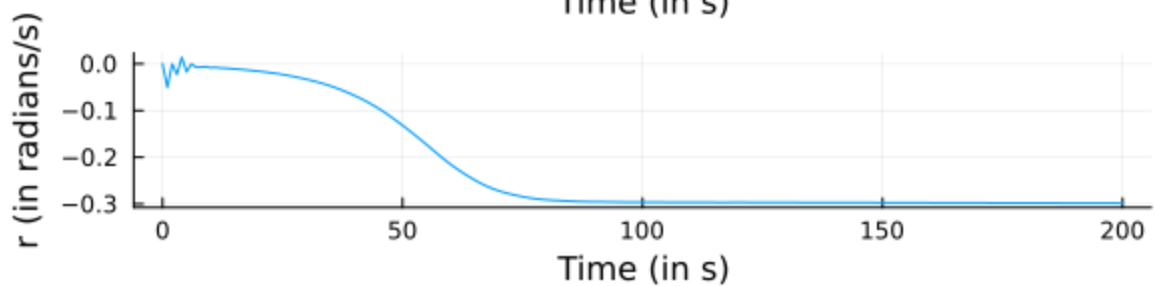
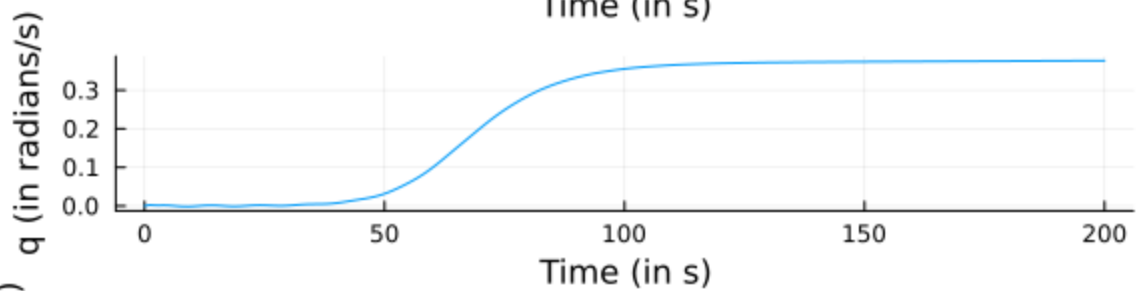
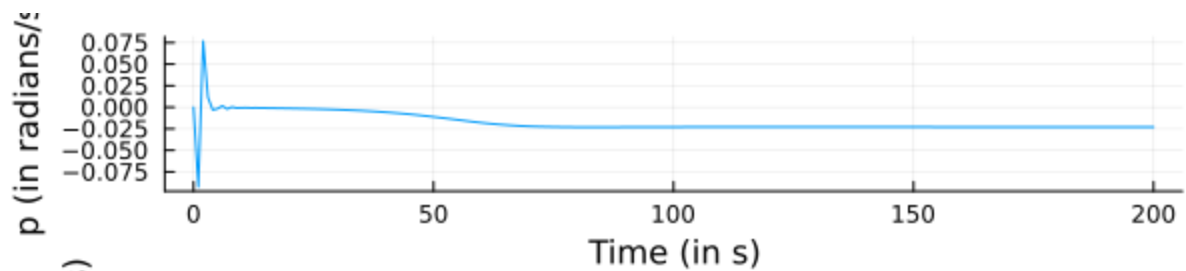


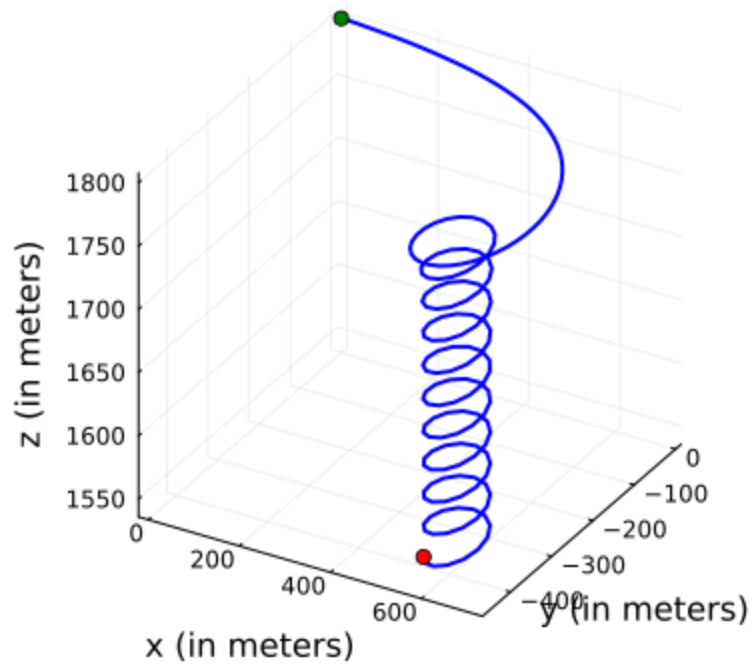
Problem 2: Part 2)

Plots are generated for 400 seconds to show the long-period behavior.





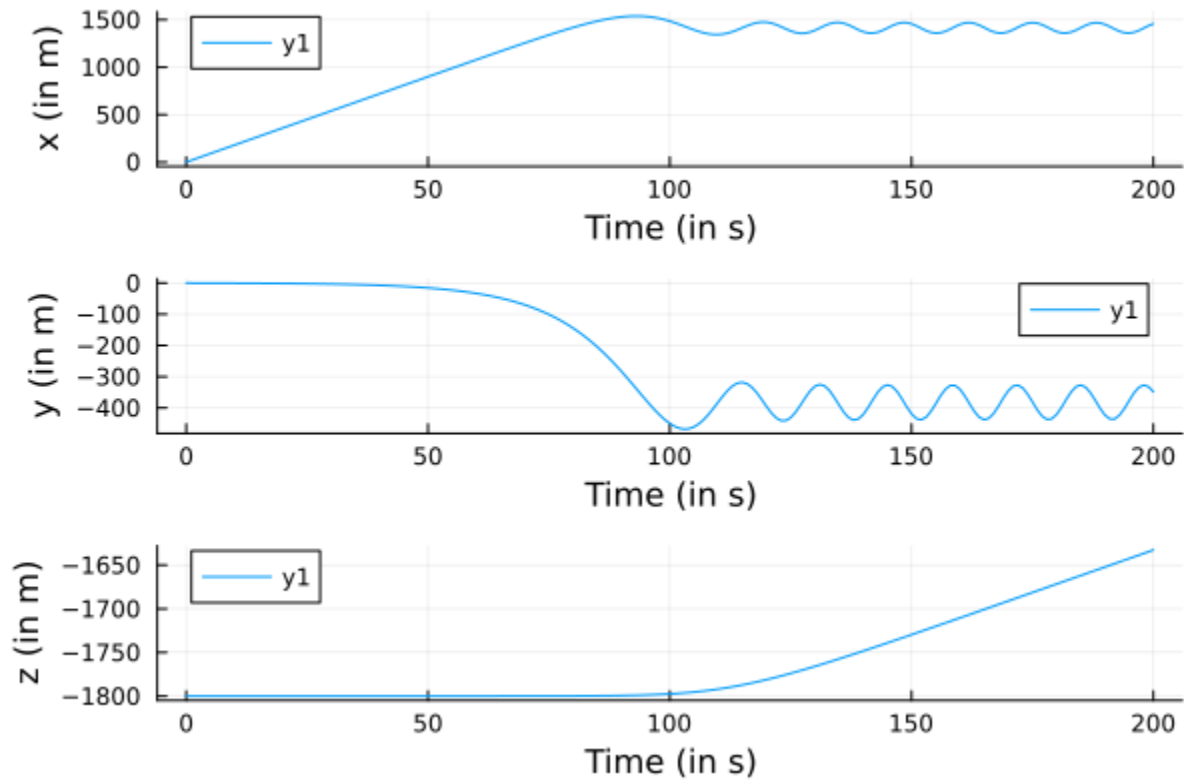


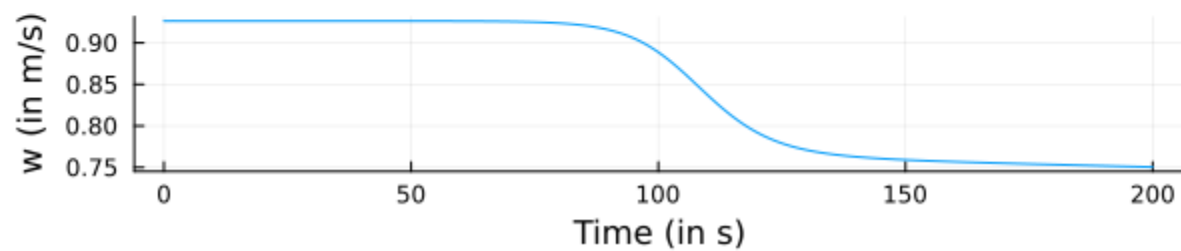
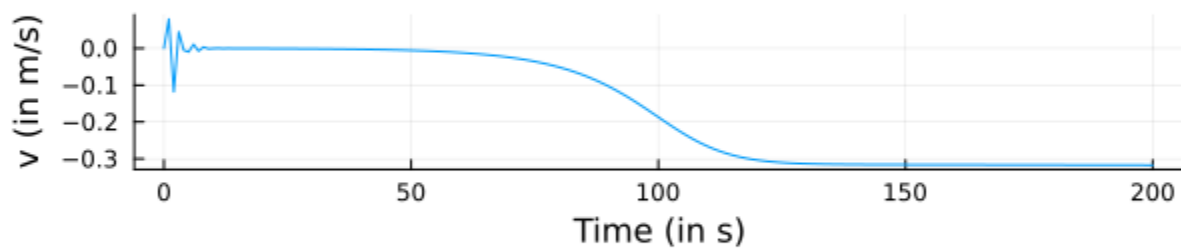
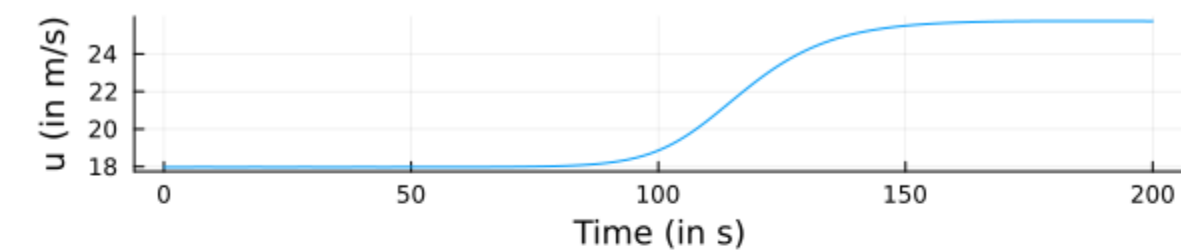
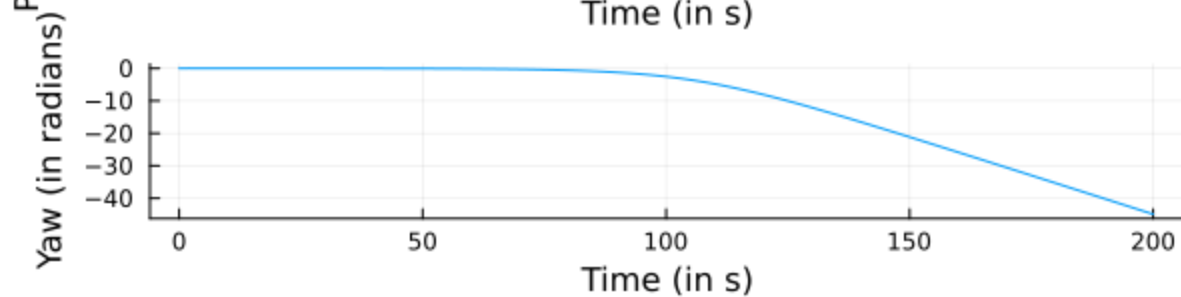
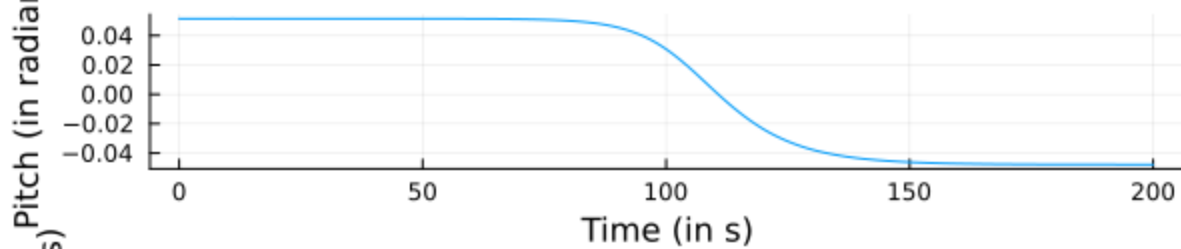
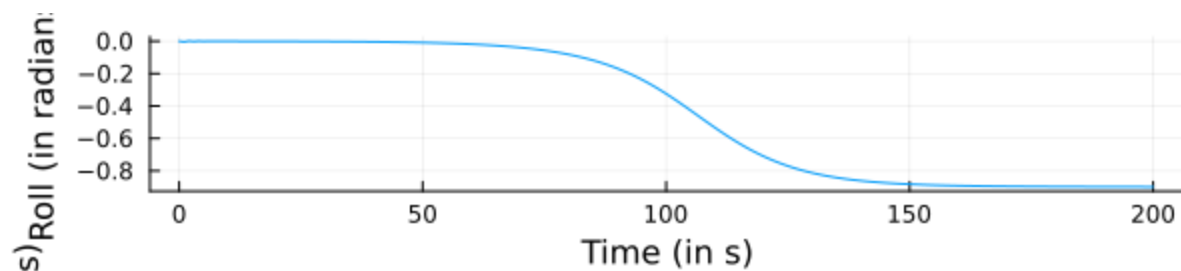


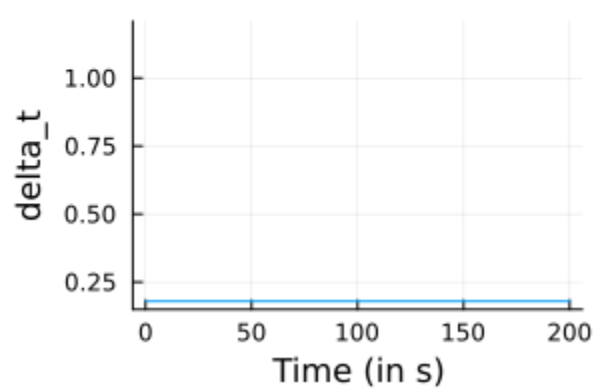
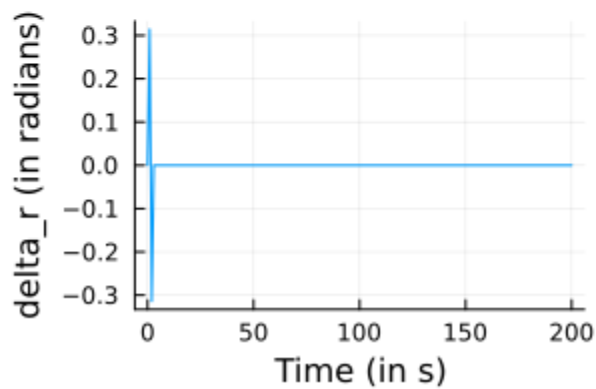
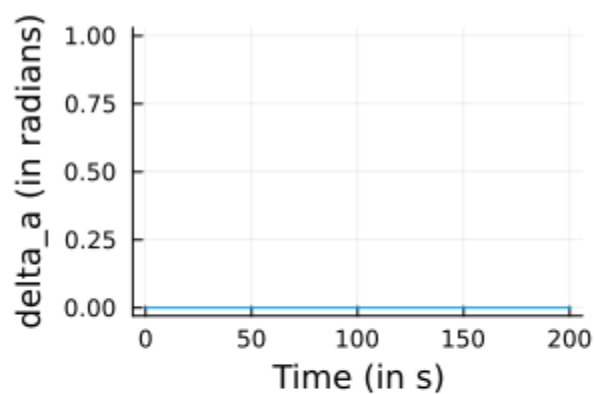
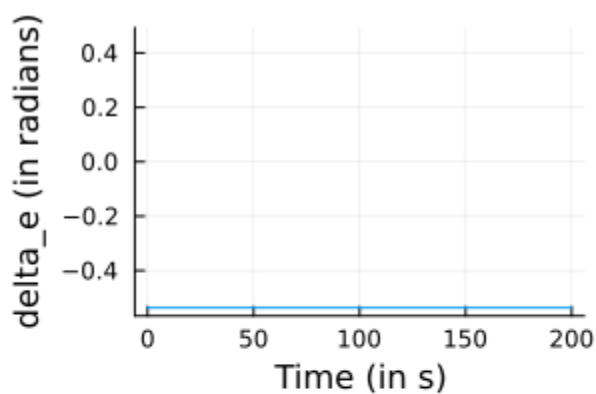
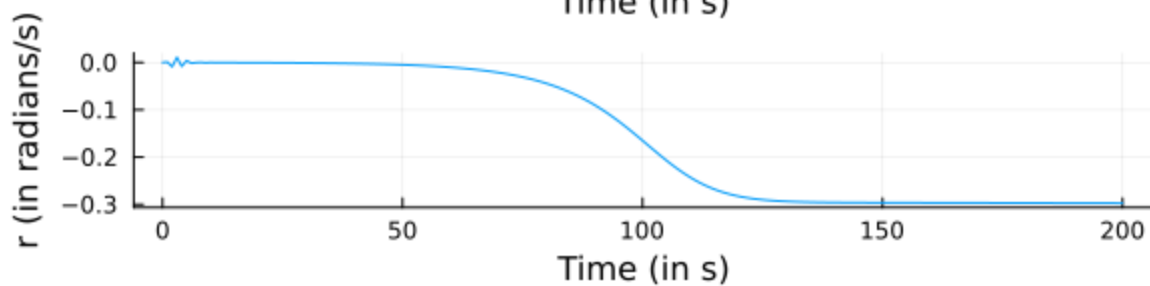
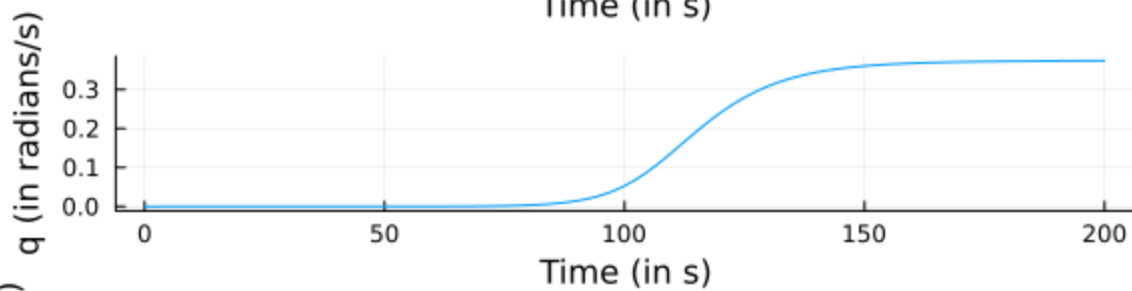
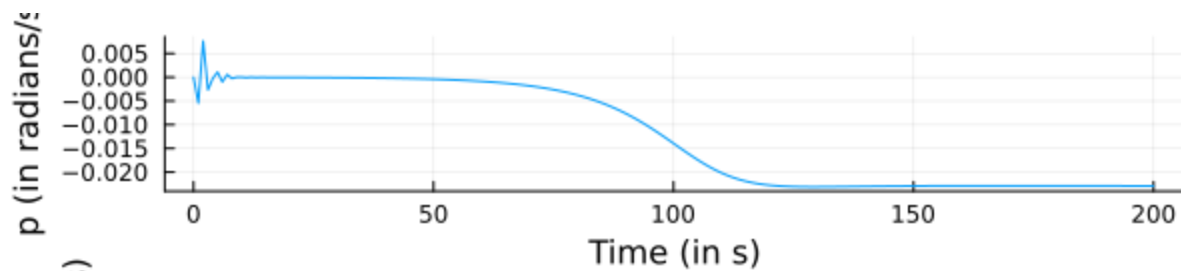
Applying a doublet to the rudder excited the lateral modes, dutch roll mode, and spiral mode. This is in accordance with the behavior observed in Problem 1. The eigenvalue corresponding to the spiral mode has a positive real part, so the system becomes unstable. The eigenvalues corresponding to the dutch roll have damping oscillations, and so the oscillations die out after some time.

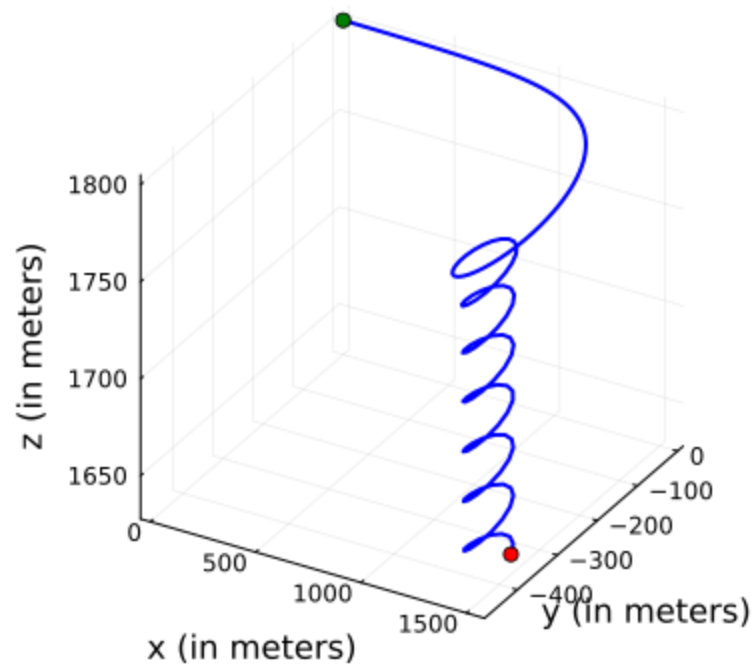
Problem 2: Part 3)

Plots are generated for 400 seconds to show the long-period behavior.









Applying a doublet to the rudder excited the lateral modes, dutch roll mode, and spiral mode. This is in accordance with the behavior observed in Problem 1. The eigenvalue corresponding to the spiral mode has a positive real part, so the system becomes unstable. The eigenvalues corresponding to the dutch roll have damping oscillations, and so the oscillations die out after some time.