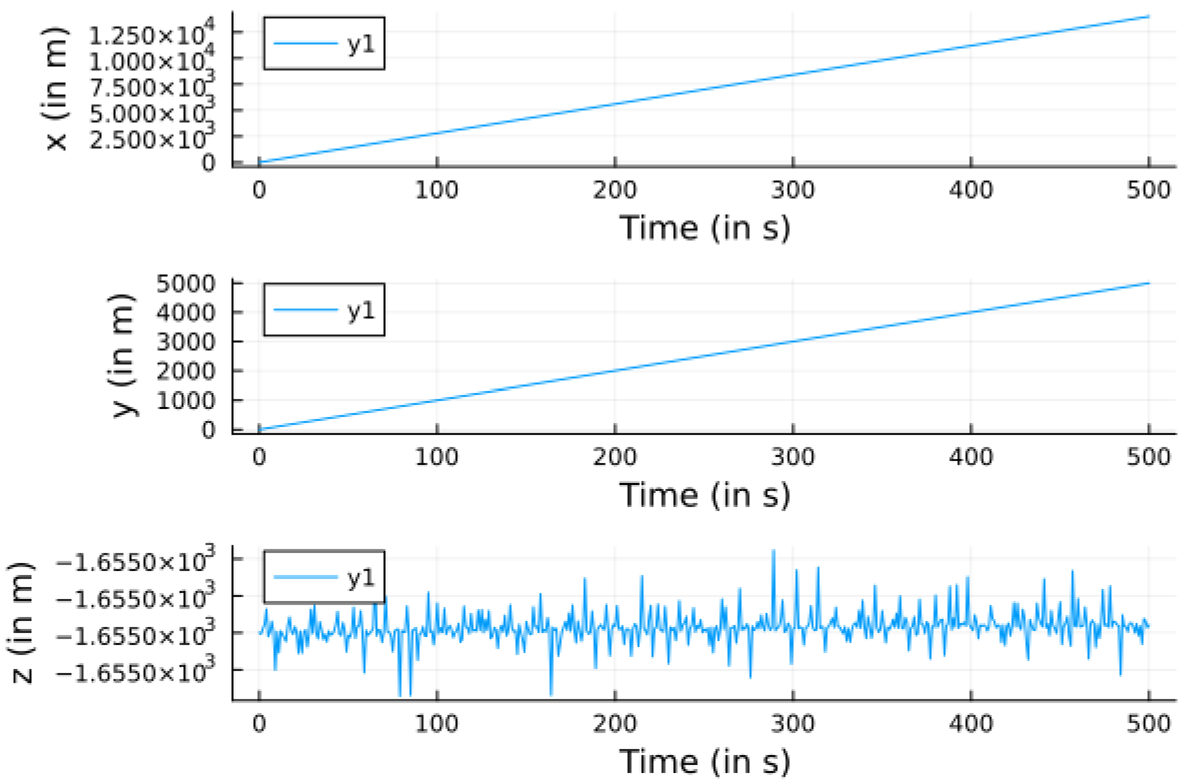
```julia
include("definitions.jl")
include("utils.jl")
include("matlab_utils.jl")
using DifferentialEquations
using Plots

function AeroForcesAndMomentsBodyStateWindCoeffs(aircraft_state::AircraftState, aircraft_surfaces::AircraftControl, wind_inertial, density, aircraft_parameters::AircraftParameters)

    rho = density #stdatmo(-aircraft_state.z)
    euler_angles = EulerAngles(aircraft_state.roll, aircraft_state.pitch, aircraft_state.yaw)
    aircraft_velocity = [aircraft_state.u,aircraft_state.v,aircraft_state.w]   #In body frame
    wind_body_frame = TransformFromInertialToBody(wind_inertial, euler_angles)
    VelocityBody = aircraft_velocity - wind_body_frame   #Va vector in body frame
    wind_angles = AirRelativeVelocityVectorToWindAngles(VelocityBody)
    Va = wind_angles.Va
    Q = 0.5*rho*Va*Va
    S = aircraft_parameters.S
    b = aircraft_parameters.b
    c = aircraft_parameters.c
    CoefficientsDict = GetCoefficients(aircraft_state, aircraft_surfaces, wind_angles, aircraft_parameters)

    #Calculate force along body X
    C_X = CoefficientsDict["CX"]
    C_T = CoefficientsDict["CT"]
    T = S*Q*C_T
    X = S*Q*C_X + T

    #Calculate force along body Y
    C_Y = CoefficientsDict["CY"]
    Y = S*Q*C_Y

    #Calculate force along body Z
    C_Z = CoefficientsDict["CZ"]
    Z = S*Q*C_Z

    #Calculate moment along body X
    C_l = CoefficientsDict["Cl"]
    G = 0.0  #As mentioned by Dr.Frew on Slack
    L = b*S*Q*C_l + G

    #Calculate moment along body Y
    C_m = CoefficientsDict["Cm"]
    M = c*S*Q*C_m

    #Calculate moment along body Z
    C_n = CoefficientsDict["Cn"]
    N = b*S*Q*C_n

    aero_force = [X,Y,Z]
    aero_moment = [L,M,N]
    return aero_force,aero_moment
end

function GravityForcesBodyState(aircraft_state::AircraftState, aircraft_parameters::AircraftParameters)

    m = aircraft_parameters.m
    g = aircraft_parameters.g
    roll = aircraft_state.roll #phi
    pitch = aircraft_state.pitch #theta

    #Calculate force along body X
    X = -m*g*sin(pitch)
    #Calculate force along body Y
    Y = m*g*cos(pitch)*sin(roll)
    #Calculate force along body Z
    Z = m*g*cos(pitch)*cos(roll)

    gravity_force = [X,Y,Z]
    return gravity_force
end

function AircraftForcesAndMoments(aircraft_state::AircraftState, aircraft_surfaces::AircraftControl, wind_inertial, density, aircraft_parameters::AircraftParameters)
    aero_force,aero_moment = AeroForcesAndMomentsBodyStateWindCoeffs(aircraft_state, aircraft_surfaces, wind_inertial, density, aircraft_parameters)
    gravity_force = GravityForcesBodyState(aircraft_state, aircraft_parameters)
    total_force = aero_force+gravity_force
    total_moment = aero_moment
    return total_force, total_moment
end

function AircraftEOM(aircraft_state,aircraft_surfaces,wind_inertial,aircraft_parameters)

    rho = stdatmo(-aircraft_state.z)
    m = aircraft_parameters.m
    euler_angles = EulerAngles(aircraft_state.roll, aircraft_state.pitch, aircraft_state.yaw)
    total_force, total_moment = AircraftForcesAndMoments(aircraft_state,aircraft_surfaces,wind_inertial,rho,aircraft_parameters)

    #Position derivatives
    velocity_vector = [aircraft_state.u, aircraft_state.v, aircraft_state.w]
    position_dot = TransformFromBodyToInertial(velocity_vector, euler_angles)

    #Euler Angle derivatives
    multiplication_matrix = GetRotationalKinematicsMatrix(euler_angles)
    roll_rate_matrix = [aircraft_state.p, aircraft_state.q, aircraft_state.r]
    euler_angles_dot = multiplication_matrix*roll_rate_matrix

    #Velocity derivatives
    u = aircraft_state.u
    v = aircraft_state.v
    w = aircraft_state.w
    u_dot = (aircraft_state.r*v) - (aircraft_state.q*w) + (total_force[1]/m)
    v_dot = (aircraft_state.p*w) - (aircraft_state.r*u) + (total_force[2]/m)
    w_dot = (aircraft_state.q*u) - (aircraft_state.p*v) + (total_force[3]/m)
    velocity_dot = [u_dot,v_dot,w_dot]

    #Rate of rotation derivatives
    Gamma = GetGammaValues(aircraft_parameters)
    p_dot = (Gamma[1]*aircraft_state.p*aircraft_state.q) - (Gamma[2]*aircraft_state.q*aircraft_state.r) +
              (Gamma[3]*total_moment[1]) + (Gamma[4]*total_moment[3])
    q_dot = (Gamma[5]*aircraft_state.p*aircraft_state.r) - (Gamma[6]*(aircraft_state.p^2 - aircraft_state.r^2)) +
              (total_moment[2]/aircraft_parameters.Iy)
    r_dot = (Gamma[7]*aircraft_state.p*aircraft_state.q) - (Gamma[1]*aircraft_state.q*aircraft_state.r) +
              (Gamma[4]*total_moment[1]) + (Gamma[8]*total_moment[3])
    rotation_rate_dot = [p_dot,q_dot,r_dot]

    x_dot = vcat(position_dot,euler_angles_dot,velocity_dot,rotation_rate_dot)
    return x_dot
end
```

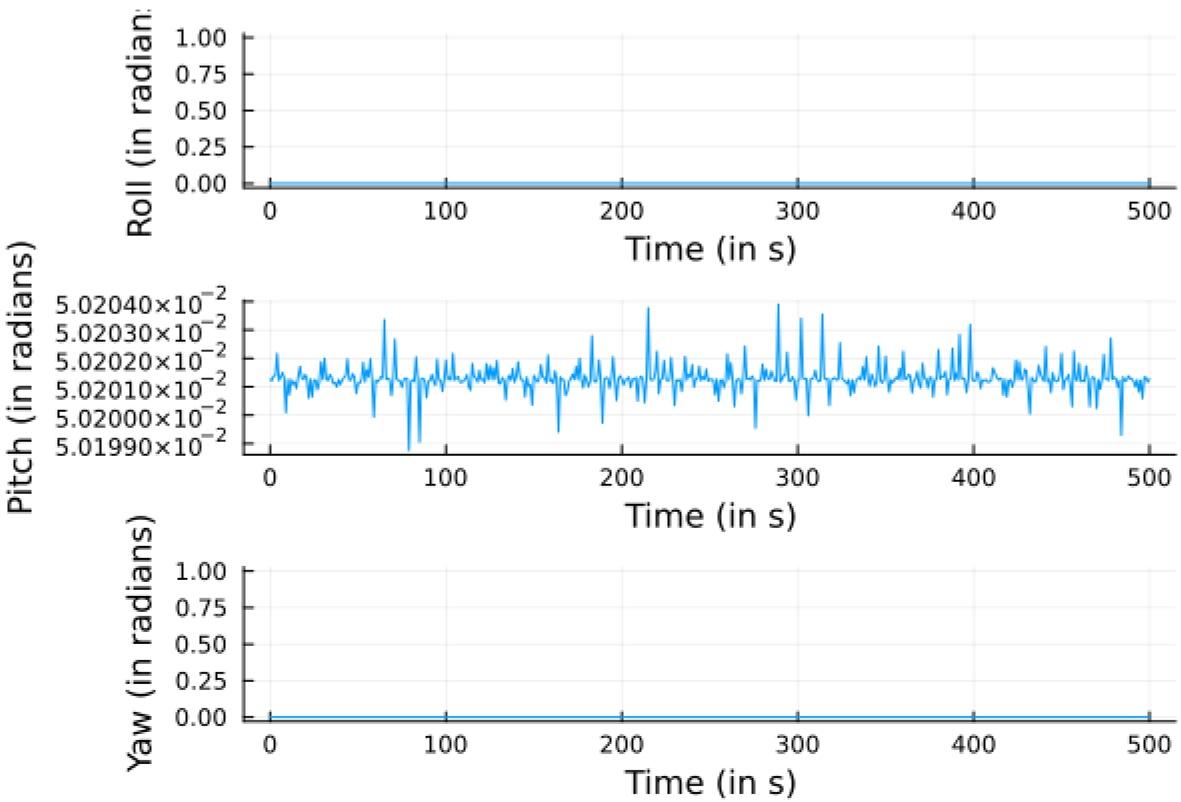
```julia
function aircraft_dynamics!(du,u,p,t)
    aircraft_state = AircraftState(u...)
    control_inputs = AircraftControl(p[1]...)
    wind_inertial = p[2]
    aircraft_parameters = p[3]
    x_dot = AircraftEOM(aircraft_state,control_inputs,wind_inertial,aircraft_parameters)
    for i in 1:length(u)
        du[i] = x_dot[i]
    end
end

function simulate(initial_state, time_interval, controls, wind_inertial, aircraft_parameters, save_at_value=1.0)
    extra_parameters = [controls, wind_inertial, aircraft_parameters]
    prob = ODEProblem(aircraft_dynamics!,initial_state,time_interval,extra_parameters)
    sol = DifferentialEquations.solve(prob,saveat=save_at_value)
    aircraft_states = []
    for i in 1:length(sol.u)
        push!(aircraft_states,AircraftState(sol.u[i]...))
    end
    return aircraft_states
end

function PlotSimulation(time, aircraft_state_array, control_input_array, col, save_plots=false)

    statefields = fieldnames(AircraftState)
    controlfields = fieldnames(AircraftControl)

    #Extract State values
    x_pos_values = [getfield(state,:x) for state in aircraft_state_array]
    y_pos_values = [getfield(state,:y) for state in aircraft_state_array]
    z_pos_values = [getfield(state,:z) for state in aircraft_state_array]
    roll_values = [getfield(state,:roll) for state in aircraft_state_array]
    pitch_values = [getfield(state,:pitch) for state in aircraft_state_array]
    yaw_values = [getfield(state,:yaw) for state in aircraft_state_array]
    u_values = [getfield(state,:u) for state in aircraft_state_array]
    v_values = [getfield(state,:v) for state in aircraft_state_array]
    w_values = [getfield(state,:w) for state in aircraft_state_array]
    p_values = [getfield(state,:p) for state in aircraft_state_array]
    q_values = [getfield(state,:q) for state in aircraft_state_array]
    r_values = [getfield(state,:r) for state in aircraft_state_array]

    #Extract Control values
    da_values = [getfield(control,:da) for control in control_input_array]
    de_values = [getfield(control,:de) for control in control_input_array]
    dr_values = [getfield(control,:dr) for control in control_input_array]
    dt_values = [getfield(control,:dt) for control in control_input_array]

    #Position plots
    px = plot( time, x_pos_values, xlabel = "Time (in s)", ylabel="x (in m)"  )
    py = plot( time, y_pos_values, xlabel = "Time (in s)", ylabel="y (in m)" )
    pz = plot( time, z_pos_values, xlabel = "Time (in s)", ylabel="z (in m)" )
    p_pos = plot(px, py, pz, layout=(3,1))
    if(save_plots)
        savefig("./plots/position.png")
    end

    #Orientation plots
    proll = plot( time, roll_values, xlabel = "Time (in s)", ylabel="Roll (in radians)", legends=false )
    ppitch = plot( time, pitch_values, xlabel = "Time (in s)", ylabel="Pitch (in radians)", legends=false )
    pyaw = plot( time, yaw_values, xlabel = "Time (in s)", ylabel="Yaw (in radians)", legends=false )
    p_orientation = plot(proll, ppitch, pyaw, layout=(3,1))
    if(save_plots)
        savefig("./plots/orientation.png")
    end

    #Velocity plots
    pu = plot( time, u_values, xlabel = "Time (in s)", ylabel="u (in m/s)", legends=false  )
    pv = plot( time, v_values, xlabel = "Time (in s)", ylabel="v (in m/s)", legends=false  )
    pw = plot( time, w_values, xlabel = "Time (in s)", ylabel="w (in m/s)", legends=false  )
    p_velocity = plot(pu, pv, pw, layout=(3,1))
    if(save_plots)
        savefig("./plots/velocity.png")
    end

    #Roll rate plots
    pp = plot( time, p_values, xlabel = "Time (in s)", ylabel="p (in radians/s)", legends=false  )
    pq = plot( time, q_values, xlabel = "Time (in s)", ylabel="q (in radians/s)", legends=false  )
    pr = plot( time, r_values, xlabel = "Time (in s)", ylabel="r (in radians/s)", legends=false  )
    p_rollrates = plot(pp, pq, pr, layout=(3,1))
    if(save_plots)
        savefig("./plots/angular_velocity.png")
    end

    #Control plots
    pde = plot( time, de_values, xlabel = "Time (in s)", ylabel="delta_e (in radians)", legends=false  )
    pda = plot( time, da_values, xlabel = "Time (in s)", ylabel="delta_a (in radians)", legends=false  )
    pdr = plot( time, dr_values, xlabel = "Time (in s)", ylabel="delta_r (in radians)", legends=false  )
    pdt = plot( time, dt_values, xlabel = "Time (in s)", ylabel="delta_t", legends=false )
    # pdt = plot( time, dt_values, xlabel = "Time (in s)", ylabel="delta_t", legends=false, ylim=(0.0, 0.25)  )
    p_control = plot(pde, pda, pdr, pdt, layout=(2,2))
    if(save_plots)
        savefig("./plots/controls.png")
    end

    #Plot Aircraft trajectory
    p_trajectory = plot3d([x_pos_values], [y_pos_values], [-z_pos_values], line=(:blue, 2), xlabel="x (in meters)", ylabel="y (in meters)", zlabel="z (in meters)", legend=false)
    scatter!([x_pos_values[1]],[y_pos_values[1]],[-z_pos_values[1]], color="green")
    scatter!([x_pos_values[end]],[y_pos_values[end]],[-z_pos_values[end]], color="red")
    if(save_plots)
        savefig("./plots/trajectory.png")
    end

    println("All the plots are generated and saved in the 'plots' folder.")
end

function GetStateAndControl(trim_definition::TrimDefinitionSL,trim_variables::TrimVariablesSL)

    #Parameters that don't matter
    x = y = ψ = 0.0
    #Parameters that are zero
    ϕ = v = p = q = r = 0.0
    #=
    Since there is no wind, γ = γ_a.
    Thus, pitch θ = flight path angle γ + angle of attack α
    Also, this is constant altitude flight. So, flight path angle, γ=0.
    =#
```

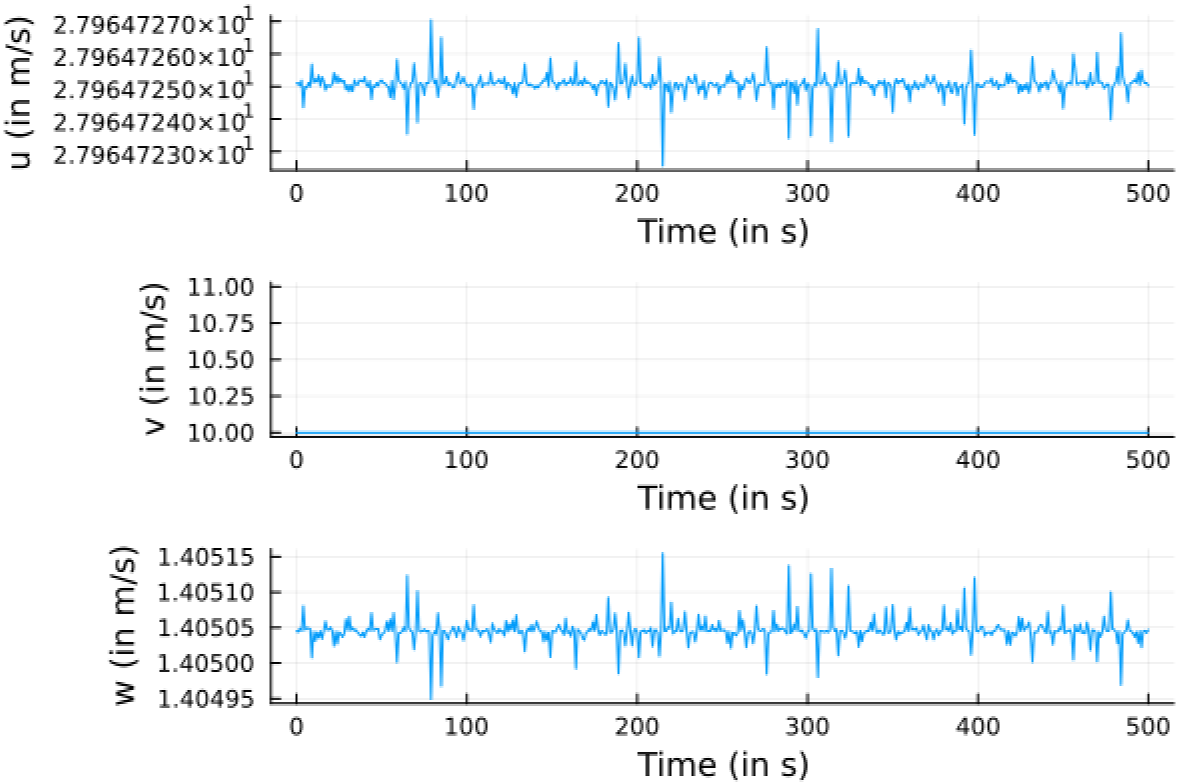
```julia
    θ = trim_definition.γ + trim_variables.α
    z = -trim_definition.h
    #For straight, wings-level, there is no side slip β
    β = 0.0
    wind_angles = WindAngles(trim_definition.Va,β,trim_variables.α)
    Va_vector = WindAnglesToAirRelativeVelocityVector(wind_angles)
    u = Va_vector[1]
    w = Va_vector[3]
    state = AircraftState(x,y,z,ϕ,θ,ψ,u,v,w,p,q,r)

    de = trim_variables.δe
    da = dr = 0.0
    dt = trim_variables.δt
    control = AircraftControl(de,da,dr,dt)

    return state,control
end

function GetCost(trim_definition::TrimDefinitionSL,trim_variables::TrimVariablesSL,aircraft_parameters::AircraftParameters)

    state,control = GetStateAndControl(trim_definition,trim_variables)
    wind_inertial = [0.0,0.0,0.0]
    rho = stdatmo(-state.z)
    force, moment = AircraftForcesAndMoments(state, control, wind_inertial, rho, aircraft_parameters)
    cost = norm(force,2)^2 + norm(moment,2)^2
    return cost
end

function OptimizerCostFunction(params::Vector{Float64},trim_definition::TrimDefinitionSL,aircraft_parameters::AircraftParameters)
    trim_variables = TrimVariablesSL(params...)
    cost = GetCost(trim_definition,trim_variables,aircraft_parameters)
    return cost
end

function GetTrimConditions(trim_definition::TrimDefinitionSL,aircraft_parameters::AircraftParameters)
    lower = [-pi/4,-pi/4,0.0]
    upper = [pi/4,pi/4,1.0]
    initial_tv = [0.5, 0.5, 0.5]
    results = optimize(x->OptimizerCostFunction(x,trim_definition,aircraft_parameters), lower, upper, initial_tv)
    trim_variables_list = results.minimizer
    trim_variables = TrimVariablesSL(trim_variables_list...)
    state, control = GetStateAndControl(trim_definition, trim_variables)
    return state, control, results
end

function GetStateAndControl(trim_definition::TrimDefinitionCT,trim_variables::TrimVariablesCT)

    #Parameters that don't matter
    x = y = ψ = 0.0

    #Parameters that matter
    ϕ = trim_variables.ϕ
    z = -trim_definition.h
    #=
    Since there is no wind, γ = γ_a.
    Thus, pitch θ = flight path angle γ + angle of attack α
    =#
    θ = trim_definition.γ + trim_variables.α
    α = trim_variables.α
    β = trim_variables.β
    wind_angles = WindAngles(trim_definition.Va,β,α)
    Va_vector = WindAnglesToAirRelativeVelocityVector(wind_angles)
    u = Va_vector[1]
    v = Va_vector[2]
    w = Va_vector[3]
    #=
    Slides have defined the p,q,r terms using the rate of change of coarse angle χ. It is called chi
    χ_dot = (velocity_perpendicular_to_the_cirle)/R
    velocity_perpendicular_to_the_cirle = Va*cos(γ), where γ is the flight path angle.
    =#

    # R = (trim_definition.Va^2)/(9.81*tan(ϕ))
    R = trim_definition.R
    χ_dot = ( trim_definition.Va*cos(trim_definition.γ) )/ R
    p = -sin(θ)*χ_dot
    q = sin(ϕ)*cos(θ)*χ_dot
    r = cos(ϕ)*cos(θ)*χ_dot
    state = AircraftState(x,y,z,ϕ,θ,ψ,u,v,w,p,q,r)

    de = trim_variables.δe
    da = trim_variables.δa
    dr = trim_variables.δr
    dt = trim_variables.δt
    control = AircraftControl(de,da,dr,dt)

    return state,control
end

function GetCost(trim_definition::TrimDefinitionCT,trim_variables::TrimVariablesCT,aircraft_parameters::AircraftParameters)
    state,control = GetStateAndControl(trim_definition,trim_variables)
    wind_inertial = [0.0,0.0,0.0]
    rho = stdatmo(-state.z)
    tangent_speed = trim_definition.Va*cos(trim_definition.γ)
    centripetal_acceleration = (tangent_speed*tangent_speed)/trim_definition.R
    a_desired_inertial_frame = [0.0, centripetal_acceleration, 0.0]
    euler_angles = EulerAngles(state.roll, state.pitch, state.yaw)
    a_desired_body_frame = TransformFromInertialToBody(a_desired_inertial_frame,euler_angles)
    desired_force = aircraft_parameters.m*a_desired_body_frame
    aero_force, aero_moment = AeroForcesAndMomentsBodyStateWindCoeffs(state, control, wind_inertial, rho, aircraft_parameters)
    total_force, total_moment = AircraftForcesAndMoments(state, control, wind_inertial, rho, aircraft_parameters)
    force = total_force - desired_force
    cost = norm(force,2)^2 + norm(total_moment,2)^2 + aero_force[2]^2
    return cost
end

function OptimizerCostFunction(params::Vector{Float64},trim_definition::TrimDefinitionCT,aircraft_parameters::AircraftParameters)
    trim_variables = TrimVariablesCT(params...)
    cost = GetCost(trim_definition,trim_variables,aircraft_parameters)
    return cost
end

function GetTrimConditions(trim_definition::TrimDefinitionCT,aircraft_parameters::AircraftParameters)
    lower = [-pi/4,-pi/4,0.0,-pi/4,-pi/4,-pi/4,-pi/4]
    upper = [pi/4,pi/4,1.0,pi/4,pi/4,pi/4,pi/4]
    initial_tv = [0.5,0.5,0.5,0.5,0.5,0.5,0.5]
    results = optimize(x->OptimizerCostFunction(x,trim_definition,aircraft_parameters), lower, upper, initial_tv)
    trim_variables_list = results.minimizer
```

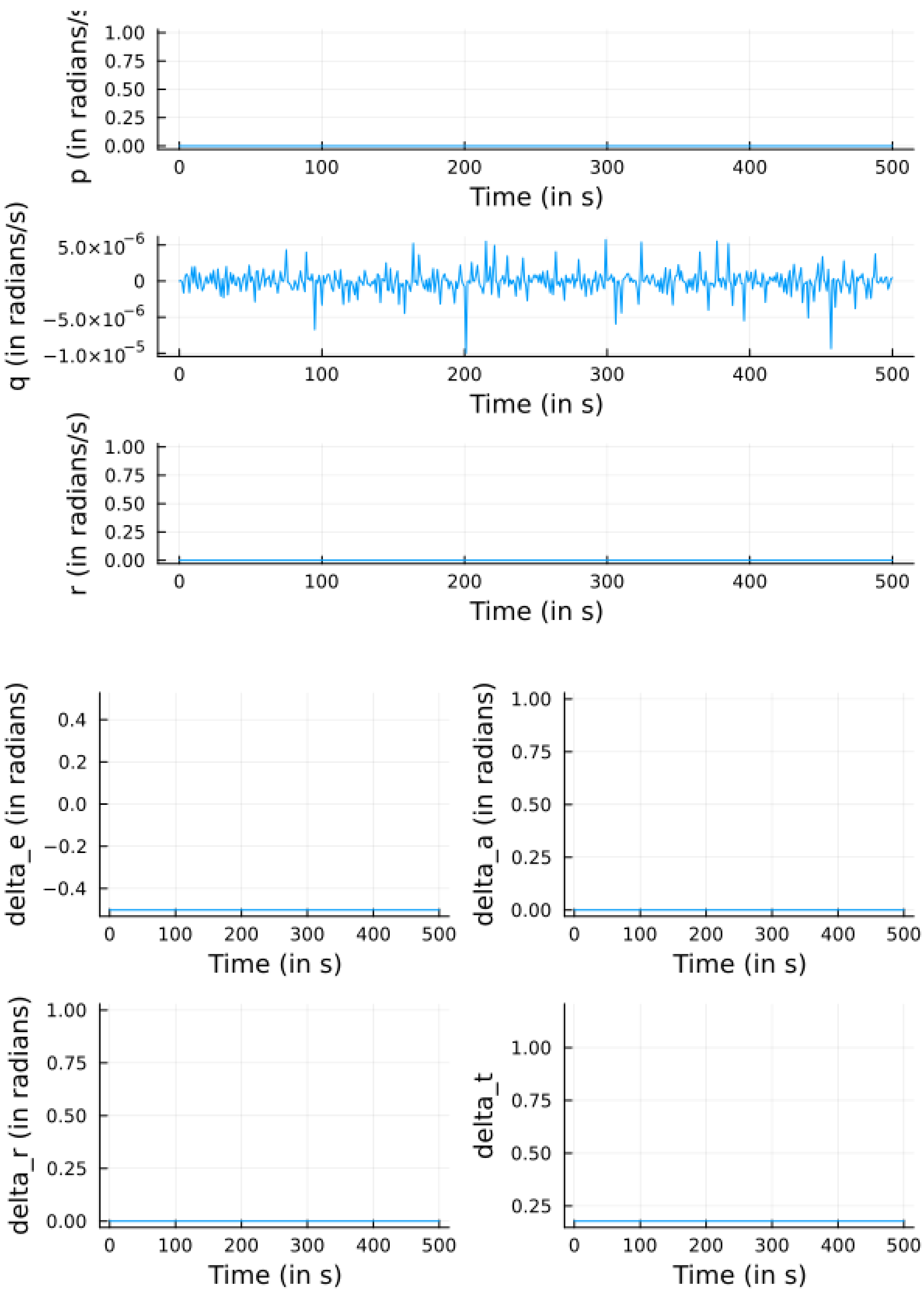
```julia
    trim_variables = TrimVariablesCT(trim_variables_list...)
    state, control = GetStateAndControl(trim_definition, trim_variables)
    return state, control, results
end

filename = "ttwistor.mat"
aircraft_parameters = AircraftParameters(filename)

#Part 1 - Verifying HW2
initial_state = [100, 200, 1000, 0.05, -0.02, 2.0, 15, -2, 3, .01, .02, .03]
s = AircraftState(initial_state...)
control_input =  [-.4, .01, .02, .3]
c = AircraftControl(control_input...)
wind_inertial = [-1, -2, -3]
rho = 1.1

af, am = AeroForcesAndMomentsBodyStateWindCoeffs(s,c, wind_inertial, rho, aircraft_parameters)
println("Aero Forces : ", af)
println("Aero Moments : ", am)
tf, tm = AircraftForcesAndMoments(s,c, wind_inertial, rho, aircraft_parameters)
println("Aircraft Forces : ", af)
println("Aircraft Moments : ", am)
xd = AircraftEOM(s, c, wind_inertial,aircraft_parameters)
println("xdot : ", xd)

#Part 2 - Verifying HW3
trim_definition = TrimDefinitionSL(18.0,0.0,1655)
state, control, results = GetTrimConditions(trim_definition, aircraft_parameters)
println("SLUF: Trim Variables: ", results.minimizer)
trim_definition = TrimDefinitionCT(18.0,0.0,1655,500.0)
state, control, results = GetTrimConditions(trim_definition, aircraft_parameters)
println("Coordinated Turn: Trim Variables: ", results.minimizer)
```

Solution Plots from Problem 3.2 in HW3

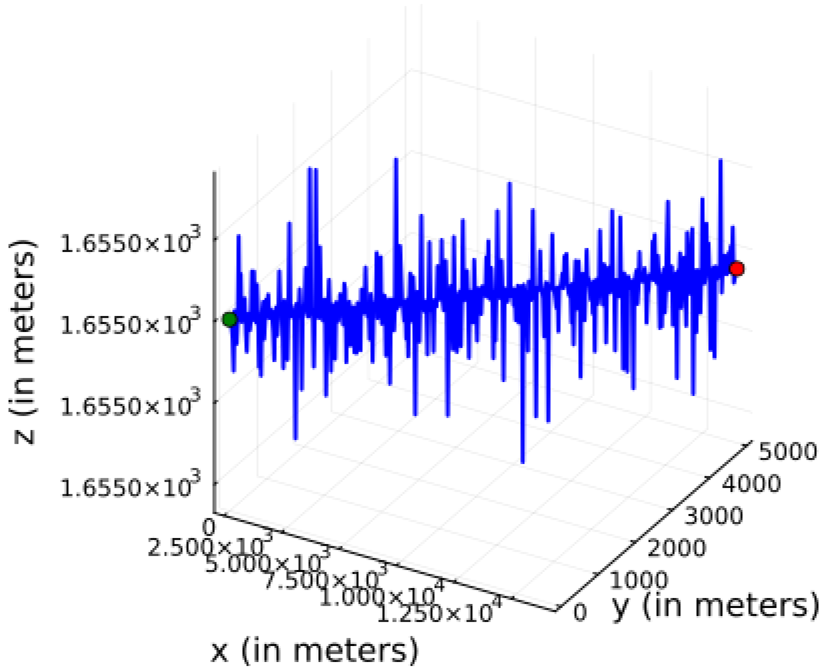



Roll (in radians)
1.00
0.75
0.50
0.25
0.00
0
100
200
300
400
500
Time (in s)
Pitch (in radians)
5.02040×10^-2
5.02030×10^-2
5.02020×10^-2
5.02010×10^-2
5.02000×10^-2
5.01990×10^-2
0
100
200
300
400
500
Time (in s)
Yaw (in radians)
1.00
0.75
0.50
0.25
0.00
0
100
200
300
400
500
Time (in s)


u (in m/s)
2.79647270×10^1
2.79647260×10^1
2.79647250×10^1
2.79647240×10^1
2.79647230×10^1
0
100
200
300
400
500
Time (in s)
v (in m/s)
11.00
10.75
10.50
10.25
10.00
0
100
200
300
400
500
Time (in s)
w (in m/s)
1.40515
1.40510
1.40505
1.40500
1.40495
0
100
200
300
400
500
Time (in s)



p (in radians/s)
1.00
0.75
0.50
0.25
0.00
0
100
200
300
400
500
Time (in s)
q (in radians/s)
5.0×10^{-6}
0
-5.0×10^{-6}
-1.0×10^{-5}
0
100
200
300
400
500
Time (in s)
r (in radians/s)
1.00
0.75
0.50
0.25
0.00
0
100
200
300
400
500
Time (in s)
delta_e (in radians)
0.4
0.2
0.0
−0.2
−0.4
0
100
200
300
400
500
Time (in s)
delta_a (in radians)
1.00
0.75
0.50
0.25
0.00
0
100
200
300
400
500
Time (in s)
delta_r (in radians)
1.00
0.75
0.50
0.25
0.00
0
100
200
300
400
500
Time (in s)
delta_t
1.00
0.75
0.50
0.25
0
100
200
300
400
500
Time (in s)



Starting (Trim) State:

0.0
0.0
-1655.0
0.0
0.050201252250689686
0.0
27.964725088966553
10.0
1.405044731858965
0.0
0.0
0.0

Trim Controls:

-0.5021558030179929
0.0
0.0
0.17764553656958398

Trim Variables:

0.050201252250689686
-0.5021558030179929
0.17764553656958398

The velocity component in the start state is augmented with the inertial wind velocity expressed in the body frame to get the inertial aircraft velocity in the body frame. This is done because the trim condition is computed using the air-relative velocity vector, not the inertial aircraft velocity.