23rd March, 2023

EXAM 1

Problem 1

1. We know that

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

Given : $q = -0.2^{\circ}/s$

$r = 0^{\circ}/s$

$\phi = -12^{\circ}$

$$\therefore \dot{\theta} = -0.2 * \cos\left(-12 \times \frac{\pi}{180}\right)$$

$$= -0.196^{\circ}/s = -0.0034 \text{ radians}/s$$

$\therefore$ time rate of change of pitch angle is $-0.196^{\circ}/s$.

2. We know $v_B^E = \begin{bmatrix} 15 \\ -3 \\ 1 \end{bmatrix}$ and

$$w_E^E = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

We know that $w_B^E = R_E^B \cdot w_E^E$

$\therefore$ $v_B^B = v_B^E - w_B^E$

We can use v_B^B to calculate α.

$$\alpha = \operatorname{atan}\left(\frac{v_{B\,z}^B}{v_{B\,x}^B}\right)$$

$\therefore$ $\alpha = 0.209$ radians $= 12.024$ degrees

3. If the thrust is applied in body-x direction, then the only forces in body-z direction are due to the aerodynamics forces or gravitational forces.

We know that

$$\dot{w}_B^E = q \cdot u_B^E - p \cdot v_B^E + \frac{1}{m} f_z$$

where $f_z = Z_{aero} + Z_{gravity}$

Given: $\dot{w}_B^E = 0.05 \ m/s^2$; $u_B^E = 15 \ m/s$

$v_B^E = -3 \ m/s$; $p = 0.08^\circ/s$; $q = -0.2^\circ/s$

$m = 10 \ kg$

$\therefore \quad f_z = 1.0236 \quad N$

Also, $Z_{gravity} = m * g * \cos(\theta) * \cos(\phi)$

$\therefore Z_{gravity} = 94.7749 \text{ N}$

$\therefore Z_{aero} = f_z - Z_{gravity}$

$= -93.7513 \text{ N}$

Problem 2

1. TRUE !

TECS will maintain a constant total of KE + PE, and it uses throttle to regulate it.

From the discussion in class, it can be inferred that to speed up the aircraft, the elevator changes to decrease drag, and similarly it can increase drag to slow its speed, all while maintaining the same altitude.
As a result, it seems the statement is True.

(However, I am not entirely convinced since there is coupling between states and so change in V_a should also have an effect on h. Need to discuss more with Dr. Frew).

2. FALSE !

The linear design model can be derived for any aircraft and doesn't depend on whether it has stable modes or not.

For ex: We have shown that the Twistor airplane that we have used so far has an unstable spiral mode. However, we have still been able to develop linear models for that aircraft.

Unstable modes can cause big changes in an aircraft's state when it encounters small perturbations from the nominal trajectory, and so controling such an aircraft can be difficult.

Problem 3

From slides and book, we know that the commanded pitch angle can be obtained from commanded height using this equation:

$$\theta^c(t) = k_{p_h}\left(h^c(t) - h(t)\right) + k_{i_h}\int_{-\infty}^{t}\left(h^c(\tau) - h(\tau)\right)\cdot d\tau$$

We also know that a desired/commanded pitch angle θ^c can be obtained by changing the elevator. The corresponding equation is

$$\delta_e(t) = k_{p_\theta}\left(\theta^c(t) - \theta(t)\right) - k_{d_\theta}\, q(t)$$

$$\therefore \theta^c(t) = \frac{1}{k_{p_\theta}} \left(\delta_e(t) + k_{d_\theta} q(t) \right) + \theta(t)$$

$\therefore$ We have

$$\frac{1}{k_{p_\theta}} \left(\delta_e(t) + k_{d_\theta} q(t) \right) + \theta(t) = k_{p_h} \left(h^c(t) - h(t) \right) + k_{i_h} \int_{-\infty}^{t} \left(h^c(\tau) - h(\tau) \right) . d\tau$$

$$\begin{aligned} \therefore \delta_e(t) = \; & k_{p_h} k_{p_\theta} \left(h^c(t) - h(t) \right) \\ & + k_{i_h} k_{p_\theta} \int_{-\infty}^{t} \left(h^c(\tau) - h(t) \right) . d\tau \\ & - k_{p_\theta} \theta(t) \\ & - k_{d_\theta} q(t) \end{aligned}$$

This is the overall law for the elevator angle for this approach.