

## Aufgabe 1

1.

$$v = \begin{pmatrix} a \\ b \\ i \end{pmatrix}$$

$$B(v) = i < n$$

$$H(v) = \begin{pmatrix} b \\ a+b \\ i+1 \end{pmatrix}$$

2.

$$INV\left(\begin{pmatrix} a \\ b \\ i \end{pmatrix}\right) = i \leq n \wedge a = \text{fib}(i) \wedge b = \text{fib}(i+1)$$

$$P(n) = n \geq 0 \wedge a = 0 \wedge b = 1 \wedge i = 0$$

$$Q(n) = \text{Fib}(n) = a$$

$$v^0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad INV\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right) = 0 \leq \text{fib}(0) \wedge 1 = \text{fib}(1) \wedge 0 \leq n \text{ ist wahr}$$

$$\text{Es gelte nun: } INV(v^j) = i^j \leq n \wedge a^j = \text{fib}(i^j) \wedge b^j = \text{fib}(i^j+1)$$

$$\text{und } B(v^j) = i^j < n$$

$$\underline{\text{z.z.}}: INV(v^{j+1}) \text{ ist wahr.}$$

$$a^{j+1} = b^j = \text{fib}(i^j+1)$$

$$b^{j+1} = a^j + b^j = \text{fib}(i^j) + \text{fib}(i^j+1) = \text{fib}(i^j+2)$$

$$B(v^j) = i^j = i^{j+1} - 1 < n \Rightarrow INV(v^{j+1}) \text{ ist wahr}$$

Am Schleifenende gilt:

$$INV(v) \wedge \neg B(v)$$

$$\Leftrightarrow i \leq n \wedge a = \text{fib}(i) \wedge b = \text{fib}(i+1) \wedge i \geq n$$

$$\Leftrightarrow i \leq n \wedge a = \text{fib}(i) \wedge b = \text{fib}(i+1)$$

$$\Leftrightarrow a = \text{fib}(n) = Q(n)$$