Problems

1. In a circle Γ_1, centered at O, AB and CD are two unequal in length chords intersecting at E inside Γ_1. A circle Γ_2, centered at I is tangent to Γ_1 internally at F, and also tangent to AB at G and CD at H. A line l through O intersects AB and CD at P and Q respectively such that $EP = EQ$. The line EF intersects l at M. Prove that the line through M parallel to AB is tangent to Γ_1.

2. A real polynomial of odd degree has all positive coefficients. Prove that there is a (possibly trivial) permutation of the coefficients such that the resulting polynomial has exactly one real zero.

3. Find all primes p and q such that $3p^{q-1} + 1$ divides $11^p + 17^p$.

Problems

1. In a circle Γ_1, centered at O, AB and CD are two unequal in length chords intersecting at E inside Γ_1. A circle Γ_2, centered at I is tangent to Γ_1 internally at F, and also tangent to AB at G and CD at H. A line l through O intersects AB and CD at P and Q respectively such that $EP = EQ$. The line EF intersects l at M. Prove that the line through M parallel to AB is tangent to Γ_1.
2. Consider a $n \times n$ board of unit squares. We place several isosceles right triangles with length 1 legs on the board such that no two intersect (except possibly on their hypotenuses), and each covers exactly half of one cell each. Each internal edge of the board is covered by exactly one right triangle. What's the maximum number of squares that don't contain triangles?
3. Find all primes p and q such that $3p^{q-1} + 1$ divides $11^p + 17^p$.

Problems

1. 10000 nonzero digits are written in a 100-by-100 table, one digit per cell. From left to right, each row forms a 100-digit integer. From top to bottom, each column forms a 100-digit integer. So the rows and columns form 200 integers (each with 100 digits), not necessarily distinct. Prove that if at least 199 of these 200 numbers are divisible by 2013, then all of them are divisible by 2013.

2. Consider a $n \times n$ board of unit squares. We place several isosceles right triangles with length 1 legs on the board such that no two intersect (except possibly on their hypotenuses), and each covers exactly half of one cell each. Each internal edge of the board is covered by exactly one right triangle. What's the maximum number of squares that don't contain triangles?

3. Suppose $ABCD$ is a parallelogram. Consider circles w_1 and w_2 such that w_1 is tangent to segments AB and AD and w_2 is tangent to segments BC and CD. Suppose that there exists a circle which is tangent to lines AD and DC and externally tangent to w_1 and w_2. Prove that there exists a circle which is tangent to lines AB and BC and also externally tangent to circles w_1 and w_2.

Problems

1. A real polynomial of odd degree has all positive coefficients. Prove that there is a (possibly trivial) permutation of the coefficients such that the resulting polynomial has exactly one real zero.
2. Find all primes p and q such that $3p^{q-1}+1$ divides 11^p+17^p.
3. Suppose $ABCD$ is a parallelogram. Consider circles w_1 and w_2 such that w_1 is tangent to segments AB and AD and w_2 is tangent to segments BC and CD. Suppose that there exists a circle which is tangent to lines AD and DC and externally tangent to w_1 and w_2. Prove that there exists a circle which is tangent to lines AB and BC and also externally tangent to circles w_1 and w_2.

Problems

1. Consider a $n \times n$ board of unit squares. We place several isosceles right triangles with length 1 legs on the board such that no two intersect (except possibly on their hypotenuses), and each covers exactly half of one cell each. Each internal edge of the board is covered by exactly one right triangle. What's the maximum number of squares that don't contain triangles?

2. Determine if the following statement is true: given any non-negative $\lambda_{i,j}$, $1 \leq i < j \leq n$, there always exists nonnegative reals $a_i, 1 \leq i \leq n$ such that $|a_i - a_j| \geq \lambda_{i,j}$ for all $1 \leq i < j \leq n$, and
$$\sum_{i=1}^{n} a_i \leq \sum_{1 \leq i < j \leq n} \lambda_{i,j}$$

3. Suppose $ABCD$ is a parallelogram. Consider circles w_1 and w_2 such that w_1 is tangent to segments AB and AD and w_2 is tangent to segments BC and CD. Suppose that there exists a circle which is tangent to lines AD and DC and externally tangent to w_1 and w_2. Prove that there exists a circle which is tangent to lines AB and BC and also externally tangent to circles w_1 and w_2.

Problems

1. 10000 nonzero digits are written in a 100-by-100 table, one digit per cell. From left to right, each row forms a 100-digit integer. From top to bottom, each column forms a 100-digit integer. So the rows and columns form 200 integers (each with 100 digits), not necessarily distinct. Prove that if at least 199 of these 200 numbers are divisible by 2013, then all of them are divisible by 2013.

2. Determine all functions $f : \mathbb{Z} \to \mathbb{Z}$ satisfying

$$f\big(f(m) + n\big) + f(m) = f(n) + f(3m) + 2014$$

for all integers m and n.

3. In a circle Γ_1, centered at O, AB and CD are two unequal in length chords intersecting at E inside Γ_1. A circle Γ_2, centered at I is tangent to Γ_1 internally at F, and also tangent to AB at G and CD at H. A line l through O intersects AB and CD at P and Q respectively such that $EP = EQ$. The line EF intersects l at M. Prove that the line through M parallel to AB is tangent to Γ_1.

Problems

1. A real polynomial of odd degree has all positive coefficients. Prove that there is a (possibly trivial) permutation of the coefficients such that the resulting polynomial has exactly one real zero.

2. Find the larget number of L-tetrominoes (reflections and rotations allowed) that can be placed (aligned with the cells) on an $n \times n$ such that there is a connected (edgewise) path of cells beginning at one corner of the board and ending at the opposite corner.

3. The sequence a_n is defined as follows: $a_1 = 1$ and for any $n \in \mathbb{N}$, the number a_{n+1} is obtained from a_n by adding 3 if n is a member of this sequence, and 2 otherwise. Show that $a_n < (1 + \sqrt{2})n$ for all n.

Problems

1. 10000 nonzero digits are written in a 100-by-100 table, one digit per cell. From left to right, each row forms a 100-digit integer. From top to bottom, each column forms a 100-digit integer. So the rows and columns form 200 integers (each with 100 digits), not necessarily distinct. Prove that if at least 199 of these 200 numbers are divisible by 2013, then all of them are divisible by 2013.

2. Let O denote the circumcentre of an acute-angled triangle ABC. Let point P on side AB be such that $\angle BOP = \angle ABC$, and let point Q on side AC be such that $\angle COQ = \angle ACB$. Prove that the reflection of BC in the line PQ is tangent to the circumcircle of triangle APQ.

3. The sequence a_n is defined as follows: $a_1 = 1$ and for any $n \in \mathbb{N}$, the number a_{n+1} is obtained from a_n by adding 3 if n is a member of this sequence, and 2 otherwise. Show that $a_n < (1 + \sqrt{2})n$ for all n.

Problems

1. Let k be a fixed positive integer. Alberto and Beralto play the following game: Given an initial number N_0 and starting with Alberto, they take turns to perform the following operation: change the number n into a number m such that $m < n$ and m and n differ, in their base-2 representations, in exactly l consecutive digits for some l such that $1 \leq l \leq k$. If someone can't play, he loses.

   We say a non-negative integer t is a winner number when the player who receives the number t has a winning strategy, that is, he can choose the next numbers in order to guarrantee his own victory, regardless the options of the other player. Else, we call it a loser.

   Prove that for every positive integer N, the total of non-negative loser integers smaller than 2^N is $2^{N-\lfloor \frac{\log(\min\{N,k\})}{\log 2} \rfloor}$.

2. Consider $m+1$ horizontal and $n+1$ vertical lines ($m, n \geq 4$) in the plane forming an $m \times n$ table. Cosider a closed path on the segments of this table such that it does not intersect itself and also it passes through all $(m-1)(n-1)$ interior vertices (each vertex is an intersection point of two lines) and it doesn't pass through any of outer vertices. Suppose A is the number of vertices such that the path passes through them straight forward, B number of the table squares that only their two opposite sides are used in the path, and C number of the table squares that none of their sides is used in the path. Prove that

$$A = B - C + m + n - 1.$$

3. A real polynomial of odd degree has all positive coefficients. Prove that there is a (possibly trivial) permutation of the coefficients such that the resulting polynomial has exactly one real zero.

Problems

1. Let k be a fixed positive integer. Alberto and Beralto play the following game: Given an initial number N_0 and starting with Alberto, they take turns to perform the following operation: change the number n into a number m such that $m < n$ and m and n differ, in their base-2 representations, in exactly l consecutive digits for some l such that $1 \leq l \leq k$. If someone can't play, he loses.

   We say a non-negative integer t is a winner number when the player who receives the number t has a winning strategy, that is, he can choose the next numbers in order to guarrantee his own victory, regardless the options of the other player. Else, we call it a loser.

   Prove that for every positive integer N, the total of non-negative loser integers smaller than 2^N is $2^{N-\lfloor \frac{\log(\min\{N,k\})}{\log 2} \rfloor}$.

2. For which positive integers k can we construct a sequence $a_0, a_1, a_2, \dots$, where for each i, we have $a_{i+1} = a_i + k$ or $a_{i+1} = a_i - k$ or $a_{i+1} = a_i \times k$ or $a_{i+1} = a_i / k$?

3. Determine if the following statement is true: given any non-negative $\lambda_{i,j}$, $1 \leq i < j \leq n$, there always exists nonnegative reals $a_i, 1 \leq i \leq n$ such that $|a_i - a_j| \geq \lambda_{i,j}$ for all $1 \leq i < j \leq n$, and
$$\sum_{i=1}^{n} a_i \leq \sum_{1 \leq i < j \leq n} \lambda_{i,j}$$

Problems

1. Let k be a fixed positive integer. Alberto and Beralto play the following game: Given an initial number N_0 and starting with Alberto, they take turns to perform the following operation: change the number n into a number m such that $m < n$ and m and n differ, in their base-2 representations, in exactly l consecutive digits for some l such that $1 \leq l \leq k$. If someone can't play, he loses.

   We say a non-negative integer t is a winner number when the player who receives the number t has a winning strategy, that is, he can choose the next numbers in order to guarrantee his own victory, regardless the options of the other player. Else, we call it a loser.

   Prove that for every positive integer N, the total of non-negative loser integers smaller than 2^N is $2^{N-\lfloor \frac{\log(\min\{N,k\})}{\log 2} \rfloor}$.

2. Find the larget number of L-tetrominoes (reflections and rotations allowed) that can be placed (aligned with the cells) on an $n \times n$ such that there is a connected (edgewise) path of cells beginning at one corner of the board and ending at the opposite corner.

3. For which positive integers k can we construct a sequence $a_0, a_1, a_2, \ldots$, where for each i, we have $a_{i+1} = a_i + k$ or $a_{i+1} = a_i - k$ or $a_{i+1} = a_i \times k$ or $a_{i+1} = a_i/k$?

Problems

1. Let k be a fixed positive integer. Alberto and Beralto play the following game: Given an initial number N_0 and starting with Alberto, they take turns to perform the following operation: change the number n into a number m such that $m < n$ and m and n differ, in their base-2 representations, in exactly l consecutive digits for some l such that $1 \le l \le k$. If someone can't play, he loses.

   We say a non-negative integer t is a winner number when the player who receives the number t has a winning strategy, that is, he can choose the next numbers in order to guarrantee his own victory, regardless the options of the other player. Else, we call it a loser.

   Prove that for every positive integer N, the total of non-negative loser integers smaller than 2^N is $2^{N-\lfloor \frac{\log(\min\{N,k\})}{\log 2} \rfloor}$.

2. A real polynomial of odd degree has all positive coefficients. Prove that there is a (possibly trivial) permutation of the coefficients such that the resulting polynomial has exactly one real zero.

3. Find the larget number of L-tetrominoes (reflections and rotations allowed) that can be placed (aligned with the cells) on an $n \times n$ such that there is a connected (edgewise) path of cells beginning at one corner of the board and ending at the opposite corner.