

IHDCB331 - Algorithmique II

Devoir 2

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1 Calcul de postconditions

1.1 Question 1

$$P \equiv \{z < y + 50\}$$

```
1 x = x - 43;
```

$$Q \equiv \{z > y * 50 \wedge x = z - 43\}$$

1.2 Question 2

$$P \equiv \{x - y \geq 0\}$$

```
1 y = y + x;
```

$$\begin{aligned} &\equiv x - y' \geq 0 \wedge y = y' + x \\ &\equiv y' = y - x \wedge x - y' \geq 0 \\ &\equiv x - y - x \geq 0 \end{aligned}$$

$$Q \equiv \{-y \geq 0\}$$

1.3 Question 3

$$P \equiv \{x \geq 3\}$$

```
1 x = x + 1;
```

$$\begin{aligned} &\equiv x = x' + 1 \wedge x' \geq 3 \\ &\equiv x - 1 = x' \wedge x' \geq 3 \\ &\equiv x \geq 4 \end{aligned}$$

```
1 x = x * x;
```

$$\begin{aligned} &\equiv x = x' * x' \wedge x' \geq 4 \\ &\equiv x' = \text{sqrt}(x) \wedge x' \geq 4 \\ &\equiv \text{sqrt}(x) \geq 4 \\ &\equiv x \geq 16 \end{aligned}$$

$$Q \equiv \{x \geq 16\}$$

1.4 Question 4

Cette question a été réalisée en cours.

2 Calcul de précondition

2.1 Question 1

$$Q \equiv \{x > 3\}$$

```
1 x = y + 3;
```

$$P \equiv \{y > 0\}$$

2.2 Question 2

$$Q \equiv \{y > 5\}$$

```
1 y = x + 2;
```

$$\equiv \{y > 7\}$$

```
1 y = y - 2;
```

$$\equiv \{x > 5\}$$

$$P \equiv \{x > 5\}$$

2.3 Question 3

$$Q \equiv \{y > 0\}$$

```
1 if (x > 2)
```

```
2   y = 1;
```

```
3 else
```

```
4   y = -1;
```

$$P \equiv \{x > 2\}$$

3 Preuves par l'invariant

3.1 Question 1

$$I \equiv 0 < i \leq n + 1 \wedge k = 2^{i-1}$$

3.1.1 Avant la boucle

$$\equiv 0 < i \leq n + 1 \wedge k = 2^{i-1} \wedge i = 1 \wedge k = 1$$

$$\equiv 0 < 1 \leq n + 1 \wedge k = 2^0 = 1 \wedge i = 1 \wedge k = 1$$

$$Q \Rightarrow I$$

3.1.2 Boucle

$$I \wedge B \equiv 0 < i \leq n + 1 \wedge k = 2^{i-1} \wedge i \leq n$$

1 $k = 2 * k;$

$$\begin{aligned} Q_1 &\equiv 0 < i \leq n + 1 \wedge k' = 2^{i-1} \wedge i \leq n \wedge k = 2 * k' \\ &\Rightarrow k' = k/2 \\ Q_1 &\equiv 0 < i \leq n + 1 \wedge k/2 = 2^{i-1} \wedge i \leq n \end{aligned}$$

1 $i++;$

$$\begin{aligned} Q_2 &\equiv 0 < i' \leq n + 1 \wedge k/2 = 2^{i'-1} \wedge i' \leq n \wedge i = i' + 1 \\ &\Rightarrow i' = i - 1 \\ Q_2 &\equiv 0 < i - 1 \leq n + 1 \wedge k/2 = 2^{i-2} \wedge i - 1 \leq n \\ Q_2 &\equiv 1 < i \leq n + 2 \wedge k/2 = 2^{i-2} \wedge i \leq n + 1 \end{aligned}$$

$$Q_2 \Rightarrow I$$

3.1.3 Terminaison de la boucle

$$V = n - i$$

$$V = 0 \equiv n - i = 0 \Rightarrow n = i$$

V diminue à chaque passage de boucle car i augmente

3.2 Question 2

$$I \equiv x \geq 0 \wedge y \geq 0 \wedge y = x_0 - x$$

3.2.1 Avant la boucle

$$\begin{aligned} &\equiv x \geq 0 \wedge y = 0 \wedge x = x_0 \\ &\equiv x \geq 0 \wedge y = 0 \wedge x = x_0 \wedge y = x_0 - x = 0 \\ Q &\Rightarrow I \end{aligned}$$

3.2.2 Boucle

$$I \wedge B \equiv x > 0 \wedge y \geq 0 \wedge x = x_0 \wedge y = x_0 - x$$

1 $x = x - 1;$

$$\begin{aligned} Q_1 &\equiv x > 0 \wedge y \geq 0 \wedge x' = x_0 \wedge y = x_0 - x' \wedge x = x' - 1 \\ &\Rightarrow x' = x + 1 \\ Q_1 &\equiv x > 0 \wedge y \geq 0 \wedge x + 1 = x_0 \wedge y = x_0 - (x + 1) \end{aligned}$$

1 $y = y + 1;$

$$\begin{aligned} Q_2 &\equiv x > 0 \wedge y \geq 0 \wedge x + 1 = x_0 \wedge y' = x_0 - (x + 1) \wedge y = y' + 1 \\ &\Rightarrow y' = y - 1; \\ Q_2 &\equiv x > 0 \wedge y \geq 0 \wedge x + 1 = x_0 \wedge y - 1 = x_0 - (x + 1) \\ Q_2 &\equiv x > 0 \wedge y \geq 0 \wedge x + 1 = x_0 \wedge y = x_0 - x + 2 \\ Q_2 &\Rightarrow I \end{aligned}$$

3.2.3 Terminaison de la boucle

Comme x est supérieur à 0 et diminue de 1 à chaque boucle, celui-ci sera un moment égale à 0 et donc la boucle se terminera.

3.3 Question 3

$$I \equiv a = a_0 \wedge 0 \leq i \leq n \wedge \exists k, \forall j : 0 \leq k \leq i \wedge 0 \leq j \leq i : a[k] \leq a[i] \wedge r = k$$

3.3.1 Avant la boucle

$$P \equiv \{n > 0\}$$

```
1 i = 0;
2 r = 0;
```

$$Q_1 \equiv n > 0 \wedge i = 0 \wedge r = 0 \wedge a = a_0$$

$$Q_1 \equiv 0 \leq 0 < n \wedge a = a_0 \wedge \exists k, \forall j : 0 \leq 0 \leq 0 : 0 \leq 0 \leq 0 : a[0] \leq a[0] \wedge r = k$$

$$Q_1 \Rightarrow I$$

3.3.2 Boucle

$$I \wedge B \equiv 0 \leq i \leq n \wedge \exists k, \forall j : 0 \leq k \leq i \wedge 0 \leq j \leq i : a[k] \leq a[i] \wedge r = k \wedge i < n$$

$$I \wedge B \equiv 0 \leq i < n \wedge \exists k, \forall j : 0 \leq k \leq i \wedge 0 \leq j \leq i : a[k] \leq a[i] \wedge r = k$$

```
1 if (a[i] < a[r]) r = i;
```

Si la condition B est vraie.

$$Q_1 \equiv 0 \leq i < n \wedge \exists k, \forall j : 0 \leq k \leq i \wedge 0 \leq j \leq i : a[k] \leq a[i] \wedge r = k \wedge a[i] < a[r]$$

$$Q_1 \equiv 0 \leq i < n \wedge \exists k, \forall j : 0 \leq k \leq i \wedge 0 \leq j \leq i : a[k] = a[i] \wedge r = k = i$$

```
1 i = i + 1;
```

$$Q_2 \equiv 0 \leq i < n \wedge \exists k, \forall j : 0 \leq k \leq i' \wedge 0 \leq j \leq i' : a[k] = a[i'] \wedge r = k = i' \wedge i = i' + 1$$

$$\Rightarrow i' = i - 1$$

$$Q_2 \equiv 0 \leq i < n \wedge \exists k, \forall j : 0 \leq k \leq i - 1 \wedge 0 \leq j \leq i - 1 : a[k] = a[i - 1] \wedge r = k = i - 1$$

$$Q_2 \Rightarrow I$$

Si la condition B est fausse.

$$Q_1 \equiv 0 \leq i < n \wedge \exists k, \forall j : 0 \leq k \leq i \wedge 0 \leq j \leq i : a[k] \leq a[i] \wedge r = k$$

```
1 i = i + 1;
```

$$Q_2 \equiv 0 \leq i < n \wedge \exists k, \forall j : 0 \leq k \leq i' \wedge 0 \leq j \leq i' : a[k] \leq a[i'] \wedge r = k \wedge i = i' + 1$$

$$\Rightarrow i' = i - 1$$

$$Q_2 \equiv 0 \leq i < n \wedge \exists k, \forall j : 0 \leq k \leq i - 1 \wedge 0 \leq j \leq i - 1 : a[k] \leq a[i - 1] \wedge r = k$$

$$Q_2 \Rightarrow I$$

3.3.3 Terminaison de la boucle

$$V = n - 1$$

$$V = 0 \equiv n - i = 0 \Rightarrow n = i$$

V diminue à chaque passage de boucle étant donné que i augmente et que n est constant.

4 Complexité

1. Simplification : $2\log_2 n + n^2 + n\log_2 4 + n\log_2 n$
 $= 2\log_2 n + n^2 + 2n + n\log_2 n$
 $\Rightarrow \mathcal{O}(n^2)$
2. Classement : $n^{1/3} < 100n < n\log n < (\log_2 n)^{100} < \log n < n^2/\log_2 n < 35n^2 < n^{10} < n^3 * 2^n < 4^n < n!$
3. $\mathcal{O}(n)$
4. $\mathcal{O}(n\log_2 n)$
5. $\mathcal{O}(n^2)$
6. $\mathcal{O}(n^3)$
7. $\mathcal{O}(n^2)$
8. $\mathcal{O}(n^2)$