Measuring the Astronomical Unit Using Ceres at Opposition

Christopher Ewasiuk and Diego Romani*
Physics Department, University of California, Santa Barbara, CA 93106

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One of the most commonly used means of measuring distances across space is known as an Astronomical Unit (AU). Based on the distance between the Earth and the Sun, this unit allows astronomers to characterize the vast distances between the Solar planets and the Sun, and between extra-solar planets and their stars. Using a high powered telescope, a nearby celestial object, and the rotation of the Earth, it is possible to calculate the astronomical unit using simple trigonometry. Our experimentation was unsuccessful, as we determined the AU to have a value of $6,691,034.176 \pm 17,133,407.19$km.

I. BACKGROUND

The concept of an internationally standardised system of units is one of the most fundamental in experimental science. Everyone uses familiar units such as kilograms, kilometres and seconds and they are indispensable in daily life. Scientists may need more exotic units such as measures of current, frequency and other scientific quantities, but the principle is the same, without an agreed scheme of measurement, scientists could not share results and there could be disastrous and costly mistakes. One of the most important of these is the astronomical unit. It is a unit of length approximating the Sun-Earth distance which is of convenient use in astronomy and calculations within our solar system.

There is evidence that the astronomical unit was first determined in ancient Greece, where Eratosthenes calculate its value to be around 149 million kilometers. Yet, its undisputed measurement came in 1877 when Scottish astronomer David Gill was able to calculate the distance from Earth to Mars during opposition in terms of AU. He was able to use the diameter of Earth at a given latitude as a baseline in the parallax determination for Mars. Through simple trigonometry and Kepler's Laws, he was able to calculate the astronomical unit to within 0.2 percent of its actual value. Currently, the AU has an accepted value of 149, 597, 870, 691 Âś 30 meters, which was determined using direct radar measurements of the distances to Venus and Mars.

Here we were able to preform an approach similar to David Gill in calculating the astronomical unit. Rather than Mars, our calculation was based on the transit of the dwarf planet Ceres at opposition and the known distance from Earth to Ceres in terms of AU. This approach yielded a value of $6,691,034.176 \pm 17,133,407.19$ km. Our uncertainty in our measurement was due to limitations in determining precise values for right ascension and declination, as we were forced to extract these values from our photographs taken of Ceres.

FIG. 1: Our observational setup for measuring the transit of Ceres. The telescope is attached to a computerized mount, where it can be oriented in any direction using the input remote. An object can be viewed through the finding lens, or a photo of the object can be captured using a camera attatched to the main eyepiece.

II. APPROACH

Our approach was based on the one described by Robert J. Vanderbei [1]. We first calculated the diameter of the Earth at our current latitude by using the radius of the Earth and the angle of our inclination *(refer to Figure 2). Our calculation of the parallax angle was to be obtained through analysis of photographs taken through our telescope. Our telescope (Meade ED APO 7" Model 178ED Telescope, Meade LXD750 Large Field Tripod and Equatorial Mount) was placed on the roof

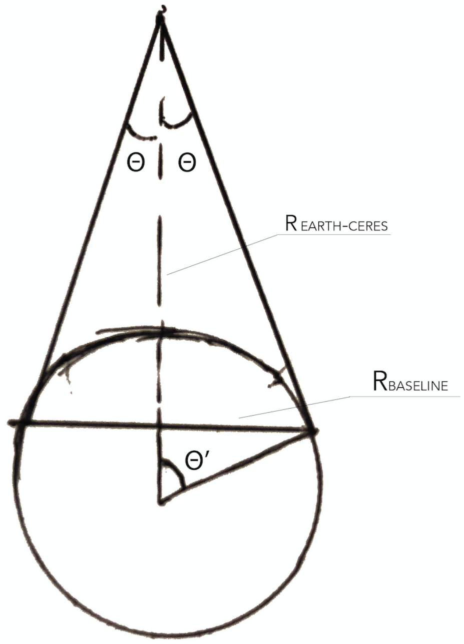

FIG. 2:
Diagram of our method for determining the astronomical unit. We calculated $R_{EARTH-CERES}$ by using simple trigonometry, relating the interior angle made by Earth and the dwarf planet to our calculated value for $R_{BASELINE}$

of the tallest nearby building to obtain a clear view of the sky. Our telescope was then auto aligned, and relevant information such as latitude, date and time were entered into the computer remote. The telescope will then focus onto a nearby star, where we confirmed it was aligned correctly. Once the telescope is properly set up, replace the eyepiece with the mounted camera, and run the Meade SkyKey program to capture images using the telescope.

Our initial step was to calculate the diameter of the Earth at our current latitude. This was done by multiplying the radius of the Earth by the sine of our latitude (given in degrees). Once obtained, we only need the parallax of the dwarf planet to calculate the distance between Earth and Ceres. Our latitude was 34.42 degrees, giving us a baseline value of 5255.5 km. This was done by using the equation

$$d - Baseline = R_{Earth} \times cos(latitude)$$

To calculate the parallax of Ceres as observed by Earth, we began by taking photographs of Ceres through our telescope at night. We took a total of nine photos per trial, taking a photo every ten minutes beginning when the sky is dark enough to observe the dwarf planet. This initial data set gives us a the total displacement of Ceres,

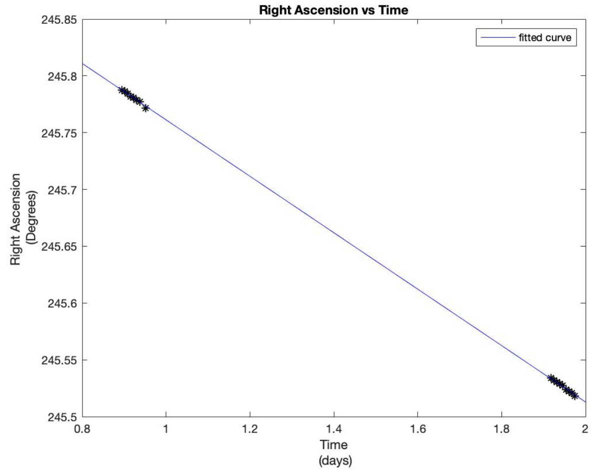

FIG. 3:
A plot of the overall motion of Ceres due to Earth's rotation and its own linear motion. We plotted a line of best fit to our data, and subtracted the coefficient for the slope from each data point to determine the motion of Ceres due to only the Earths rotational motion.

which is due to a combination of both the linear motion of Ceres and the rotation of the Earth. To obtain the parallax of Ceres, we must subtract the uniform motion of Ceres from the total displacement of Ceres.

To find the uniform motion of Ceres, we took a second data set as close to twenty four hours apart from our initial data set. This allows us to ignore the Earths rotation and find the displacement of Ceres due entirely to its uniform motion. Once obtained, we can subtract the uniform motion from our total displacement, and find its motion due solely to the rotation of the Earth. A quick plot of our data, fitted with a sinusoidal curve (since Earths rotation is modeled by circular motion), allows us to determine a value for the parallax of Ceres.

Sinusoidal motion due to Earths rotation is plotted in Figure 4. The difference between the maximum and minimum yields our value for the parallax of Ceres in units of degrees.

Now our determination of the astronomical unit only requires a simple trigonometric calculation. Using the illustration in Figure 2, the distance from Earth to Ceres can be found using the equation

$$R_{EARTH-CERES} = \frac{R_{BASELINE}}{2tan(\frac{\theta}{2})}$$

Once the distance between Earth and Ceres is obtained we can simply divide our calculated value by the known distance of Earth to Ceres (in our case it was 1.76 AU) in

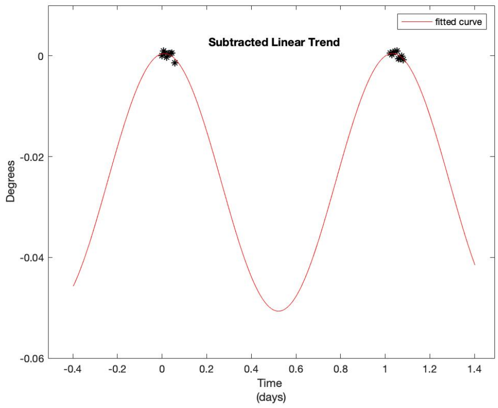

FIG. 4:
A plot of the sinusoidal motion due to Earth's rotation. We used the difference in peaks to calculate the parallax for Ceres. Since one full period on this graph represents 1 day, the difference in peaks represents the angle between our initial position of observation and our observation position twelve hours later.

terms of AU to get a value for 1 AU in units of kilometers.

III. ANALYSIS

Our calculation from the AU was based off of two values. We needed to use simple trigonometry and a baseline measurement. We calculated θ by plotting a graph of Ceres location in RA taken 24 hours apart, in order to see its overall linear motion. We could then find its motion due to the Earths rotation by subtracting the values obtained by a line of best fit from our data. Plotting our data minus the linear motion would hopefully give us a graph that looked approximately like a sinusoidal function. However this was not the case, and no matter what way me or my partner tried to orient or configure our data, we were unable to make any clear deductions from our data. This was likely due to the inaccuracy of calculating our RA, and inability to accurately subtract the linear trend from our data.

Using the sinusoidal graph in Fig 4, we found that the difference between peaks was roughly 0.05 degrees. Yet, to obtain an accurate value, our expected value would have to be a magnitude smaller with a value of approximately 0.001 degrees. Still, we calculated our value for the astronomical unit using the baseline calculated above and a value of 0.02557 degrees (this was $\frac{\theta}{2}$). Using the equation

$$d_{Earth-Ceres} = \frac{baseline}{tan(\theta)} = \frac{5255.5}{tan(0.02557)} =$$

$$11{,}776{,}220.15 \text{ km}$$

This distance was known to be 1.76AU, so our final value for the AU was calculated to be:

$$\frac{11{,}776{,}220.15}{1.76} = 6{,}691{,}034.176 \text{ km} = \text{AU}$$

The true value for the astronomical unit, determined to be roughly 150 million km, is far larger than our experimentally determined value. Our result is inconsistent with measurements reported by Vanderbei and Belikov [1], who used a setup nearly identical to our own, except with a smaller asteroid instead of Ceres. However in their case, they had a third data trial that fell in between the twenty four hour time span between our two data trials. I believe that this difference in procedure was the main factor in our erroneous value. When fitting the sinusoidal graph to our data it was nearly impossible to get an accurate amplitude, and therefore value for our parallax.

IV. UNCERTAINTY

We chose to ignore our change in declination during our experiment, since it was fairly negligible in the overall motion of Ceres. The uncertainty for our value would therefore be based entirely on our calculated right ascension values. Since we were unable to figure out how to correctly use Astroart7, we were forced to use Fiji to extract our Right Ascension values.. This posed multiple obstacles, as right ascension and declination are angles and should be analyzed in polar coordinates, whereas we analyzed our values in cartesian coordinates. Since our field of view was roughly a 1x1 square with units of $degrees^2$, we operated on the assumption that the curvature of our field was negligible, and that the distances were roughly straight lines. Additionally, instead of getting exact RA values we had to deal with the uncertainty in the center of our reference stars as well as the different orientation of our images to the true axis of orientation for right ascension and declination. Regardless of the obstacles, our calculations for our uncertainty are in the section below.

Our estimate for our uncertainty in finding the center of our reference star was about 20 pixels, due to the limitations in our instruments clarity.

$$\delta_{Pixels} = \frac{20pixels}{\sqrt{12}} = 5.77pixels$$

We then converted to minutes and seconds so that we could add distances calculated in pixels to the known RA and Dec of our reference stars. The conversion for pixels to seconds is 0.05 seconds/pixel. With these conversions the uncertainty in RA, in units of seconds, becomes:

$$\delta_S = \delta_{Pixels} \times 0.5s/pixel = 0.41s$$

Our RA used in our calculations was an average of the calculated RA from two separate reference stars. Our uncertainty in RA therefore became:

$$\delta_{RA} = \sqrt{\delta_{S1}^2 + \delta_{S2}^2} = \sqrt{2}\delta_S$$

$$\delta_{RA} = 0.41seconds$$

We then converted RA to degrees to plot our data and calculate a value for the AU.

1 hour=15 degrees

1 minute=0.25 degrees

1 second=0.0042 degrees

With δ_{RA}= 0.001722 degrees, the uncertainty in our value for the astronomical unit could then be calculated from the equation for adding uncertainties in quadrature. Since we only had one source of uncertainty, we calculated our uncertainty in AU by the following equation.

$$\delta_{d_{Earth-Ceres}} = \sqrt{((\frac{\partial}{\partial\theta})(f) \times \delta_{RA})^2}$$

$$\text{with } f = \frac{baseline}{tan(\frac{\theta}{2})}$$

$$\delta_{d_{Earth-Ceres}} = \sqrt{(\frac{-d_{baseline} \times sec^2(\frac{\theta}{2})}{2tan^2(\frac{\theta}{2})} \times \delta_{RA})^2}$$

Therefore our calculated uncertainty for our value was equal to

$$\delta_{d_{Earth-Ceres}} = \pm 17,133,407.19km$$

Our value for uncertainty is significantly higher than our calculated value. This is likely due to the high uncertainty in determining the exact right ascension of Ceres using the software Fiji. Since we were unable to locate the exact center of the star due to blurriness of the reference stars and relative large size compared to pixel size, it was expected that our value would have a large uncertainty. As for the reasoning that our value was so far off, it was likely due to us not taking a third set of data between the twenty four hour time span. Had we done this, we likely would have gotten a more accurate amplitude for the sinusoidal rotation of the Earth, and therefore accurate value for our parallax.

V. CONCLUSION

In summary, we have measured the astronomical unit to be $6,691,034.176 \pm 17,133,407.19km$. Our measurement suffered multiple systematic errors that completely hindered any hope of an accurate measurement. In conclusion, my partner and I were unsuccessful in calculating the astronomical unit. I believe that there are three errors in our procedures that resulted in an inaccuracy of this magnitude. The first, and most significant, would be our failure to get a third set of data that does not fall on the same position of our sinusoidal curve. Since we only had our two reference points, it was extremely difficult to determine the amplitude of our sine curve. Since our entire measurement was dependent on the value extracted from this curve, this error essentially doomed our project from the beginning.

A second error would be our inaccuracy in determining the exact Right Ascension from our images. This would have been avoided if we were able to use the software Astroart7, however our limited time constraint resulted in us having to compromise accuracy for results. Had we extracted the exact RA and Dec at a given moment, our data would have fit a curve much easier, and likely resulted in us having better luck in obtaining an accurate result.

Our third major difficulty in procedure came with the subtraction of the linear trend from our data. I cannot say with complete certainty that me or my partner obtained accurate data after our subtraction, as both he and I were unsure as to how we would approach this obstacle. The project presented many difficulties that included, but were not limited to, software difficulties, extreme time constraints, observing the wrong celestial objects, inability to see desired celestial objects, errors in procedure, sleep deprivation and unintentional erroneous data analysis.

I would like to thank my lab partner, Diego Romani, for insightful conversations and insight into astronomy that would have made any attempt to measure the unit even more unsuccessful. I am grateful to our TA Ari Kaplan who spent hours of his own personal time to help us gather data on the rooftop of Broida hall until the wee hours of the morning. I am especially thankful to Scott Taylor, who went out of his way to bring his own personal telescope to help us gather data once we realized that our initial telescope would not be a viable option for this experiment. This work was supported by Physics 25L lab fees.

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