Measuring the Coefficient of Restitution of a Ping Pong Ball

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27 January 2021

In this study, we were able to measure the coefficient of restitution for a partially elastic ping pong ball using the differences in times between each bounce. We were able to record data using the Phyphox acoustic measurement tool at various heights with repeated trials. Using this method, we were able to determine a coefficient of restitution of 0.863 ± 0.024. This value constitutes an error range of around 2.80%.

INTRODUCTION

The coefficient of restitution is defined as the ratio between an objects initial and final velocity when dropped vertically, and is dependant on the composition of the object. This can be described mathematically as

$$\epsilon = \frac{v_f}{v_0}, \tag{1}$$

and describes the amount of energy lost from the collision with the ground. The corresponding velocities can be found using simple kinematic equations. By simplifying the equation $h = v_0 t - \frac{1}{2}gt^2$ we found that the consecutive velocities are governed by the time intervals, or

$$v(t) = \frac{gt}{2}, \tag{2}$$

with h being zero. By measuring the time between each of the bounces, we are able to determine the velocity immediately after the collision with the floor, and calculate our coefficient of restitution.

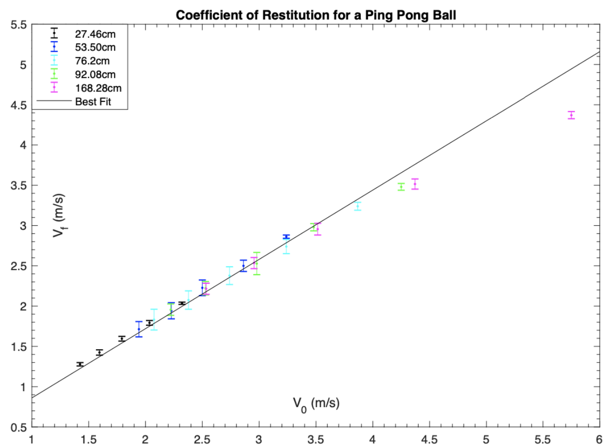

FIG. 1: *A plot illustrating the average v_f versus v_0 for all the varying heights tested. The slope of the line of best fit represents the average value for our coefficient of restitution for a ping pong ball, which we found to be 0.863 ± 0.024. From this data we can see that air resistance plays a significant role at larger heights.*

METHODS

The setup for our experiment was fairly rudimentary, and only consisted of a ping pong ball, our cell phones and various set dropping heights. Within our trials, we used five various heights ranging between around 25cm to 170cm. For each of our separate drop heights, we allowed for a total of five bounces per trial for a total of ten trials. The time between bounces was measured using the app Phyphox, where we were able to record acoustically when the ball hit the ground and determine the time between bounces. This resulted in our group attaining a total of 500 data points, each with a corresponding coefficient of restitution. The coefficients for each trial were calculated using equation (2) for the initial and final velocities. The error associated with our calculations is solely dependent on the variable t in equation (2). We are limited in our measurement for our value for t due to the precision of the equipment. An important note is that our initial velocity for each trial was computed differently, and was done so by setting the kinetic and potential energies equal to one another and solving for v_0.

$$v_0 = \sqrt{2gh} \tag{3}$$

This often resulted in a higher calculated velocity than the one recorded, and was neglected in higher heights due to the effects of air resistance on the ping pong ball, which were non-negligible.

ANALYSIS/RESULTS

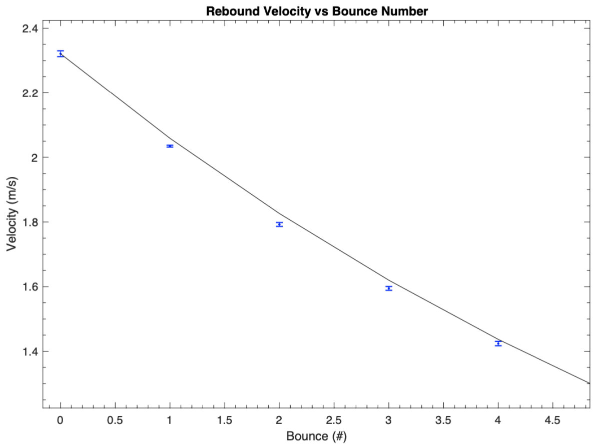

FIG. 2: *A comparison of the measured velocities per bounce with the predictive function,* $v(n) = (2.321m/s)(\epsilon)^n$ *which appears very similar to the plot of the averages, with a squared correlation coefficient of* $R^2 = 0.868$.

Bounce	Bounce Time (s)	Velocity (m/s)	Coefficient
1	0.414	2.03 ± 0.014	0.877 ± 0.014
2	0.365	1.79 ± 0.029	0.881 ± 0.029
3	0.325	1.59 ±0.029	0.889 ± 0.029
4	0.290	1.42 ± 0.033	0.893 ± 0.033
5	0.261	1.28 ±0.021	0.897 ± 0.021

TABLE I: A table illustrating the average bounce time, velocity and coefficient of restitution for a ping pong ball being dropped from a height of 27.5cm over a total of ten different trials.

Using the above methods we were able to calculate the coefficient of restitution to be 0.863 ± 0.024. Comparing this to an accepted range for a ping pong ball being between 0.800 and 0.694 [1], we find that our value is slightly higher than expected. This could be due to a variance in experimental method, environmental settings or equipment used.

The value for our overall coefficient of restitution was calculated by taking the average of all of the coefficients calculated for each bounce within each trial. The standard deviation was calculated using

$$\sigma = \sqrt{(\frac{\sum x - \bar{x})^2}{N})} \quad (4)$$

where we included each calculation for the coefficient of restitution for each individual trial. We intended to include the uncertainty due to the limitation in measurement of Phyphox. The precision of our instrument was to the order of 0.001s. However the dependence on the microphone within our cellphones resulted in systematic error of much larger magnitude than the limitations of the instrument. The influence of external acoustics played a significant role which we were unable to fully eliminated within our limited settings. For this reason we chose to ignore the uncertainty associated with our instrumentation and solely focus on the random deviation of the actual experiment. The analysis on our fourth trial, where air resistance had the least significant effect, can be seen in Figure 2 and Table I. This data is likely the most accurate, as it was dropped from the lowest height and had the least influence of air resistance. It is important to note that the value for the coefficient of restitution for this height was 0.888 ± 0.009, which is roughly 3% higher with a significantly lower standard deviation. This solidifies our initial statement that air resistance plays a significant role in the precision of our experiment and, had we only experimented with smaller heights, would likely have seen a much smaller error than we currently do.

DISCUSSION

This experiment allowed us to use the app Phyphox to calculate the coefficient of restitution for a specified object and predict its overall trajectory with each bounce. We found an overall coefficient value of 0.863 ± 0.024. This is fairly larger than the accepted range of 0.694 to 0.800. As our values for the coefficient of restitution varied within a range of 3% of our average for 500 separate bounces, I believe we can confidently disagree with the accepted values.

As observed in Figure 1, our assumptions about our system held for relatively small heights but were greatly effected by air resistance at larger heights. The initial velocity of each trial was computed using energies rather than being directly measured. For this reason we chose to analyze subsequent bounces, since air resistance usually had the most significant impact during the first bounce. During subsequent trials, the ball was governed by roughly predictable motion, losing roughly 13% of its velocity with each bounce.

Our experiment can likely be refined further. Using a more dense ball, accurate microphone and smoother floor would likely give a smaller standard deviation for our experiment. A denser ball would combat the air resistance and random error associated with the experiment, whereas a smoother floor and more accurate microphone would help with the systematic error. Regardless of the overall flaws within our experiment, I believe that our procedure and result were both replicable and gave valid results.

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