Effect of a Coriolis Force due to a Planetary Rotation

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1 Introduction

The purpose of this project was to illustrate the change in position and velocity to a frictionless object, given a set of initial conditions, due to a planetary coriolis force. The equations of motion used as the basis for this program were derived using Newtons Second law in a rotating frame. The equation used to derive the x and y positions of the object is:

$$\mathbf{F}_{cor} = 2m\dot{\mathbf{r}} \times \Omega = 2m\mathbf{v} \times \Omega$$

This project requires no additional software to be downloaded, as it was animated using standard matplotlib programs. Solutions to the coriolis equation are included at the end.

2 Project Description

A large part of this project was solving the mathematical equations relevant to the Coriolis Force. The program begins by illustrating a diagram to allow the user to understand the layout of the specific problem that I have chosen to solve. Once the image has been closed the terminal will prompt the user to pick between an animated or simple plot. After specifying the desired plot style the terminal will prompt for all input parameters relevant to the problem, include initial velocities, angular velocity, latitude and desired timeframe.

For the simple plot the program works by plugging in the initial parameters into the solved equations for both velocity and position. I use a loop to solve for the position and velocities at each time interval. I operated under the assumption that since the position was dependant on the current velocity, the velocity within the equations for position were updated every iteration. This is not a completely accurate assumption, as the integrals used to solve this for position treated those velocities as constants. This does, however, allow for a more accurate answer to these equations than the integrals allowed. Once I had the data for position I simply sent this data to a plot. The simple plot was included to get quick answers, which is far more useful when larger timeframes were used.

The animated plot was by far more interesting. For this portion of the project the

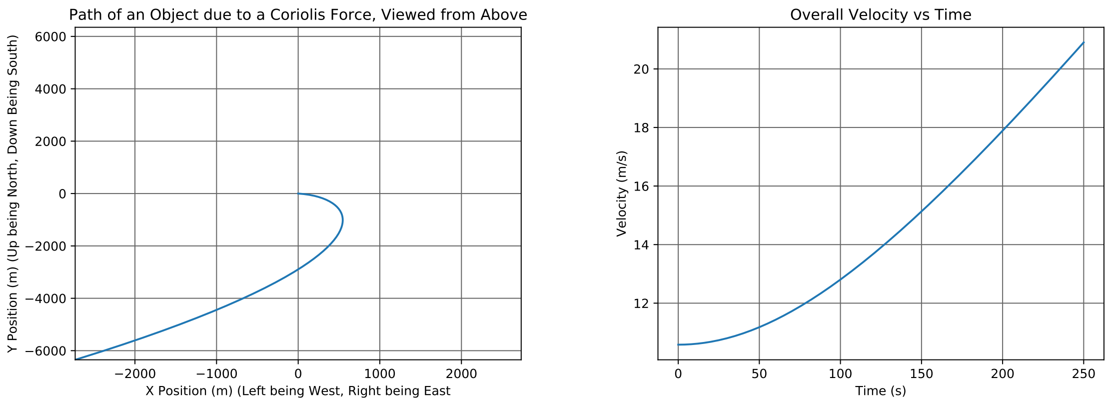


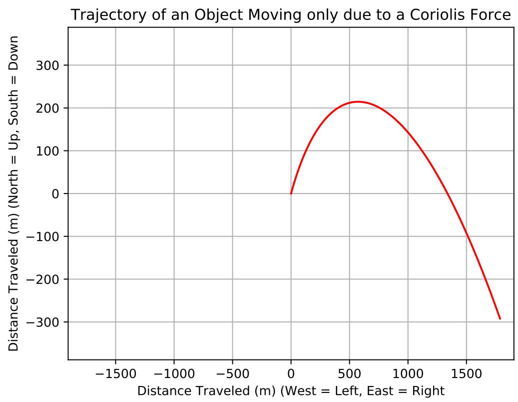
Figure 1: Left: Illustration of the effects of a Coriolis force. As can be expected for very large values of Ω, the velocity gained through the Coriolis force dominates the trajectory. Right: Depiction of overall velocity as a function of time.

data was updated for each frame of the animation using the matplotlib.animation. This required that the axes be updated so that the plot did not end up off of the page. This was done within the update function, and done so that the axis stop updating at the specified time interval. The animated portion of the graph is more useful when trying to view the effects of the Coriolis force. For specific initial conditions some motion gets washed out within the simple plot and can only be viewed while watching the object move in real time.

A plot of the overall velocity is included to illustrate the acceleration of the object due to a Coriolis Force. This only depicts the magnitude of the velocity, as it is more interesting than any individual component velocity.

3 Results

Some of the underlying assumptions used to solve this problem using Newtonian mechanics result in inaccuracies in the influence due to a coriolis force. One such example is the balancing of v_x and v_y for small timescales and low values of omega. An example of this can be seen when setting $v_x = v_y$ when defining the inital conditions for the simulation with the angular velocity set to the norm for Earth. Yet, using this simulation, we can still see particles with almost eliptical motion with a large enough timeframe. An example of this can be illustrated using the initial conditions $v_x = 10$ m/s, $v_y = -2$ m/s, $\Omega = 0.005$ rad/s, latitude = 45 deg and timeframe ≈ 250 s.For all problems assume a value for the angular velocity equal or less than 0.005 rad/s, as this allows for some normal motion before the coriolis force dominates the objects trajectory.



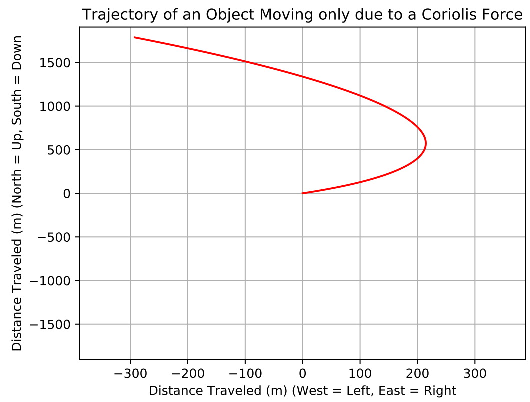


Figure 2: Northern vs Sothern Hemispheres: Two plots showing identical initial conditions, except one being at a latitude of 45 deg and the other being -45 deg.

Another interesting example is when $v_x = 10$ m/s, $v_y = 10$ m/s, $\Omega = 0.005$ rad/s, latitude = 45 deg and timeframe $\approx$ 400 s. This example illustrates the motion of the object being similar to that of a fibonacci spiral, as the motion of the particle will begin to curl outward with time. This motion is what contributes to the formation of hurricanes by the acceleration of air particles due to the coriolis force formed by the Earth. Interestingly enough, if you were to invert the latitude from 45 deg to -45 deg we would witness the direction of rotation flip. Lastly, on the equator we would witness zero influence from the coriolis force due to the mathematical dependance on a sine function. This results in the phenomena of direction of hurricane rotation being opposite across the equator, or the myth that drains and toilets flow in the opposite direction in Australia.

4 Solutions to Newtonian Equations

X Position

$$\Delta x = \Omega \dot{y} sin(\lambda) t^2 + v_{x,i} t$$

Y Position

$$\Delta y = -\Omega \dot{x} sin(\lambda) t^2 + v_{y,i} t$$

X Velocity

$$v_x = 2\Omega \dot{y} sin(\lambda) t + v_{x,i}$$

Y Velocity

$$v_y = -2\Omega \dot{x} sin(\lambda) t + v_{y,i}$$