Measuring the Index of Refraction for Air Using a Michelson Interferometer and a Vacuum Chamber

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In this report, I observed the change in the number of fringes within the interference pattern of a Michelson interferometer. This was done by adjusting the phase of one leg of the laser using a hand pump to vary the pressure for a small section of the beam. Using the change in fringe count, I was able to calculate the index of refraction for room temperature air to be 1.000263 ± 0.000055. This value agrees with the established value of 1.000273. I will also go over improvements to our methods which could decrease uncertainty and serve as a valuable guide for other future projects.

INTRODUCTION

The index of refraction is a dimensionless scale factor that describes how fast light can travel through a given medium. It is defined as:

$$n = \frac{c}{v} \tag{1}$$

where c is the speed of light in a vacuum and v is the speed of light through a given media. The index of refraction of specific gasses can also be determined using a Michelson interferometer. An interferometer is a system composed of two intersecting light beams of equal length being directed into an extremely small detector, as shown in Figure (1). These beams happen to be in phase, so upon reaching a detector the power associated with the beam is a constant value. If there is a slight disturbance in the environment through a shift in medium, the beams will be slightly out of phase resulting in a noticeable disturbance picked up by the detector. The interference pattern displayed on the screen, shown as the detector in Figure (1), will depict a series of 'fringe patterns'. By placing a vacuum chamber of length L and variable pressure P, measured in inches of mercury, at the end of one of the beam legs, the index of refraction can be calculated using the relation:

$$n = \frac{mP_{atm}\lambda}{2PL} + 1 \tag{2}$$

where λ is the wavelength of the laser, m is the fringe count and P_{atm} is the current pressure of the environment. This relation was taken from work done by Kimberly Belmesand Carly E Stauffer [1]. Air reduces the speed of light and effectively increases the overall path length. Whenever the path length of one of the arms is increased by one wavelength, a fringe can be seen to shift on screen. By counting the number of shifted fringes I was able to deduce the overall index of refraction for the medium in which it passes. The benefit of using a Michelson interferometer to measure the index of refraction is the sensitivity of the beam arms, which can be used to detect extremely small differences between the density of air.

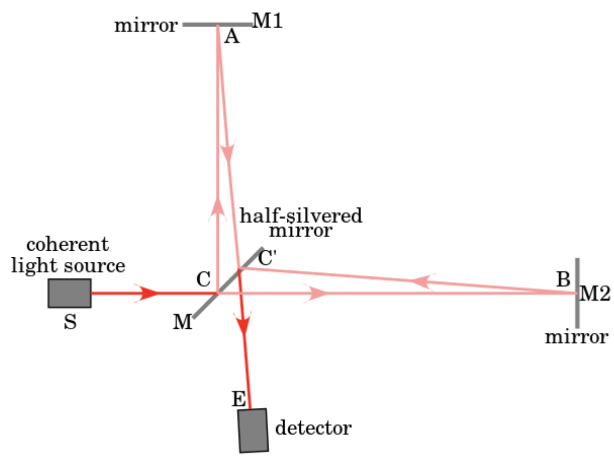

FIG. 1: *Diagram depicting a Michelson interferometer in its most basic form.*

METHODS

To measure the index of refraction of air, I setup a Michelson interferometer, loaned to me by the University of California Santa Barbara, with a beam of wavelength $\lambda = 635$nm. The beam was placed in front of a roughly 50% opaque mirror, where the beam would then split into two separate legs. Both beams would then hit separate mirrors, seen as M1 and M2 in Figure (1), and then would return to the

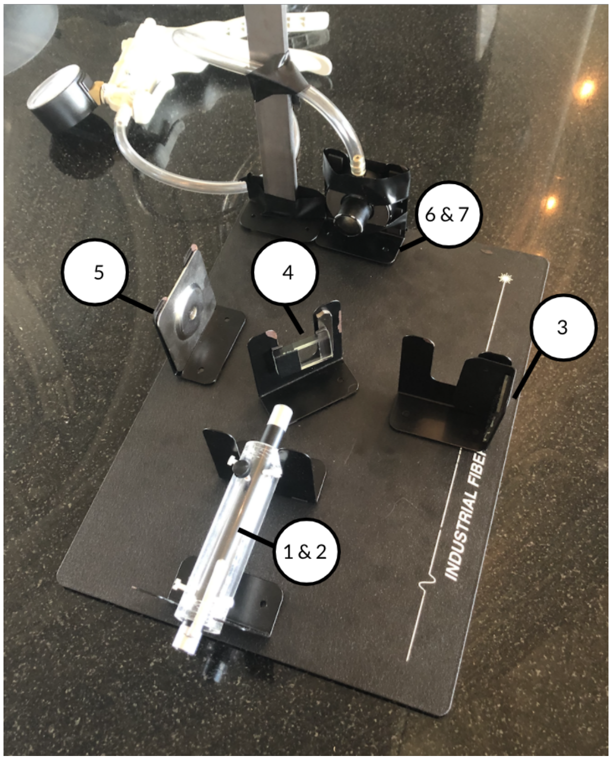

FIG. 2: *Diagram depicting my actual setup. Labels are as follows. 1 & 2 : Laser and Stabilizer, 3: Reflective Mirrors, 4: 50% Opaque Mirror, 5: Diverging Lens, 6 & 7: Vacuum Pump and Chamber, with stabilizing rod.*

partially opaque mirror and become centered on a diverging lens. The diverging lens would then magnify the interference patterns of the beams, allowing for the counting of fringes on a screen placed behind the diverging lens. In our specific setup, all materials were anchored to a metal platform with magnets to improve stability.

In order to actually measure the number of fringes as a function of varying pressure I attached a MV8255 transparent hand vacuum to the end of M1. Since the vacuum relied on manually pumping out the air inside the chamber, I attached the pump to a table via a metal rod to stabilize the entire apparatus. This minimized the vibrations of the system, but they were ultimately still present when removing the air. When counting fringes, I let the system settle completely. The length of the vacuum chamber measured 0.0033 ± 0.0001 m, and allowed us to vary the pressure for that segment to be anywhere between 0 and approximately 25 in.Hg. By varying the amount of air in the vacuum, I was able to record the number of shifted fringes as a function of pressure.

Vacuum Pressure (in.Hg)	Average Count (m)
0	0
2.5	2.0
5	4.4
7.5	6.6
10	8.8
12.5	11.8
15	14.2
17.5	16.8
20	18.6
22.5	21.2
25	24.0

TABLE I: A table summarizing how the number of fringe shifts changes with vacuum pressure. It is important to note that the number of shifted fringes counted were to the closest whole integer.

When taking data I began by completely filling the vacuum chamber with air and counting the number of fringes on the screen. From an internal pressure of 0 in.Hg, I slowly removed the air from the chamber in 2.5 in.Hg intervals and recorded the number of fringes that shift on the screen. This gave us a total of 10 data points per trial, which I then went on to repeat for five separate trials. A summary of our data can be seen in Table (I).

The number of fringes on the screen were limited to what I could observe with the human eye. This is likely a source of error due to the reliance on the eyesight of a specific individual. Additionally I am limited in the precision of our hand vacuum, which can only measure the pressure up to the nearest half inch of mercury. The final limitation in our overall measurement of the index of refraction of air lies in the assumption of an atmospheric pressure of 29.92 in.Hg, which may vary with the conditions of the day and the environment where the data was collected.

ANALYSIS/RESULTS

Using the methods described above, I was able to calculate a value for the index of refraction for air to be 1.000263 ± 0.000055. This value compares to the actual value of 1.000273 fairly well, and lies within one standard deviation of our measured value. However, the standard deviation is rather large, and allows the calculated index of refraction to encompass the indices of refraction for other gasses such as Oxygen and Nitrogen. This is fairly intuitive since air is

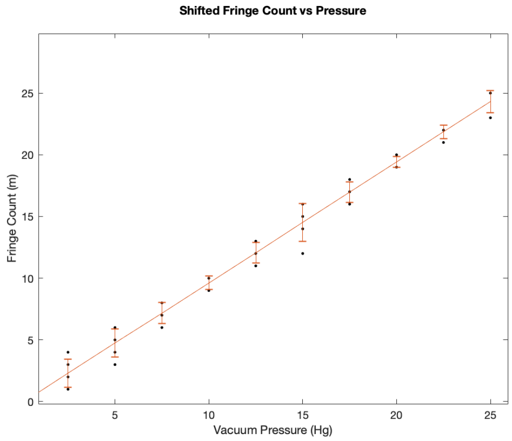

FIG. 3: *This graph depicts the entirety of our data for fringe count against vacuum pressure. Due to the fact that the fringes must be whole integers, a large number of the data points overlap. The line of best fit has a chi-square value of 0.9993 and a RMSE of 0.1986.*

composed of Nitrogen Oxygen and Argon, yet points to some inaccuracies in our data collection methods.

Once the data for the fringe count as a function of pressure was collected, I was able to calculate the index of refraction for each pressure using Equation (2). For all 50 data points collected I was able to find a specific index of refraction for the surrounding air. This was done by running an array of all the data points through Equation (2). Now with 50 separate values for the index of air, I simply took the average to find our final value. Using this method, I was able to calculate a value of 1.000263.

The statistical uncertainty of our experiment was calculated using the equation:

$$\delta_{n_{stat}} = \sum \sqrt{\frac{(x_i - \bar{x})^2}{N}} \tag{3}$$

Using this method, I found the statistical uncertainty of our measurement to be $\pm$ 0.000052.

The limitations of our measurement were primarily in the measurements made for the pressure, shifted fringe count, and length of the vacuum chamber. It was our assumption that the atmospheric pressure and wavelength of the laser were accurate, since I was unable to physically measure them. When measuring pressure, there was some difficulty aligning the pressure dial specifically with the desired pressure. Since the vacuum pump was accurate to the nearest 0.5 in.Hg, I claimed this as our systematic uncertainty for P. Our measurement of fringe count was limited by the visibility of fringes on the screen. Our count was only accurate to the nearest whole number, so our δ_m was $\pm$ 1. Lastly, the measurement of the length of the vacuum chamber was limited by our ruler, which was accurate to the nearest millimeter. Therefore our uncertainty in L was $\pm$ 0.001m.

Collecting all of these uncertainties, I was able to propagate these values in quadrature to find the uncertainty in our value for the index of refraction. I found the systematic uncertainty in n by using the equation:

$$\delta_{n_{sys}} = \sqrt{(\frac{\partial n}{\partial P}\delta_P)^2 + (\frac{\partial n}{\partial m}\delta_m)^2 + (\frac{\partial n}{\partial L}\delta_L)^2} \tag{4}$$

When calculating the value for the systematic uncertainty in n, I did so using the average values for fringe count against pressure, seen in Table (I). Using this method I calculated a total of 10 uncertainties, and simply took the average to find our overall uncertainty in our measurement. Ultimately, I report $\delta_{n_{sys}}$ to be $\pm$ 0.000019.

The statistical and systematic uncertainties add in quadrature. Using the equation

$$\delta_n = \sqrt{\delta^2_{n_{sys}} + \delta^2_{n_{stat}}} \tag{5}$$

I found the uncertainty in n to be $\pm$ 0.000055. Thus, I report the index of refraction of air to be

$$\boxed{n = 1.000263 \pm 0.000055}$$

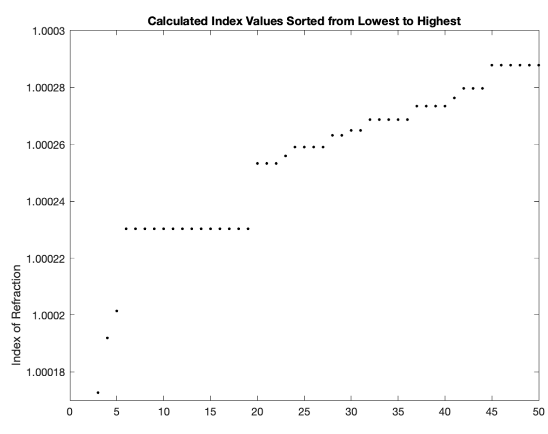

FIG. 4: *All of the calculated indices of refraction sorted from lowest to highest. It is important to note that the three values that were significantly lower all occurred within the first data point of each trial.*

Vac Pressure (in.Hg)	Avg Index (n)	Uncertainty ($\pm$)
2.5	1.000230	0.000091
5	1.000253	0.000085
7.5	1.000253	0.000071
10	1.000253	0.000057
12.5	1.000272	0.000049
15	1.000273	0.000048
17.5	1.000276	0.000033
20	1.000268	0.000027
22.5	1.000271	0.000030
25	1.000276	0.000031

TABLE II: A table summarizing how the index of refraction changes with vacuum pressure, with uncertainties included.

DISCUSSION

The index of refraction serves an important purpose in the study of light and optics and allows for a fundamental understanding of the behavior of light in media. The index of refraction for air has a known value of 1.000273 at standard temperature and pressure, which falls within the range of our index value of 1.000263 $\pm$ 0.000055.

The use of the Michelson interferometer allows for the precise measurement of slight disturbances within the environment surrounding the beams. Unfortunately, a consequence of this is that our instruments have to be extremely fine tuned in order to take accurate data. The existence of this problem largely contributes to our somewhat larger uncertainty value. All data for this lab was taken outside of a laboratory on rather unstable furniture. This would cause the entire apparatus to sway and oscillate, ultimately interfering with the data. Additionally, the magnetic strips used to hold our setup were quite weak, and would occasionally move when the table was hit, causing us to reset on our data collection. To improve our data collection methods, further experimentation would have to be done with a more rigid apparatus. An anchored table and stronger magnets would likely lower our statistical uncertainty.

An additional source of systematic error was the type of vacuum pump used. The pump was only accurate to the nearest half inch of mercury, which lead to a disparity between the shifted fringe counts at what were supposed to be constant pressure increments. When left to rest, I also found that air leaked into the vacuum chamber at a fairly slow rate. Still, this made attaining a constant pressure almost impossible. By being able to record data at a constant pressure I would be able to drop the magnitude of both our statistical and systematic uncertainty further.

The data collected was fairly similar to what I predicted with Equation (2), illustrating a linear relationship between pressure and fringe count. Still I encountered some outliers within our data, seen as the lowest data points in Figure (3). The most non-linear correlations between pressure and shifted fringe count occurred between the range of zero and 2.5 in.Hg. This random error was likely due to the initial conditions of our setup, where a fringe may be too faint to detect. The unavoidable variability of our initial conditions primarily contributed to our statistical uncertainty.

Overall numerous adjustments could be made to refine our project. I am happy to report that the values obtained are fairly accurate, although not as precise as I had desired. The ability to operate in a controlled environment would help improve our the precision of our experiment greatly. I would like to thank our TA Keegan Downham for helping keep our project on track and reaching out the the physics department on my behalf to obtain a vacuum pump. Additionally I would like to thank the physics department for putting together a cohesive lab during these difficult times.

BIBLIOGRAPHY

[1] Belmes, Kimberly B. and Stauffer, Carly E. (2018) "Using a Michelson Interferometer to Measure the Index of Refraction of Air,"Journal of Advanced Undergraduate Physics Laboratory Investigation: Vol. 3 , Article